FH Vorarlberg
University of Applied Sciences

Mathematics 2: Ordinary Differential Equations

Summary

Mathematics 2

Vorarlberg University of Applied Sciences

Bachelors's in Mechatronics

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1 Ordinary Differential Equation of 1^{st} Order

Ordinary Differential Equations (or short ODEs) are equations where a derivative of one variable is present. The order of an ODE is defined by the largest derivative in the equation.

1.1 Seperable ODE

These are equations of the form

$$y' = f(x)g(y) \tag{1.1}$$

The functions $f(x)$ and $g(y)$ can be seperated in a way that all y are on one side and all x are on the other side. It is benificial to leave negative signs on the right side.

1. bring $f(x)$ and $g(y)$ to each side: $y'\frac{1}{g(y)} = f(x)$
2. rewrite differential expression: $\frac{dy}{dx}\frac{1}{g(y)} = f(x)$
3. integrate both sides $\int \frac{1}{g(y)}dy = \int f(x)dx$
4. if necessary, bring y on one side: $y(x) = ...$

1.2 Euler-Homogeneous ODE

This equations look like

$$y' = g\left(\frac{y}{x}\right) \tag{1.2}$$

By using an appropriate substitution, the Euler-Homogeneous ODE can be transformed into a seperable ODE and than solved accordingly. The challenge is to find the right substitution. This largely depends on the right hand side of the equation. Some important substitutions are:

form	$y' = g\left(\frac{x}{y}\right)$	$y' = g(x+y)$
substitution	$u = \frac{x}{y}$	$u = x + y$
derivative	$y' = u'x - u$	$y' = u' - 1$
example	$y' = \frac{1}{2}\left(\frac{y}{x} - \frac{x}{y}\right)$	$y' = sin(x+y)$

In general, the process of solving this type of equation is

1. find substitution $u = ...$
2. calculate the derivative $y' = ...$
3. insert into the original equation
4. solve the now seperable ODE
5. resubstitute $y = ...$

1.3 Linear ODE of 1_{st} Order

These are equations of the form

$$y' + p(x)y = r(x) \tag{1.3}$$

Here, $p(x)$ and $r(x)$ can be an arbitrary function of x. In the special case where $r(x) = 0$, the ODE is seperable and can be solved according to chapter 1.1

1.4 Bernoulli ODE

1.5 Transformation to New Coordinate System

1.6 Transformation to Polar Coordinates

1.7 Exact ODE and Total Differential

1.8 Excursion: Constructing Trajectories

1.8.1 Orthogonal Trajectories

1.8.2 Isogonal Trajectories

2 Ordinary Differential Equation of 2$^{\text{nd}}$ Order

2.1 Converstion to 1$^{\text{st}}$ Order ODE

3 Ordinary Differential Equation of N$^{\text{th}}$ Order