CATEGORY THEORY MIDLANDS GRADUATE SCHOOL 2023

EXERCISE 2 (3 APRIL)

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REMINDER

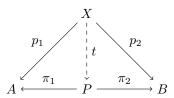
Definition 1 (product diagram). For $A, B \in C_0$, a product diagram is an object $P \in C_0$ together with morphisms $\pi_1 \in C(P, A)$ and $\pi_2 \in C(P, B)$,

$$A \longleftarrow^{\pi_1} P \longrightarrow^{\pi_2} B$$

with the following universal property: Given any other diagram of this shape,

$$A \longleftarrow^{p_1} X \longrightarrow^{p_2} B$$

there is a unique morphism $t \in \mathcal{C}(X, P)$ such that $p_1 = \pi_1 \circ t$ and $p_2 = \pi_2 \circ t$. One expresses these equations by saying that the following two triangles commute:



EXERCISE 3: COPRODUCTS

Using the definition of a product diagram given in the reminder above, re-construct the definition of a coproduct diagram by "reversing all arrows". Check that, in the category SET, your definition describes the disjoint union of two sets. What objects are characterised by your definition in the other categories discussed in the lecture?

EXERCISE 4: INTERACTION OF PRODUCTS, COPRODUCTS, INITIAL AND TERMINAL OBJECTS

Assume that the category \mathcal{C} has a terminal object, for which we write 1, and an initial object, for which we write 0. Moreover, assume that \mathcal{C} has all products and coproducts. This means that, for every pair of objects $A, B \in \mathcal{C}_0$, we have a product diagram and a coproduct diagram. We write $A \times B$ for the "middle object" of the product diagram, and A + B for the "middle object" of the coproduct diagram.

Recall that we write $X \cong Y$ if there is an isomorphism in $\mathcal{C}(X,Y)$. For each of the following isomorphisms, either prove them in \mathcal{C} or find a category \mathcal{C} in which they don't hold:

- **a.** $A \times 1 \cong A$
- **b.** $A \times 0 \cong A$
- c. $A \times 0 \cong 0$
- **d.** $A + 0 \cong A$
- **e.** $A \times B \cong B \times A$
- **f.** $A \times (B+C) \cong (A \times B) + (A \times C)$

g. Can you write down an isomorphism that never holds, i.e., that doesn't hold in any category \mathcal{C} ?