Group coherence

Using the coefficient of variation (CV) to estimate changes in range shape and fragmentation

N. Love and S. Otto (2026)

Individuals distributed according to a bivariate normal

Commands for simplifying terms [ENTER]

This section gives the list of constraints and summarizes the results to avoid having to reenter them:

In[∘]:=
```
constraints = {σx > 0, σy > 0, σ > 0, ρ ≥ 0, Δx^2 > 0,
    Δy^2 > 0, Δz > 0, c > 0, Element[{σx, σy, ρ, Δx, Δy, Δz, c}, Reals]};
```

Summarizing the results:

In[∘]:=
$$\text{soln} = \frac{2\,\sigma x\,\text{EllipticE}\left[1-\frac{\sigma y^2}{\sigma x^2}\right]}{\sqrt{\pi}};$$

In[∘]:=
$$\text{solnsq} = 2\left(\sigma x^2+\sigma y^2\right);$$

In[∘]:=
$$\text{CVherd} = \frac{\sqrt{\frac{1}{2}\,\pi\left(\sigma x^2+\sigma y^2\right)-\sigma x^2\,\text{EllipticE}\left[1-\frac{\sigma y^2}{\sigma x^2}\right]^2}}{\sigma x\,\text{EllipticE}\left[1-\frac{\sigma y^2}{\sigma x^2}\right]};$$

The results below assume *σ*x=*σ*y=*σ*:

In[∘]:=
```
indsoln = 1.7724538509056207` σ;
indsolnsq = 3.3412233051393185` σ^2;
```

In[∘]:=
```
CVind = 0.2520801657678077`; (*Symmetric single patch*)
```
$$\text{CVind1} = \sqrt{-1+\frac{1}{12}\left(6\sqrt{3}+\pi\right)}\;;\text{(*Extremely elongated single patch*)}$$

$$\text{solnC} = (1-f)\,\sqrt{\pi}\,\sigma+\frac{1}{4}\,e^{-\frac{c_S^2}{8}}\,f\,\sqrt{\pi}\,\sigma\left(\text{BesselI}\left[1,\frac{c_S^2}{8}\right]c_S^2+\text{BesselI}\left[0,\frac{c_S^2}{8}\right]\left(4+c_S^2\right)\right);$$

In[∘]:=
$$\text{solnsqC} = 4\,(1-f)\,\sigma^2+f\,\sigma^2\left(4+c_S^2\right);$$

In[ ]:= CVherdC =

$$\text{Sqrt}\left[\frac{16\, e^{\frac{c_S^2}{4}}\left(4+f\, c_S^2\right)}{\pi\left(4\, e^{\frac{c_S^2}{8}}(1-f)+f\,\text{BesselI}\left[1,\frac{c_S^2}{8}\right]c_S^2+f\,\text{BesselI}\left[0,\frac{c_S^2}{8}\right]\left(4+c_S^2\right)\right)^2}-1\right]\ /.\ c_S\to\frac{c}{\sigma};$$

A herd split in two with a chance f of drawing from different patches has a CVind of:

In[ ]:= CVindfarsplit = $\sqrt{\frac{\left(\frac{1}{2}-f\right)}{f}}$;

A very elongated herd:

In[ ]:= CVherd1 = Limit[CVherd, σx → Infinity]

Out[ ]=

$$\sqrt{\frac{1}{2}(-2+\pi)}$$

For comparison, we can numerically integrate the function for a given parameter set.

For the herd-level CV, we can use analytical results where available:

mean = :

$$(1-f)\,\text{soln}+f\,\text{NIntegrate}\left[\frac{e^{-\frac{(\text{tryc}-\Delta x)^2}{4\,\text{try}\sigma x^2}}\,\text{try}\sigma y\,\text{HypergeometricU}\left[-\frac{1}{2},0,\frac{\Delta x^2}{4\,\text{try}\sigma y^2}\right]}{\sqrt{\pi}\,\text{try}\sigma x},\{\Delta x,-\text{tryLIM},\text{tryLIM}\}\right]\ /.\ \sigma x\to\text{try}\sigma x\ /.\ \sigma y\to\text{try}\sigma y\ /.\ f\to\text{tryf}\ /.\ c\to\text{tryc};$$

sqH =

$$(1-f)\,\text{solnsq}+f\left(c^2+2\left(\sigma x^2+\sigma y^2\right)\right)\ /.\ \sigma x\to\text{try}\sigma x\ /.\ \sigma y\to\text{try}\sigma y\ /.\ f\to\text{tryf}\ /.\ c\to\text{tryc};$$

The individual-level CV first requires integration of the average distance of an individual to any other. Analytical results are only available for the average distance in the symmetric Gaussian case (σx=σy):

In[ ]:= Solve[2 p (1 - p) == f, p]

Out[ ]=

$$\left\{\left\{p\to\frac{1}{2}\left(1-\sqrt{1-2f}\right)\right\},\left\{p\to\frac{1}{2}\left(1+\sqrt{1-2f}\right)\right\}\right\}$$

```
In[•]:= avedist[x2_?NumericQ, y2_?NumericQ, σ_?NumericQ, c_?NumericQ] :=
         Sqrt[π/2] σ LaguerreL[1/2, -((c + x2)^2 + y2^2)/(2 σ^2)]; (*c is displacement along x-axis*)
        sqI[σ_?NumericQ, c_?NumericQ, f_?NumericQ] :=
         NIntegrate[((p1 * (p1 * avedist[x2, y2, σ, 0] + p2 * avedist[x2, y2, σ, c])^2 +
               p2 * (p1 * avedist[x2, y2, σ, -c] + p2 * avedist[x2, y2, σ, 0])^2) /. p2 → 1 - p1 /.
            p1 → 1/2 (1 - Sqrt[1 - 2 f])) PDF[MultinormalDistribution[{0, 0}, {{σ^2, 0}, {0, σ^2}}],
           {x2, y2}], {x2, -tryLIM, tryLIM}, {y2, -tryLIM, tryLIM}];
```

A much slower integration is needed to first find the average distance in the asymmetrical case:

```
In[•]:= avedist[x2_?NumericQ, y2_?NumericQ, σx_?NumericQ, σy_?NumericQ, c_?NumericQ] :=
         NIntegrate[Sqrt[((x2 - x1)^2 + (y2 - y1)^2)]
           PDF[MultinormalDistribution[{c, 0}, {{σx^2, 0}, {0, σy^2}}], {x1, y1}],
          {x1, -tryLIM, tryLIM}, {y1, -tryLIM, tryLIM}];
        sqI[σx_?NumericQ, σy_?NumericQ, c_?NumericQ, f_?NumericQ] :=
         NIntegrate[((p1 * (p1 * avedist[x2, y2, σx, σy, 0] + p2 * avedist[x2, y2, σx, σy, c])^2 +
               p2 * (p1 * avedist[x2, y2, σx, σy, -c] + p2 * avedist[x2, y2, σx, σy, 0])^2) /.
             p2 → 1 - p1 /. p1 → 1/2 (1 - Sqrt[1 - 2 f])) *
           PDF[MultinormalDistribution[{0, 0}, {{σx^2, 0}, {0, σy^2}}], {x2, y2}],
          {x2, -tryLIM, tryLIM}, {y2, -tryLIM, tryLIM}];
```

When c=0 (no split), we can speed up the integration:

```
In[•]:= sqI[σx_?NumericQ, σy_?NumericQ] := NIntegrate[(avedist[x2, y2, σx, σy, 0])^2 *
           PDF[MultinormalDistribution[{0, 0}, {{σx^2, 0}, {0, σy^2}}], {x2, y2}],
          {x2, -tryLIM, tryLIM}, {y2, -tryLIM, tryLIM}];
```

When f=1/2 (even split), we can also speed up the integration (recognizing that the average distance for -c and for +c are equal):

```
In[•]:= sqI[σx_?NumericQ, σy_?NumericQ, c_?NumericQ] :=
         NIntegrate[(1/2 * avedist[x2, y2, σx, σy, 0] + 1/2 * avedist[x2, y2, σx, σy, c])^2 *
           PDF[MultinormalDistribution[{0, 0}, {{σx^2, 0}, {0, σy^2}}], {x2, y2}],
          {x2, -tryLIM, tryLIM}, {y2, -tryLIM, tryLIM}];
```

Analysis

E[X]: Average distance among all pairs

Consider a simplified range that is Gaussian in shape with mean 0, standard deviation σx and σy, and correlation structure ρ:

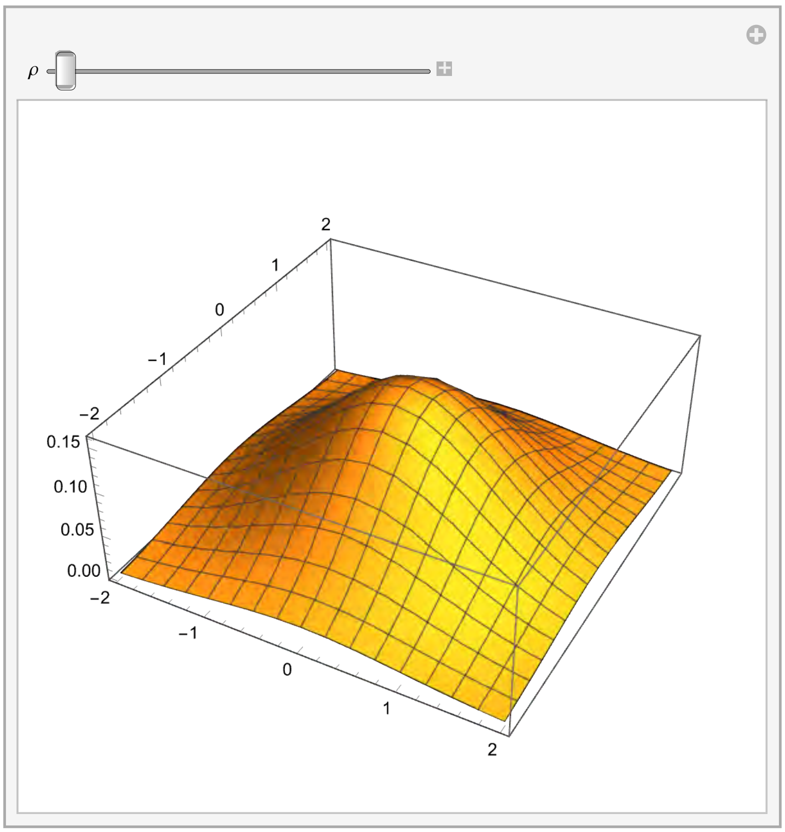
In[ ]:=
```
PDF[MultinormalDistribution[{0, 0}, {{σx², ρ σx σy}, {ρ σx σy, σy²}}], {x, y}]
```

Out[ ]=

$$\frac{e^{\frac{1}{2}\left(-\frac{x\,(y\,\rho\,\sigma x-x\,\sigma y)}{(-1+\rho^2)\,\sigma x^2\,\sigma y}-\frac{y\,(-y\,\sigma x+x\,\rho\,\sigma y)}{(-1+\rho^2)\,\sigma x\,\sigma y^2}\right)}}{2\,\pi\,\sqrt{\sigma x^2\,\sigma y^2-\rho^2\,\sigma x^2\,\sigma y^2}}$$

In[ ]:=
```
Manipulate[
 Plot3D[PDF[MultinormalDistribution[{0, 0}, {{1, ρ }, {ρ , 1}}], {x, y}],
  {x, -2, 2}, {y, -2, 2}],
 {ρ, 0, 1}]
```

Out[ ]=



The average pairwise distance is obtained by integrating over all positions of one individual {x1,y1} and then all positions of the second individual {x2,y2}, all of which are real numbers within the range.

In[∘]:=
```
Simplify[Sqrt[(x2 - x1)^2 + (y2 - y1)^2]
  PDF[MultinormalDistribution[{0, 0}, {{σx^2, ρ σx σy}, {ρ σx σy, σy^2}}], {x1, y1}] ×
  PDF[MultinormalDistribution[{0, 0}, {{σx^2, ρ σx σy}, {ρ σx σy, σy^2}}],
   {x2, y2}]] /. ρ → 0
```

Out[∘]=

$$\frac{e^{-\frac{y1^2\,\sigma x^2+y2^2\,\sigma x^2+\left(x1^2+x2^2\right)\sigma y^2}{2\,\sigma x^2\,\sigma y^2}}\sqrt{(x1-x2)^2+(y1-y2)^2}}{4\,\pi^2\,\sigma x^2\,\sigma y^2}$$

Unfortunately, this does not integrate directly:

In[∘]:=
```
Integrate[%, x1]
```

Out[∘]=

$$\frac{\int e^{-\frac{y1^2\,\sigma x^2+y2^2\,\sigma x^2+\left(x1^2+x2^2\right)\sigma y^2}{2\,\sigma x^2\,\sigma y^2}}\sqrt{(x1-x2)^2+(y1-y2)^2}\,dx1}{4\,\pi^2\,\sigma x^2\,\sigma y^2}$$

We work instead with Δx=x2-x1 and Δy=y2-y1, which simplifies the integration:

In[∘]:=
```
Simplify[Sqrt[Δx^2 + Δy^2]
  PDF[MultinormalDistribution[{0, 0}, {{σx^2, ρ σx σy}, {ρ σx σy, σy^2}}], {x1, y1}] ×
  PDF[MultinormalDistribution[{0, 0}, {{σx^2, ρ σx σy}, {ρ σx σy, σy^2}}],
   {Δx + x1, Δy + y1}]] /. ρ → 0
```

Out[∘]=

$$\frac{e^{-\frac{2\,y1^2\,\sigma x^2+2\,y1\,\Delta y\,\sigma x^2+\Delta y^2\,\sigma x^2+\left(2\,x1^2+2\,x1\,\Delta x+\Delta x^2\right)\sigma y^2}{2\,\sigma x^2\,\sigma y^2}}\sqrt{\Delta x^2+\Delta y^2}}{4\,\pi^2\,\sigma x^2\,\sigma y^2}$$

In[∘]:=
```
Simplify[Integrate[%, {x1, -Infinity, Infinity}], constraints]
```

Out[∘]=

$$\frac{e^{\frac{1}{4}\left(-\frac{\Delta x^2}{\sigma x^2}-\frac{2\left(2\,y1^2+2\,y1\,\Delta y+\Delta y^2\right)}{\sigma y^2}\right)}\sqrt{\Delta x^2+\Delta y^2}}{4\,\pi^{3/2}\,\sigma x\,\sigma y^2}$$

In[∘]:=
```
Simplify[Integrate[%, {y1, -Infinity, Infinity}], constraints]
```

Out[∘]=

$$\frac{e^{-\frac{\Delta x^2}{4\,\sigma x^2}-\frac{\Delta y^2}{4\,\sigma y^2}}\sqrt{\Delta x^2+\Delta y^2}}{4\,\pi\,\sigma x\,\sigma y}$$

In[∘]:=
```
Simplify[Integrate[%, {Δy, -Infinity, Infinity}], constraints]
```

Out[∘]=

$$\frac{e^{-\frac{\Delta x^2}{4\,\sigma x^2}}\,\sigma y\,\mathrm{HypergeometricU}\left[-\frac{1}{2},\,0,\,\frac{\Delta x^2}{4\,\sigma y^2}\right]}{\sqrt{\pi}\,\sigma x}$$

[Note that the order was reversed, first Δy and then Δx, to get the term in the EllipticE to be between 0 and 1 when *σ*x is assumed to be the major axis.]

In[∘]:=
```
soln = Simplify[Integrate[%, {Δx, -Infinity, Infinity}], constraints]
```
Out[∘]=

$$\frac{2\,\sigma x\,\text{EllipticE}\left[1-\frac{\sigma y^2}{\sigma x^2}\right]}{\sqrt{\pi}}$$

where EllipticE is the complete integral over the ellipse of $\int_0^{\pi/2}(1-m\sin^2(\theta))^{1/2}\,d\theta$

In[∘]:=

$$\text{soln} = \frac{2\,\sigma x\,\text{EllipticE}\left[1-\frac{\sigma y^2}{\sigma x^2}\right]}{\sqrt{\pi}};$$

In[∘]:=
```
soln /. σx → σ /. σy → σ // Simplify
```
Out[∘]=

$$\sqrt{\pi}\ \sigma$$

In[∘]:=
```
soln /. ρ → 0 /. σx → 0.5 /. σy → 2.
```
Out[∘]=
```
2.41993
```

In[∘]:=
```
Simplify[Sqrt[(x2 - x1)^2 + (y2 - y1)^2]
    PDF[MultinormalDistribution[{0, 0}, {{σx^2, ρ σx σy}, {ρ σx σy, σy^2}}], {x1, y1}] ×
    PDF[MultinormalDistribution[{0, 0}, {{σx^2, ρ σx σy}, {ρ σx σy, σy^2}}],
     {x2, y2}]] /. ρ → 0 /. σx → 1 / 2 /. σy → 2;
NIntegrate[%, {x1, -Infinity, Infinity}, {x2, -Infinity, Infinity},
 {y1, -Infinity, Infinity}, {y2, -Infinity, Infinity}]
```

NIntegrate: Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small.

NIntegrate: The global error of the strategy GlobalAdaptive has increased more than 2000 times. The global error is expected to decrease monotonically after a number of integrand evaluations. Suspect one of the following: the working precision is insufficient for the specified precision goal; the integrand is highly oscillatory or it is not a (piecewise) smooth function; or the true value of the integral is 0. Increasing the value of the GlobalAdaptive option MaxErrorIncreases might lead to a convergent numerical integration. NIntegrate obtained 2.419837695756608` and 0.0006003289543609284` for the integral and error estimates.

Out[∘]=
```
2.41984
```

Significance: By knowing the area and shape of herd home ranges (or schools of fish, flocks of birds, etc.), we can estimate the average distance between individuals, here assuming a Gaussian distribution.

E[X^2]: Average distance among all pairs

In[ ]:=
```
Simplify[(Δx^2 + Δy^2)
   PDF[MultinormalDistribution[{0, 0}, {{σx^2, ρ σx σy}, {ρ σx σy, σy^2}}], {x1, y1}] ×
   PDF[MultinormalDistribution[{0, 0}, {{σx^2, ρ σx σy}, {ρ σx σy, σy^2}}],
    {Δx + x1, Δy + y1}]] /. ρ → 0
```

Out[ ]=

$$\frac{e^{-\frac{2\,y1^2\,\sigma x^2+2\,y1\,\Delta y\,\sigma x^2+\Delta y^2\,\sigma x^2+\left(2\,x1^2+2\,x1\,\Delta x+\Delta x^2\right)\sigma y^2}{2\,\sigma x^2\,\sigma y^2}}\left(\Delta x^2+\Delta y^2\right)}{4\,\pi^2\,\sigma x^2\,\sigma y^2}$$

In[ ]:=
```
Simplify[Integrate[%, {x1, -Infinity, Infinity}], constraints]
```

Out[ ]=

$$\frac{e^{\frac{1}{4}\left(-\frac{\Delta x^2}{\sigma x^2}-\frac{2\left(2\,y1^2+2\,y1\,\Delta y+\Delta y^2\right)}{\sigma y^2}\right)}\left(\Delta x^2+\Delta y^2\right)}{4\,\pi^{3/2}\,\sigma x\,\sigma y^2}$$

In[ ]:=
```
Simplify[Integrate[%, {y1, -Infinity, Infinity}], constraints]
```

Out[ ]=

$$\frac{e^{-\frac{\Delta x^2}{4\,\sigma x^2}-\frac{\Delta y^2}{4\,\sigma y^2}}\left(\Delta x^2+\Delta y^2\right)}{4\,\pi\,\sigma x\,\sigma y}$$

In[ ]:=
```
Simplify[Integrate[%, {Δx, -Infinity, Infinity}], constraints]
```

Out[ ]=

$$\frac{e^{-\frac{\Delta y^2}{4\,\sigma y^2}}\left(\Delta y^2+2\,\sigma x^2\right)}{2\,\sqrt{\pi}\,\sigma y}$$

```
solnsq = Simplify[Integrate[%, {Δy, -Infinity, Infinity}], constraints]
```

Out[ ]=

$$2\left(\sigma x^2+\sigma y^2\right)$$

```
solnsq /. ρ → 0 /. σx → 0.5 /. σy → 2.
```

Out[ ]=

```
8.5
```

In[ ]:=
```
Simplify[((x2 - x1)^2 + (y2 - y1)^2)
    PDF[MultinormalDistribution[{0, 0}, {{σx^2, ρ σx σy}, {ρ σx σy, σy^2}}], {x1, y1}] ×
    PDF[MultinormalDistribution[{0, 0}, {{σx^2, ρ σx σy}, {ρ σx σy, σy^2}}],
     {x2, y2}]] /. ρ → 0 /. σx → 0.5 /. σy → 2;
NIntegrate[%, {x1, -Infinity, Infinity}, {x2, -Infinity, Infinity},
 {y1, -Infinity, Infinity}, {y2, -Infinity, Infinity}]
```

NIntegrate: Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small.

NIntegrate: The global error of the strategy GlobalAdaptive has increased more than 2000 times. The global error is expected to decrease monotonically after a number of integrand evaluations. Suspect one of the following: the working precision is insufficient for the specified precision goal; the integrand is highly oscillatory or it is not a (piecewise) smooth function; or the true value of the integral is 0. Increasing the value of the GlobalAdaptive option MaxErrorIncreases might lead to a convergent numerical integration. NIntegrate obtained 8.49997058974396` and 0.0013316117066312645` for the integral and error estimates.

Out[ ]=

8.49997

Herd CV : Coefficient of variation among all pairs

Considering all pairwise distances measured in the population (ignoring individual identity), the herd CV for pairwise distances is:

$$\text{CVherd} = \text{Simplify}\left[\frac{\text{Sqrt}\left[\text{solnsq} - \text{soln}^2\right]}{\text{soln}}, \text{constraints}\right]$$

Out[ ]=

$$\frac{\sqrt{\frac{1}{2}\pi\left(\sigma x^2 + \sigma y^2\right) - \sigma x^2\,\text{EllipticE}\left[1 - \frac{\sigma y^2}{\sigma x^2}\right]^2}}{\sigma x\,\text{EllipticE}\left[1 - \frac{\sigma y^2}{\sigma x^2}\right]}$$

Alternatively, we can write the CV as $\text{Sqrt}\left[\frac{\text{solnsq}}{\text{soln}^2} - 1\right]$. Next we show that the fraction $\frac{\text{solnsq}}{\text{soln}^2}$ (and hence CV) doesn't depend on the scale, as long as the ratio of ymax/xmax remains constant (call this "α"):

$$\text{Simplify}\left[\frac{\text{solnsq}}{\text{soln}^2} \;/.\; \sigma y \to \alpha\,\sigma x, \text{constraints}\right]$$

Out[ ]=

$$\frac{\pi\left(1 + \alpha^2\right)}{2\,\text{EllipticE}\left[1 - \alpha^2\right]^2}$$

Thus, the CV can be written as:

In[ ]:=

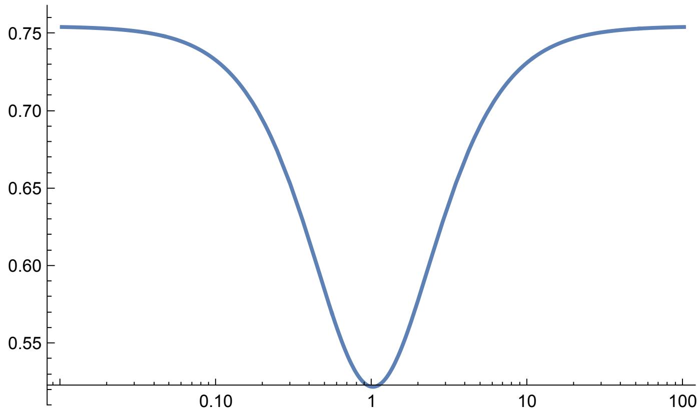
$$\text{CVherdALT} = \text{Sqrt}\left[\frac{\pi\left(1 + \alpha^2\right)}{2\,\text{EllipticE}\left[1 - \alpha^2\right]^2} - 1\right];$$

Thus, the CV is independent of the scale (σx,σy), as expected.

As the range elongates (either becoming broader α<1 or taller α>1), CV rises:

In[∘]:=
```
LogLinearPlot[CVherdALT, {α, 1 / 100, 100}]
```

Out[∘]=



The CV is $\sqrt{\frac{4}{\pi} - 1}$ (=0.523) when the Gaussian is symmetrical (α=1) and rises as the herd elongates to a maximum of $\sqrt{\frac{\pi}{2} - 1}$ (=0.756)

In[∘]:=
$$\text{Limit}\left[\text{Sqrt}\left[\frac{\pi\,(1+\alpha^2)}{2\,\alpha^2\,\text{EllipticE}\left[1-\frac{1}{\alpha^2}\right]^2}-1\right],\ \alpha\to 1\right]$$

```
% // N
```

Out[∘]=
$$\sqrt{-1+\frac{4}{\pi}}$$

Out[∘]=
```
0.522723
```

In[∘]:=
$$\text{Limit}\left[\text{Sqrt}\left[\frac{\pi\,(1+\alpha^2)}{2\,\alpha^2\,\text{EllipticE}\left[1-\frac{1}{\alpha^2}\right]^2}-1\right],\ \alpha\to 0\right]$$

```
% // N
```

Out[∘]=
$$\sqrt{\frac{1}{2}\,(-2+\pi)}$$

Out[∘]=
```
0.755511
```

Herd split patches: Two Gaussian patches at a distance c from one another [σx = σy]

Here we split the range in two and move one patch to the right by an amount c. For the intra-individual distances, half will be in the same patch (given above) and half will be in the other patch, which we now calculate. For clarity, we assume that individual 2 is from the right-most patch, displaced by c on the x axis.

In[∘]:=
```
Simplify[Sqrt[Δx^2 + Δy^2]
    PDF[MultinormalDistribution[{0, 0}, {{σx^2, ρ σx σy}, {ρ σx σy, σy^2}}], {x1, y1}] ×
    PDF[MultinormalDistribution[{c, 0}, {{σx^2, ρ σx σy}, {ρ σx σy, σy^2}}],
     {Δx + x1, Δy + y1}]] /. ρ → 0
```

Out[∘]=

$$\frac{e^{-\frac{2\,y1^2\,\sigma x^2+2\,y1\,\Delta y\,\sigma x^2+\Delta y^2\,\sigma x^2+\left(c^2+2\,x1^2+2\,x1\,\Delta x+\Delta x^2-2\,c\,(x1+\Delta x)\right)\,\sigma y^2}{2\,\sigma x^2\,\sigma y^2}}\,\sqrt{\Delta x^2+\Delta y^2}}{4\,\pi^2\,\sigma x^2\,\sigma y^2}$$

In[∘]:=
```
Simplify[Integrate[%, {x1, -Infinity, Infinity}], constraints]
```

Out[∘]=

$$\frac{e^{\frac{1}{4}\left(-\frac{(c-\Delta x)^2}{\sigma x^2}-\frac{2\left(2\,y1^2+2\,y1\,\Delta y+\Delta y^2\right)}{\sigma y^2}\right)}\,\sqrt{\Delta x^2+\Delta y^2}}{4\,\pi^{3/2}\,\sigma x\,\sigma y^2}$$

In[∘]:=
```
Simplify[Integrate[%, {y1, -Infinity, Infinity}], constraints]
```

Out[∘]=

$$\frac{e^{-\frac{(c-\Delta x)^2}{4\,\sigma x^2}-\frac{\Delta y^2}{4\,\sigma y^2}}\,\sqrt{\Delta x^2+\Delta y^2}}{4\,\pi\,\sigma x\,\sigma y}$$

In[∘]:=
```
integrateme = Simplify[Integrate[%, {Δy, -Infinity, Infinity}], constraints]
```

Out[∘]=

$$\frac{e^{-\frac{(c-\Delta x)^2}{4\,\sigma x^2}}\,\sigma y\,\text{HypergeometricU}\left[-\frac{1}{2},\,0,\,\frac{\Delta x^2}{4\,\sigma y^2}\right]}{\sqrt{\pi}\,\sigma x}$$

This does not integrate directly:

In[∘]:=
```
Integrate[integrateme, Δx]
```

Out[∘]=

$$\frac{\sigma y\int e^{-\frac{(c-\Delta x)^2}{4\,\sigma x^2}}\,\text{HypergeometricU}\left[-\frac{1}{2},\,0,\,\frac{\Delta x^2}{4\,\sigma y^2}\right]\,d\Delta x}{\sqrt{\pi}\,\sigma x}$$

In[∘]:=
```
Integrate[integrateme, {Δx, -Infinity, Infinity}]
```

Out[∘]=

$$\int_{-\infty}^{\infty}\frac{e^{-\frac{(c-\Delta x)^2}{4\,\sigma x^2}}\,\sigma y\,\text{HypergeometricU}\left[-\frac{1}{2},\,0,\,\frac{\Delta x^2}{4\,\sigma y^2}\right]}{\sqrt{\pi}\,\sigma x}\,d\Delta x$$

However, in the symmetric case (σx=σy=σy), it does:

In[∘]:=
```
FullSimplify[Integrate[integrateme /. σy → σ /. σx → σ,
   {Δx, -Infinity, Infinity}, Assumptions → constraints], constraints]
```

Out[∘]=

$$\frac{e^{-\frac{c^2}{8\,\sigma^2}}\,\sqrt{\pi}\,\left(\left(c^2+4\,\sigma^2\right)\,\text{BesselI}\left[0,\,\frac{c^2}{8\,\sigma^2}\right]+c^2\,\text{BesselI}\left[1,\,\frac{c^2}{8\,\sigma^2}\right]\right)}{4\,\sigma}$$

The distance c only enters in terms of c/σ (the scaled distance between the two populations), so we switch to working in terms of this scaled distance c_S (the “c” presented in the paper):

In[∘]:=
```
partsolnC = FullSimplify[% /. c → c_S σ, constraints]
```

Out[∘]=

$$\frac{1}{4} e^{-\frac{c_S^2}{8}} \sqrt{\pi}\, \sigma \left(\text{BesselI}\left[1, \frac{c_S^2}{8}\right] c_S^2 + \text{BesselI}\left[0, \frac{c_S^2}{8}\right] \left(4 + c_S^2\right)\right)$$

The expected value of the distance squared is much simpler to calculate:

In[∘]:=
```
Simplify[(Δx^2 + Δy^2)
   PDF[MultinormalDistribution[{0, 0}, {{σx^2, ρ σx σy}, {ρ σx σy, σy^2}}], {x1, y1}] ×
   PDF[MultinormalDistribution[{c, 0}, {{σx^2, ρ σx σy}, {ρ σx σy, σy^2}}],
    {Δx + x1, Δy + y1}]] /. ρ → 0
```

Out[∘]=

$$\frac{e^{-\frac{2\, y1^2\, \sigma x^2 + 2\, y1\, \Delta y\, \sigma x^2 + \Delta y^2\, \sigma x^2 + \left(c^2 + 2\, x1^2 + 2\, x1\, \Delta x + \Delta x^2 - 2\, c\, (x1 + \Delta x)\right) \sigma y^2}{2\, \sigma x^2\, \sigma y^2}} \left(\Delta x^2 + \Delta y^2\right)}{4\, \pi^2\, \sigma x^2\, \sigma y^2}$$

In[∘]:=
```
Simplify[Integrate[%, {x1, -Infinity, Infinity}], constraints]
```

Out[∘]=

$$\frac{e^{\frac{1}{4}\left(-\frac{(c - \Delta x)^2}{\sigma x^2} - \frac{2\left(2\, y1^2 + 2\, y1\, \Delta y + \Delta y^2\right)}{\sigma y^2}\right)} \left(\Delta x^2 + \Delta y^2\right)}{4\, \pi^{3/2}\, \sigma x\, \sigma y^2}$$

In[∘]:=
```
Simplify[Integrate[%, {y1, -Infinity, Infinity}], constraints]
```

Out[∘]=

$$\frac{e^{-\frac{(c - \Delta x)^2}{4\, \sigma x^2} - \frac{\Delta y^2}{4\, \sigma y^2}} \left(\Delta x^2 + \Delta y^2\right)}{4\, \pi\, \sigma x\, \sigma y}$$

In[∘]:=
```
Simplify[Integrate[%, {Δy, -Infinity, Infinity}], constraints]
```

Out[∘]=

$$\frac{e^{-\frac{(c - \Delta x)^2}{4\, \sigma x^2}} \left(\Delta x^2 + 2\, \sigma y^2\right)}{2\, \sqrt{\pi}\, \sigma x}$$

In[∘]:=
```
Simplify[Integrate[%, {Δx, -Infinity, Infinity}], constraints]
```

Out[∘]=

$$c^2 + 2\left(\sigma x^2 + \sigma y^2\right)$$

In the symmetric case measuring c in terms of standard deviations:

```
partsolnsqC = Factor[% /. c → c_S σ /. σx → σ /. σy → σ]
```

Out[∘]=

$$\sigma^2 \left(4 + c_S^2\right)$$

Recall that we need to assume *σ*x=*σ*y=*σ* and average over individuals chosen from the same patch or from different patches. Here we allow a fraction 1-f to be chosen from within a patch (either one) and f between the patches:

In[∘]:=
```
solnC = (1 - f) soln + f partsolnC /. σx → σ /. σy → σ
solnsqC = (1 - f) solnsq + f partsolnsqC /. σx → σ /. σy → σ
```

Out[∘]=

$$(1-f)\sqrt{\pi}\,\sigma + \frac{1}{4} e^{-\frac{c_S^2}{8}} f \sqrt{\pi}\,\sigma \left(\text{BesselI}\left[1, \frac{c_S^2}{8}\right] c_S^2 + \text{BesselI}\left[0, \frac{c_S^2}{8}\right] \left(4 + c_S^2\right)\right)$$

Out[∘]=

$$4\,(1-f)\,\sigma^2 + f\,\sigma^2\,\left(4 + c_S^2\right)$$

Relative to the unsplit case:

In[∘]:=

$$\text{FullSimplify}\left[\frac{\text{solnsqC}}{\text{solnsq}^2} \text{ /. } \sigma x \to \sigma \text{ /. } \sigma y \to \sigma,\ \text{constraints}\right]$$

Out[∘]=

$$\frac{4 + f\,c_S^2}{16\,\sigma^2}$$

Calculating the distance squared over the mean squared for the CV:

In[∘]:=

$$\text{Simplify}\left[\frac{\text{solnsqC}}{\text{solnC}^2} \text{ /. } \sigma x \to \sigma \text{ /. } \sigma y \to \sigma\right]$$

Out[∘]=

$$\frac{16\, e^{\frac{c_S^2}{4}} \left(4 + f\,c_S^2\right)}{\pi \left(-4\, e^{\frac{c_S^2}{8}} (-1+f) + 4 f\, \text{BesselI}\left[0, \frac{c_S^2}{8}\right] + f\left(\text{BesselI}\left[0, \frac{c_S^2}{8}\right] + \text{BesselI}\left[1, \frac{c_S^2}{8}\right]\right) c_S^2\right)^2}$$

Note that the above depends only on $\frac{c}{\sigma}$ and not on c itself (as expected). That is, the amount of split relative to the standard deviation of each patch is what matters.

In[∘]:=

$$\text{CVherdC} = \text{Sqrt}\left[\frac{16\, e^{\frac{c_S^2}{4}} \left(4 + f\,c_S^2\right)}{\pi \left(4\, e^{\frac{c_S^2}{8}} (1-f) + \left(4 + c_S^2\right) f\, \text{BesselI}\left[0, \frac{c_S^2}{8}\right] + f\,c_S^2\, \text{BesselI}\left[1, \frac{c_S^2}{8}\right]\right)^2} - 1\right];$$

Note that the CVherd goes to $\sqrt{\frac{1}{f} - 1}$ in the limit as c becomes very large relative to σ, because a fraction f of the time pairs are at distance c and 1-f the time near 0 so CVherdC approaches $\text{Sqrt}\left[\frac{((1-f)\,0 + f\,c_S{}^2)}{((1-f)\,0 + f\,c_S)^2} - 1\right] = 1$

In[∘]:=

$$\text{Sqrt}\left[\frac{((1-f)\,0 + f\,c_S{}^2)}{((1-f)\,0 + f\,c_S)^2} - 1\right]$$

Out[∘]=

$$\sqrt{-1 + \frac{1}{f}}$$

In[∘]:=
```
Limit[CVherdC, cS → Infinity, Assumptions → {0 < f < 1, cS > 0, σ > 0}]
```

Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[∘]=

$$\sqrt{-1 + \frac{1}{f}}$$

and goes to $\sqrt{\frac{4}{\pi} - 1}$ as $c_S \to 0$, as in the case of a single population:

In[ ]:= `Limit[Simplify[CVherdC], cS → 0]`

Out[ ]=

$$\sqrt{-1 + \frac{64}{(-4\ (-1 + f)\ + 4\ f)^2\ \pi}}$$

Individual CV: Average pairwise distance from individual i [$\sigma x = \sigma y$]

We now focus on an individual at position {x1,y1} and determine its average pairwise distance to all other individuals measured (not yet using the PDF for {x1,y1}). This would then be used as the basis for calculating the CV among individuals.

Unfortunately, there appears to be no closed form solution, although the Rice Distribution describes the average distance from a point to a Gaussian PDF, but only for $\sigma x = \sigma y$:

In[ ]:= `Mean[RiceDistribution[Sqrt[x1^2 + y2^2], σ]]`

Out[ ]=

$$\sqrt{\frac{\pi}{2}}\ \sigma\ \text{LaguerreL}\left[\frac{1}{2}, -\frac{x1^2 + y2^2}{2\ \sigma^2}\right]$$

In[ ]:=
```
tryx1 = 0.3; tryy1 = 0.7; tryσ = 2;
Simplify[Sqrt[(x2 - tryx1)^2 + (y2 - tryy1)^2] PDF[MultinormalDistribution[{0, 0},
        {{σx^2, ρ σx σy}, {ρ σx σy, σy^2}}], {x2, y2}]] /. ρ → 0 /. σx → tryσ /. σy → tryσ;
NIntegrate[%, {x2, -Infinity, Infinity}, {y2, -Infinity, Infinity}]
```

Out[ ]=

2.59668

In[ ]:= `Mean[RiceDistribution[Sqrt[tryx1^2 + tryy1^2], tryσ]] // N`

Out[ ]=

2.59668

Unfortunately, the Laguerre function in the mean is not readily integrable:

In[ ]:=
```
Simplify[Mean[RiceDistribution[Sqrt[x1^2 + y1^2], σ]] ×
    PDF[MultinormalDistribution[{0, 0}, {{σx^2, ρ σx σy}, {ρ σx σy, σy^2}}], {x1, y1}] /.
   ρ → 0 /. σx → σ /. σy → σ, constraints]
```

Out[ ]=

$$\frac{e^{-\frac{x1^2+y1^2}{2\sigma^2}}\ \text{LaguerreL}\left[\frac{1}{2}, -\frac{x1^2+y1^2}{2\sigma^2}\right]}{2\ \sqrt{2\ \pi}\ \sigma}$$

In[ ]:= `Simplify[Integrate[%, x1], constraints]`

Out[ ]=

$$\frac{\int e^{-\frac{x1^2+y1^2}{2\sigma^2}}\ \text{LaguerreL}\left[\frac{1}{2}, -\frac{x1^2+y1^2}{2\sigma^2}\right] dx1}{2\ \sqrt{2\ \pi}\ \sigma}$$

Note that we can simplify this, using r to denote the radius between {x1,y1} and the centre of the Gaussian, the probability of being at that point r is then given by the normal distribution multiplied by

2*r (the circumference around the bivariate Gaussian at distance r from the mean) and divided by the area:

In[ ]:=
```
Simplify[
 Integrate[(2 r PDF[ NormalDistribution[0, σ], r]), {r, 0, Infinity}], constraints]
```

Out[ ]=

$$\sqrt{\frac{2}{\pi}}\ \sigma$$

In[ ]:=
```
Mean[RiceDistribution[r, σ]] (2 r PDF[ NormalDistribution[0, σ], r] / (Sqrt[2/π] σ))
```

Out[ ]=

$$\frac{e^{-\frac{r^2}{2\sigma^2}}\ \sqrt{\frac{\pi}{2}}\ r\ \text{LaguerreL}\left[\frac{1}{2},\ -\frac{r^2}{2\sigma^2}\right]}{\sigma}$$

In[ ]:=
```
Simplify[Integrate[%, r], constraints]
```

Out[ ]=

$$\frac{\sqrt{\frac{\pi}{2}}\ \int e^{-\frac{r^2}{2\sigma^2}}\ r\ \text{LaguerreL}\left[\frac{1}{2},\ -\frac{r^2}{2\sigma^2}\right]\, dr}{\sigma}$$

Rescaling r to σ ($rS = \frac{r}{\sigma}$) and noting that dr becomes $\sigma\, d rS$:

$$\sigma\ e^{-\frac{rS^2}{2}}\ \sqrt{\frac{\pi}{2}}\ rS\ \text{LaguerreL}\left[\frac{1}{2},\ -\frac{rS^2}{2}\right]$$

While the above cannot be integrated analytically, we only need a numerical solution (factoring the σ out):

In[ ]:=
```
indsoln = σ NIntegrate[e^(-rS^2/2) Sqrt[π/2] rS LaguerreL[1/2, -rS^2/2], {rS, 0, Infinity}]
```

Out[ ]=

$1.77245\ \sigma$

As expected, when integrated over r, we get the mean distance found above:

In[ ]:=
```
soln /. σx → σ /. σy → σ
% // N
```

Out[ ]=

$\sqrt{\pi}\ \sigma$

Out[ ]=

$1.77245\ \sigma$

What we really want though is E[X^2]:

In[∘]:= `Mean[RiceDistribution[r, σ]]`

Out[∘]=

$$\sqrt{\frac{\pi}{2}}\,\sigma\,\text{LaguerreL}\left[\frac{1}{2}, -\frac{r^2}{2\,\sigma^2}\right]$$

In[∘]:=

$$\text{Mean[RiceDistribution[r, }\sigma\text{]]}^2\left(\frac{2\,r\,\text{PDF[NormalDistribution[0, }\sigma\text{], r]}}{\sqrt{\frac{2}{\pi}}\,\sigma}\right)$$

Out[∘]=

$$\frac{1}{2}\,e^{-\frac{r^2}{2\sigma^2}}\,\pi\,r\,\text{LaguerreL}\left[\frac{1}{2}, -\frac{r^2}{2\,\sigma^2}\right]^2$$

In[∘]:= `Simplify[Integrate[%, r], constraints]`

Out[∘]=

$$\frac{1}{2}\,\pi\int e^{-\frac{r^2}{2\sigma^2}}\,r\,\text{LaguerreL}\left[\frac{1}{2}, -\frac{r^2}{2\,\sigma^2}\right]^2 dr$$

Rescaling r to σ ($rS = \frac{r}{\sigma}$) and noting that dr becomes $\sigma\, drS$:

In[∘]:=

$$\frac{1}{2}\,e^{-\frac{rS^2}{2}}\,\pi\,rS\,\sigma^2\,\text{LaguerreL}\left[\frac{1}{2}, -\frac{rS^2}{2}\right]^2;$$

In[∘]:= `Integrate[ %, rS]`

Out[∘]=

$$\frac{1}{2}\,\pi\,\sigma^2\int e^{-\frac{rS^2}{2}}\,rS\,\text{LaguerreL}\left[\frac{1}{2}, -\frac{rS^2}{2}\right]^2 drS$$

While the above cannot be integrated analytically, we only need a numerical solution:

In[∘]:=

$$\text{indsolnsq} = \frac{\sigma^2\,\pi}{2}\,\text{NIntegrate}\left[e^{-\frac{rS^2}{2}}\;rS\;\text{LaguerreL}\left[\frac{1}{2}, -\frac{rS^2}{2}\right]^2, \{rS, 0, \text{Infinity}\}\right]$$

Out[∘]=

$$3.34122\,\sigma^2$$

It could also be approximated (not used):

In[∘]:=

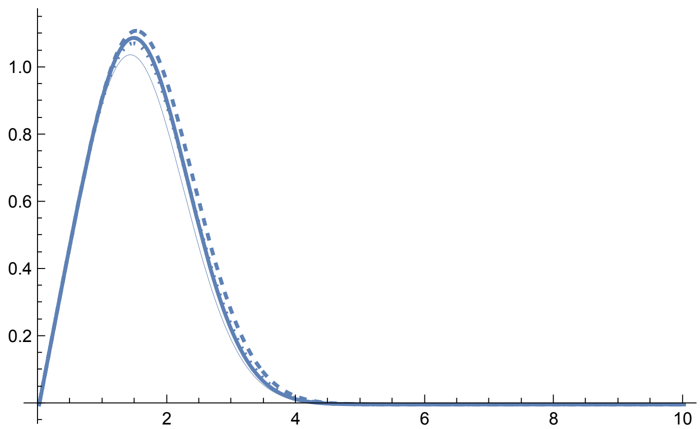
$$\text{Series}\left[\text{LaguerreL}\left[\frac{1}{2}, -\frac{rS^2}{2}\right]^2, \{rS, 0, 5\}\right]$$

Out[∘]=

$$1 + \frac{rS^2}{2} + \frac{rS^4}{32} + O[rS]^6$$

In[∘]:=
```
Show[
 Plot[ⅇ^(-rS^2/2) rS LaguerreL[1/2, -rS^2/2]^2, {rS, 0, 10}],
 Plot[ⅇ^(-rS^2/2) rS (1 + rS^2/2), {rS, 0, 10}, PlotStyle → Thin],
 Plot[ⅇ^(-rS^2/2) rS (1 + rS^2/2 + rS^4/64), {rS, 0, 10}, PlotStyle → Dotted],
 Plot[ⅇ^(-rS^2/2) rS (1 + rS^2/2 + rS^4/32), {rS, 0, 10}, PlotStyle → Dashed], PlotRange → All
]
```

Out[∘]=



Half-way between the quadratic and quartic approximation works best when weighted by $e^{-\frac{rS^2}{2}}\ rS$

In[∘]:=
```
{NIntegrate[ⅇ^(-rS^2/2) rS LaguerreL[1/2, -rS^2/2]^2, {rS, 0, Infinity}],
 NIntegrate[ⅇ^(-rS^2/2) rS (1 + rS^2/2), {rS, 0, Infinity}],
 NIntegrate[ⅇ^(-rS^2/2) rS (1 + rS^2/2 + rS^4/64), {rS, 0, Infinity}],
 NIntegrate[ⅇ^(-rS^2/2) rS (1 + rS^2/2 + rS^4/32), {rS, 0, Infinity}]}
```

Out[∘]=

```
{2.12709, 2., 2.125, 2.25}
```

For the $\sigma x = \sigma y = \sigma$ case, individual CV can be written as Sqrt$\left[\frac{\text{indsolnsq}}{\text{soln}^2} - 1\right]$, which again does not depend on the scale (σ):

In[∘]:=
```
CVind = Simplify[Sqrt[indsolnsq/soln^2 - 1] /. σx → σ /. σy → σ]
```

Out[∘]=

```
0.25208
```

1D Analysis - For comparison, a very elongated distribution would be equivalent to the 1D case:

In[∘]:=
```
Sqrt[(x2 - x1)^2] PDF[NormalDistribution[0, σx], x2]
```

Out[∘]=

$$\frac{e^{-\frac{x2^2}{2\,\sigma x^2}}\sqrt{(-x1+x2)^2}}{\sqrt{2\,\pi}\,\sigma x}$$

We take the focal individual to be at position x1 and calculate its distance to other individuals [it's easier to just take the distance as the positive value, breaking the integral into x2>x1 and x2<x1 (switching the sign in the latter case)]

In[∘]:=
```
Integrate[(x2 - x1) PDF[NormalDistribution[0, σx], x2],
 x2, Assumptions → {x1 > 0, σx > 0}]
```

Out[∘]=

$$\frac{-e^{-\frac{x2^2}{2\,\sigma x^2}}\,\sigma x^2-\sqrt{\frac{\pi}{2}}\,x1\,\sigma x\,\mathrm{Erf}\left[\frac{x2}{\sqrt{2}\,\sigma x}\right]}{\sqrt{2\,\pi}\,\sigma x}$$

In[∘]:=
```
Simplify[Simplify[Limit[%, x2 → Infinity, Assumptions → {x1 > 0, σx > 0}] -
    Limit[%, x2 → x1, Assumptions → {x1 > 0, σx > 0}], {x1 > 0, σx > 0}] +
  Simplify[Limit[-%, x2 → x1, Assumptions → {x1 > 0, σx > 0}] - Limit[-%,
     x2 → -Infinity, Assumptions → {x1 > 0, σx > 0}], {x1 > 0, σx > 0}], constraints]
```

Out[∘]=

$$e^{-\frac{x1^2}{2\,\sigma x^2}}\sqrt{\frac{2}{\pi}}\,\sigma x+x1\,\mathrm{Erf}\left[\frac{x1}{\sqrt{2}\,\sigma x}\right]$$

The above gives the average distance of individuals at position x1 to all others. We then calculate the average distance and then the average distance squared among focal individuals:

In[∘]:=
```
Integrate[(e^(-x1^2/(2 σx^2)) Sqrt[2/π] σx + x1 Erf[x1/(Sqrt[2] σx)]) PDF[NormalDistribution[0, σx], x1],
 x1, Assumptions → constraints]
```

Out[∘]=

$$\frac{\sigma x\,\mathrm{Erf}\left[\frac{x1}{\sigma x}\right]}{\sqrt{\pi}}-\frac{e^{-\frac{x1^2}{2\,\sigma x^2}}\,\sigma x\,\mathrm{Erf}\left[\frac{x1}{\sqrt{2}\,\sigma x}\right]}{\sqrt{2\,\pi}}$$

In[∘]:=
```
indsoln1 =
 Simplify[Simplify[Limit[%, x1 → Infinity, Assumptions → {x1 > 0, σx > 0}] - Limit[%,
     x1 → -Infinity, Assumptions → {x1 > 0, σx > 0}], {x1 > 0, σx > 0}], constraints]
```

Limit: Warning: Assumptions that involve the limit variable are ignored.

Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[∘]=

$$\frac{2\,\sigma x}{\sqrt{\pi}}$$

Repeating for the average distance squared turns out to be easier with the limits of integration:

In[∘]:= indsolnsq1 =

$$\text{Integrate}\left[\left(e^{-\frac{x1^2}{2\,\sigma x^2}}\sqrt{\frac{2}{\pi}}\,\sigma x + x1\,\text{Erf}\left[\frac{x1}{\sqrt{2}\,\sigma x}\right]\right)^2 \text{PDF}[\text{NormalDistribution}[0, \sigma x], x1], \{x1, -\text{Infinity}, \text{Infinity}\}, \text{Assumptions} \to \text{constraints}\right]$$

Out[∘]=

$$\frac{\left(6\sqrt{3}+\pi\right)\sigma x^2}{3\,\pi}$$

In[∘]:= CVind1 = Simplify$\left[\text{Sqrt}\left[\frac{\frac{\left(6\sqrt{3}+\pi\right)\sigma x^2}{3\pi}}{\left(\frac{2\,\sigma x}{\sqrt{\pi}}\right)^2} - 1\right]\right]$

Out[∘]=

$$\sqrt{-1+\frac{1}{12}\left(6\sqrt{3}+\pi\right)}$$

```
% // N
```

Out[∘]=

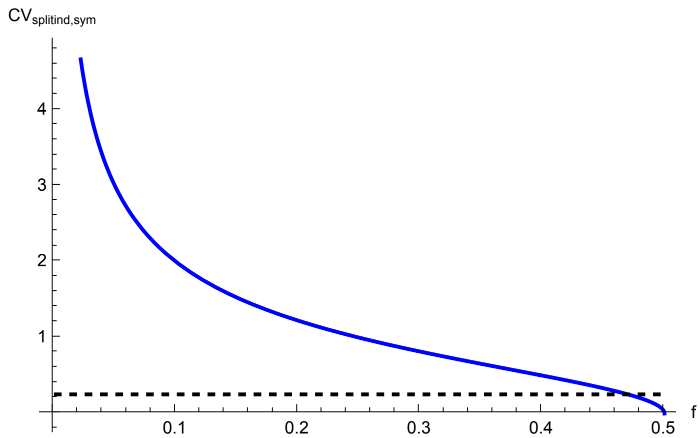
0.357526

Given the challenges of integrating the individual-level average distance, attempts to analyse the case of a split herd did not bear fruit. That said, the extreme cases are known. When the two herds overlap completely (c=0), we already have the non-split result above (CVind). At the other extreme, where the two patches are very far apart (c -> ∞), the individual mean μ_{IID} approaches $p_2\,c$ for individuals drawn from patch 1 (with ~0 distance to other individuals within the patch and a distance c when drawn from the other patch) and similarly $p_1\,c$ for individuals drawn from patch 2; altogether, this gives an average distance of $E[\mu_{\text{IID}}] = 2\,p_1\,p_2\,c = f\,c$ and an $E\left[\mu_{\text{IID}}{}^2\right] = p_1\,(p_2\,c)^2 + p_2\,(p_1\,c)^2 = \frac{f}{2}\,c^2$, an individual-level variance of $\text{Var}[\mu_{\text{IID}}] = \frac{f}{2}\,c^2 - (f\,c)^2 = f\left(\frac{1}{2}-f\right)c^2$, and so a CV of $\text{Sqrt}\left[\frac{f\left(\frac{1}{2}-f\right)c^2}{(f\,c)^2}\right] = \sqrt{\frac{\left(\frac{1}{2}-f\right)}{f}}$. When the two patches are equal in size (f=1/2), all individuals have the same average distance to all others (~c/2) and there is no variance (CVind=0), but when the two patches are unequal in size, CVind can rise because individuals in the smaller patch have a much higher average IID to all other individuals, since most other individuals are in the larger patch.

In[∘]:= CVindfarsplit = $\sqrt{\frac{\left(\frac{1}{2}-f\right)}{f}}$;

In[ ]:=
```
Show[
 Plot[Sqrt[(1/2 - f)/f], {f, 0, 1/2}, PlotStyle → Blue],
 Plot[CVind, {f, 0, 1/2}, PlotStyle → {Dashed, Black}],
 AxesLabel → {"f", "CV_splitind,sym"}
]
```

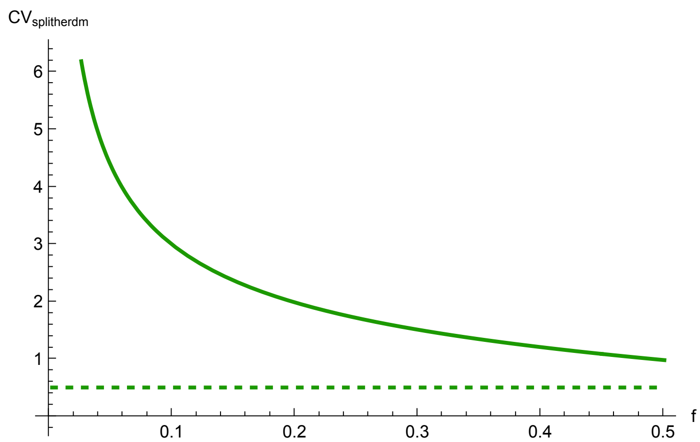
Out[ ]=



By comparison, a plot of CV_{herd} in the limit of very distant patches (solid) as a function of f (the chance individuals are drawn from different patches):

In[ ]:=
```
Show[
 Plot[Sqrt[1/f - 1] (*Asymmetrical split case*),
  {f, 0, 1/2}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
 Plot[Sqrt[4/π - 1] (*Unsplit case*), {f, 0, 1/2},
  PlotStyle → {Dashed, RGBColor[0.11, 0.6, 0.]}],
 AxesLabel → {"f", "CV_splitherdm"}, PlotRange → All, AxesOrigin → {0, 0}
]
```

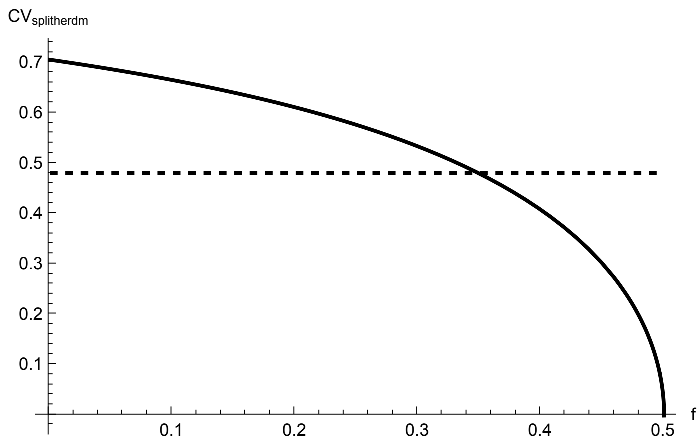
Out[ ]=



Plot of CV_{herd}/CV_{ind} in the limit of very distant patches (solid) as a function of f (the chance individuals are drawn from different patches):

```
Show[
 Plot[Sqrt[(1/2 - f)/f]/Sqrt[1/f - 1] (*Asymmetrical split case*), {f, 0, 1/2}, PlotStyle → Black],
 Plot[CVind/Sqrt[4/π - 1] (*Unsplit case*), {f, 0, 1/2}, PlotStyle → {Dashed, Black}],
 AxesLabel → {"f", "CV_ratio"}, PlotRange → All, AxesOrigin → {0, 0}
]
```

Out[ ]=



Thus, CV_{ratio} falls for distant patches only when f is small enough (roughly f=0.35, with a 35% chance of drawing individuals from different patches).

Simulation check

1000 individuals were simulated from the given population distribution. CV_{herd} and CV_{ind} match the predictions to the first two digits of precision.

Herd with a Gaussian range (CV_{herd} ~ 0.525, CV_{ind} ~ 0.255)

In[ ]:=

```
SeedRandom[28 645];
```

Assume a random distribution of individuals (blue):

```
tryσx = 1;
tryσy = 1;
table = Table[{Random[NormalDistribution[0, tryσx]],
    Random[NormalDistribution[0, tryσy]]}, {i, 1, 1000}];
```

Calculating the mean Euclidean distance for all pairwise distances (blue)

This table includes all of the pairwise distances:

In[∘]:=
```
disttable = Flatten[
   Table[EuclideanDistance[table⟦i⟧, table⟦j⟧], {i, 1, 1000}, {j, i + 1, 1000}]];
```

The mean and SD:

In[∘]:=
```
{ Mean[disttable], StandardDeviation[disttable]}
```

Out[∘]=
```
{1.74168, 0.914825}
```

The coefficient of variation across the herd (CV_{herd}):

In[∘]:=
```
StandardDeviation[disttable] / Mean[disttable]
```

Out[∘]=
```
0.525253
```

This matches the expectation:

```
Simplify[CVherd /. σx → tryσx /. σy → tryσy, constraints] // N
```

Out[∘]=
```
0.522723
```

Looking instead at how far, on average, each individual is to all others, then calculating the CV in this value among individuals (CV_{ind}):

In[∘]:=
```
disttable2 =
  Table[1 / (Length[table] - 1) (Sum[EuclideanDistance[table⟦i⟧, table⟦j⟧], {j, 1, i - 1}] +
      Sum[EuclideanDistance[table⟦i⟧, table⟦j⟧], {j, i + 1, 1000}]), {i, 1, 1000}];
```

The mean and SD:

In[∘]:=
```
{Mean[disttable2], StandardDeviation[disttable2]}
```

Out[∘]=
```
{1.74168, 0.443523}
```

Because of the averaging that occurs for each individual’s average pairwise distance, we get a lower CV_{ind}:

```
StandardDeviation[disttable2] / Mean[disttable2]
```

Out[∘]=
```
0.254652
```

This matches the expectation:

In[∘]:=
```
Simplify[CVind, constraints] // N
```

Out[∘]=
```
0.25208
```

Moderately elongated range ($\sigma y = \sigma x/2$; $CV_{herd} \sim 0.587$, $CV_{ind} \sim 0.285$)

In[∘]:=
```
SeedRandom[29543];
```

Assume a random distribution of individuals (blue):

In[∘]:=
```
tryσx = 1;
tryσy = 1 / 2;
```

In[∘]:=
```
table = Table[{Random[NormalDistribution[0, tryσx]],
    Random[NormalDistribution[0, tryσy]]}, {i, 1, 1000}];
```

Calculating the mean Euclidean distance for all pairwise distances (blue)

This table includes all of the pairwise distances:

In[∘]:=
```
disttable = Flatten[
   Table[EuclideanDistance[table〚i〛, table〚j〛], {i, 1, 1000}, {j, i + 1, 1000}]];
```

The mean and SD:

In[∘]:=
```
{ Mean[disttable], StandardDeviation[disttable]}
```

Out[∘]=
```
{1.40677, 0.825723}
```

The coefficient of variation across the herd (CV_{herd}):

In[∘]:=
$$\frac{\texttt{StandardDeviation[disttable]}}{\texttt{Mean[disttable]}}$$

Out[∘]=
```
0.586963
```

This matches the expectation:

In[∘]:=
```
Simplify[CVherd /. σx → tryσx /.  σy → tryσy, constraints] // N
```

Out[∘]=
```
0.582027
```

Looking instead at how far, on average, each individual is to all others, then calculating the CV in this value among individuals (CV_{ind}):

In[∘]:=
```
disttable2 =
  Table[1/(Length[table] - 1) (Sum[EuclideanDistance[table〚i〛, table〚j〛], {j, 1, i - 1}] +
      Sum[EuclideanDistance[table〚i〛, table〚j〛], {j, i + 1, 1000}]), {i, 1, 1000}];
```

The mean and SD:

In[∘]:=
```
{Mean[disttable2], StandardDeviation[disttable2]}
```

Out[∘]=
```
{1.40677, 0.401356}
```

Because of the averaging that occurs for each individual's average pairwise distance, we get a lower CV_{ind}:

```
StandardDeviation[disttable2]
-----------------------------
      Mean[disttable2]
```

Out[ ]=

```
0.285303
```

Very elongated range ($\sigma y = \sigma x/100$; $CV_{herd} \sim 0.752$, $CV_{ind} \sim 0.354$)

In[ ]:=

```
SeedRandom[87 772];
```

Assume a random distribution of individuals (blue):

```
tryσx = 1;
tryσy = 1 / 100;
```

```
table = Table[{Random[NormalDistribution[0, tryσx]],
    Random[NormalDistribution[0, tryσy]]}, {i, 1, 1000}];
```

Calculating the mean Euclidean distance for all pairwise distances (blue)

This table includes all of the pairwise distances:

In[ ]:=

```
disttable = Flatten[
   Table[EuclideanDistance[table⟦i⟧, table⟦j⟧], {i, 1, 1000}, {j, i + 1, 1000}]];
```

The mean and SD:

In[ ]:=

```
{ Mean[disttable], StandardDeviation[disttable]}
```

Out[ ]=

```
{1.14843, 0.863437}
```

The coefficient of variation across the herd (CV_{herd}):

In[ ]:=

```
StandardDeviation[disttable]
----------------------------
      Mean[disttable]
```

Out[ ]=

```
0.751843
```

This matches the expectation:

```
Simplify[CVherd /. σx → tryσx /. σy → tryσy, constraints] // N
```

Out[ ]=

```
0.755044
```

Looking instead at how far, on average, each individual is to all others, then calculating the CV in this value among individuals (CV_{ind}):

In[ ]:=

```
disttable2 =
  Table[        1
        ----------------- (Sum[EuclideanDistance[table⟦i⟧, table⟦j⟧], {j, 1, i - 1}] +
        Length[table] - 1
      Sum[EuclideanDistance[table⟦i⟧, table⟦j⟧], {j, i + 1, 1000}]), {i, 1, 1000}];
```

The mean and SD:

In[∘]:=
```
{Mean[disttable2], StandardDeviation[disttable2]}
```
Out[∘]=
```
{1.14843, 0.406857}
```

Because of the averaging that occurs for each individual's average pairwise distance, we get a lower CV_{ind}:

In[∘]:=
$$\frac{\texttt{StandardDeviation[disttable2]}}{\texttt{Mean[disttable2]}}$$

Out[∘]=
```
0.354273
```

This matches the expectation:

In[∘]:=
```
Simplify[CVind1, constraints] // N
```
Out[∘]=
```
0.357526
```

Figure 1 - numerical integration

Blue: Individual level CV (dots based on simulations, lines based on symmetrical case).
Green: Herd level CV (dots based on simulation, lines based on symmetrical case).
Black: CV_{ind}/CV_{herd} (dots based on simulations for CV_{ind}, lines based on symmetrical case).

Integrations - Previously evaluated [ENTER]

$$\texttt{intplottingherdB} = \left\{\{0, 0.5227232008770635`\}, \left\{\frac{1}{10}, 0.5293407474740066`\right\}, \left\{\frac{1}{5}, 0.5503767552787215`\right\}, \left\{\frac{3}{10}, 0.5848351377204023`\right\}, \left\{\frac{2}{5}, 0.62782586044906`\right\}, \left\{\frac{1}{2}, 0.6719299336710343`\right\}, \left\{\frac{3}{5}, 0.7097154906842982`\right\}, \left\{\frac{7}{10}, 0.7362030893671205`\right\}, \left\{\frac{4}{5}, 0.750347311420061`\right\}, \left\{\frac{9}{10}, 0.7550437399359384`\right\}\right\}$$

$$\texttt{intplottingindB} = \left\{\{0, 0.2520803544905147`\}, \left\{\frac{1}{10}, 0.25518000579634537`\right\}, \left\{\frac{1}{5}, 0.26500879705698094`\right\}, \left\{\frac{3}{10}, 0.28104222825868547`\right\}, \left\{\frac{2}{5}, 0.30089248432425053`\right\}, \left\{\frac{1}{2}, 0.32100396904321005`\right\}, \left\{\frac{3}{5}, 0.3379153430671249`\right\}, \left\{\frac{7}{10}, 0.34947066393942994`\right\}, \left\{\frac{4}{5}, 0.35543883559280276`\right\}, \left\{\frac{9}{10}, 0.3573447344676925`\right\}\right\};$$

```
In[•]:= intplottingherdC = {{0, 0.5227232009873112`}, {1/10, 0.5227239329075928`},
    {1/5, 0.5228035469998735`}, {3/10, 0.524138853351632`}, {2/5, 0.5331526008906213`},
    {1/2, 0.564706069452737`}, {3/5, 0.6287045924514781`}, {7/10, 0.7160813351874343`},
    {4/5, 0.8123470005583199`}, {17/20, 0.8607613089836089`},
    {9/10, 0.9084236045853212`}, {19/20, 0.9549220148114106`}};

In[•]:= intplottingindC =
   {{0, 0.2520801165942636`}, {1/10, 0.25200522607365977`}, {1/5, 0.25045324939015384`},
    {3/10, 0.24214702204367256`}, {2/5, 0.2196389599705442`}, {1/2, 0.1824561380264057`},
    {3/5, 0.13975272658817786`}, {7/10, 0.09890672312111803`},
    {4/5, 0.06196960807245782`}, {17/20, 0.04504547458329638`},
    {9/10, 0.02911028854877327`}, {19/20, 0.014115108294420907`}};

In[•]:= intplottingherdD = {{0, 0.5227232009167528`}, {1/10, 0.522765342177266`},
    {1/5, 0.5237411178502892`}, {3/10, 0.5303806786463606`},
    {2/5, 0.5568407057211127`}, {1/2, 0.6290112526942427`}, {3/5, 0.7709046690326646`},
    {7/10, 0.9898655921091033`}, {4/5, 1.285715663381551`}, {17/20, 1.4635528845278027`},
    {9/10, 1.6629036118663814`}, {19/20, 1.885663601432402`}};

In[•]:= intplottingindD = {{0, 0.25208011647204787`}, {1/10, 0.2521252663731851`},
    {1/5, 0.2531597393007574`}, {3/10, 0.2599886855698586`},
    {2/5, 0.28536759406129264`}, {1/2, 0.3469263473303876`}, {3/5, 0.4535365545965457`},
    {7/10, 0.6039566130027363`}, {4/5, 0.797324070459304`}, {17/20, 0.9111044114964313`},
    {9/10, 1.0374447025752387`}, {19/20, 1.1776331930939605`}};
```

```
In[•]:= intplottingherdE = {{0, 0.6930602128358243`}, {1/10, 0.6948904347790711`},
    {1/5, 0.6985044096686023`}, {3/10, 0.6966414046157752`}, {2/5, 0.6902150082664524`},
    {1/2, 0.7009810157250438`}, {3/5, 0.7433856557817154`}, {7/10, 0.8052669243278289`},
    {4/5, 0.8725072815481681`}, {17/20, 0.9058183616725517`},
    {9/10, 0.9383112518124032`}, {19/20, 0.9697485747084602`}};

In[•]:= intplottingindE = {{0, 0.3305091378599227`}, {1/10, 0.3311328217296327`},
    {1/5, 0.32901781580559153`}, {3/10, 0.31061300951506027`},
    {2/5, 0.2657695141011268`}, {1/2, 0.20557280709498166`},
    {3/5, 0.14918433456574715`}, {7/10, 0.10202850796401869`}};

In[•]:= intplottingherdF = {{0, 0.6930602123134018`}, {1/10, 0.693862097938722`},
    {1/5, 0.6980850105248394`}, {3/10, 0.713045444952277`}, {2/5, 0.7563161059997862`},
    {1/2, 0.8528248405288947`}, {3/5, 1.0150758204965915`}, {7/10, 1.2343819546078152`},
    {4/5, 1.4983253202659426`}, {17/20, 1.6445273457921878`},
    {9/10, 1.7994518487401867`}, {19/20, 1.9627864846282088`}};

In[•]:= intplottingindF = {{0, 0.33050913703905305`}, {1/10, 0.3309342481823283`},
    {1/5, 0.33427240136835296`}, {3/10, 0.3494190470482914`}, {2/5, 0.3937162387004852`},
    {1/2, 0.48014277552932955`}, {3/5, 0.604581513129078`}, {7/10, 0.7567573245973087`}};
```

```
In[ ]:= intplottingherdG = {{0, 0.693060212015846`}, {1/10, 0.6909930062529258`},
   {1/5, 0.6829132007857964`}, {3/10, 0.6652837114870311`}, {2/5, 0.6345135606116789`},
   {1/2, 0.5900257015584462`}, {3/5, 0.5404849669297872`}, {7/10, 0.5148716694424429`},
   {4/5, 0.5679102580542682`}, {17/20, 0.6379907350895276`},
   {9/10, 0.7366346897317203`}, {19/20, 0.8591757553792789`}};

In[ ]:= intplottingindG = {{0, 0.3305070397971567`}, {1/10, 0.32958054824689326`},
   {1/5, 0.32592369336767346`}, {3/10, 0.31772906558774194`},
   {2/5, 0.30257026103488976`}, {1/2, 0.27779181354490734`},
   {3/5, 0.24108205847797515`}, {7/10, 0.19102717033238367`}};

In[ ]:= intplottingherdH = {{0, 0.6930602120182102`}, {1/10, 0.6923158078070205`},
   {1/5, 0.689402355177155`}, {3/10, 0.6830261797738891`}, {2/5, 0.671856507590081`},
   {1/2, 0.6557918659982612`}, {3/5, 0.6393680779919669`}, {7/10, 0.6423135312150211`},
   {4/5, 0.7315501716253063`}, {17/20, 0.8569435601543314`},
   {9/10, 1.0823639522911108`}, {19/20, 1.4682982329072491`}};

In[ ]:= intplottingindH =
  {{0, 0.33050703980087154`}, {1/10, 0.3301743864655322`}, {1/5, 0.3288795654495974`},
   {3/10, 0.3260900099882`}, {2/5, 0.3214051608033421`}, {1/2, 0.31549620210759655`},
   {3/5, 0.3128443626830841`}, {7/10, 0.3297041622472292`}};
```

Panel A: Symmetric range contraction [σ along x axis, decreasing from left to right]

```
In[ ]:= SeedRandom[12 241];
```

We seek to examine a range of σx=σy=σ from 0 to 1. [We expect no effect.] In our plots, we'll show σ decreasing along the x-axis varying x from 0 to 1. To do so, we define $\sigma = (1 - x)^V$, which allows us to vary σ from 1 at x=0 to 0 at x=1. V allows us to change the midpoint, which we set to 2, so that a value of x=1/2 corresponds to σ=1/4. To convert the x-axis to σ, we use $1 - \sigma^{\frac{1}{V}}$:

In[•]:=
```
tryV = 2;
```

In[•]:=
```
Solve[σ == (1 - x)^V, x]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[•]=

$$\left\{\left\{x \to 1 - \sigma^{\frac{1}{V}}\right\}\right\}$$

In[•]:=
```
tab = Join[Table[i, {i, 0, 9/10, 1/10}], {{19/20}}] // Flatten
```

Out[•]=

$$\left\{0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{9}{10}, \frac{19}{20}\right\}$$

In[•]:=
```
numind = 100;
```

In[•]:=
```
For[t = 1, t ≤ Length[tab], t++,
 table = Table[{Random[NormalDistribution[0, (1 - tab[[t]])^tryV]],
    Random[NormalDistribution[0, (1 - tab[[t]])^tryV]]}, {i, 1, numind}];
 disttable = Flatten[
   Table[EuclideanDistance[table[[i]], table[[j]]], {i, 1, numind}, {j, i + 1, numind}]];
 CVherdsim[tab[[t]], tab[[t]]] = StandardDeviation[disttable] / Mean[disttable];
 disttable2 = Table[
   1/(Length[table] - 1) (Sum[EuclideanDistance[table[[i]], table[[j]]], {j, 1, i - 1}] + Sum[
       EuclideanDistance[table[[i]], table[[j]]], {j, i + 1, numind}]), {i, 1, numind}];
 CVindsim[tab[[t]], tab[[t]]] = StandardDeviation[disttable2] / Mean[disttable2]
]
```

In[•]:=
```
forplottingherdA = Table[{tab[[t]], CVherdsim[tab[[t]], tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[•]=

$$\left\{\{0, 0.512284\}, \left\{\frac{1}{10}, 0.503982\right\}, \left\{\frac{1}{5}, 0.513253\right\}, \left\{\frac{3}{10}, 0.49795\right\}, \left\{\frac{2}{5}, 0.525723\right\}, \left\{\frac{1}{2}, 0.513991\right\}, \left\{\frac{3}{5}, 0.525625\right\}, \left\{\frac{7}{10}, 0.504365\right\}, \left\{\frac{4}{5}, 0.53337\right\}, \left\{\frac{9}{10}, 0.509055\right\}, \left\{\frac{19}{20}, 0.504871\right\}\right\}$$

In[ ]:=
```
forplottingindA = Table[{tab[[t]], CVindsim[tab[[t]], tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$$\left\{\{0, 0.247301\}, \left\{\frac{1}{10}, 0.226619\right\}, \left\{\frac{1}{5}, 0.241133\right\}, \left\{\frac{3}{10}, 0.225003\right\}, \left\{\frac{2}{5}, 0.260052\right\}, \left\{\frac{1}{2}, 0.246679\right\}, \left\{\frac{3}{5}, 0.261796\right\}, \left\{\frac{7}{10}, 0.234893\right\}, \left\{\frac{4}{5}, 0.275199\right\}, \left\{\frac{9}{10}, 0.236232\right\}, \left\{\frac{19}{20}, 0.236838\right\}\right\}$$

To label the x axis with σ values:

In[ ]:=
```
{x, (1 - x)^tryV} /. x → tab // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & \frac{1}{10} & \frac{1}{5} & \frac{3}{10} & \frac{2}{5} & \frac{1}{2} & \frac{3}{5} & \frac{7}{10} & \frac{4}{5} & \frac{9}{10} & \frac{19}{20} \\ 1 & \frac{81}{100} & \frac{16}{25} & \frac{49}{100} & \frac{9}{25} & \frac{1}{4} & \frac{4}{25} & \frac{9}{100} & \frac{1}{25} & \frac{1}{100} & \frac{1}{400} \end{pmatrix}$$

In[ ]:=
```
Select[x /. Solve[(1 - x)^tryV == 1 / 10, x], # < 1 &]
```

Out[ ]=

$$\left\{\frac{1}{10}\left(10-\sqrt{10}\right)\right\}$$

In[ ]:=
```
xticks = {{0.01, "1"}, {1/2 (2 - √2), "1/2"}, {1 / 2, "1/4"},
   {1/10 (10 - √10), "1/10"}, {4 / 5, "1/25"}, {9 / 10, "1/100"}, {1, "1/∞"}}
```

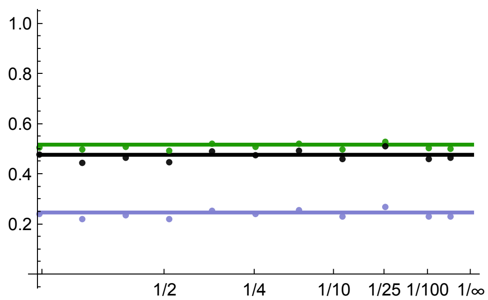
Out[ ]=

$$\left\{\{0.01, 1\}, \left\{\frac{1}{2}\left(2-\sqrt{2}\right), 1/2\right\}, \left\{\frac{1}{2}, 1/4\right\}, \left\{\frac{1}{10}\left(10-\sqrt{10}\right), 1/10\right\}, \left\{\frac{4}{5}, 1/25\right\}, \left\{\frac{9}{10}, 1/100\right\}, \{1, 1/\infty\}\right\}$$

In[ ]:=
```
maxc = 1;
```

In[∘]:=
```
plotA = Show[
  Plot[CVherd /. σx → σy /. σy → (1 - x)^tryV,
   {x, 0, 1}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  Plot[CVind, {x, 0, 1}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[(CVind / CVherd /. σx → σy /. σy → (1 - x)^tryV), {x, 0, 1}, PlotStyle → Black],
  ListPlot[forplottingherdA, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[forplottingindA, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[Table[{forplottingindA[[i, 1]], forplottingindA[[i, 2]] / forplottingherdA[[i, 2]]},
    {i, 1, Length[forplottingindA]}], PlotStyle → {Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, 1}, {0, 1}},
  Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[∘]=



Panel B: Asymmetric contraction leading to elongated ranges [σx/σy along x axis]

In[∘]:=
```
SeedRandom[21 773];
```

We seek to examine a range of σy from 0 to 1, holding σx=1. In our plots, we'll show σx/σy along the x-axis using values of x from 0 to 1. To do so, we define $\sigma y = (1 - x)^V$, which allows us to vary σx/σy from 1 at x=0 to infinity at x=1. V allows us to change the midpoint, which we set to 2, so that a value of x=1/2 corresponds to σx/σy=4. To convert the x-axis to σy, we use $1 - \sigma y^{\frac{1}{V}}$:

In[∘]:=
```
tryV = 2;
```

In[∘]:=
```
Solve[σy == (1 - x)^V, x]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[∘]=

$$\left\{\left\{x \to 1 - \sigma y^{\frac{1}{V}}\right\}\right\}$$

In[∘]:=
```
tab = Table[i, {i, 0, 9/10, 1/10}] // Flatten
```
Out[∘]=

$$\left\{0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{9}{10}\right\}$$

In[∘]:=
```
tryσx = 1;
tryf = 0; (*No split*)
```
In[∘]:=
```
numind = 100;
```
In[∘]:=
```
For[t = 1, t ≤ Length[tab], t++,
 table = Table[{Random[NormalDistribution[0, tryσx]],
    Random[NormalDistribution[0, (1 - tab[[t]])^tryV]]}, {i, 1, numind}];
 disttable = Flatten[
   Table[EuclideanDistance[table[[i]], table[[j]]], {i, 1, numind}, {j, i + 1, numind}]];
 CVherdsim[tryσx, tab[[t]]] = StandardDeviation[disttable] / Mean[disttable];
 disttable2 = Table[
   1 / (Length[table] - 1) (Sum[EuclideanDistance[table[[i]], table[[j]]], {j, 1, i - 1}] + Sum[
       EuclideanDistance[table[[i]], table[[j]]], {j, i + 1, numind}]), {i, 1, numind}];
 CVindsim[tryσx, tab[[t]]] = StandardDeviation[disttable2] / Mean[disttable2]
]
```
In[∘]:=
```
forplottingherdB = Table[{tab[[t]], CVherdsim[tryσx, tab[[t]]]}, {t, 1, Length[tab]}]
```
Out[∘]=

$$\left\{\{0, 0.491624\}, \left\{\frac{1}{10}, 0.509512\right\}, \left\{\frac{1}{5}, 0.567223\right\}, \left\{\frac{3}{10}, 0.616676\right\}, \left\{\frac{2}{5}, 0.60762\right\}, \left\{\frac{1}{2}, 0.644016\right\}, \left\{\frac{3}{5}, 0.716753\right\}, \left\{\frac{7}{10}, 0.724227\right\}, \left\{\frac{4}{5}, 0.710472\right\}, \left\{\frac{9}{10}, 0.73793\right\}\right\}$$

In[∘]:=
```
forplottingindB = Table[{tab[[t]], CVindsim[tryσx, tab[[t]]]}, {t, 1, Length[tab]}]
```
Out[∘]=

$$\left\{\{0, 0.215466\}, \left\{\frac{1}{10}, 0.240091\right\}, \left\{\frac{1}{5}, 0.282201\right\}, \left\{\frac{3}{10}, 0.332271\right\}, \left\{\frac{2}{5}, 0.288243\right\}, \left\{\frac{1}{2}, 0.28341\right\}, \left\{\frac{3}{5}, 0.374266\right\}, \left\{\frac{7}{10}, 0.328504\right\}, \left\{\frac{4}{5}, 0.283964\right\}, \left\{\frac{9}{10}, 0.347029\right\}\right\}$$

Below, we'll want the ratio of $\frac{\text{CVind}}{\text{CVherd}}$ in the limit for a very elongated range:

In[∘]:=
```
limratio = Limit[CVind1 / CVherd /. σx → tryσx, σy → Infinity]
```
Out[∘]=

$$\sqrt{\frac{6\left(-2+\sqrt{3}\right)+\pi}{6\,(-2+\pi)}}$$

To label the x axis:

In[∘]:=
```
{x, tryσx / (1 - x)^tryV} /. x → tab // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & \frac{1}{10} & \frac{1}{5} & \frac{3}{10} & \frac{2}{5} & \frac{1}{2} & \frac{3}{5} & \frac{7}{10} & \frac{4}{5} & \frac{9}{10} \\ 1 & \frac{100}{81} & \frac{25}{16} & \frac{100}{49} & \frac{25}{9} & 4 & \frac{25}{4} & \frac{100}{9} & 25 & 100 \end{pmatrix}$$

In[∘]:=
```
Select[x /. Solve[1 / (1 - x)^tryV == 10, x], # < 1 &]
```

Out[∘]=

$$\left\{\frac{1}{10}\left(10-\sqrt{10}\right)\right\}$$

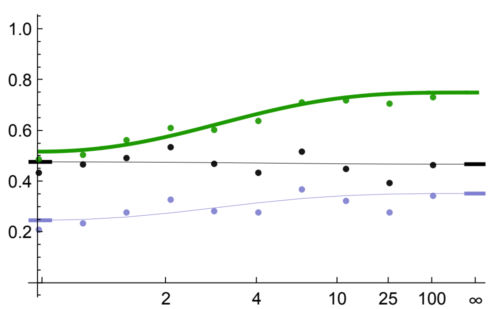
In[∘]:=
```
xticks = {{0.01, "1"}, {1/2 (2 - √2), "2"}, {1 / 2, "4"},
   {1/10 (10 - √10), "10"}, {4 / 5, "25"}, {9 / 10, "100"}, {1, "∞"}}
```

Out[∘]=

$$\left\{\{0.01, 1\}, \left\{\frac{1}{2}\left(2-\sqrt{2}\right), 2\right\}, \left\{\frac{1}{2}, 4\right\}, \left\{\frac{1}{10}\left(10-\sqrt{10}\right), 10\right\}, \left\{\frac{4}{5}, 25\right\}, \left\{\frac{9}{10}, 100\right\}, \{1, \infty\}\right\}$$

In[ ]:=
```
plotB = Show[
  Plot[CVherd /. σx → tryσx /. σy → (1 - x)^tryV,
   {x, 0, 1}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  Plot[CVind, {x, -0.025, 0.025}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVind1, {x, 0.975, 1.025}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[(CVind / CVherd /. σx → tryσx /. σy → tryσx),
   {x, -0.025, 0.025}, PlotStyle → Black],
  Plot[limratio, {x, 0.975, 1.025}, PlotStyle → Black],
  ListPlot[forplottingherdB, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[forplottingindB, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[Join[intplottingindB, {{1, CVind1}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}],
  ListPlot[Table[{forplottingindB[[i, 1]], forplottingindB[[i, 2]] / forplottingherdB[[i, 2]]},
    {i, 1, Length[forplottingindB]}], PlotStyle → {Black}],
  ListPlot[
   Join[Table[{intplottingindB[[i, 1]], intplottingindB[[i, 2]] / (CVherd /. σx → tryσx /. σy → (1 - tab[[i]])^tryV)},
     {i, 1, Length[intplottingindB]}], {{1, limratio}}],
   Joined → True, PlotStyle → {Thin, Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, 1}, {0, 1}},
  Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[ ]=



Dividing CVind by CVherd gives very similar answers in the 1D and symmetrical 2D cases (at opposite ends of this “elongation” axis). While the dashed line looks flat, the values are not exactly the same.

In[ ]:=
```
{CVind / (CVherd /. σx → 1 /. σy → 1), CVind1 / CVherd1} // N
```

Out[ ]=
```
{0.482244, 0.473224}
```

Panel B: Numerical integration [SLOW - Don't enter]

For comparison, we can numerically integrate:

```
In[◦]:= tryLIM = Infinity;
tryc = 0;
tryf = 0;
tryσx = 1;

For[t = 1, t ≤ Length[tab], t++,
 tryσy = (1 - tab⟦t⟧)^tryV;
 mean = (1 - f) soln + f NIntegrate[(E^(-(tryc - Δx)^2/(4 tryσx^2)) tryσy HypergeometricU[-1/2, 0, Δx^2/(4 tryσy^2)])/(Sqrt[π] tryσx),
     {Δx, -tryLIM, tryLIM}] /. σx → tryσx /. σy → tryσy /. f → tryf /. c → tryc;
 sqH =
  (1 - f) solnsq + f (c^2 + 2 (σx^2 + σy^2)) /. σx → tryσx /. σy → tryσy /. f → tryf /. c → tryc;
 CVherdintC[tryσx, tab⟦t⟧] = Sqrt[sqH - mean^2] / mean;
 Print[{CVherdintC[tryσx, tab⟦t⟧]}]
]
```

```
{0.522723}

{0.529341}

{0.550377}

{0.584835}

{0.627826}

{0.67193}

{0.709715}

{0.736203}

{0.750347}

{0.755044}
```

In[ ]:=
```
For[t = 1, t ≤ Length[tab], t++,
 tryσy = (1 - tab[[t]])^tryV;
 mean =
  N[(1 - f) soln + f NIntegrate[(E^(-(tryc - Δx)^2/(4 tryσx^2)) tryσy HypergeometricU[-1/2, 0, Δx^2/(4 tryσy^2)])/(Sqrt[π] tryσx), {Δx,
       -tryLIM, tryLIM}] /. σx → tryσx /. σy → tryσy /. f → tryf /. c → tryc];

 CVindintC[tryσx, tab[[t]]] = Sqrt[sqI[tryσx, tryσy] - mean^2] / mean;
 Print[{CVindintC[tryσx, tab[[t]]]}]
]
```

{0.25208}

NIntegrate: Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small.

{0.25518}

{0.265009}

{0.281042}

{0.300892}

{0.321004}

{0.337915}

{0.349471}

{0.355439}

{0.357345}

In[ ]:=
```
intplottingherdB = Table[{tab[[t]], CVherdintC[tryσx, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$\left\{\{0, 0.522723\}, \left\{\frac{1}{10}, 0.529341\right\}, \left\{\frac{1}{5}, 0.550377\right\}, \left\{\frac{3}{10}, 0.584835\right\}, \left\{\frac{2}{5}, 0.627826\right\}, \left\{\frac{1}{2}, 0.67193\right\}, \left\{\frac{3}{5}, 0.709715\right\}, \left\{\frac{7}{10}, 0.736203\right\}, \left\{\frac{4}{5}, 0.750347\right\}, \left\{\frac{9}{10}, 0.755044\right\}\right\}$

In[ ]:=
```
intplottingindB = Table[{tab[[t]], CVindintC[tryσx, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$\left\{\{0, 0.25208\}, \left\{\frac{1}{10}, 0.25518\right\}, \left\{\frac{1}{5}, 0.265009\right\}, \left\{\frac{3}{10}, 0.281042\right\}, \left\{\frac{2}{5}, 0.300892\right\}, \left\{\frac{1}{2}, 0.321004\right\}, \left\{\frac{3}{5}, 0.337915\right\}, \left\{\frac{7}{10}, 0.349471\right\}, \left\{\frac{4}{5}, 0.355439\right\}, \left\{\frac{9}{10}, 0.357345\right\}\right\}$

Panel C: Even herd split [midpoint at c=4]

In[ ]:=
```
SeedRandom[32912];
```

We seek to examine a range of distances, c, between the peaks. To do so, we use c/(V+c) along the x-axis, which allows us to vary c from 0 at x=0 to infinity at x=1, within a single plot (we use V = 4 so that patches 4 sd apart are half-way along the plot). To convert the x-axis to c, we use:

In[∘]:=
```
tryV = 4;
```

In[∘]:=
```
Solve[c / (tryV + c) == x, c]
```

Out[∘]=

$$\left\{\left\{c \to -\frac{4\,x}{-1+x}\right\}\right\}$$

In[∘]:=
```
tab =
 Join[Table[i, {i, 0 / 10, 7 / 10, 1 / 10}], Table[i, {i, 8 / 10, 19 / 20, 1 / 20}]] // Flatten
```

Out[∘]=

$$\left\{0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{17}{20}, \frac{9}{10}, \frac{19}{20}\right\}$$

In[∘]:=
```
numind1 = 50;
numind2 = 50;
numind = numind1 + numind2;
```

In[∘]:=
```
tryf = 2 (numind2 / numind) (numind1 / numind)
  (*Unequal split with 95% in one patch so f = 0.095*)
```

Out[∘]=

$$\frac{1}{2}$$

In[∘]:=
```
tryσ = 1;
tryσx = tryσ;
tryσy = tryσ;
```

In[ ]:=
```
For[t = 1, t ≤ Length[tab], t++,
 table = Join[Table[{Random[NormalDistribution[0, tryσx]],
     Random[NormalDistribution[0, tryσy]]}, {i, 1, numind1}],
   Table[{Random[NormalDistribution[tryV tab[[t]]/(1 - tab[[t]]), tryσx]],
     Random[NormalDistribution[0, tryσy]]}, {i, 1, numind2}]];
 disttable = Flatten[
   Table[EuclideanDistance[table[[i]], table[[j]]], {i, 1, numind}, {j, i + 1, numind}]];
 CVherdsimC[tryσx, tab[[t]]] = StandardDeviation[disttable]/Mean[disttable];
 disttable2 = Table[
   1/(Length[table] - 1) (Sum[EuclideanDistance[table[[i]], table[[j]]], {j, 1, i - 1}] + Sum[
       EuclideanDistance[table[[i]], table[[j]]], {j, i + 1, numind}]), {i, 1, numind}];
 CVindsimC[tryσx, tab[[t]]] = StandardDeviation[disttable2]/Mean[disttable2]
]
```

In[ ]:=
```
forplottingherdC = Table[{tab[[t]], CVherdsimC[tryσx, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$$\left\{\{0, 0.539929\}, \left\{\frac{1}{10}, 0.511008\right\}, \left\{\frac{1}{5}, 0.497931\right\}, \left\{\frac{3}{10}, 0.515449\right\}, \left\{\frac{2}{5}, 0.509044\right\}, \left\{\frac{1}{2}, 0.567113\right\}, \left\{\frac{3}{5}, 0.62758\right\}, \left\{\frac{7}{10}, 0.707042\right\}, \left\{\frac{4}{5}, 0.800881\right\}, \left\{\frac{17}{20}, 0.852974\right\}, \left\{\frac{9}{10}, 0.903404\right\}, \left\{\frac{19}{20}, 0.943065\right\}\right\}$$

In[ ]:=
```
forplottingindC = Table[{tab[[t]], CVindsimC[tryσx, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$$\left\{\{0, 0.280179\}, \left\{\frac{1}{10}, 0.243007\right\}, \left\{\frac{1}{5}, 0.22465\right\}, \left\{\frac{3}{10}, 0.24886\right\}, \left\{\frac{2}{5}, 0.185671\right\}, \left\{\frac{1}{2}, 0.15517\right\}, \left\{\frac{3}{5}, 0.143541\right\}, \left\{\frac{7}{10}, 0.0960585\right\}, \left\{\frac{4}{5}, 0.0705833\right\}, \left\{\frac{17}{20}, 0.0401585\right\}, \left\{\frac{9}{10}, 0.0276361\right\}, \left\{\frac{19}{20}, 0.0142774\right\}\right\}$$

In[ ]:=
```
maxc = 1;
```

Below, we'll want the ratio of $\frac{\text{CVind}}{\text{CVherd}}$ in the limit for very distant patches when the patches are uneven:

In[ ]:=
```
limratio = Limit[CVindfarsplit/CVherdC /. σ → tryσ /. f → tryf, c → Infinity]
```

Out[ ]=

0

We also plot CVherd in the limit for very distant patches when the patches are uneven:

In[ ]:=
```
limratioHERD = Limit[CVherdC /. σ → tryσ /. f → tryf, c → Infinity]
```

Out[ ]=

1

Choosing x-axis tick positions:

In[ ]:=
$$\left\{x, \frac{\text{tryV } x}{1 - x}\right\} \text{ /. } x \to \text{tab // Transpose}$$

Out[ ]=
$$\left\{\{0, 0\}, \left\{\frac{1}{10}, \frac{4}{9}\right\}, \left\{\frac{1}{5}, 1\right\}, \left\{\frac{3}{10}, \frac{12}{7}\right\}, \left\{\frac{2}{5}, \frac{8}{3}\right\}, \left\{\frac{1}{2}, 4\right\}, \left\{\frac{3}{5}, 6\right\}, \left\{\frac{7}{10}, \frac{28}{3}\right\}, \left\{\frac{4}{5}, 16\right\}, \left\{\frac{17}{20}, \frac{68}{3}\right\}, \left\{\frac{9}{10}, 36\right\}, \left\{\frac{19}{20}, 76\right\}\right\}$$

In[ ]:=
$$\text{Solve}\left[\frac{\text{tryV } x}{1 - x} == 2, x\right]$$

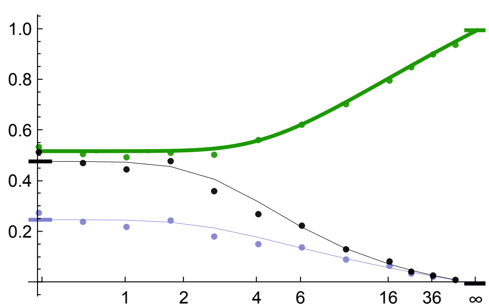
Out[ ]=
$$\left\{\left\{x \to \frac{1}{3}\right\}\right\}$$

In[ ]:=
```
xticks = {{0.01, "0"}, {1 / 5, "1"}, {1 / 3, "2"},
   {1 / 2, "4"}, {3 / 5, "6"}, {4 / 5, "16"}, {9 / 10, "36"}, {1, "∞"}}
```

Out[ ]=
$$\left\{\{0.01, 0\}, \left\{\frac{1}{5}, 1\right\}, \left\{\frac{1}{3}, 2\right\}, \left\{\frac{1}{2}, 4\right\}, \left\{\frac{3}{5}, 6\right\}, \left\{\frac{4}{5}, 16\right\}, \left\{\frac{9}{10}, 36\right\}, \{1, \infty\}\right\}$$

In[∘]:=
```
plotC = Show[
  Plot[CVherdC /. σ → tryσ /. f → tryf /. c → (tryV x)/(1 - x), {x, 0, maxc},
   WorkingPrecision → 50, PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
  Plot[limratioHERD, {x, 0.975, 1.05},
   PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
  Plot[CVind /. σx → tryσx /. σy → tryσy,
   {x, -0.025, 0.025}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVindfarsplit /. σx → tryσx /. σy → tryσy /. f → tryf,
   {x, 0.975, 1.05}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVind/CVherd /. σx → tryσx /. σy → tryσy, {x, -0.025, 0.025}, PlotStyle → Black],
  Plot[limratio, {x, 0.975, 1.05}, PlotStyle → Black],
  ListPlot[forplottingherdC, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[forplottingindC, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[intplottingindC,
   Joined → True, PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}],
  ListPlot[Table[{forplottingherdC[[i, 1]], forplottingindC[[i, 2]]/forplottingherdC[[i, 2]]},
    {i, 1, Length[forplottingindC]}], PlotStyle → {Black}],
  ListPlot[Table[{intplottingindC[[i, 1]],
     intplottingindC[[i, 2]]/N[(CVherdC /. σ → tryσ /. f → tryf /. c → (tryV x)/(1-x) /. x → tab[[i]]), 20]},
    {i, 1, Length[forplottingindC]}], Joined → True, PlotStyle → {Thin, Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, maxc}, {0, 1}}, Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[∘]=



Panel C: Numerical integration [SLOW - Don’t enter]

For comparison, we can numerically integrate:

In[ ]:= tryLIM = Infinity;

$$\text{For}\Big[t = 1,\ t \le \text{Length}[tab],\ t{+}{+},$$

$$\text{tryc} = \frac{\text{tryV}\ \text{tab}[[t]]}{1 - \text{tab}[[t]]};$$

$$\text{mean} = (1-f)\ \text{soln} + f\ \text{NIntegrate}\Big[\frac{e^{-\frac{(\text{tryc}-\Delta x)^2}{4\ \text{try}\sigma x^2}}\ \text{try}\sigma y\ \text{HypergeometricU}\Big[-\frac{1}{2}, 0, \frac{\Delta x^2}{4\ \text{try}\sigma y^2}\Big]}{\sqrt{\pi}\ \text{try}\sigma x},$$

$$\{\Delta x, -\text{tryLIM}, \text{tryLIM}\}\Big]\ /.\ \sigma x \to \text{try}\sigma x\ /.\ \sigma y \to \text{try}\sigma y\ /.\ f \to \text{tryf}\ /.\ c \to \text{tryc};$$

$$\text{sqH} = (1-f)\ \text{solnsq} + f\left(c^2 + 2\left(\sigma x^2 + \sigma y^2\right)\right)\ /.\ \sigma x \to \text{try}\sigma x\ /.\ \sigma y \to \text{try}\sigma y\ /.\ f \to \text{tryf}\ /.\ c \to \text{tryc};$$

$$\text{CVherdintC}[\text{try}\sigma x, \text{tab}[[t]]] = \text{Sqrt}\big[\text{sqH} - \text{mean}^2\big] \big/ \text{mean};$$

Print[{CVherdintC[tryσx, tab[[t]]]}]

$\Big]$

{0.522723}

{0.522724}

{0.522804}

{0.524139}

{0.533153}

{0.564706}

{0.628705}

{0.716081}

{0.812347}

{0.860761}

{0.908424}

{0.954922}

$$\text{For}\Big[t = 1,\ t \le \text{Length}[tab],\ t{+}{+},$$

$$\text{tryc} = \frac{\text{tryV}\ \text{tab}[[t]]}{1 - \text{tab}[[t]]};$$

$$\text{mean} = (1-f)\ \text{soln} + f\ \text{NIntegrate}\Big[\frac{e^{-\frac{(\text{tryc}-\Delta x)^2}{4\ \text{try}\sigma x^2}}\ \text{try}\sigma y\ \text{HypergeometricU}\Big[-\frac{1}{2}, 0, \frac{\Delta x^2}{4\ \text{try}\sigma y^2}\Big]}{\sqrt{\pi}\ \text{try}\sigma x},$$

$$\{\Delta x, -\text{tryLIM}, \text{tryLIM}\}\Big]\ /.\ \sigma x \to \text{try}\sigma x\ /.\ \sigma y \to \text{try}\sigma y\ /.\ f \to \text{tryf}\ /.\ c \to \text{tryc};$$

$$\text{CVindintC}[\text{try}\sigma, \text{tab}[[t]]] = \text{Sqrt}\big[\text{sqI}[\text{try}\sigma, \text{tryc}, \text{tryf}] - \text{mean}^2\big] \big/ \text{mean};$$

Print[{CVindintC[tryσ, tab[[t]]]}]

$\Big]$

```
{0.25208}

{0.252005}

{0.250453}

{0.242147}

{0.219639}

{0.182456}

{0.139753}

{0.0989067}

{0.0619696}

{0.0450455}

{0.0291103}

{0.0141151}
```

In[∘]:=
```
intplottingherdC = Table[{tab⟦t⟧, CVherdintC[tryσx, tab⟦t⟧]}, {t, 1, Length[tab]}]
```

Out[∘]=

$$\left\{\{0, 0.522723\}, \left\{\frac{1}{10}, 0.522724\right\}, \left\{\frac{1}{5}, 0.522804\right\}, \left\{\frac{3}{10}, 0.524139\right\}, \left\{\frac{2}{5}, 0.533153\right\}, \left\{\frac{1}{2}, 0.564706\right\}, \left\{\frac{3}{5}, 0.628705\right\}, \left\{\frac{7}{10}, 0.716081\right\}, \left\{\frac{4}{5}, 0.812347\right\}, \left\{\frac{17}{20}, 0.860761\right\}, \left\{\frac{9}{10}, 0.908424\right\}, \left\{\frac{19}{20}, 0.954922\right\}\right\}$$

In[∘]:=
```
intplottingindC = Table[{tab⟦t⟧, CVindintC[tryσx, tab⟦t⟧]}, {t, 1, Length[tab]}]
```

Out[∘]=

$$\left\{\{0, 0.25208\}, \left\{\frac{1}{10}, 0.252005\right\}, \left\{\frac{1}{5}, 0.250453\right\}, \left\{\frac{3}{10}, 0.242147\right\}, \left\{\frac{2}{5}, 0.219639\right\}, \left\{\frac{1}{2}, 0.182456\right\}, \left\{\frac{3}{5}, 0.139753\right\}, \left\{\frac{7}{10}, 0.0989067\right\}, \left\{\frac{4}{5}, 0.0619696\right\}, \left\{\frac{17}{20}, 0.0450455\right\}, \left\{\frac{9}{10}, 0.0291103\right\}, \left\{\frac{19}{20}, 0.0141151\right\}\right\}$$

Panel D: Uneven herd split [midpoint at c=4]

In[∘]:=
```
SeedRandom[43 812];
```

We seek to examine a range of distances, c, between the peaks. To do so, we use c/(V+c) along the x-axis, which allows us to vary c from 0 at x=0 to infinity at x=1, within a single plot (we use V = 4 so that patches 4 sd apart are half-way along the plot). To convert the x-axis to c, we use:

In[∘]:=
```
tryV = 4;
```

In[∘]:=
```
Solve[c / (tryV + c) == x, c]
```

Out[∘]=

$$\left\{\left\{c \to -\frac{4\,x}{-1+x}\right\}\right\}$$

In[•]:=
```
tab =
 Join[Table[i, {i, 0 / 10, 7 / 10, 1 / 10}], Table[i, {i, 8 / 10, 19 / 20, 1 / 20}]] // Flatten
```
Out[•]=

$$\left\{0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{17}{20}, \frac{9}{10}, \frac{19}{20}\right\}$$

In[•]:=
```
numind1 = 90;
numind2 = 10;
numind = numind1 + numind2;
```
In[•]:=
```
tryf = 2 (numind2 / numind) (numind1 / numind)
  (*Unequal split with 90% in one patch so f = 0.18*)
```
Out[•]=

$$\frac{9}{50}$$

In[•]:=
```
tryσ = 1;
tryσx = tryσ;
tryσy = tryσ;
```
In[•]:=
```
For[t = 1, t ≤ Length[tab], t++,
 table = Join[Table[{Random[NormalDistribution[0, tryσx]],
     Random[NormalDistribution[0, tryσy]]}, {i, 1, numind1}],
   Table[{Random[NormalDistribution[tryV tab[[t]] / (1 - tab[[t]]), tryσx]],
     Random[NormalDistribution[0, tryσy]]}, {i, 1, numind2}]];
 disttable = Flatten[
   Table[EuclideanDistance[table[[i]], table[[j]]], {i, 1, numind}, {j, i + 1, numind}]];
 CVherdsimC[tryσx, tab[[t]]] = StandardDeviation[disttable] / Mean[disttable];
 disttable2 = Table[
   1 / (Length[table] - 1) (Sum[EuclideanDistance[table[[i]], table[[j]]], {j, 1, i - 1}] + Sum[
       EuclideanDistance[table[[i]], table[[j]]], {j, i + 1, numind}]), {i, 1, numind}];
 CVindsimC[tryσx, tab[[t]]] = StandardDeviation[disttable2] / Mean[disttable2]
]
```

In[∘]:=
```
forplottingherdD = Table[{tab⟦t⟧, CVherdsimC[tryσx, tab⟦t⟧]}, {t, 1, Length[tab]}]
```
Out[∘]=

$$\left\{\{0, 0.557435\}, \left\{\frac{1}{10}, 0.514321\right\}, \left\{\frac{1}{5}, 0.524728\right\}, \left\{\frac{3}{10}, 0.539212\right\}, \left\{\frac{2}{5}, 0.52205\right\}, \left\{\frac{1}{2}, 0.611301\right\}, \left\{\frac{3}{5}, 0.772606\right\}, \left\{\frac{7}{10}, 0.975436\right\}, \left\{\frac{4}{5}, 1.24874\right\}, \left\{\frac{17}{20}, 1.48845\right\}, \left\{\frac{9}{10}, 1.66103\right\}, \left\{\frac{19}{20}, 1.88208\right\}\right\}$$

In[∘]:=
```
forplottingindD = Table[{tab⟦t⟧, CVindsimC[tryσx, tab⟦t⟧]}, {t, 1, Length[tab]}]
```
Out[∘]=

$$\left\{\{0, 0.297448\}, \left\{\frac{1}{10}, 0.246918\right\}, \left\{\frac{1}{5}, 0.257451\right\}, \left\{\frac{3}{10}, 0.276005\right\}, \left\{\frac{2}{5}, 0.258947\right\}, \left\{\frac{1}{2}, 0.327936\right\}, \left\{\frac{3}{5}, 0.458483\right\}, \left\{\frac{7}{10}, 0.595828\right\}, \left\{\frac{4}{5}, 0.782364\right\}, \left\{\frac{17}{20}, 0.93942\right\}, \left\{\frac{9}{10}, 1.04646\right\}, \left\{\frac{19}{20}, 1.18934\right\}\right\}$$

These are visually indistinguishable.

In[∘]:=
```
maxc = 1;
```

Below, we’ll want the ratio of $\frac{\text{CVind}}{\text{CVherd}}$ in the limit for very distant patches when the patches are uneven:

In[∘]:=
$$\text{limratio} = \text{Limit}\left[\frac{\text{CVindfarsplit}}{\text{CVherdC}} \text{ /. } \sigma \to \text{try}\sigma \text{ /. } f \to \text{tryf}, c \to \text{Infinity}\right]$$
Out[∘]=

$$\frac{4}{\sqrt{41}}$$

We also plot CVherd in the limit for very distant patches when the patches are uneven:

In[∘]:=
```
limratioHERD = Limit[CVherdC /. σ → tryσ /. f → tryf, c → Infinity]
```
Out[∘]=

$$\frac{\sqrt{41}}{3}$$

Choosing x-axis tick positions:

In[∘]:=
$$\left\{x, \frac{\text{tryV}\, x}{1 - x}\right\} \text{ /. } x \to \text{tab // Transpose}$$
Out[∘]=

$$\left\{\{0, 0\}, \left\{\frac{1}{10}, \frac{4}{9}\right\}, \left\{\frac{1}{5}, 1\right\}, \left\{\frac{3}{10}, \frac{12}{7}\right\}, \left\{\frac{2}{5}, \frac{8}{3}\right\}, \left\{\frac{1}{2}, 4\right\}, \left\{\frac{3}{5}, 6\right\}, \left\{\frac{7}{10}, \frac{28}{3}\right\}, \left\{\frac{4}{5}, 16\right\}, \left\{\frac{17}{20}, \frac{68}{3}\right\}, \left\{\frac{9}{10}, 36\right\}, \left\{\frac{19}{20}, 76\right\}\right\}$$

In[ ]:= Solve$\left[\dfrac{\text{tryV}\,x}{1-x} == 2, x\right]$

Out[ ]=

$\left\{\left\{x \to \frac{1}{3}\right\}\right\}$

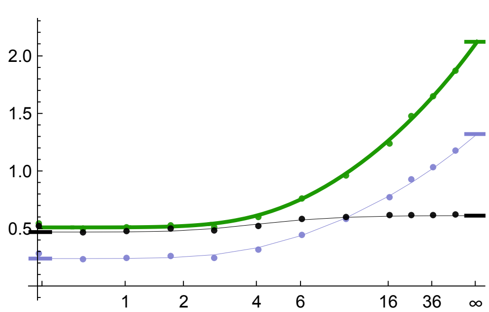
In[ ]:=
```
xticks = {{0.01, "0"}, {1/5, "1"}, {1/3, "2"},
  {1/2, "4"}, {3/5, "6"}, {4/5, "16"}, {9/10, "36"}, {1, "∞"}}
```

Out[ ]=

$\left\{\{0.01, 0\}, \left\{\frac{1}{5}, 1\right\}, \left\{\frac{1}{3}, 2\right\}, \left\{\frac{1}{2}, 4\right\}, \left\{\frac{3}{5}, 6\right\}, \left\{\frac{4}{5}, 16\right\}, \left\{\frac{9}{10}, 36\right\}, \{1, \infty\}\right\}$

In[∘]:=
```
plotD = Show[
  Plot[CVherdC /. σ → tryσ /. f → tryf /. c → (tryV x)/(1 - x), {x, 0, maxc},
   WorkingPrecision → 50, PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
  Plot[limratioHERD, {x, 0.975, 1.05},
   PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
  Plot[CVind /. σx → tryσx /. σy → tryσy,
   {x, -0.025, 0.025}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVindfarsplit /. f → tryf,
   {x, 0.975, 1.05}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVind/CVherd /. σx → tryσx /. σy → tryσy, {x, -0.025, 0.025}, PlotStyle → Black],
  Plot[limratio, {x, 0.975, 1.05}, PlotStyle → Black],
  ListPlot[forplottingherdD, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[forplottingindD, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[Join[intplottingindD, {{1, CVindfarsplit /. f → tryf}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}],
  ListPlot[Table[{forplottingherdD[[i, 1]], forplottingindD[[i, 2]]/forplottingherdD[[i, 2]]},
    {i, 1, Length[forplottingindD]}], PlotStyle → {Black}],
  ListPlot[Join[Table[{intplottingindD[[i, 1]], intplottingindD[[i, 2]]/intplottingherdD[[i, 2]]},
     {i, 1, Length[forplottingindD]}], {{1, limratio}}],
   Joined → True, PlotStyle → {Thin, Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, maxc}, {0, 2.2}}, Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[∘]=



Panel D: Numerical integration [SLOW - Don’t enter]

For comparison, we can numerically integrate:

```
In[•]:= tryLIM = Infinity;

For[t = 1, t ≤ Length[tab], t++,
 tryc = (tryV tab[[t]])/(1 - tab[[t]]);
 mean = (1 - f) soln + f NIntegrate[(E^(-(tryc - Δx)^2/(4 tryσx^2)) tryσy HypergeometricU[-1/2, 0, Δx^2/(4 tryσy^2)])/(Sqrt[π] tryσx),
     {Δx, -tryLIM, tryLIM}] /. σx → tryσx /. σy → tryσy /. f → tryf /. c → tryc;
 sqH =
  (1 - f) solnsq + f (c^2 + 2 (σx^2 + σy^2)) /. σx → tryσx /. σy → tryσy /. f → tryf /. c → tryc;
 CVherdintC[tryσx, tab[[t]]] = Sqrt[sqH - mean^2] / mean;
 Print[{CVherdintC[tryσx, tab[[t]]]}]
]
{0.522723}
{0.522765}
{0.523741}
{0.530381}
{0.556841}
{0.629011}
{0.770905}
{0.989866}
{1.28572}
{1.46355}
{1.6629}
{1.88566}
For[t = 1, t ≤ Length[tab], t++,
 tryc = (tryV tab[[t]])/(1 - tab[[t]]);
 mean = (1 - f) soln + f NIntegrate[(E^(-(tryc - Δx)^2/(4 tryσx^2)) tryσy HypergeometricU[-1/2, 0, Δx^2/(4 tryσy^2)])/(Sqrt[π] tryσx),
     {Δx, -tryLIM, tryLIM}] /. σx → tryσx /. σy → tryσy /. f → tryf /. c → tryc;

 CVindintC[tryσ, tab[[t]]] = Sqrt[sqI[tryσ, tryc, tryf] - mean^2] / mean;
 Print[{CVindintC[tryσ, tab[[t]]]}]
]
```

```
{0.25208}
{0.252125}
{0.25316}
{0.259989}
{0.285368}
{0.346926}
{0.453537}
{0.603957}
{0.797324}
{0.911104}
{1.03744}
{1.17763}
```

In[∘]:=
```
intplottingherdD = Table[{tab⟦t⟧, CVherdintC[tryσx, tab⟦t⟧]}, {t, 1, Length[tab]}]
```

Out[∘]=

$$\left\{\{0, 0.522723\}, \left\{\frac{1}{10}, 0.522765\right\}, \left\{\frac{1}{5}, 0.523741\right\}, \left\{\frac{3}{10}, 0.530381\right\}, \left\{\frac{2}{5}, 0.556841\right\}, \left\{\frac{1}{2}, 0.629011\right\}, \left\{\frac{3}{5}, 0.770905\right\}, \left\{\frac{7}{10}, 0.989866\right\}, \left\{\frac{4}{5}, 1.28572\right\}, \left\{\frac{17}{20}, 1.46355\right\}, \left\{\frac{9}{10}, 1.6629\right\}, \left\{\frac{19}{20}, 1.88566\right\}\right\}$$

In[∘]:=
```
intplottingindD = Table[{tab⟦t⟧, CVindintC[tryσx, tab⟦t⟧]}, {t, 1, Length[tab]}]
```

Out[∘]=

$$\left\{\{0, 0.25208\}, \left\{\frac{1}{10}, 0.252125\right\}, \left\{\frac{1}{5}, 0.25316\right\}, \left\{\frac{3}{10}, 0.259989\right\}, \left\{\frac{2}{5}, 0.285368\right\}, \left\{\frac{1}{2}, 0.346926\right\}, \left\{\frac{3}{5}, 0.453537\right\}, \left\{\frac{7}{10}, 0.603957\right\}, \left\{\frac{4}{5}, 0.797324\right\}, \left\{\frac{17}{20}, 0.911104\right\}, \left\{\frac{9}{10}, 1.03744\right\}, \left\{\frac{19}{20}, 1.17763\right\}\right\}$$

Panel E: Even herd split with elongated patches [σy short]

In[∘]:=
```
SeedRandom[59 238];
```

We seek to examine a range of distances, c, between the peaks, now allowing each patch to be elongated. To do so, we use c/(V+c) along the x-axis, which allows us to vary c from 0 at x=0 to infinity at x=1, within a single plot (we use V = 4 so that patches 4 sd apart are half-way along the plot). To convert the x-axis to c, we use:

In[∘]:=
```
tryV = 4;
```

```
In[]:= Solve[c / (tryV + c) == x, c]
Out[]=
```

$$\left\{\left\{c \to -\frac{4\,x}{-1+x}\right\}\right\}$$

```
In[]:= tab =
  Join[Table[i, {i, 0 / 10, 7 / 10, 1 / 10}], Table[i, {i, 8 / 10, 19 / 20, 1 / 20}]] // Flatten
Out[]=
```

$$\left\{0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{17}{20}, \frac{9}{10}, \frac{19}{20}\right\}$$

```
In[]:= numind1 = 50;
numind2 = 50;
numind = numind1 + numind2;

In[]:= tryf = 2 (numind2 / numind) (numind1 / numind)
   (*Equal split with 50% in one patch so f = 1/2*)
Out[]=
```

$$\frac{1}{2}$$

```
In[]:= tryσx = 1;
tryσy = 1 / 5;

In[]:= For[t = 1, t ≤ Length[tab], t++,
 table = Join[Table[{Random[NormalDistribution[0, tryσx]],
      Random[NormalDistribution[0, tryσy]]}, {i, 1, numind1}],
    Table[{Random[NormalDistribution[tryV tab[[t]] / (1 - tab[[t]]), tryσx]],
      Random[NormalDistribution[0, tryσy]]}, {i, 1, numind2}]];
 disttable = Flatten[
    Table[EuclideanDistance[table[[i]], table[[j]]], {i, 1, numind}, {j, i + 1, numind}]];
 CVherdsimC[tryσx, tab[[t]]] = StandardDeviation[disttable] / Mean[disttable];
 disttable2 = Table[
    1 / (Length[table] - 1) (Sum[EuclideanDistance[table[[i]], table[[j]]], {j, 1, i - 1}] + Sum[
        EuclideanDistance[table[[i]], table[[j]]], {j, i + 1, numind}]), {i, 1, numind}];
 CVindsimC[tryσx, tab[[t]]] = StandardDeviation[disttable2] / Mean[disttable2]
]
```

In[∘]:=
```
forplottingherdE = Table[{tab⟦t⟧, CVherdsimC[tryσx, tab⟦t⟧]}, {t, 1, Length[tab]}]
```
Out[∘]=

$\left\{\{0, 0.662284\}, \left\{\frac{1}{10}, 0.731637\right\}, \left\{\frac{1}{5}, 0.659794\right\}, \left\{\frac{3}{10}, 0.705657\right\}, \left\{\frac{2}{5}, 0.700424\right\}, \left\{\frac{1}{2}, 0.686867\right\}, \left\{\frac{3}{5}, 0.731116\right\}, \left\{\frac{7}{10}, 0.802803\right\}, \left\{\frac{4}{5}, 0.867401\right\}, \left\{\frac{17}{20}, 0.893992\right\}, \left\{\frac{9}{10}, 0.931899\right\}, \left\{\frac{19}{20}, 0.960352\right\}\right\}$

In[∘]:=
```
forplottingindE = Table[{tab⟦t⟧, CVindsimC[tryσx, tab⟦t⟧]}, {t, 1, Length[tab]}]
```
Out[∘]=

$\left\{\{0, 0.318859\}, \left\{\frac{1}{10}, 0.384075\right\}, \left\{\frac{1}{5}, 0.265289\right\}, \left\{\frac{3}{10}, 0.330758\right\}, \left\{\frac{2}{5}, 0.301587\right\}, \left\{\frac{1}{2}, 0.224402\right\}, \left\{\frac{3}{5}, 0.154839\right\}, \left\{\frac{7}{10}, 0.0955232\right\}, \left\{\frac{4}{5}, 0.0606599\right\}, \left\{\frac{17}{20}, 0.0510101\right\}, \left\{\frac{9}{10}, 0.0256701\right\}, \left\{\frac{19}{20}, 0.0146037\right\}\right\}$

Below, we’ll want the ratio of $\frac{\text{CVind}}{\text{CVherd}}$ in the limit for very distant patches when the patches are uneven:

In[∘]:=
```
limratio = Limit[CVindfarsplit / CVherdC /. σ → tryσx /. f → tryf, c → Infinity]
```
Out[∘]=

0

We also plot CVherd in the limit for very distant patches when the patches are uneven:

In[∘]:=
```
CVherdfar = Limit[CVherdC /. σ → tryσx /. f → tryf, c → Infinity]
```
Out[∘]=

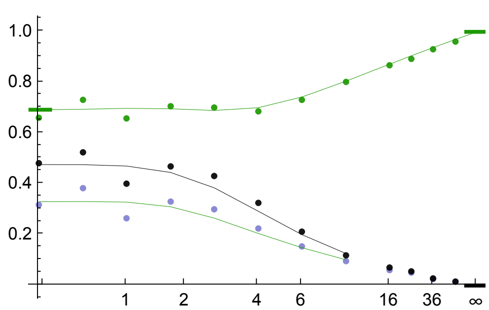
1

In[∘]:=
```
maxc = 1;
```
In[∘]:=
```
xticks = {{0.01, "0"}, {1 / 5, "1"}, {1 / 3, "2"},
   {1 / 2, "4"}, {3 / 5, "6"}, {4 / 5, "16"}, {9 / 10, "36"}, {1, "∞"}}
```
Out[∘]=

$\left\{\{0.01, 0\}, \left\{\frac{1}{5}, 1\right\}, \left\{\frac{1}{3}, 2\right\}, \left\{\frac{1}{2}, 4\right\}, \left\{\frac{3}{5}, 6\right\}, \left\{\frac{4}{5}, 16\right\}, \left\{\frac{9}{10}, 36\right\}, \{1, \infty\}\right\}$

In[ ]:=
```
plotE = Show[
  Plot[CVherdfar, {x, 0.975, 1.05},
   PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
  Plot[CVindfarsplit /. σx → tryσx /. σy → tryσy /. f → tryf,
   {x, 0.975, 1.05}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVherd /. σx → tryσx /. σy → tryσy,
   {x, -0.025, 0.025}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  Plot[limratio, {x, 0.975, 1.05}, PlotStyle → Black],
  ListPlot[forplottingherdE, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[Join[intplottingherdE, {{1, CVherdfar}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.11, 0.6, 0.]}],
  ListPlot[forplottingindE, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[intplottingindE,
   Joined → True, PlotStyle → {Thin, RGBColor[0.11, 0.6, 0.]}],
  ListPlot[Table[{forplottingherdE[[i, 1]], forplottingindE[[i, 2]] / forplottingherdE[[i, 2]]},
    {i, 1, Length[forplottingindE]}], PlotStyle → {Black}],
  ListPlot[Table[{intplottingherdE[[i, 1]], intplottingindE[[i, 2]] / intplottingherdE[[i, 2]]},
    {i, 1, Length[intplottingindE]}], Joined → True, PlotStyle → {Thin, Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, maxc}, {0, 1}}, Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[ ]=



Panel E: Numerical integration [SLOW - Don’t enter]

For comparison, we can numerically integrate:

In[ ]:=
```
tryLIM = Infinity;

For[t = 1, t ≤ Length[tab], t++,
 tryc = (tryV tab[[t]])/(1 - tab[[t]]);
 mean = (1 - f) soln + f NIntegrate[(E^(-(tryc - Δx)^2/(4 tryσx^2)) tryσy HypergeometricU[-1/2, 0, Δx^2/(4 tryσy^2)])/(Sqrt[π] tryσx),
     {Δx, -tryLIM, tryLIM}] /. σx → tryσx /. σy → tryσy /. f → tryf /. c → tryc;
 sqH =
  (1 - f) solnsq + f (c^2 + 2 (σx^2 + σy^2)) /. σx → tryσx /. σy → tryσy /. f → tryf /. c → tryc;
 CVherdintC[tryσx, tab[[t]]] = Sqrt[sqH - mean^2] / mean;
 Print[{CVherdintC[tryσx, tab[[t]]]}]
]
```

```
{0.69306}
{0.69489}
{0.698504}
{0.696641}
{0.690215}
{0.700981}
{0.743386}
{0.805267}
{0.872507}
{0.905818}
{0.938311}
{0.969749}
```

Uses slightly faster code for an even split. Errors become too large in the numerical integration for values of c above ~10, so we stop there:

```
tryLIM = Infinity;

For[t = 1, t ≤ 9, t++,
 tryc = (tryV tab[[t]])/(1 - tab[[t]]);

 mean = (1 - f) soln + f NIntegrate[(E^(-(tryc-Δx)^2/(4 tryσx^2)) tryσy HypergeometricU[-1/2, 0, Δx^2/(4 tryσy^2)])/(Sqrt[π] tryσx),
      {Δx, -tryLIM, tryLIM}] /. σx → tryσx /. σy → tryσy /. f → tryf /. c → tryc;

 CVindintC[tryσx, tab[[t]]] = Sqrt[sqI[tryσx, tryσy, tryc] - mean^2] / mean;
 Print[{CVindintC[tryσx, tab[[t]]]}]
]
```

{0.330509}

{0.331133}

{0.329018}

{0.310613}

{0.26577}

{0.205573}

{0.149184}

{0.102029}

{0. + 0.844342 i}

In[◦]:=
```
intplottingherdE = Table[{tab[[t]], CVherdintC[tryσx, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[◦]=

$$\left\{\{0, 0.69306\}, \left\{\frac{1}{10}, 0.69489\right\}, \left\{\frac{1}{5}, 0.698504\right\}, \left\{\frac{3}{10}, 0.696641\right\}, \left\{\frac{2}{5}, 0.690215\right\}, \left\{\frac{1}{2}, 0.700981\right\}, \left\{\frac{3}{5}, 0.743386\right\}, \left\{\frac{7}{10}, 0.805267\right\}, \left\{\frac{4}{5}, 0.872507\right\}, \left\{\frac{17}{20}, 0.905818\right\}, \left\{\frac{9}{10}, 0.938311\right\}, \left\{\frac{19}{20}, 0.969749\right\}\right\}$$

In[◦]:=
```
intplottingindE = Table[{tab[[t]], CVindintC[tryσx, tab[[t]]]}, {t, 1, 8}]
```

Out[◦]=

$$\left\{\{0, 0.330509\}, \left\{\frac{1}{10}, 0.331133\right\}, \left\{\frac{1}{5}, 0.329018\right\}, \left\{\frac{3}{10}, 0.310613\right\}, \left\{\frac{2}{5}, 0.26577\right\}, \left\{\frac{1}{2}, 0.205573\right\}, \left\{\frac{3}{5}, 0.149184\right\}, \left\{\frac{7}{10}, 0.102029\right\}\right\}$$

Panel F: Uneven herd split with elongated patches [σy short]

In[◦]:=
```
SeedRandom[66127];
```

We seek to examine a range of distances, c, between the peaks, now allowing each patch to be elongated. To do so, we use c/(V+c) along the x-axis, which allows us to vary c from 0 at x=0 to infinity at x=1,

within a single plot (we use V = 4 so that patches 4 sd apart are half-way along the plot). To convert the x-axis to c, we use:

In[•]:=
```
tryV = 4;
```

In[•]:=
```
Solve[c / (tryV + c) == x, c]
```

Out[•]=

$$\left\{\left\{c \to -\frac{4\,x}{-1 + x}\right\}\right\}$$

In[•]:=
```
tab =
 Join[Table[i, {i, 0 / 10, 7 / 10, 1 / 10}], Table[i, {i, 8 / 10, 19 / 20, 1 / 20}]] // Flatten
```

Out[•]=

$$\left\{0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{17}{20}, \frac{9}{10}, \frac{19}{20}\right\}$$

In[•]:=
```
numind1 = 90;
numind2 = 10;
numind = numind1 + numind2;
```

In[•]:=
```
tryf = 2 (numind2 / numind) (numind1 / numind)
  (*Unequal split with 90% in one patch so f = 0.18*)
```

Out[•]=

$$\frac{9}{50}$$

In[•]:=
```
tryσx = 1;
tryσy = 1 / 5;
```

In[•]:=
```
For[t = 1, t ≤ Length[tab], t++,
 table = Join[Table[{Random[NormalDistribution[0, tryσx]],
     Random[NormalDistribution[0, tryσy]]}, {i, 1, numind1}],
   Table[{Random[NormalDistribution[(tryV tab[[t]])/(1 - tab[[t]]), tryσx]],
     Random[NormalDistribution[0, tryσy]]}, {i, 1, numind2}]];
 disttable = Flatten[
   Table[EuclideanDistance[table[[i]], table[[j]]], {i, 1, numind}, {j, i + 1, numind}]];
 CVherdsimC[tryσx, tab[[t]]] = StandardDeviation[disttable] / Mean[disttable];
 disttable2 = Table[
   1/(Length[table] - 1) (Sum[EuclideanDistance[table[[i]], table[[j]]], {j, 1, i - 1}] + Sum[
       EuclideanDistance[table[[i]], table[[j]]], {j, i + 1, numind}]), {i, 1, numind}];
 CVindsimC[tryσx, tab[[t]]] = StandardDeviation[disttable2] / Mean[disttable2]
]
```

In[∘]:=
```
forplottingherdF = Table[{tab⟦t⟧, CVherdsimC[tryσx, tab⟦t⟧]}, {t, 1, Length[tab]}]
```
Out[∘]=

$$\left\{\{0, 0.691802\}, \left\{\frac{1}{10}, 0.69979\right\}, \left\{\frac{1}{5}, 0.691527\right\}, \left\{\frac{3}{10}, 0.689225\right\}, \left\{\frac{2}{5}, 0.710581\right\}, \left\{\frac{1}{2}, 0.875459\right\}, \left\{\frac{3}{5}, 1.00076\right\}, \left\{\frac{7}{10}, 1.24493\right\}, \left\{\frac{4}{5}, 1.50389\right\}, \left\{\frac{17}{20}, 1.63601\right\}, \left\{\frac{9}{10}, 1.80324\right\}, \left\{\frac{19}{20}, 1.93481\right\}\right\}$$

In[∘]:=
```
forplottingindF = Table[{tab⟦t⟧, CVindsimC[tryσx, tab⟦t⟧]}, {t, 1, Length[tab]}]
```
Out[∘]=

$$\left\{\{0, 0.336235\}, \left\{\frac{1}{10}, 0.354119\right\}, \left\{\frac{1}{5}, 0.32242\right\}, \left\{\frac{3}{10}, 0.32635\right\}, \left\{\frac{2}{5}, 0.351786\right\}, \left\{\frac{1}{2}, 0.498425\right\}, \left\{\frac{3}{5}, 0.603531\right\}, \left\{\frac{7}{10}, 0.77143\right\}, \left\{\frac{4}{5}, 0.944348\right\}, \left\{\frac{17}{20}, 1.02885\right\}, \left\{\frac{9}{10}, 1.13972\right\}, \left\{\frac{19}{20}, 1.22167\right\}\right\}$$

Below, we'll want the ratio of $\frac{\text{CVind}}{\text{CVherd}}$ in the limit for very distant patches when the patches are uneven:

In[∘]:=
$$\text{limratio} = \text{Limit}\left[\frac{\text{CVindfarsplit}}{\text{CVherdC}} \text{ /. } \sigma \to \text{try}\sigma\text{x /. f} \to \text{tryf, c} \to \text{Infinity}\right]$$
Out[∘]=

$$\frac{4}{\sqrt{41}}$$

We also plot CVherd in the limit for very distant patches when the patches are uneven:

In[∘]:=
```
CVherdfar = Limit[CVherdC /. σ → tryσx /. f → tryf, c → Infinity]
```
Out[∘]=

$$\frac{\sqrt{41}}{3}$$

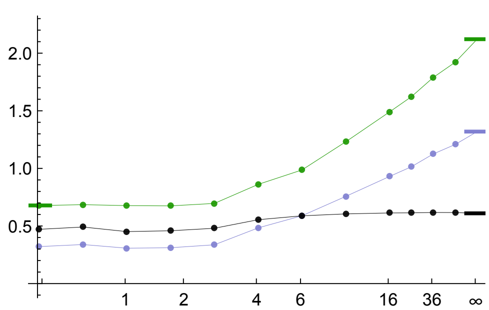
In[∘]:=
```
maxc = 1;
```

In[∘]:=
```
xticks = {{0.01, "0"}, {1/5, "1"}, {1/3, "2"},
   {1/2, "4"}, {3/5, "6"}, {4/5, "16"}, {9/10, "36"}, {1, "∞"}}
```
Out[∘]=

$$\left\{\{0.01, 0\}, \left\{\frac{1}{5}, 1\right\}, \left\{\frac{1}{3}, 2\right\}, \left\{\frac{1}{2}, 4\right\}, \left\{\frac{3}{5}, 6\right\}, \left\{\frac{4}{5}, 16\right\}, \left\{\frac{9}{10}, 36\right\}, \{1, \infty\}\right\}$$

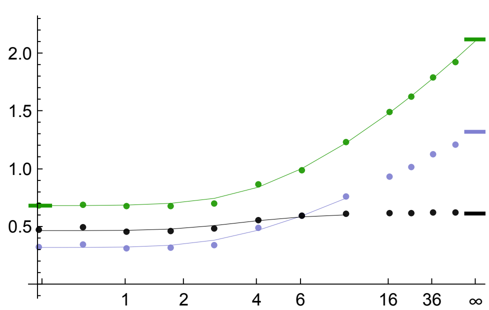
In[ ]:=
```
plotF = Show[
  Plot[CVherdfar, {x, 0.975, 1.05},
   PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
  Plot[CVindfarsplit /. f → tryf,
   {x, 0.975, 1.05}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVherd /. σx → tryσx /. σy → tryσy,
   {x, -0.025, 0.025}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  Plot[limratio, {x, 0.975, 1.05}, PlotStyle → Black],
  ListPlot[forplottingherdF, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[Join[forplottingherdF, {{1, CVherdfar}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.11, 0.6, 0.]}],
  ListPlot[forplottingindF, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[Join[forplottingindF, {{1, CVindfarsplit /. f → tryf}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}],
  ListPlot[Table[{forplottingherdF[[i, 1]], forplottingindF[[i, 2]] / forplottingherdF[[i, 2]]},
    {i, 1, Length[forplottingindF]}], PlotStyle → {Black}],
  ListPlot[Join[Table[{forplottingherdF[[i, 1]], forplottingindF[[i, 2]] / forplottingherdF[[i, 2]]},
     {i, 1, Length[forplottingindF]}], {{1, limratio}}],
   Joined → True, PlotStyle → {Thin, Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, maxc}, {0, 2.2}}, Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[ ]=



In[ ]:=
```
plotF = Show[
  Plot[CVherdfar, {x, 0.975, 1.05},
   PlotStyle → {RGBColor[0.11, 0.6, 0.]}, PlotRange → All],
  Plot[CVindfarsplit /. f → tryf,
   {x, 0.975, 1.05}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVherd /. σx → tryσx /. σy → tryσy,
   {x, -0.025, 0.025}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  Plot[limratio, {x, 0.975, 1.05}, PlotStyle → Black],
  ListPlot[forplottingherdF, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[Join[intplottingherdF, {{1, CVherdfar}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.11, 0.6, 0.]}],
  ListPlot[forplottingindF, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[intplottingindF, Joined → True,
   PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}, PlotRange → All],
  ListPlot[Table[{forplottingherdF[[i, 1]], forplottingindF[[i, 2]] / forplottingherdF[[i, 2]]},
    {i, 1, Length[forplottingindF]}], PlotStyle → {Black}],
  ListPlot[Table[{intplottingherdF[[i, 1]], intplottingindF[[i, 2]] / intplottingherdF[[i, 2]]},
    {i, 1, Length[intplottingindF]}], Joined → True, PlotStyle → {Thin, Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, maxc}, {0, 2.2}}, Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[ ]=



Panel F: Numerical integration [SLOW - Don’t enter]

For comparison, we can numerically integrate:

In[ ]:=
```
tryLIM = Infinity;

For[t = 1, t ≤ Length[tab], t++,
 tryc = tryV tab[[t]] / (1 - tab[[t]]);
 mean = (1 - f) soln + f NIntegrate[(E^(-(tryc - Δx)^2/(4 tryσx^2)) tryσy HypergeometricU[-1/2, 0, Δx^2/(4 tryσy^2)])/(Sqrt[π] tryσx),
     {Δx, -tryLIM, tryLIM}] /. σx → tryσx /. σy → tryσy /. f → tryf /. c → tryc;
 sqH =
  (1 - f) solnsq + f (c^2 + 2 (σx^2 + σy^2)) /. σx → tryσx /. σy → tryσy /. f → tryf /. c → tryc;
 CVherdintC[tryσx, tab[[t]]] = Sqrt[sqH - mean^2] / mean;
 Print[{CVherdintC[tryσx, tab[[t]]]}]
]
```

```
{0.69306}
{0.693862}
{0.698085}
{0.713045}
{0.756316}
{0.852825}
{1.01508}
{1.23438}
{1.49833}
{1.64453}
{1.79945}
{1.96279}
```

Errors become too large in the numerical integration for values of c above ~10, so we stop there:

In[∘]:=
```
tryLIM = Infinity;

For[t = 1, t ≤ 9, t++,
 tryc = tryV tab[[t]] / (1 - tab[[t]]);
 mean = (1 - f) soln + f NIntegrate[(E^(-(tryc - Δx)^2/(4 tryσx^2)) tryσy HypergeometricU[-1/2, 0, Δx^2/(4 tryσy^2)]) / (Sqrt[π] tryσx),
     {Δx, -tryLIM, tryLIM}] /. σx → tryσx /. σy → tryσy /. f → tryf /. c → tryc;

 CVindintC[tryσx, tab[[t]]] = Sqrt[sqI[tryσx, tryσy, tryc, tryf] - mean^2] / mean;
 Print[{CVindintC[tryσx, tab[[t]]]}]
]
```

{0.330509}

{0.330934}

{0.334272}

{0.349419}

{0.393716}

{0.480143}

{0.604582}

{0.756757}

{0. + 0.646899 ⅈ}

In[∘]:=
```
intplottingherdF = Table[{tab[[t]], CVherdintC[tryσx, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[∘]=

$$\left\{\{0, 0.69306\}, \left\{\frac{1}{10}, 0.693862\right\}, \left\{\frac{1}{5}, 0.698085\right\}, \left\{\frac{3}{10}, 0.713045\right\}, \left\{\frac{2}{5}, 0.756316\right\}, \left\{\frac{1}{2}, 0.852825\right\}, \left\{\frac{3}{5}, 1.01508\right\}, \left\{\frac{7}{10}, 1.23438\right\}, \left\{\frac{4}{5}, 1.49833\right\}, \left\{\frac{17}{20}, 1.64453\right\}, \left\{\frac{9}{10}, 1.79945\right\}, \left\{\frac{19}{20}, 1.96279\right\}\right\}$$

In[∘]:=
```
intplottingindF = Table[{tab[[t]], CVindintC[tryσx, tab[[t]]]}, {t, 1, 8}]
```

Out[∘]=

$$\left\{\{0, 0.330509\}, \left\{\frac{1}{10}, 0.330934\right\}, \left\{\frac{1}{5}, 0.334272\right\}, \left\{\frac{3}{10}, 0.349419\right\}, \left\{\frac{2}{5}, 0.393716\right\}, \left\{\frac{1}{2}, 0.480143\right\}, \left\{\frac{3}{5}, 0.604582\right\}, \left\{\frac{7}{10}, 0.756757\right\}\right\}$$

Panel G: Even herd split with elongated patches [σy long]

In[∘]:=
```
SeedRandom[77 212];
```

We seek to examine a range of distances, c, between the peaks, now allowing each patch to be elongated. To do so, we use c/(V+c) along the x-axis, which allows us to vary c from 0 at x=0 to infinity at x=1,

within a single plot (we use V = 4 so that patches 4 sd apart are half-way along the plot). To convert the x-axis to c, we use:

In[ ]:=
```
tryV = 4;
```

In[ ]:=
```
Solve[c / (tryV + c) == x, c]
```

Out[ ]=

$$\left\{\left\{c \to -\frac{4\,x}{-1 + x}\right\}\right\}$$

In[ ]:=
```
tab =
 Join[Table[i, {i, 0 / 10, 7 / 10, 1 / 10}], Table[i, {i, 8 / 10, 19 / 20, 1 / 20}]] // Flatten
```

Out[ ]=

$$\left\{0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{17}{20}, \frac{9}{10}, \frac{19}{20}\right\}$$

In[ ]:=
```
numind1 = 50;
numind2 = 50;
numind = 100;
```

In[ ]:=
```
tryf = 2 (numind2 / numind) (numind1 / numind)
  (*Equal split with 50% in one patch so f = 1/2*)
```

Out[ ]=

$$\frac{1}{2}$$

In[ ]:=
```
tryσx = 1;
tryσy = 5;
```

In[ ]:=
```
For[t = 1, t ≤ Length[tab], t++,
 table = Join[Table[{Random[NormalDistribution[0, tryσx]],
     Random[NormalDistribution[0, tryσy]]}, {i, 1, numind1}],
   Table[{Random[NormalDistribution[tryV tab[[t]] / (1 - tab[[t]]), tryσx]],
     Random[NormalDistribution[0, tryσy]]}, {i, 1, numind2}]];
 disttable = Flatten[
   Table[EuclideanDistance[table[[i]], table[[j]]], {i, 1, numind}, {j, i + 1, numind}]];
 CVherdsimC[tryσx, tab[[t]]] = StandardDeviation[disttable] / Mean[disttable];
 disttable2 = Table[
   1 / (Length[table] - 1) (Sum[EuclideanDistance[table[[i]], table[[j]]], {j, 1, i - 1}] + Sum[
       EuclideanDistance[table[[i]], table[[j]]], {j, i + 1, numind}]), {i, 1, numind}];
 CVindsimC[tryσx, tab[[t]]] = StandardDeviation[disttable2] / Mean[disttable2]
]
```

In[∘]:= forplottingherdG = Table[{tab⟦t⟧, CVherdsimC[tryσx, tab⟦t⟧]}, {t, 1, Length[tab]}]

Out[∘]=

$\left\{\{0, 0.686409\}, \left\{\frac{1}{10}, 0.644523\right\}, \left\{\frac{1}{5}, 0.704123\right\}, \left\{\frac{3}{10}, 0.656835\right\}, \left\{\frac{2}{5}, 0.617756\right\}, \left\{\frac{1}{2}, 0.558788\right\}, \left\{\frac{3}{5}, 0.554038\right\}, \left\{\frac{7}{10}, 0.500392\right\}, \left\{\frac{4}{5}, 0.582835\right\}, \left\{\frac{17}{20}, 0.641793\right\}, \left\{\frac{9}{10}, 0.731081\right\}, \left\{\frac{19}{20}, 0.839928\right\}\right\}$

In[∘]:= forplottingindG = Table[{tab⟦t⟧, CVindsimC[tryσx, tab⟦t⟧]}, {t, 1, Length[tab]}]

Out[∘]=

$\left\{\{0, 0.318933\}, \left\{\frac{1}{10}, 0.287707\right\}, \left\{\frac{1}{5}, 0.352326\right\}, \left\{\frac{3}{10}, 0.341636\right\}, \left\{\frac{2}{5}, 0.289254\right\}, \left\{\frac{1}{2}, 0.255619\right\}, \left\{\frac{3}{5}, 0.266793\right\}, \left\{\frac{7}{10}, 0.188248\right\}, \left\{\frac{4}{5}, 0.117176\right\}, \left\{\frac{17}{20}, 0.0912151\right\}, \left\{\frac{9}{10}, 0.0579121\right\}, \left\{\frac{19}{20}, 0.0308411\right\}\right\}$

Below, we'll want the ratio of $\frac{\text{CVind}}{\text{CVherd}}$ in the limit for very distant patches when the patches are uneven:

In[∘]:= $\text{limratio} = \text{Limit}\left[\frac{\text{CVindfarsplit}}{\text{CVherdC}} \text{ /. } \sigma \to \text{try}\sigma\text{x /. f} \to \text{tryf, c} \to \text{Infinity}\right]$

Out[∘]=

0

We also plot CVherd in the limit for very distant patches when the patches are uneven:

In[∘]:= CVherdfar = Limit[CVherdC /. σ → tryσx /. f → tryf, c → Infinity]

Out[∘]=

1

In[∘]:= maxc = 1;

In[∘]:= $\text{Solve}\left[\frac{\text{tryV x}}{1 - \text{x}} == 6, \text{x}\right]$

Out[∘]=

$\left\{\left\{x \to \frac{3}{5}\right\}\right\}$

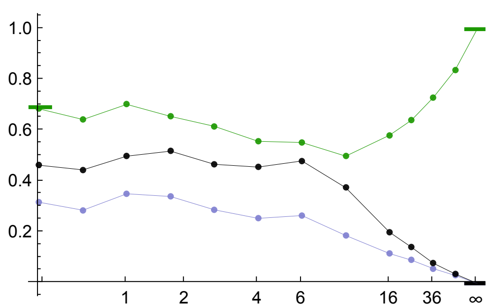
In[∘]:= xticks = {{0.01, "0"}, {1/5, "1"}, {1/3, "2"},
{1/2, "4"}, {3/5, "6"}, {4/5, "16"}, {9/10, "36"}, {1, "∞"}}

Out[∘]=

$\left\{\{0.01, 0\}, \left\{\frac{1}{5}, 1\right\}, \left\{\frac{1}{3}, 2\right\}, \left\{\frac{1}{2}, 4\right\}, \left\{\frac{3}{5}, 6\right\}, \left\{\frac{4}{5}, 16\right\}, \left\{\frac{9}{10}, 36\right\}, \{1, \infty\}\right\}$

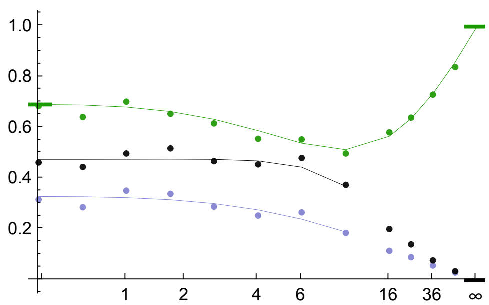
In[•]:=
```
plotG = Show[
  Plot[CVherdfar, {x, 0.975, 1.05},
   PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
  Plot[CVindfarsplit /. f → tryf,
   {x, 0.975, 1.05}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVherd /. σx → tryσx /. σy → tryσy,
   {x, -0.025, 0.025}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  Plot[limratio, {x, 0.975, 1.05}, PlotStyle → Black],
  ListPlot[forplottingherdG, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[Join[forplottingherdG, {{1, CVherdfar}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.11, 0.6, 0.]}],
  ListPlot[forplottingindG, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[Join[forplottingindG, {{1, CVindfarsplit /. f → tryf}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}],
  ListPlot[Table[{forplottingherdG[[i, 1]], forplottingindG[[i, 2]] / forplottingherdG[[i, 2]]},
    {i, 1, Length[forplottingindG]}], PlotStyle → {Black}],
  ListPlot[Join[Table[{forplottingherdG[[i, 1]], forplottingindG[[i, 2]] / forplottingherdG[[i, 2]]},
     {i, 1, Length[forplottingindG]}], {{1, limratio}}],
   Joined → True, PlotStyle → {Thin, Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, maxc}, {0, 1}}, Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[•]=



In[ ]:=
```
plotG = Show[
  Plot[CVherdfar, {x, 0.975, 1.05},
   PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
  Plot[CVindfarsplit /. f → tryf,
   {x, 0.975, 1.05}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVherd /. σx → tryσx /. σy → tryσy,
   {x, -0.025, 0.025}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  Plot[limratio, {x, 0.975, 1.05}, PlotStyle → Black],
  ListPlot[forplottingherdG, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[Join[intplottingherdG, {{1, CVherdfar}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.11, 0.6, 0.]}],
  ListPlot[forplottingindG, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[Join[intplottingindG],
   Joined → True, PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}],
  ListPlot[Table[{forplottingherdG[[i, 1]], forplottingindG[[i, 2]] / forplottingherdG[[i, 2]]},
    {i, 1, Length[forplottingindG]}], PlotStyle → {Black}],
  ListPlot[Join[Table[{intplottingherdG[[i, 1]], intplottingindG[[i, 2]] / intplottingherdG[[i, 2]]},
     {i, 1, Length[intplottingindG]}]], Joined → True, PlotStyle → {Thin, Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, maxc}, {0, 1}}, Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[ ]=



Panel G: Numerical integration [SLOW - Don’t enter]

For comparison, we can numerically integrate:

```
In[•]:= tryLIM = Infinity;

For[t = 1, t ≤ Length[tab], t++,
 tryc = (tryV tab[[t]])/(1 - tab[[t]]);
 mean = (1 - f) soln + f NIntegrate[(E^(-(tryc - Δx)^2/(4 tryσx^2)) tryσy HypergeometricU[-1/2, 0, Δx^2/(4 tryσy^2)])/(Sqrt[π] tryσx),
       {Δx, -tryLIM, tryLIM}] /. σx → tryσx /. σy → tryσy /. f → tryf /. c → tryc;
 sqH =
  (1 - f) solnsq + f (c^2 + 2 (σx^2 + σy^2)) /. σx → tryσx /. σy → tryσy /. f → tryf /. c → tryc;
 CVherdintC[tryσx, tab[[t]]] = Sqrt[sqH - mean^2] / mean;
 Print[{CVherdintC[tryσx, tab[[t]]]}]
]
```

```
{0.69306}
{0.690993}
{0.682913}
{0.665284}
{0.634514}
{0.590026}
{0.540485}
{0.514872}
{0.56791}
{0.637991}
{0.736635}
{0.859176}
```

Uses slightly faster code for an even split. Errors become too large in the numerical integration for values of c above ~10, so we stop there:

Errors become too large in the numerical integration for values of c above ~10, so we stop there:

```
tryLIM = Infinity;

For[t = 1, t ≤ 9, t++,
 tryc = tryV tab[[t]] / (1 - tab[[t]]);
 mean = (1 - f) soln + f NIntegrate[(E^(-(tryc - Δx)^2/(4 tryσx^2)) tryσy HypergeometricU[-1/2, 0, Δx^2/(4 tryσy^2)])/(Sqrt[π] tryσx),
      {Δx, -tryLIM, tryLIM}] /. σx → tryσx /. σy → tryσy /. f → tryf /. c → tryc;

 CVindintC[tryσx, tab[[t]]] = Sqrt[sqI[tryσx, tryσy, tryc] - mean^2] / mean;
 Print[{CVindintC[tryσx, tab[[t]]]}]
]
```

```
{0.330507}
{0.329581}
{0.325924}
{0.317729}
{0.30257}
{0.277792}
{0.241082}
{0.191027}
{0. + 0.754737 i}
```

In[ ]:= `intplottingherdG = Table[{tab[[t]], CVherdintC[tryσx, tab[[t]]]}, {t, 1, Length[tab]}]`

Out[ ]=

$\{\{0, 0.69306\}, \{\frac{1}{10}, 0.690993\}, \{\frac{1}{5}, 0.682913\}, \{\frac{3}{10}, 0.665284\}, \{\frac{2}{5}, 0.634514\}, \{\frac{1}{2}, 0.590026\}, \{\frac{3}{5}, 0.540485\}, \{\frac{7}{10}, 0.514872\}, \{\frac{4}{5}, 0.56791\}, \{\frac{17}{20}, 0.637991\}, \{\frac{9}{10}, 0.736635\}, \{\frac{19}{20}, 0.859176\}\}$

In[ ]:= `intplottingindG = Table[{tab[[t]], CVindintC[tryσx, tab[[t]]]}, {t, 1, 8}]`

Out[ ]=

$\{\{0, 0.330507\}, \{\frac{1}{10}, 0.329581\}, \{\frac{1}{5}, 0.325924\}, \{\frac{3}{10}, 0.317729\}, \{\frac{2}{5}, 0.30257\}, \{\frac{1}{2}, 0.277792\}, \{\frac{3}{5}, 0.241082\}, \{\frac{7}{10}, 0.191027\}\}$

Panel H: Uneven herd split with elongated patches [σy long]

In[ ]:= `SeedRandom[827 523];`

We seek to examine a range of distances, c, between the peaks, now allowing each patch to be elongated. To do so, we use c/(V+c) along the x-axis, which allows us to vary c from 0 at x=0 to infinity at x=1,

within a single plot (we use V = 4 so that patches 4 sd apart are half-way along the plot). To convert the x-axis to c, we use:

In[•]:=
```
tryV = 4;
```

In[•]:=
```
Solve[c / (tryV + c) == x, c]
```

Out[•]=

$$\left\{\left\{c \to -\frac{4\,x}{-1+x}\right\}\right\}$$

In[•]:=
```
tab =
 Join[Table[i, {i, 0 / 10, 7 / 10, 1 / 10}], Table[i, {i, 8 / 10, 19 / 20, 1 / 20}]] // Flatten
```

Out[•]=

$$\left\{0,\ \frac{1}{10},\ \frac{1}{5},\ \frac{3}{10},\ \frac{2}{5},\ \frac{1}{2},\ \frac{3}{5},\ \frac{7}{10},\ \frac{4}{5},\ \frac{17}{20},\ \frac{9}{10},\ \frac{19}{20}\right\}$$

In[•]:=
```
numind1 = 90;
numind2 = 10;
numind = numind1 + numind2;
```

In[•]:=
```
tryf = 2 (numind2 / numind) (numind1 / numind)
  (*Unequal split with 90% in one patch so f = 0.18*)
```

Out[•]=

$$\frac{9}{50}$$

In[•]:=
```
tryσx = 1;
tryσy = 5;
```

In[•]:=
```
For[t = 1, t ≤ Length[tab], t++,
 table = Join[Table[{Random[NormalDistribution[0, tryσx]],
     Random[NormalDistribution[0, tryσy]]}, {i, 1, numind1}],
   Table[{Random[NormalDistribution[tryV tab[[t]] / (1 - tab[[t]]), tryσx]],
     Random[NormalDistribution[0, tryσy]]}, {i, 1, numind2}]];
 disttable = Flatten[
   Table[EuclideanDistance[table[[i]], table[[j]]], {i, 1, numind}, {j, i + 1, numind}]];
 CVherdsimC[tryσx, tab[[t]]] = StandardDeviation[disttable] / Mean[disttable];
 disttable2 = Table[
   1 / (Length[table] - 1) (Sum[EuclideanDistance[table[[i]], table[[j]]], {j, 1, i - 1}] + Sum[
       EuclideanDistance[table[[i]], table[[j]]], {j, i + 1, numind}]), {i, 1, numind}];
 CVindsimC[tryσx, tab[[t]]] = StandardDeviation[disttable2] / Mean[disttable2]
]
```

In[ ]:= `forplottingherdH = Table[{tab⟦t⟧, CVherdsimC[tryσx, tab⟦t⟧]}, {t, 1, Length[tab]}]`

Out[ ]=

$$\left\{\{0, 0.694576\}, \left\{\frac{1}{10}, 0.716758\right\}, \left\{\frac{1}{5}, 0.677128\right\}, \left\{\frac{3}{10}, 0.660936\right\}, \left\{\frac{2}{5}, 0.678201\right\}, \left\{\frac{1}{2}, 0.696807\right\}, \left\{\frac{3}{5}, 0.641276\right\}, \left\{\frac{7}{10}, 0.632392\right\}, \left\{\frac{4}{5}, 0.778978\right\}, \left\{\frac{17}{20}, 0.856287\right\}, \left\{\frac{9}{10}, 1.08728\right\}, \left\{\frac{19}{20}, 1.44195\right\}\right\}$$

In[ ]:= `forplottingindH = Table[{tab⟦t⟧, CVindsimC[tryσx, tab⟦t⟧]}, {t, 1, Length[tab]}]`

Out[ ]=

$$\left\{\{0, 0.338501\}, \left\{\frac{1}{10}, 0.380425\right\}, \left\{\frac{1}{5}, 0.304561\right\}, \left\{\frac{3}{10}, 0.316123\right\}, \left\{\frac{2}{5}, 0.334335\right\}, \left\{\frac{1}{2}, 0.368381\right\}, \left\{\frac{3}{5}, 0.31681\right\}, \left\{\frac{7}{10}, 0.334846\right\}, \left\{\frac{4}{5}, 0.460827\right\}, \left\{\frac{17}{20}, 0.518812\right\}, \left\{\frac{9}{10}, 0.675009\right\}, \left\{\frac{19}{20}, 0.903865\right\}\right\}$$

Below, we'll want the ratio of $\frac{\text{CVind}}{\text{CVherd}}$ in the limit for very distant patches when the patches are uneven:

In[ ]:= $\text{limratio} = \text{Limit}\left[\frac{\text{CVindfarsplit}}{\text{CVherdC}} \text{ /. } \sigma \to \text{try}\sigma\text{x /. f} \to \text{tryf, c} \to \text{Infinity}\right]$

Out[ ]=

$$\frac{4}{\sqrt{41}}$$

We also plot CVherd in the limit for very distant patches when the patches are uneven:

In[ ]:= `CVherdfar = Limit[CVherdC /. σ → tryσx /. f → tryf, c → Infinity]`

Out[ ]=

$$\frac{\sqrt{41}}{3}$$

In[ ]:= `maxc = 1;`

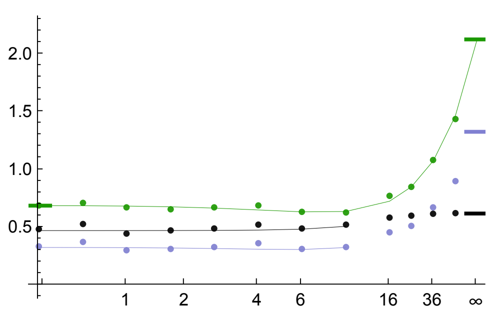
In[ ]:= `xticks = {{0.01, "0"}, {1/5, "1"}, {1/3, "2"}, {1/2, "4"}, {3/5, "6"}, {4/5, "16"}, {9/10, "36"}, {1, "∞"}}`

Out[ ]=

$$\left\{\{0.01, 0\}, \left\{\frac{1}{5}, 1\right\}, \left\{\frac{1}{3}, 2\right\}, \left\{\frac{1}{2}, 4\right\}, \left\{\frac{3}{5}, 6\right\}, \left\{\frac{4}{5}, 16\right\}, \left\{\frac{9}{10}, 36\right\}, \{1, \infty\}\right\}$$

In[∘]:=
```
plotH = Show[
  Plot[CVherdfar, {x, 0.975, 1.05},
   PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
  Plot[CVindfarsplit /. f → tryf,
   {x, 0.975, 1.05}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVherd /. σx → tryσx /. σy → tryσy,
   {x, -0.025, 0.025}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  Plot[limratio, {x, 0.975, 1.05}, PlotStyle → Black],
  ListPlot[forplottingherdH, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[Join[intplottingherdH, {{1, CVherdfar}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.11, 0.6, 0.]}],
  ListPlot[forplottingindH, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[intplottingindH, Joined → True,
   PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}, PlotRange → All],
  ListPlot[Table[{forplottingherdH[[i, 1]], forplottingindH[[i, 2]] / forplottingherdH[[i, 2]]},
    {i, 1, Length[forplottingindH]}], PlotStyle → {Black}],
  ListPlot[Table[{intplottingherdH[[i, 1]], intplottingindH[[i, 2]] / intplottingherdH[[i, 2]]},
    {i, 1, Length[intplottingindH]}], Joined → True, PlotStyle → {Thin, Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, maxc}, {0, 2.2}}, Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[∘]=



Panel H: Numerical integration [SLOW - Don’t enter]

For comparison, we can numerically integrate:

In[ ]:=
```
tryLIM = Infinity;

For[t = 1, t ≤ Length[tab], t++,
 tryc = (tryV tab[[t]])/(1 - tab[[t]]);
 mean = (1 - f) soln + f NIntegrate[(E^(-(tryc - Δx)^2/(4 tryσx^2)) tryσy HypergeometricU[-1/2, 0, Δx^2/(4 tryσy^2)])/(Sqrt[π] tryσx),
      {Δx, -tryLIM, tryLIM}] /. σx → tryσx /. σy → tryσy /. f → tryf /. c → tryc;
 sqH =
  (1 - f) solnsq + f (c^2 + 2 (σx^2 + σy^2)) /. σx → tryσx /. σy → tryσy /. f → tryf /. c → tryc;
 CVherdintC[tryσx, tab[[t]]] = Sqrt[sqH - mean^2] / mean;
 Print[{CVherdintC[tryσx, tab[[t]]]}]
]
```

```
{0.69306}
{0.692316}
{0.689402}
{0.683026}
{0.671857}
{0.655792}
{0.639368}
{0.642314}
{0.73155}
{0.856944}
{1.08236}
{1.4683}
```

Errors become too large in the numerical integration for values of c above ~10, so we stop there:

```
tryLIM = Infinity;

For[t = 1, t ≤ 9, t++,
 tryc = tryV tab[[t]] / (1 - tab[[t]]);

 mean = (1 - f) soln + f NIntegrate[(e^(-(tryc-Δx)^2/(4 tryσx^2)) tryσy HypergeometricU[-1/2, 0, Δx^2/(4 tryσy^2)]) / (Sqrt[π] tryσx),
      {Δx, -tryLIM, tryLIM}] /. σx → tryσx /. σy → tryσy /. f → tryf /. c → tryc;

 CVindintC[tryσx, tab[[t]]] = Sqrt[sqI[tryσx, tryσy, tryc, tryf] - mean^2] / mean;
 Print[{CVindintC[tryσx, tab[[t]]]}]
]
```

```
{0.330507}
{0.330174}
{0.32888}
{0.32609}
{0.321405}
{0.315496}
{0.312844}
{0.329704}
{0. + 0.528792 i}
```

In[∘]:=
```
intplottingherdH = Table[{tab[[t]], CVherdintC[tryσx, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[∘]=

$$\left\{\{0, 0.69306\}, \left\{\frac{1}{10}, 0.692316\right\}, \left\{\frac{1}{5}, 0.689402\right\}, \left\{\frac{3}{10}, 0.683026\right\}, \left\{\frac{2}{5}, 0.671857\right\}, \left\{\frac{1}{2}, 0.655792\right\}, \left\{\frac{3}{5}, 0.639368\right\}, \left\{\frac{7}{10}, 0.642314\right\}, \left\{\frac{4}{5}, 0.73155\right\}, \left\{\frac{17}{20}, 0.856944\right\}, \left\{\frac{9}{10}, 1.08236\right\}, \left\{\frac{19}{20}, 1.4683\right\}\right\}$$

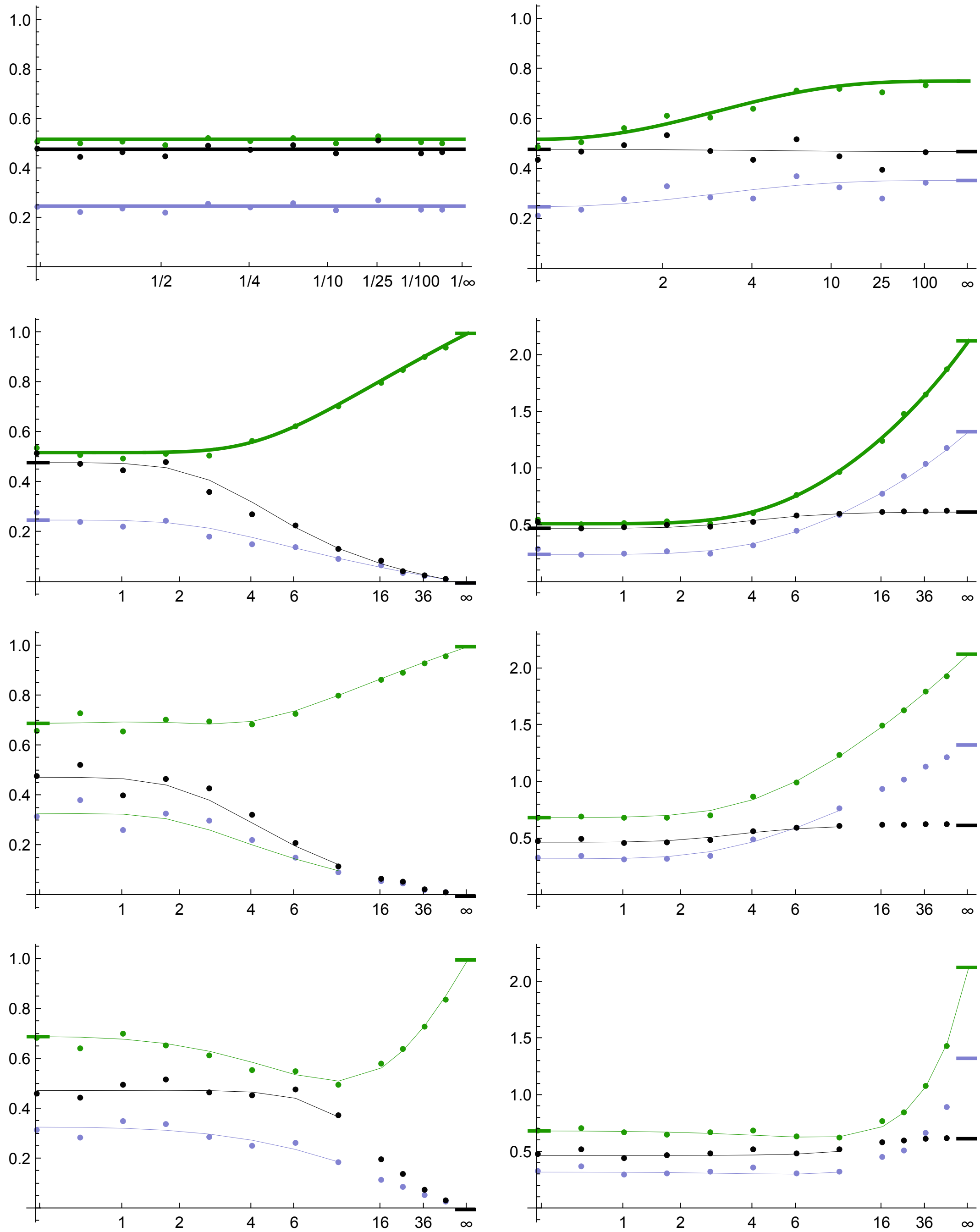
In[∘]:=
```
intplottingindH = Table[{tab[[t]], CVindintC[tryσx, tab[[t]]]}, {t, 1, Length[tab] - 4}]
```

Out[∘]=

$$\left\{\{0, 0.330507\}, \left\{\frac{1}{10}, 0.330174\right\}, \left\{\frac{1}{5}, 0.32888\right\}, \left\{\frac{3}{10}, 0.32609\right\}, \left\{\frac{2}{5}, 0.321405\right\}, \left\{\frac{1}{2}, 0.315496\right\}, \left\{\frac{3}{5}, 0.312844\right\}, \left\{\frac{7}{10}, 0.329704\right\}\right\}$$

Altogether

In[ ]:=
```
GraphicsGrid[{{plotA, plotB}, {plotC, plotD}, {plotE, plotF}, {plotG, plotH}}]
```
Out[ ]=



Individuals distributed uniformly across a rectangle

Commands for simplifying terms [ENTER]

Assume that the positions are positive and real values:

In[ ]:=
```
fullconditions = {0 < x1 < xmax, 0 < y1 < ymax, 0 < x2 < xmax, 0 < y2 < ymax,
   -xmax < dx < xmax, -ymax < dy < ymax, 0 < dymax < ymax, _Symbol ∈ Reals};
```

Suppress error messages when some of the above are irrelevant:

In[ ]:=
```
Off[Limit::alimv]
```

Log terms can be gathered together using the code suggested by Alexei Boulbitch in StackExchange (https://mathematica.stackexchange.com/questions/22705/simplify-expressions-with-log)

In[ ]:=
```
collectLog[expr_] :=
  Module[{rule1, rule2, a, b, x}, rule1 = Log[a_] + Log[b_] → Log[a * b];
   rule2 = x_ * Log[a_] → Log[a ^ x];
   (expr /. rule1) /. rule2 /. rule1 /. rule2];
```

In[ ]:=
```
collectLog2[expr_] :=
  Module[{rule1, rule2, a, b, x}, rule1 = -x_ Log[a_] + x_ Log[b_] → x Log[a / b];
   (expr /. rule1)];
```

In[ ]:=
```
collectLog3[expr_] :=
  Module[{rule1, rule2, a, b, x}, rule1 = x_ Log[a_] + x_ Log[b_] → x Log[a b];
   (expr /. rule1)];
```

In[ ]:=
```
collectLog4[expr_] :=
  Module[{rule1, rule2, a, b, x}, rule1 = x_ Log[a_] + y_ Log[b_] → Log[a^x b^y];
   (expr /. rule1)];
```

Summarizing the results:

In[ ]:=
$$\text{soln} = \frac{(\text{xmax}^5 + \text{ymax}^5)}{15\ \text{xmax}^2\ \text{ymax}^2} + \frac{1}{15} H \left(3 - \frac{\text{xmax}^2}{\text{ymax}^2} - \frac{\text{ymax}^2}{\text{xmax}^2}\right) + \frac{\text{xmax}^3 \text{Log}\left[\frac{\text{ymax}+H}{\text{xmax}}\right] + \text{ymax}^3 \text{Log}\left[\frac{\text{xmax}+H}{\text{ymax}}\right]}{6\ \text{xmax ymax}} \;/.\; H \to \sqrt{\text{xmax}^2 + \text{ymax}^2};$$

In[ ]:=
$$\text{FullSimplify}\left[\text{soln} - \frac{\text{xmax}}{30\ \alpha^2} \text{solncapt} \;/.\; \text{ymax} \to \alpha\ \text{xmax} \;/.\; \text{solncapt} \to 2\ (1 + \alpha^5) - 2\ H\ (1 - 3\ \alpha^2 + \alpha^4) + \text{Log}\left[\left(\frac{1 + H}{\alpha}\right)^{5\alpha^4} (H + \alpha)^{5\alpha}\right] \;/.\; H \to \sqrt{1 + \alpha^2},\ \{\text{xmax} > 0,\ \alpha > 0\}\right]$$

Out[ ]=
0

In[ ]:=
$$\text{solnX2} = \frac{\text{xmax}^2 + \text{ymax}^2}{6};$$

In[∘]:=
$$\text{CVherd} = \text{Sqrt}\left[\frac{150\,\alpha^4\,(1+\alpha^2)}{\text{solncapt}^2} - 1\right] \text{ /. solncapt} \to 2 + 2\,\alpha^5 - 2\,H\,(1 - 3\,\alpha^2 + \alpha^4) + \text{Log}\left[\left(\frac{1+H}{\alpha}\right)^{5\,\alpha^4}\,(H+\alpha)^{5\,\alpha}\right] \text{ /. } H \to \sqrt{1+\alpha^2} \text{ /. } \alpha \to \text{ymax} / \text{xmax};$$

where α->ymax/xmax gives the asymmetry in the range.

For herds separated by a distance of $c_s = \frac{c}{\text{xmax}}$ along the x-axis, the

In[∘]:=
$$\text{solnc} = \text{soln} + \frac{f\,\text{max}}{30\,\alpha^2}\,\text{distcapt};$$

In[∘]:=
$$\text{solncX2} = \frac{\text{xmax}^2 + \text{ymax}^2}{6} + c^2\,f;$$

In[∘]:=
$$\text{backtransform2} = \left\{H \to \sqrt{1+\alpha^2},\ H1 \to \sqrt{\alpha^2 + (-1+c_s)^2},\ H2 \to \sqrt{\alpha^2 + c_s^2},\ H3 \to \sqrt{\alpha^2 + (1+c_s)^2},\ H4 \to \sqrt{(-1+c_s)^2}\right\};$$

In[∘]:=
$$\begin{aligned}\text{distcapt} = {} & \Big(\Big((1+c_s)^5 - 2\,(1+\alpha^5+c_s^5)\Big) + 2\,H\,(1-3\,\alpha^2+\alpha^4) + 2\,H2\,(\alpha^2 - \alpha\,c_s - c_s^2)\,(\alpha^2 + \alpha\,c_s - c_s^2) - \\ & H1\,(-1+\alpha+\alpha^2-(-2+\alpha)\,c_s - c_s^2)\,(-1-\alpha+\alpha^2+(2+\alpha)\,c_s - c_s^2) + \\ & H3\,(-1+\alpha+\alpha^2+(-2+\alpha)\,c_s - c_s^2)\,(1+\alpha-\alpha^2+(2+\alpha)\,c_s + c_s^2) + H4^5\Big) + \\ & \frac{5}{4}\,\alpha\,\text{Log}\Big[(H+\alpha)^{-4}\left(\frac{H2+\alpha}{c_s}\right)^{-4\,c_s^4}\left(\frac{H1+\alpha}{H4}\right)^{2\,(-1+c_s)^4}\left(\frac{H3+\alpha}{1+c_s}\right)^{2\,(1+c_s)^4} \\ & \left(\left(\frac{\alpha}{1+H}\right)^2\frac{1+H3+c_s}{-1+H1+c_s}\right)^{2\,\alpha^3}\left(\frac{(-1+H1+c_s)\,(1+H3+c_s)}{(H2+c_s)^2}\right)^{2\,\alpha^3\,c_s}\Big];\end{aligned}$$

In[∘]:=
$$\begin{aligned}\text{CVherdC} = {} & \text{Sqrt}\left[\frac{\alpha^4\,150\,(1+\alpha^2+6\,f\,c_s^2)}{(\text{solncapt} + f\,\text{distcapt})^2} - 1\right] \text{ /. } \\ & \text{solncapt} \to 2 + 2\,\alpha^5 - 2\,H\,(1-3\,\alpha^2+\alpha^4) + \text{Log}\left[\left(\frac{1+H}{\alpha}\right)^{5\,\alpha^4}(H+\alpha)^{5\,\alpha}\right] \text{ /. } \\ & H \to \sqrt{1+\alpha^2} \text{ /. backtransform2 /. } \alpha \to \text{ymax}/\text{xmax} \text{ /. } c_s \to \frac{c}{\text{xmax}};\end{aligned}$$

For individual-level CV:

In[ ]:= $$\text{intraind} = \frac{1}{3}\frac{1}{\text{xmax ymax}}\Big(\text{x1 y1}\sqrt{\text{x1}^2+\text{y1}^2} + (\text{xmax}-\text{x1})\,\text{y1}\sqrt{(\text{xmax}-\text{x1})^2+\text{y1}^2} + (\text{ymax}-\text{y1})\,\text{x1}\sqrt{\text{x1}^2+(\text{ymax}-\text{y1})^2} + (\text{xmax}-\text{x1})\,(\text{ymax}-\text{y1})\sqrt{(\text{xmax}-\text{x1})^2+(\text{ymax}-\text{y1})^2}\Big) + \frac{1}{6}\frac{1}{\text{xmax ymax}}\,\text{Log}\Big[\left(\frac{\text{xmax}-\text{x1}+\sqrt{(\text{xmax}-\text{x1})^2+\text{y1}^2}}{\sqrt{\text{x1}^2+\text{y1}^2}-\text{x1}}\right)^{\text{y1}^3}\left(\frac{\text{ymax}-\text{y1}+\sqrt{\text{x1}^2+(\text{ymax}-\text{y1})^2}}{\sqrt{\text{x1}^2+\text{y1}^2}-\text{y1}}\right)^{\text{x1}^3}\left(\frac{\sqrt{(\text{xmax}-\text{x1})^2+\text{y1}^2}-\text{y1}}{\text{ymax}-\text{y1}+\sqrt{(\text{xmax}-\text{x1})^2+(\text{ymax}-\text{y1})^2}}\right)^{(\text{x1}-\text{xmax})^3}\left(\frac{\sqrt{(\text{ymax}-\text{y1})^2+\text{x1}^2}-\text{x1}}{\text{xmax}-\text{x1}+\sqrt{(\text{xmax}-\text{x1})^2+(\text{ymax}-\text{y1})^2}}\right)^{(\text{y1}-\text{ymax})^3}\Big];$$

The results below assume xmax=ymax=max:

In[ ]:= $$\text{indsoln} = \frac{1}{15}\,\text{max}\left(2+\sqrt{2}+5\,\text{Log}\left[1+\sqrt{2}\right]\right);$$

indsolnsq = 0.27892463475631724` max^4;

CVind = 0.16116021233464142`;

$$\text{CVind1} = \frac{1}{2\sqrt{5}};$$

A herd split in two with a chance f of drawing from different patches has a CVind of:

In[ ]:= $$\text{CVindfarsplit} = \sqrt{\frac{\left(\frac{1}{2}-\text{f}\right)}{\text{f}}};$$

A very elongated herd has:

$$\text{CVherd1} = \frac{1}{\sqrt{2}};$$

$$\text{CVind1} = \frac{1}{2\sqrt{5}};$$

For comparison, we can numerically integrate the function for a given parameter set.

For the herd-level CV, we can use analytical results where available:

```
mean =
  soln + (f xmax)/(30 α^2) (((((1 + cs)^5 - 2 (1 + α^5 + cs^5)) + 2 H (1 - 3 α^2 + α^4) + 2 H2 (α^2 - α cs -
             cs^2) (α^2 + α cs - cs^2) - H1 (-1 + α + α^2 - (-2 + α) cs - cs^2)
           (-1 - α + α^2 + (2 + α) cs - cs^2) + H3 (-1 + α + α^2 + (-2 + α) cs - cs^2)
           (1 + α - α^2 + (2 + α) cs + cs^2) + H4^5) +
         5/4 α Log[(H + α)^-4 ((H1+α)/H4)^(2 (-1+cs)^4) (cs/(H2+α))^(4 cs^4) ((H3+α)/(1+cs))^(2 (1+cs)^4)
           ((α/(1+H))^2 (1+H3+cs)/(-1+H1+cs))^(2 α^3) (((-1+H1+cs) (1+H3+cs))/(H2+cs)^2)^(2 α^3 cs)]) /.
       {H → Sqrt[1 + α^2], H1 → Sqrt[α^2 + (-1 + cs)^2], H2 → Sqrt[α^2 + cs^2],
        H3 → Sqrt[α^2 + (1 + cs)^2], H4 → Sqrt[(-1 + cs)^2]} /.
      cs → c / xmax /. α → ymax / xmax /. xmax → tryxmax /.
    ymax → tryymax /. f → tryf /. c → tryc;
sqH =
  (xmax^2+ymax^2)/6 + c^2 f /. xmax → tryxmax /. ymax → tryymax /. f → tryf /. c → tryc;
```

The individual-level CV first requires integration of the average distance of an individual to any other. Analytical results are only available for the average distance in the symmetric Gaussian case (xmax=ymax):

In[•]:=

```
Solve[2 p (1 - p) == f, p]
```

Out[•]=

$$\left\{\left\{p \to \frac{1}{2}\left(1 - \sqrt{1 - 2 f}\right)\right\}, \left\{p \to \frac{1}{2}\left(1 + \sqrt{1 - 2 f}\right)\right\}\right\}$$

In[ ]:=
```
avedist[x1_?NumericQ, y1_?NumericQ, xmax_?NumericQ, ymax_?NumericQ] :=
  1/3 1/(xmax ymax) (x1 y1 Sqrt[x1^2 + y1^2] +
      (xmax - x1) y1 Sqrt[(xmax - x1)^2 + y1^2] + (ymax - y1) x1 Sqrt[x1^2 + (ymax - y1)^2] +
      (xmax - x1) (ymax - y1) Sqrt[(xmax - x1)^2 + (ymax - y1)^2]) + 1/6 1/(xmax ymax)
   Log[ ((xmax - x1 + Sqrt[(xmax - x1)^2 + y1^2])/(Sqrt[x1^2 + y1^2] - x1))^(y1^3) ((ymax - y1 + Sqrt[x1^2 + (ymax - y1)^2])/(Sqrt[x1^2 + y1^2] - y1))^(x1^3)
     ((Sqrt[(xmax - x1)^2 + y1^2] - y1)/(ymax - y1 + Sqrt[(xmax - x1)^2 + (ymax - y1)^2]))^((x1-xmax)^3)
     ((Sqrt[(ymax - y1)^2 + x1^2] - x1)/(xmax - x1 + Sqrt[(xmax - x1)^2 + (ymax - y1)^2]))^((y1-ymax)^3) ];
sqI[xmax_?NumericQ, ymax_?NumericQ, c_?NumericQ, f_?NumericQ] := NIntegrate[
   ((p1 * (p1 * avedist[x1, y1, xmax, ymax] + p2 * avedist[x1 + c, y1, xmax, ymax])^2 + p2 *
          (p1 * avedist[x1 - c, y1, xmax, ymax] + p2 * avedist[x1, y1, xmax, ymax])^2) /.
       p2 -> 1 - p1 /. p1 -> 1/2 (1 - Sqrt[1 - 2 f])) (1/(xmax ymax)), {x1, 0, xmax},
   {y1, 0, ymax}];(*c is displacement along x-axis*)
```

When c=0 (no split), we can speed up the integration:

In[ ]:=
```
sqI[xmax_?NumericQ, ymax_?NumericQ] :=
  NIntegrate[
   (avedist[x1, y1, xmax, ymax, 0])^2 (1/(xmax ymax)), {x1, 0, xmax}, {y1, 0, ymax}];
```

When xmax=ymax, we can speed up the integration:

In[ ]:=

```
avedist[x1_?NumericQ, y1_?NumericQ, max_?NumericQ] :=
```

$$\frac{1}{6\,\text{max}^2}\left(2\left((\text{max}-\text{x1})\sqrt{(\text{max}-\text{x1})^2+(\text{max}-\text{y1})^2}\,(\text{max}-\text{y1})+\text{x1}\sqrt{\text{x1}^2+(\text{max}-\text{y1})^2}\,(\text{max}-\text{y1})+(\text{max}-\text{x1})\,\text{y1}\sqrt{(\text{max}-\text{x1})^2+\text{y1}^2}+\text{x1}\,\text{y1}\sqrt{\text{x1}^2+\text{y1}^2}\right)+\text{Log}\left[\left(\frac{-\text{x1}+\sqrt{\text{x1}^2+(\text{max}-\text{y1})^2}}{\text{max}-\text{x1}+\sqrt{(\text{max}-\text{x1})^2+(\text{max}-\text{y1})^2}}\right)^{(-\text{max}+\text{y1})^3}\left(\frac{-\text{y1}+\sqrt{(\text{max}-\text{x1})^2+\text{y1}^2}}{\text{max}+\sqrt{(\text{max}-\text{x1})^2+(\text{max}-\text{y1})^2}-\text{y1}}\right)^{(-\text{max}+\text{x1})^3}\left(\frac{\text{max}-\text{x1}+\sqrt{(\text{max}-\text{x1})^2+\text{y1}^2}}{-\text{x1}+\sqrt{\text{x1}^2+\text{y1}^2}}\right)^{\text{y1}^3}\left(\frac{\text{max}+\sqrt{\text{x1}^2+(\text{max}-\text{y1})^2}-\text{y1}}{-\text{y1}+\sqrt{\text{x1}^2+\text{y1}^2}}\right)^{\text{x1}^3}\right]\right);$$

```
sqI[max_?NumericQ, c_?NumericQ, f_?NumericQ] :=
```

$$\text{NIntegrate}\left[\left(\left(\text{p1}*(\text{p1}*\text{avedist}[\text{x1},\text{y1},\text{max}]+\text{p2}*\text{avedist}[\text{x1}+\text{c},\text{y1},\text{max}])^2+\text{p2}*(\text{p1}*\text{avedist}[\text{x1}-\text{c},\text{y1},\text{max}]+\text{p2}*\text{avedist}[\text{x1},\text{y1},\text{max}])^2\right)\ /.\ \text{p2}\to 1-\text{p1}\ /.\ \text{p1}\to\frac{1}{2}\left(1-\sqrt{1-2\,\text{f}}\right)\right)\left(\frac{1}{\text{max max}}\right),\{\text{x1},0,\text{max}\},\{\text{y1},0,\text{max}\}\right];$$

```
(*c is displacement along x-axis*)
```

The above were not sufficiently accurate for large c values, but accuracy could be improved by first taking the limit in avedist to the distant x1 value:

In[ ]:=
```
avedist[x1_?NumericQ, y1_?NumericQ, xmax_?NumericQ, ymax_?NumericQ] :=
  Limit[1/3 1/(xmax ymax) (xx1 y1 Sqrt[xx1^2 + y1^2] +
       (xmax - xx1) y1 Sqrt[(xmax - xx1)^2 + y1^2] + (ymax - y1) xx1 Sqrt[xx1^2 + (ymax - y1)^2] +
       (xmax - xx1) (ymax - y1) Sqrt[(xmax - xx1)^2 + (ymax - y1)^2]) + 1/6 1/(xmax ymax)
     Log[((xmax - xx1 + Sqrt[(xmax - xx1)^2 + y1^2])/(Sqrt[xx1^2 + y1^2] - xx1))^(y1^3) ((ymax - y1 + Sqrt[xx1^2 + (ymax - y1)^2])/(Sqrt[xx1^2 + y1^2] - y1))^(xx1^3)
       ((Sqrt[(xmax - xx1)^2 + y1^2] - y1)/(ymax - y1 + Sqrt[(xmax - xx1)^2 + (ymax - y1)^2]))^((xx1-xmax)^3)
       ((Sqrt[(ymax - y1)^2 + xx1^2] - xx1)/(xmax - xx1 + Sqrt[(xmax - xx1)^2 + (ymax - y1)^2]))^((y1-ymax)^3)], xx1 -> x1];
sqI[xmax_?NumericQ, ymax_?NumericQ, c_?NumericQ, f_?NumericQ] := NIntegrate[
   ((p1 * (p1 * avedist[x1, y1, xmax, ymax] + p2 * avedist[x1 + c, y1, xmax, ymax])^2 + p2 *
          (p1 * avedist[x1 - c, y1, xmax, ymax] + p2 * avedist[x1, y1, xmax, ymax])^2) /.
       p2 -> 1 - p1 /. p1 -> 1/2 (1 - Sqrt[1 - 2 f])) (1/(xmax ymax)), {x1, 0, xmax},
   {y1, 0, ymax}];(*c is displacement along x-axis*)
```

Also when xmax=ymax:

In[ ]:=
```
avedist[x1_?NumericQ, y1_?NumericQ, max_?NumericQ] := Limit[
```
$$\frac{1}{6\,\text{max}^2}\Bigg(2\Big((\text{max}-\text{xx1})\sqrt{(\text{max}-\text{xx1})^2+(\text{max}-\text{y1})^2}\,(\text{max}-\text{y1})+\text{xx1}\sqrt{\text{xx1}^2+(\text{max}-\text{y1})^2}\,(\text{max}-\text{y1})+(\text{max}-\text{xx1})\,\text{y1}\sqrt{(\text{max}-\text{xx1})^2+\text{y1}^2}+\text{xx1}\,\text{y1}\sqrt{\text{xx1}^2+\text{y1}^2}\Big)+$$
$$\text{Log}\Bigg[\left(\frac{-\text{xx1}+\sqrt{\text{xx1}^2+(\text{max}-\text{y1})^2}}{\text{max}-\text{xx1}+\sqrt{(\text{max}-\text{xx1})^2+(\text{max}-\text{y1})^2}}\right)^{(-\text{max}+\text{y1})^3}\left(\frac{-\text{y1}+\sqrt{(\text{max}-\text{xx1})^2+\text{y1}^2}}{\text{max}+\sqrt{(\text{max}-\text{xx1})^2+(\text{max}-\text{y1})^2}-\text{y1}}\right)^{(-\text{max}+\text{xx1})^3}$$
$$\left(\frac{\text{max}-\text{xx1}+\sqrt{(\text{max}-\text{xx1})^2+\text{y1}^2}}{-\text{xx1}+\sqrt{\text{xx1}^2+\text{y1}^2}}\right)^{\text{y1}^3}\left(\frac{\text{max}+\sqrt{\text{xx1}^2+(\text{max}-\text{y1})^2}-\text{y1}}{-\text{y1}+\sqrt{\text{xx1}^2+\text{y1}^2}}\right)^{\text{xx1}^3}\Bigg]\Bigg),\ \text{xx1}\to\text{x1}\Big];$$
```
sqI[max_?NumericQ, c_?NumericQ, f_?NumericQ] :=
```
$$\text{NIntegrate}\Big[\Big(\big(\text{p1}*(\text{p1}*\text{avedist}[\text{x1},\text{y1},\text{max}]+\text{p2}*\text{avedist}[\text{x1}+\text{c},\text{y1},\text{max}])^2+\text{p2}*(\text{p1}*\text{avedist}[\text{x1}-\text{c},\text{y1},\text{max}]+\text{p2}*\text{avedist}[\text{x1},\text{y1},\text{max}])^2\big)\ /.\ \text{p2}\to 1-\text{p1}\ /.\ \text{p1}\to\frac{1}{2}\left(1-\sqrt{1-2\,\text{f}}\right)\Big)\left(\frac{1}{\text{max max}}\right),\ \{\text{x1},0,\text{max}\},\ \{\text{y1},0,\text{max}\}\Big];$$
```
(*c is displacement along x-axis*)
```

Analysis

E[X]: Average distance among all pairs

Consider a simplified range that is rectangular in shape from {0,xmax} and {0,ymax}, with uniform density of individuals across the range with probability density function g[x1,y1] = $\frac{1}{\text{xmax ymax}}$.

The average pairwise distance is obtained by integrating over all positions of one individual {x1,y1} and then all positions of the second individual {x2,y2}, all of which are real numbers within the range.

We carry out these integrations in steps (it is faster to use the indefinite integrals and simplify when applying the limits of integration), first rewriting g[x1,y1] g[x2,y2] Sqrt[(x2-x1)^2+(y2-y1)^2] in terms of the distances dx=x2-x1 and dy=y2-y1 and then factoring out g[x1,y1] g[x2,y2], which we’ll handle at the end.

Step 1: Integrating over dx

In[∘]:=
```
Integrate[Sqrt[dx^2 + dy^2], dx];
Collect[collectLog[% // TrigToExp], Log, Simplify[#, Assumptions → fullconditions] &]
```

Out[∘]=

$$\frac{1}{4}\left(2\,dx\,\sqrt{dx^2+dy^2}-dy^2\,\text{Log}\left[\frac{-dx+\sqrt{dx^2+dy^2}}{dx+\sqrt{dx^2+dy^2}}\right]\right)$$

We evaluate the integrals starting at the left-most point (x1,y1), which means that we are ignoring the half of the space where (x2,y2) is left most (this halves the area that we are looking over, accounted for below). That is, we assume here that x2>x1, so the smallest that dx can be is zero.

In[∘]:=
```
FullSimplify[(% /. dx → xmax - x1) - (% /. dx → 0), fullconditions]
```

Out[∘]=

$$\frac{1}{4}\left(2\,\sqrt{dy^2+(x1-xmax)^2}\,(-x1+xmax)-dy^2\,\text{Log}\left[\frac{x1+\sqrt{dy^2+(x1-xmax)^2}-xmax}{-x1+\sqrt{dy^2+(x1-xmax)^2}+xmax}\right]\right)$$

In[∘]:=
```
FullSimplify[Integrate[%, x1], fullconditions]
```

Out[∘]=

$$\frac{1}{6}\left(2\,dy^2-(x1-xmax)^2\right)\sqrt{dy^2+(x1-xmax)^2}+$$
$$\frac{1}{4}\,dy^2\,(-x1+xmax)\,\text{Log}\left[\frac{x1+\sqrt{dy^2+(x1-xmax)^2}-xmax}{-x1+\sqrt{dy^2+(x1-xmax)^2}+xmax}\right]$$

In[∘]:=
```
FullSimplify[
 (% /. x1 → xmax) - Limit[%, x1 → 0, Assumptions → fullconditions], fullconditions]
```

Out[∘]=

$$\frac{1}{12}\left(2\left(-2\,dy^2+xmax^2\right)\sqrt{dy^2+xmax^2}+\right.$$
$$\left.4\,\text{Abs}[dy]^3-3\,dy^2\,xmax\,\text{Log}\left[1+\frac{2\,xmax\left(xmax-\sqrt{dy^2+xmax^2}\right)}{dy^2}\right]\right)$$

The $\text{Abs}[dy]^3$ term is coming from terms like $\left(dy^2\right)^{3/2}$ but cause Mathematica problems when integrating (determined by relaxing the assumptions given by fullconditions). Replacing the Abs[] terms with explicit functions:

In[∘]:=
```
% /. Abs[dy]^3 → (dy^2)^(3/2);
```

Step 2: Integrating over dy

In[∘]:=
```
FullSimplify[TrigToExp[Integrate[%, dy]], fullconditions]
```

Out[∘]=

$$\frac{1}{24}\left(\mathrm{dy}\,\sqrt{\mathrm{dy}^2+\mathrm{xmax}^2}\,\left(-2\,\mathrm{dy}^2+3\,\mathrm{xmax}^2\right)-\mathrm{xmax}^4\,\mathrm{Log}\left[\mathrm{xmax}^2\left(-\mathrm{dy}+\sqrt{\mathrm{dy}^2+\mathrm{xmax}^2}\right)\right]+ 2\,\mathrm{dy}^3\left(\mathrm{Abs}[\mathrm{dy}]-\mathrm{xmax}\,\mathrm{Log}\left[1+\frac{2\,\mathrm{xmax}\left(\mathrm{xmax}-\sqrt{\mathrm{dy}^2+\mathrm{xmax}^2}\right)}{\mathrm{dy}^2}\right]\right)\right)$$

Again replacing the Abs[] terms with explicit functions:

In[∘]:=
```
% /. Abs[dy] → (dy^2)^(1/2);
```

We evaluate the integrals starting at the left-most point (x1,y1), which means that we are ignoring the half of the space where (x2,y2) is left most (this halves the area that we are looking over, accounted for below). That is, we assume here that y2>y1, so the smallest that dy can be is zero.

In[∘]:=
```
FullSimplify[
  (% /. dy → ymax - y1) - Limit[%, dy → 0, Assumptions → fullconditions], fullconditions]
```

Out[∘]=

$$\frac{1}{24}\left(\left(3\,\mathrm{xmax}^2-2\,(\mathrm{y1}-\mathrm{ymax})^2\right)\sqrt{\mathrm{xmax}^2+(\mathrm{y1}-\mathrm{ymax})^2}\,(-\mathrm{y1}+\mathrm{ymax})+3\,\mathrm{xmax}^4\,\mathrm{Log}[\mathrm{xmax}]+ 2\,(\mathrm{y1}-\mathrm{ymax})^3\left(\mathrm{y1}-\mathrm{ymax}+\mathrm{xmax}\,\mathrm{Log}\left[1+\frac{2\,\mathrm{xmax}\left(\mathrm{xmax}-\sqrt{\mathrm{xmax}^2+(\mathrm{y1}-\mathrm{ymax})^2}\right)}{(\mathrm{y1}-\mathrm{ymax})^2}\right]\right)- \mathrm{xmax}^4\,\mathrm{Log}\left[\mathrm{xmax}^2\left(\mathrm{y1}+\sqrt{\mathrm{xmax}^2+(\mathrm{y1}-\mathrm{ymax})^2}-\mathrm{ymax}\right)\right]\right)$$

In[∘]:=
```
FullSimplify[Integrate[%, y1], fullconditions]
```

Out[∘]=

$$\frac{1}{24}\left(\frac{2}{5}\left(\mathrm{xmax}^4\sqrt{\mathrm{xmax}^2+(\mathrm{y1}-\mathrm{ymax})^2}-3\,\mathrm{xmax}^2\sqrt{\mathrm{xmax}^2+(\mathrm{y1}-\mathrm{ymax})^2}\,(\mathrm{y1}-\mathrm{ymax})^2+(\mathrm{y1}-\mathrm{ymax})^4\left(\mathrm{y1}+\sqrt{\mathrm{xmax}^2+(\mathrm{y1}-\mathrm{ymax})^2}-\mathrm{ymax}\right)\right)+\mathrm{xmax}^4\,(\mathrm{y1}+2\,\mathrm{ymax})\,\mathrm{Log}[\mathrm{xmax}]+ \frac{1}{2}\,\mathrm{xmax}\,(\mathrm{y1}-\mathrm{ymax})^4\,\mathrm{Log}\left[1+\frac{2\,\mathrm{xmax}\left(\mathrm{xmax}-\sqrt{\mathrm{xmax}^2+(\mathrm{y1}-\mathrm{ymax})^2}\right)}{(\mathrm{y1}-\mathrm{ymax})^2}\right]+ \mathrm{xmax}^4\,(-\mathrm{y1}+\mathrm{ymax})\,\mathrm{Log}\left[\mathrm{y1}+\sqrt{\mathrm{xmax}^2+(\mathrm{y1}-\mathrm{ymax})^2}-\mathrm{ymax}\right]\right)$$

In[ ]:=
```
FullSimplify[Limit[%, y1 → ymax, Assumptions → fullconditions] -
  Limit[%, y1 → 0, Assumptions → fullconditions], fullconditions]
```

Out[ ]=

$$\frac{1}{240}\left(4\left(xmax^5+ymax^5-xmax^4\sqrt{xmax^2+ymax^2}+3\,xmax^2\,ymax^2\sqrt{xmax^2+ymax^2}-ymax^4\sqrt{xmax^2+ymax^2}\right)+10\,xmax^4\,ymax\,\text{Log}\left[\frac{ymax+\sqrt{xmax^2+ymax^2}}{xmax}\right]-5\,xmax\,ymax^4\,\text{Log}\left[1+\frac{2\,xmax\left(xmax-\sqrt{xmax^2+ymax^2}\right)}{ymax^2}\right]\right)$$

This has to be multiplied by the uniform probability density for the two individuals, g[x1,y1] g[x2,y2] = $\frac{1}{(xmax\,ymax)^2}$, and multiplied by 4 to account for the fact that we have only looked at half of the x region (x1<x2) and half of the y region (y1<y2):

In[ ]:=
$$\text{Collect}\left[\text{collectLog4}\left[\%\,\frac{4}{(xmax\;ymax)^2}\right],\left\{\text{Log},\sqrt{xmax^2+ymax^2}\right\},\text{Simplify}\right]$$

Out[ ]=

$$\frac{1}{15}\sqrt{xmax^2+ymax^2}\left(3-\frac{xmax^2}{ymax^2}-\frac{ymax^2}{xmax^2}\right)+\frac{4\left(xmax^5+ymax^5\right)+\text{Log}\left[\left(\frac{ymax+\sqrt{xmax^2+ymax^2}}{xmax}\right)^{10\,xmax^4\,ymax}\left(1+\frac{2\,xmax\left(xmax-\sqrt{xmax^2+ymax^2}\right)}{ymax^2}\right)^{-5\,xmax\,ymax^4}\right]}{60\,xmax^2\,ymax^2}$$

Recognizing that $\left(1+\frac{2\,xmax\left(xmax-\sqrt{xmax^2+ymax^2}\right)}{ymax^2}\right)=\left(\frac{ymax^2}{\left(xmax+\sqrt{ymax^2+xmax^2}\right)^2}\right)$ [multiplying top and bottom by $\left(xmax+\sqrt{ymax^2+xmax^2}\right)^2$] and simplifying, the average pairwise distance can be written as:

In[ ]:=
$$\text{soln}=\frac{\left(xmax^5+ymax^5\right)}{15\,xmax^2\,ymax^2}+\frac{1}{15}\sqrt{xmax^2+ymax^2}\left(3-\frac{xmax^2}{ymax^2}-\frac{ymax^2}{xmax^2}\right)+\frac{\text{Log}\left[\left(\frac{ymax+\sqrt{xmax^2+ymax^2}}{xmax}\right)^{xmax^3}\left(\frac{xmax+\sqrt{ymax^2+xmax^2}}{ymax}\right)^{ymax^3}\right]}{6\,xmax\,ymax};$$

We can rewrite the squareroots as the hypotenuse:

In[ ]:=
$$\text{soln}=\frac{\left(xmax^5+ymax^5\right)}{15\,xmax^2\,ymax^2}+\frac{1}{15}H\left(3-\frac{xmax^2}{ymax^2}-\frac{ymax^2}{xmax^2}\right)+\frac{xmax^3\,\text{Log}\left[\frac{ymax+H}{xmax}\right]+ymax^3\,\text{Log}\left[\frac{xmax+H}{ymax}\right]}{6\,xmax\,ymax}\;/.\;H\to\sqrt{xmax^2+ymax^2};$$

In the square case, the mean distance scales with the length (xmax):

In[ ]:=
```
Simplify[soln /. ymax → xmax, xmax > 0, Trig → False]
```
Out[ ]=

$$\frac{1}{15}\,\text{xmax}\,\left(2+\sqrt{2}+5\,\text{Log}\left[1+\sqrt{2}\right]\right)$$

In[ ]:=
```
% // N
```
Out[ ]=
```
0.521405 xmax
```

Significance: By knowing the area and shape of herd home ranges (or schools of fish, flocks of birds, etc.), we can estimate the average distance between individuals, here assuming a rectangular distribution with uniform density.

E[X^2]: Average distance squared among all pairs

We repeat the above to calculate (x2-x1)^2+(y2-y1)^2, first rewriting in terms of the distances dx=x2-x1 and dy=y2-y1.

In[ ]:=
```
fullconditions = {0 < x1 < xmax, 0 < y1 < ymax, 0 < x2 < xmax,
    0 < y2 < ymax, -xmax < dx < xmax, -ymax < dy < ymax, _Symbol ∈ Reals};
```

Step 1: Integrating over dx

In[ ]:=
```
Integrate[dx^2 + dy^2, dx]
```
Out[ ]=

$$\frac{\text{dx}^3}{3}+\text{dx}\,\text{dy}^2$$

We evaluate the integrals starting at the left-most point (x1,y1), which means that we are ignoring the half of the space where (x2,y2) is left most (this halves the area that we are looking over, accounted for below). That is, we assume here that x2>x1, so the smallest that dx can be is zero.

In[ ]:=
```
FullSimplify[(% /. dx → xmax - x1) - (% /. dx → 0), fullconditions]
```
Out[ ]=

$$\text{dy}^2\,(-\text{x1}+\text{xmax})+\frac{1}{3}\,(-\text{x1}+\text{xmax})^3$$

In[ ]:=
```
FullSimplify[Integrate[%, x1], fullconditions]
```
Out[ ]=

$$-\frac{1}{12}\,\text{x1}\,(\text{x1}-2\,\text{xmax})\,\left(6\,\text{dy}^2+\text{x1}^2-2\,\text{x1}\,\text{xmax}+2\,\text{xmax}^2\right)$$

In[ ]:=
```
FullSimplify[
  (% /. x1 → xmax) - Limit[%, x1 → 0, Assumptions → fullconditions], fullconditions]
```
Out[ ]=

$$\frac{1}{12}\,\text{xmax}^2\,\left(6\,\text{dy}^2+\text{xmax}^2\right)$$

In[∘]:=
```
FullSimplify[Integrate[%, dy], fullconditions]
```

Out[∘]=

$$\frac{1}{12}\,dy\,xmax^2\,\left(2\,dy^2+xmax^2\right)$$

Step 2: Integrating over dy

We evaluate the integrals starting at the left-most point (x1,y1), which means that we are ignoring the half of the space where (x2,y2) is left most (this halves the area that we are looking over, accounted for below). That is, we assume here that y2>y1, so the smallest that dy can be is zero.

In[∘]:=
```
FullSimplify[
  (% /. dy → ymax - y1) - Limit[%, dy → 0, Assumptions → fullconditions], fullconditions]
```

Out[∘]=

$$\frac{1}{12}\,xmax^2\,\left(xmax^2+2\,(y1-ymax)^2\right)\,(-y1+ymax)$$

In[∘]:=
```
FullSimplify[Integrate[%, y1], fullconditions]
```

Out[∘]=

$$-\frac{1}{24}\,xmax^2\,y1\,(y1-2\,ymax)\,\left(xmax^2+y1^2-2\,y1\,ymax+2\,ymax^2\right)$$

In[∘]:=
```
FullSimplify[Limit[%, y1 → ymax, Assumptions → fullconditions] -
   Limit[%, y1 → 0, Assumptions → fullconditions], fullconditions]
```

Out[∘]=

$$\frac{1}{24}\,xmax^2\,ymax^2\,\left(xmax^2+ymax^2\right)$$

This has to be multiplied by the uniform probability density for the two individuals, g[x1,y1] g[x2,y2] = $\frac{1}{(xmax\,ymax)^2}$, and multiplied by 4 to account for the fact that we have only looked at half of the x region (x1<x2) and half of the y region (y1<y2):

In[∘]:=

$$\%\,\frac{4}{(xmax\,ymax)^2}$$

Out[∘]=

$$\frac{1}{6}\,\left(xmax^2+ymax^2\right)$$

In[∘]:=

$$solnX2=\frac{xmax^2+ymax^2}{6};$$

Herd CV : Coefficient of variation among all pairs

Considering all pairwise distances measured in the population (ignoring individual identity), the herd CV for pairwise distances is:

In[∘]:= $\text{CVherd} = \dfrac{\text{Sqrt}[\text{solnX2} - \text{soln}^2]}{\text{soln}}$

Out[∘]=

$$\left(\sqrt{\left(\frac{1}{6}\left(\text{xmax}^2+\text{ymax}^2\right)-\left(\frac{1}{15}\sqrt{\text{xmax}^2+\text{ymax}^2}\left(3-\frac{\text{xmax}^2}{\text{ymax}^2}-\frac{\text{ymax}^2}{\text{xmax}^2}\right)+\frac{\text{xmax}^5+\text{ymax}^5}{15\,\text{xmax}^2\,\text{ymax}^2}+\frac{\text{ymax}^3\,\text{Log}\left[\frac{\text{xmax}+\sqrt{\text{xmax}^2+\text{ymax}^2}}{\text{ymax}}\right]+\text{xmax}^3\,\text{Log}\left[\frac{\text{ymax}+\sqrt{\text{xmax}^2+\text{ymax}^2}}{\text{xmax}}\right]}{6\,\text{xmax}\,\text{ymax}}\right)^2\right)}\right)\Big/$$

$$\left(\frac{1}{15}\sqrt{\text{xmax}^2+\text{ymax}^2}\left(3-\frac{\text{xmax}^2}{\text{ymax}^2}-\frac{\text{ymax}^2}{\text{xmax}^2}\right)+\frac{\text{xmax}^5+\text{ymax}^5}{15\,\text{xmax}^2\,\text{ymax}^2}+\frac{\text{ymax}^3\,\text{Log}\left[\frac{\text{xmax}+\sqrt{\text{xmax}^2+\text{ymax}^2}}{\text{ymax}}\right]+\text{xmax}^3\,\text{Log}\left[\frac{\text{ymax}+\sqrt{\text{xmax}^2+\text{ymax}^2}}{\text{xmax}}\right]}{6\,\text{xmax}\,\text{ymax}}\right)$$

Alternatively, we can write the CV as $\text{Sqrt}\left[\frac{\text{solnX2}}{\text{soln}^2}-1\right]$. Next we show that the fraction $\frac{\text{solnX2}}{\text{soln}^2}$ (and hence CV) doesn't depend on the scale, as long as the ratio of ymax/xmax remains constant (call this "α"):

In[∘]:= $\text{Simplify}\left[\frac{\text{solnX2}}{\text{soln}^2}\ /.\ \text{ymax}\to\alpha\,\text{xmax},\ \{\text{xmax}>0,\ \alpha>0\},\ \text{Trig}\to\text{False}\right]$

Out[∘]=

$$\frac{150\,\alpha^4\left(1+\alpha^2\right)}{\left(2+2\,\alpha^5-2\,\sqrt{1+\alpha^2}+6\,\alpha^2\,\sqrt{1+\alpha^2}-2\,\alpha^4\,\sqrt{1+\alpha^2}+5\,\alpha^4\,\text{Log}\left[\frac{1+\sqrt{1+\alpha^2}}{\alpha}\right]+5\,\alpha\,\text{Log}\left[\alpha+\sqrt{1+\alpha^2}\right]\right)^2}$$

The denominator can be simplified using the hypotenuse:

In[∘]:= `Simplify[soln /. xmax → max /. ymax → α max, fullconditions]`

Out[∘]=

$$\frac{\text{max}\left(2+2\,\alpha^5-2\,\sqrt{1+\alpha^2}+6\,\alpha^2\,\sqrt{1+\alpha^2}-2\,\alpha^4\,\sqrt{1+\alpha^2}+5\,\alpha^4\,\text{Log}\left[\frac{1+\sqrt{1+\alpha^2}}{\alpha}\right]+5\,\alpha\,\text{Log}\left[\alpha+\sqrt{1+\alpha^2}\right]\right)}{30\,\alpha^2}$$

In[∘]:= $\text{Simplify}\left[\text{collectLog}\left[\%\Big/\frac{\text{max}}{30\,\alpha^2}\ /.\ \sqrt{1+\alpha^2}\to\text{H}\right]\right]$

Out[∘]=

$$2+2\,\alpha^5-2\,\text{H}\left(1-3\,\alpha^2+\alpha^4\right)+\text{Log}\left[\left(\frac{1+\text{H}}{\alpha}\right)^{5\,\alpha^4}(\text{H}+\alpha)^{5\,\alpha}\right]$$

After a bit of cleaning, the CV can be written as:

In[∘]:=
```
CVherdALT = Sqrt[150 α^4 (1 + α^2)/solncapt^2 - 1] /.
    solncapt → 2 + 2 α^5 - 2 H (1 - 3 α^2 + α^4) + Log[((1 + H)/α)^(5 α^4) (H + α)^(5 α)] /. H → Sqrt[1 + α^2];
```

$$\text{CVherdALT} = \text{Sqrt}\left[\frac{150\,\alpha^4\,(1+\alpha^2)}{\text{solncapt}^2} - 1\right] \text{/.}\ \text{solncapt} \to 2 + 2\,\alpha^5 - 2\,H\,(1 - 3\,\alpha^2 + \alpha^4) + \text{Log}\left[\left(\frac{1+H}{\alpha}\right)^{5\,\alpha^4} (H+\alpha)^{5\,\alpha}\right] \text{/.}\ H \to \sqrt{1+\alpha^2};$$

Thus, the CV is independent of the scale, as expected.

In the square case, the expected CV is:

In[∘]:=
```
Simplify[CVherdALT /. α → 1, Trig → False]
```

Out[∘]=

$$\sqrt{-1 + \frac{75}{\left(2 + \sqrt{2} + 5\,\text{Log}\left[1 + \sqrt{2}\right]\right)^2}}$$

In[∘]:=
```
% // N
```

Out[∘]=

0.475505

As the range elongates (either becoming broader α<1 or taller α>1), CV rises towards 0.707:

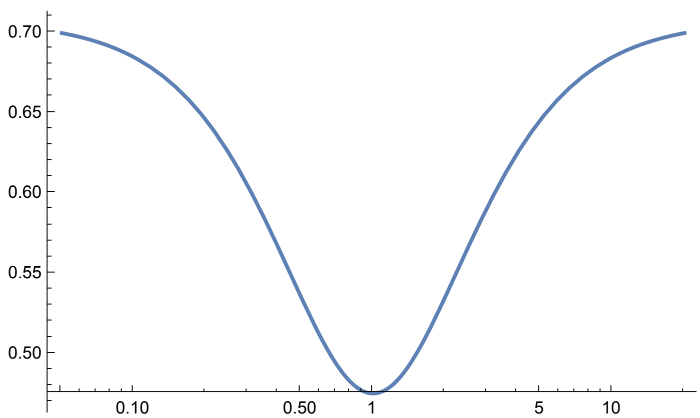
```
CVherd1 = Limit[CVherdALT, α → 0]
```

Out[∘]=

$$\frac{1}{\sqrt{2}}$$

In[∘]:=
```
LogLinearPlot[CVherdALT, {α, 0.05, 20}]
```

Out[∘]=



In[∘]:=
```
CVherd /. ymax → 1 /. xmax → {0.2, 1.}
```

Out[∘]=

{0.645688, 0.475505}

Herd split patches: E[X] for two patches at a distance c from one another

Here we split the range in two and move one patch to the right by an amount c. For the intra-individual distances, a fraction f of pairs will be drawn from different patches and the remainder from the same patch (given above). For clarity, we assume that individual 2 is from the right-most patch, displaced by c on the x axis.

Care must be taken, however, when considering $dx^2 = ((x2 + c) - x1)^2$. we cannot just take x2>x1 because that overestimates the distance compared to the case where x2<x1. Below we integrate directly with respect to x2 and x1 over the full range, but we start by integrating with respect to y1, where we again can consider only those cases where y2>y1 in the integration, accounting for this restriction afterwards.

Note that we will deal with the probability density function for the position of individuals in two patches at the end, which we can do because these distributions are uniform within each patch.

Updating full conditions:

```
In[∘]:= fullconditions = {0 < x1 < xmax, 0 < y1 < ymax, 0 < x2 < xmax, 0 < y2 < ymax, 0 < dy < ymax,
    c > 0, -xmax + c < dx < xmax + c, Δy > 0, max > 0, α > 0, cs > 0, _Symbol ∈ Reals};
```

We need to integrate:

Integrate[Sqrt[((x2+c)-x1)^2 + (y2 - y1)^2], x2]

We carry out these integrations in steps (it is faster to use the indefinite integrals and simplify when applying the limits of integration), first rewriting Sqrt[(x2-x1)^2+(y2-y1)^2] in terms of the distances dx=(x2+c)-x1 and dy=y2-y1.

We start with the simpler of the integrals, along the y-axis:

Step 1: Integrating over dy

```
In[∘]:= Integrate[Sqrt[dx^2 + dy^2], dy];
Collect[collectLog[% // TrigToExp], Log, Simplify[#, Assumptions → fullconditions] &]
```

Out[∘]=

$$\frac{1}{4}\left(2\,dy\,\sqrt{dx^2+dy^2}-dx^2\,\mathrm{Log}\left[\frac{-dy+\sqrt{dx^2+dy^2}}{dy+\sqrt{dx^2+dy^2}}\right]\right)$$

We evaluate the integrals assuming that the top-most point within a patch is (x2,y2), which means that we are ignoring the half of the space where (x2,y2) is bottom most (this halves the area that we are looking over, accounted for below).

```
In[∘]:= Simplify[collectLog[Simplify[(% /. dy → ymax - y1) - (% /. dy → 0), fullconditions]],
  fullconditions]
```

Out[∘]=

$$\frac{1}{4}\left(2\sqrt{dx^2+(y1-ymax)^2}\,(-y1+ymax)-dx^2\,\mathrm{Log}\left[\frac{y1+\sqrt{dx^2+(y1-ymax)^2}-ymax}{-y1+\sqrt{dx^2+(y1-ymax)^2}+ymax}\right]\right)$$

We can simplify the fraction in the log by noting that

$$\left(y1+\sqrt{dx^2+(y1-ymax)^2}-ymax\right)\left(-y1+\sqrt{dx^2+(y1-ymax)^2}+ymax\right)\ //\ \mathrm{Expand}$$

gives dx^2.

Allowing us to rewrite the above as:

In[ ]:=
$$\frac{1}{4}\left(2\sqrt{dx^2+(y1-ymax)^2}\,(-y1+ymax)-dx^2\,\mathrm{Log}\left[\frac{dx^2}{\left(ymax-y1+\sqrt{dx^2+(y1-ymax)^2}\right)^2}\right]\right);$$

or as:

[Avoid taking the square out of the log, as numerical errors creep in when dx can be negative.]

$$\frac{1}{4}\left(2\sqrt{dx^2+(y1-ymax)^2}\,(-y1+ymax)+dx^2\,\mathrm{Log}\left[\frac{\left(ymax-y1+\sqrt{dx^2+(y1-ymax)^2}\right)^2}{dx^2}\right]\right);$$

In[ ]:=
```
Collect[Simplify[collectLog[Integrate[%, y1]], fullconditions],
 Log, Simplify[#, fullconditions] &]
```

Out[ ]=
$$\frac{1}{6}\left(2\,dx^2-(y1-ymax)^2\right)\sqrt{dx^2+(y1-ymax)^2}+\frac{1}{4}dx^2\,(-y1+ymax)\,\mathrm{Log}\left[\frac{dx^2}{\left(-y1+\sqrt{dx^2+(y1-ymax)^2}+ymax\right)^2}\right]$$

In[ ]:=
```
Simplify[
 collectLog[Simplify[(% /. y1 → ymax) - (% /. y1 → 0), fullconditions]], fullconditions]
```

Out[ ]=
$$\frac{1}{12}\left(2\left(-2\,dx^2+ymax^2\right)\sqrt{dx^2+ymax^2}+4\,\mathrm{Abs}[dx]^3-3\,dx^2\,ymax\,\mathrm{Log}\left[\frac{dx^2}{\left(ymax+\sqrt{dx^2+ymax^2}\right)^2}\right]\right)$$

Step 2: Integrating over dx

Because we need to consider $dx^2 = ((x2+c)-x1)^2$ for both x2<x1 and x2>x1, we proceed with integrating with respect to x2 and x1 directly:

[Converting $\mathrm{Abs}[dx]^3$ to $\left(dx^2\right)^{3/2}$ allows Mathematica to integrate.]

In[∘]:= $$\frac{1}{12}\left(2\left(-2\,dx^2+ymax^2\right)\sqrt{dx^2+ymax^2}+4\left(dx^2\right)^{3/2}-3\,dx^2\,ymax\,\mathrm{Log}\left[\frac{dx^2}{\left(ymax+\sqrt{dx^2+ymax^2}\right)^2}\right]\right)\;/.$$

dx → x2 + c − x1;

```
Simplify[TrigToExp[Integrate[%, x2]], fullconditions]
```

Out[∘]=

$$\frac{1}{24}\left(2\,(c-x1+x2)\,ymax^2\sqrt{(c-x1+x2)^2+ymax^2}-(c-x1+x2)\left(2\,c^2-4\,c\,x1+2\,x1^2+4\,c\,x2-4\,x1\,x2+2\,x2^2-ymax^2\right)\sqrt{(c-x1+x2)^2+ymax^2}+2\,(c-x1+x2)\,\mathrm{Abs}[c-x1+x2]^3-3\,ymax^4\,\mathrm{Log}\left[-c+x1-x2+\sqrt{(c-x1+x2)^2+ymax^2}\right]+ymax^4\left(\mathrm{Log}\left[-c+x1-x2+\sqrt{(c-x1+x2)^2+ymax^2}\right]-\mathrm{Log}\left[c-x1+x2+\sqrt{(c-x1+x2)^2+ymax^2}\right]\right)-2\,(c-x1+x2)^3\,ymax\,\mathrm{Log}\left[\frac{(c-x1+x2)^2}{\left(ymax+\sqrt{(c-x1+x2)^2+ymax^2}\right)^2}\right]\right)$$

In[∘]:= `collectLog[collectLog[collectLog4[%]]]`

Out[∘]=

$$\frac{1}{24}\left(2\,(c-x1+x2)\,ymax^2\sqrt{(c-x1+x2)^2+ymax^2}-(c-x1+x2)\left(2\,c^2-4\,c\,x1+2\,x1^2+4\,c\,x2-4\,x1\,x2+2\,x2^2-ymax^2\right)\sqrt{(c-x1+x2)^2+ymax^2}+2\,(c-x1+x2)\,\mathrm{Abs}[c-x1+x2]^3+\mathrm{Log}\left[\left(-c+x1-x2+\sqrt{(c-x1+x2)^2+ymax^2}\right)^{-3\,ymax^4}\left(\frac{-c+x1-x2+\sqrt{(c-x1+x2)^2+ymax^2}}{c-x1+x2+\sqrt{(c-x1+x2)^2+ymax^2}}\right)^{ymax^4}\left(\frac{(c-x1+x2)^2}{\left(ymax+\sqrt{(c-x1+x2)^2+ymax^2}\right)^2}\right)^{-2\,(c-x1+x2)^3\,ymax}\right]\right)$$

In[∘]:= $$\frac{1}{24}\left(2\,(c-x1+x2)\,ymax^2\sqrt{(c-x1+x2)^2+ymax^2}-(c-x1+x2)\left(2\,c^2-4\,c\,x1+2\,x1^2+4\,c\,x2-4\,x1\,x2+2\,x2^2-ymax^2\right)\sqrt{(c-x1+x2)^2+ymax^2}+2\,(c-x1+x2)\,\mathrm{Abs}[c-x1+x2]^3+\mathrm{Log}\left[\left(\frac{c-x1+x2+\sqrt{(c-x1+x2)^2+ymax^2}}{ymax^4}\right)^{ymax^4}\left(\frac{(c-x1+x2)^2}{\left(ymax+\sqrt{(c-x1+x2)^2+ymax^2}\right)^2}\right)^{-2\,(c-x1+x2)^3\,ymax}\right]\right);$$

In[∘]:=
```
Simplify[
 collectLog[Simplify[(% /. x2 → xmax) - (% /. x2 → 0), fullconditions]], fullconditions]
```

Out[∘]=

$$\frac{1}{24}\Bigg(2\,(c-x1+xmax)^4-2\,(c-x1)\,ymax^2\,\sqrt{c^2-2\,c\,x1+x1^2+ymax^2}+(c-x1)\,\left(2\,c^2-4\,c\,x1+2\,x1^2-ymax^2\right)\,\sqrt{c^2-2\,c\,x1+x1^2+ymax^2}+2\,(c-x1+xmax)\,ymax^2\,\sqrt{(c-x1+xmax)^2+ymax^2}-(c-x1+xmax)\left(2\,c^2-4\,c\,x1+2\,x1^2+4\,c\,xmax-4\,x1\,xmax+2\,xmax^2-ymax^2\right)\sqrt{(c-x1+xmax)^2+ymax^2}-2\,(c-x1)\,\mathrm{Abs}[c-x1]^3+\mathrm{Log}\Bigg[\left(\frac{(c-x1)^2}{\left(ymax+\sqrt{(c-x1)^2+ymax^2}\right)^2}\right)^{2\,(c-x1)^3\,ymax}\left(\frac{c-x1+\sqrt{(c-x1)^2+ymax^2}}{c-x1+xmax+\sqrt{(c-x1+xmax)^2+ymax^2}}\right)^{-ymax^4}\left(\frac{(c-x1+xmax)^2}{\left(ymax+\sqrt{(c-x1+xmax)^2+ymax^2}\right)^2}\right)^{-2\,(c-x1+xmax)^3\,ymax}\Bigg]\Bigg)$$

In[∘]:=
```
Collect[Simplify[collectLog[Integrate[% /. Abs[c - x1]^3 → ((c - x1)^2)^(3/2), x1]],
   fullconditions], Log, Simplify[#, fullconditions] &];
```

In[∘]:=

```
Simplify[
 collectLog[Simplify[(% /. x1 → xmax) - (% /. x1 → 0), fullconditions]], fullconditions]
```

Out[∘]=

$$\frac{1}{240}\Bigg(-4 c^5+20 c^4 xmax+40 c^3 xmax^2+40 c^2 xmax^3+20 c\, xmax^4+4 xmax^5+8 c^4 \sqrt{c^2+ymax^2}-24 c^2 ymax^2 \sqrt{c^2+ymax^2}+8 ymax^4 \sqrt{c^2+ymax^2}-4 c^4 \sqrt{c^2-2 c\, xmax+xmax^2+ymax^2}+16 c^3 xmax \sqrt{c^2-2 c\, xmax+xmax^2+ymax^2}-24 c^2 xmax^2 \sqrt{c^2-2 c\, xmax+xmax^2+ymax^2}+16 c\, xmax^3 \sqrt{c^2-2 c\, xmax+xmax^2+ymax^2}-4 xmax^4 \sqrt{c^2-2 c\, xmax+xmax^2+ymax^2}+12 c^2 ymax^2 \sqrt{c^2-2 c\, xmax+xmax^2+ymax^2}-24 c\, xmax\, ymax^2 \sqrt{c^2-2 c\, xmax+xmax^2+ymax^2}+12 xmax^2 ymax^2 \sqrt{c^2-2 c\, xmax+xmax^2+ymax^2}-4 ymax^4 \sqrt{c^2-2 c\, xmax+xmax^2+ymax^2}-4 c^4 \sqrt{(c+xmax)^2+ymax^2}-16 c^3 xmax \sqrt{(c+xmax)^2+ymax^2}-24 c^2 xmax^2 \sqrt{(c+xmax)^2+ymax^2}-16 c\, xmax^3 \sqrt{(c+xmax)^2+ymax^2}-4 xmax^4 \sqrt{(c+xmax)^2+ymax^2}+12 c^2 ymax^2 \sqrt{(c+xmax)^2+ymax^2}+24 c\, xmax\, ymax^2 \sqrt{(c+xmax)^2+ymax^2}+12 xmax^2 ymax^2 \sqrt{(c+xmax)^2+ymax^2}-4 ymax^4 \sqrt{(c+xmax)^2+ymax^2}+4 \text{Abs}[c-xmax]^5+$$

$$\text{Log}\Bigg[\Bigg(\left(\frac{c^2}{\left(ymax+\sqrt{c^2+ymax^2}\right)^2}\right)^{-2 c^3 ymax}\left(\frac{c-xmax+\sqrt{c^2-2 c\, xmax+xmax^2+ymax^2}}{c+\sqrt{c^2+ymax^2}}\right)^{-ymax^4}\left(\frac{(c-xmax)^2}{\left(ymax+\sqrt{c^2-2 c\, xmax+xmax^2+ymax^2}\right)^2}\right)^{2 (c-xmax)^3 ymax}\Bigg)^{-10 c}\Bigg]+$$

$$\text{Log}\Bigg[\Bigg(\left(\frac{c^2}{\left(ymax+\sqrt{c^2+ymax^2}\right)^2}\right)^{-2 c^3 ymax}\left(\frac{c-xmax+\sqrt{c^2-2 c\, xmax+xmax^2+ymax^2}}{c+\sqrt{c^2+ymax^2}}\right)^{-ymax^4}\left(\frac{(c-xmax)^2}{\left(ymax+\sqrt{c^2-2 c\, xmax+xmax^2+ymax^2}\right)^2}\right)^{2 (c-xmax)^3 ymax}\Bigg)^{10\, xmax}\Bigg]+$$

$$\text{Log}\Bigg[\left(\frac{c}{ymax+\sqrt{c^2+ymax^2}}\right)^{-20 c^3 (c-2\, xmax)\, ymax}\left(ymax+\sqrt{c^2-2 c\, xmax+xmax^2+ymax^2}\right)^{-30 (c-xmax)^4 ymax}\left(\frac{c+\sqrt{c^2+ymax^2}}{c+xmax+\sqrt{(c+xmax)^2+ymax^2}}\right)^{-10 (c+xmax)\, ymax^4}\left(\frac{c+xmax}{ymax+\sqrt{(c+xmax)^2+ymax^2}}\right)^{-10 (c+xmax)^4 ymax}\text{Abs}[c-xmax]^{30 (c-xmax)^4 ymax}\Bigg]\Bigg)$$

In[∘]:=
```
collectLog[collectLog[collectLog4[%]]];
Collect[%, Log, Simplify[#, fullconditions] &] /. Abs[c - xmax] → ((c - xmax)^2)^(1/2)
```

Out[∘]=

$$\frac{1}{240}\Bigg(4\left((c-xmax)^2\right)^{5/2}+4\Big(-c^5+xmax^5+2\,c^3\,xmax\left(5\,xmax+2\sqrt{c^2-2\,c\,xmax+xmax^2+ymax^2}-2\sqrt{c^2+2\,c\,xmax+xmax^2+ymax^2}\right)+ymax^4\left(2\sqrt{c^2+ymax^2}-\sqrt{c^2-2\,c\,xmax+xmax^2+ymax^2}-\sqrt{c^2+2\,c\,xmax+xmax^2+ymax^2}\right)+c^4\left(5\,xmax+2\sqrt{c^2+ymax^2}-\sqrt{c^2-2\,c\,xmax+xmax^2+ymax^2}-\sqrt{c^2+2\,c\,xmax+xmax^2+ymax^2}\right)-xmax^4\left(\sqrt{c^2-2\,c\,xmax+xmax^2+ymax^2}+\sqrt{c^2+2\,c\,xmax+xmax^2+ymax^2}\right)+3\,xmax^2\,ymax^2\left(\sqrt{c^2-2\,c\,xmax+xmax^2+ymax^2}+\sqrt{c^2+2\,c\,xmax+xmax^2+ymax^2}\right)+c\left(5\,xmax^4+4\,xmax^3\left(\sqrt{c^2-2\,c\,xmax+xmax^2+ymax^2}-\sqrt{c^2+2\,c\,xmax+xmax^2+ymax^2}\right)+6\,xmax\,ymax^2\left(-\sqrt{c^2-2\,c\,xmax+xmax^2+ymax^2}+\sqrt{c^2+2\,c\,xmax+xmax^2+ymax^2}\right)\right)+c^2\left(10\,xmax^3-6\,xmax^2\left(\sqrt{c^2-2\,c\,xmax+xmax^2+ymax^2}+\sqrt{c^2+2\,c\,xmax+xmax^2+ymax^2}\right)+3\,ymax^2\left(-2\sqrt{c^2+ymax^2}+\sqrt{c^2-2\,c\,xmax+xmax^2+ymax^2}+\sqrt{c^2+2\,c\,xmax+xmax^2+ymax^2}\right)\right)\Big)+\mathrm{Log}\Bigg[\left((c-xmax)^2\right)^{15\,(c-xmax)^4\,ymax}\left(\frac{c}{ymax+\sqrt{c^2+ymax^2}}\right)^{20\,c^4\,ymax}\left(\left((c-xmax)^2\right)^{2\,(c-xmax)^3\,ymax}\left(\frac{c-xmax+\sqrt{c^2-2\,c\,xmax+xmax^2+ymax^2}}{c+\sqrt{c^2+ymax^2}}\right)^{-ymax^4}\right)^{10\,(-c+xmax)}\left(ymax+\sqrt{c^2-2\,c\,xmax+xmax^2+ymax^2}\right)^{10\,(c-xmax)^4\,ymax}\left(\frac{c+\sqrt{c^2+ymax^2}}{c+xmax+\sqrt{(c+xmax)^2+ymax^2}}\right)^{-10\,(c+xmax)\,ymax^4}\left(\frac{c+xmax}{ymax+\sqrt{(c+xmax)^2+ymax^2}}\right)^{-10\,(c+xmax)^4\,ymax}\Bigg]\Bigg)$$

In[ ]:=

```
Collect[4 ((c - xmax)^2)^(5/2) + 4 (-c^5 + xmax^5 +
    2 c^3 xmax (5 xmax + 2 Sqrt[c^2 - 2 c xmax + xmax^2 + ymax^2] - 2 Sqrt[c^2 + 2 c xmax + xmax^2 + ymax^2]) +
    ymax^4 (2 Sqrt[c^2 + ymax^2] - Sqrt[c^2 - 2 c xmax + xmax^2 + ymax^2] - Sqrt[c^2 + 2 c xmax + xmax^2 + ymax^2]) +
    c^4 (5 xmax + 2 Sqrt[c^2 + ymax^2] - Sqrt[c^2 - 2 c xmax + xmax^2 + ymax^2] -
        Sqrt[c^2 + 2 c xmax + xmax^2 + ymax^2]) -
    xmax^4 (Sqrt[c^2 - 2 c xmax + xmax^2 + ymax^2] + Sqrt[c^2 + 2 c xmax + xmax^2 + ymax^2]) +
    3 xmax^2 ymax^2 (Sqrt[c^2 - 2 c xmax + xmax^2 + ymax^2] + Sqrt[c^2 + 2 c xmax + xmax^2 + ymax^2]) +
    c (5 xmax^4 + 4 xmax^3 (Sqrt[c^2 - 2 c xmax + xmax^2 + ymax^2] - Sqrt[c^2 + 2 c xmax + xmax^2 + ymax^2]) +
        6 xmax ymax^2 (-Sqrt[c^2 - 2 c xmax + xmax^2 + ymax^2] + Sqrt[c^2 + 2 c xmax + xmax^2 + ymax^2])) +
    c^2 (10 xmax^3 - 6 xmax^2 (Sqrt[c^2 - 2 c xmax + xmax^2 + ymax^2] + Sqrt[c^2 + 2 c xmax + xmax^2 + ymax^2]) +
        3 ymax^2 (-2 Sqrt[c^2 + ymax^2] + Sqrt[c^2 - 2 c xmax + xmax^2 + ymax^2] +
            Sqrt[c^2 + 2 c xmax + xmax^2 + ymax^2]))),
  {Sqrt[c^2 - 2 c xmax + xmax^2 + ymax^2], Sqrt[c^2 + ymax^2], Sqrt[c^2 + 2 c xmax + xmax^2 + ymax^2],
   ((c - xmax)^2)^(5/2)}, Factor]
```

Out[ ]=

$$
\begin{aligned}
&4\left((c-\text{xmax})^2\right)^{5/2}-4\left(c^5-5c^4\,\text{xmax}-10c^3\,\text{xmax}^2-10c^2\,\text{xmax}^3-5c\,\text{xmax}^4-\text{xmax}^5\right)+\\
&\;8\left(c^2-c\,\text{ymax}-\text{ymax}^2\right)\left(c^2+c\,\text{ymax}-\text{ymax}^2\right)\sqrt{c^2+\text{ymax}^2}-\\
&\;4\left(c^2-2c\,\text{xmax}+\text{xmax}^2+c\,\text{ymax}-\text{xmax}\,\text{ymax}-\text{ymax}^2\right)\\
&\quad\left(c^2-2c\,\text{xmax}+\text{xmax}^2-c\,\text{ymax}+\text{xmax}\,\text{ymax}-\text{ymax}^2\right)\sqrt{c^2-2c\,\text{xmax}+\text{xmax}^2+\text{ymax}^2}-\\
&\;4\left(c^2+2c\,\text{xmax}+\text{xmax}^2-c\,\text{ymax}-\text{xmax}\,\text{ymax}-\text{ymax}^2\right)\\
&\quad\left(c^2+2c\,\text{xmax}+\text{xmax}^2+c\,\text{ymax}+\text{xmax}\,\text{ymax}-\text{ymax}^2\right)\sqrt{c^2+2c\,\text{xmax}+\text{xmax}^2+\text{ymax}^2}
\end{aligned}
$$

In[ ]:=
```
FullSimplify[((c - xmax)^2)^(15 (c-xmax)^4 ymax) (c/(ymax + Sqrt[c^2 + ymax^2]))^(20 c^4 ymax)
 (((c - xmax)^2)^(2 (c-xmax)^3 ymax) ((c - xmax + Sqrt[c^2 - 2 c xmax + xmax^2 + ymax^2])/(c + Sqrt[c^2 + ymax^2]))^(-ymax^4))^(10 (-c+xmax))
 (ymax + Sqrt[c^2 - 2 c xmax + xmax^2 + ymax^2])^(10 (c-xmax)^4 ymax)
 ((c + Sqrt[c^2 + ymax^2])/(c + xmax + Sqrt[(c + xmax)^2 + ymax^2]))^(-10 (c+xmax) ymax^4)
 ((c + xmax)/(ymax + Sqrt[(c + xmax)^2 + ymax^2]))^(-10 (c+xmax)^4 ymax), fullconditions]
```

Out[ ]=

$$\left(\frac{c}{ymax+\sqrt{c^2+ymax^2}}\right)^{20\,c^4\,ymax}\left(ymax+\sqrt{(c-xmax)^2+ymax^2}\right)^{10\,(c-xmax)^4\,ymax}$$
$$\left(\frac{c+\sqrt{c^2+ymax^2}}{c+xmax+\sqrt{(c+xmax)^2+ymax^2}}\right)^{-10\,(c+xmax)\,ymax^4}$$
$$\left(\frac{c+xmax}{ymax+\sqrt{(c+xmax)^2+ymax^2}}\right)^{-10\,(c+xmax)^4\,ymax}\ \mathrm{Abs}[c-xmax]^{30\,(c-xmax)^4\,ymax}$$
$$\left(\left(\frac{c-xmax+\sqrt{(c-xmax)^2+ymax^2}}{c+\sqrt{c^2+ymax^2}}\right)^{-ymax^4}\ \mathrm{Abs}[c-xmax]^{4\,(c-xmax)^3\,ymax}\right)^{10\,(-c+xmax)}$$

Cleaning up a bit by hand (rearranging the log terms):

In[ ]:=

$$\frac{1}{240}\Big(4\left((c-xmax)^2\right)^{5/2}-4\left(c^5-5c^4\,xmax-10c^3\,xmax^2-10c^2\,xmax^3-5c\,xmax^4-xmax^5\right)+$$
$$8\left(c^2-c\,ymax-ymax^2\right)\left(c^2+c\,ymax-ymax^2\right)\sqrt{c^2+ymax^2}-$$
$$4\left(c^2-2c\,xmax+xmax^2+c\,ymax-xmax\,ymax-ymax^2\right)$$
$$\left(c^2-2c\,xmax+xmax^2-c\,ymax+xmax\,ymax-ymax^2\right)\sqrt{c^2-2c\,xmax+xmax^2+ymax^2}-$$
$$4\left(c^2+2c\,xmax+xmax^2-c\,ymax-xmax\,ymax-ymax^2\right)$$
$$\left(c^2+2c\,xmax+xmax^2+c\,ymax+xmax\,ymax-ymax^2\right)\sqrt{c^2+2c\,xmax+xmax^2+ymax^2}+$$
$$\text{Log}\Big[\left(\frac{c}{ymax+\sqrt{c^2+ymax^2}}\right)^{20c^4\,ymax}\left(\frac{ymax+\sqrt{(c-xmax)^2+ymax^2}}{\text{Abs}[c-xmax]}\right)^{10(c-xmax)^4\,ymax}$$
$$\left(\frac{ymax+\sqrt{(c+xmax)^2+ymax^2}}{c+xmax}\right)^{10(c+xmax)^4\,ymax}$$
$$\left(\frac{c+xmax+\sqrt{(c+xmax)^2+ymax^2}}{c+\sqrt{c^2+ymax^2}}\right)^{10(c+xmax)\,ymax^4}$$
$$\left(\frac{c-xmax+\sqrt{(c-xmax)^2+ymax^2}}{c+\sqrt{c^2+ymax^2}}\right)^{10(c-xmax)\,ymax^4}\Big]\Big);$$

This has to be multiplied by the probability density, which depends on the patch sizes. Given that f is the probability of drawing two individuals from different patches, we have $g[x1, y1] \times g[x2, y2] = \frac{f}{(xmax\,ymax)^2}$, which accounts for the total probability of choosing the first individual from the left or the right patch. Recalling also that we only looked at half of the y region (y1<y2) and so doubling, we get :

In[ ]:=

$$\text{ExpXwithc} = 2\,\frac{f}{\text{xmax}^2\,\text{ymax}^2}\left(\frac{1}{240}\left(4\left((c-\text{xmax})^2\right)^{5/2} - 4\left(c^5 - 5c^4\,\text{xmax} - 10c^3\,\text{xmax}^2 - 10c^2\,\text{xmax}^3 - 5c\,\text{xmax}^4 - \text{xmax}^5\right) + 8\left(c^2 - c\,\text{ymax} - \text{ymax}^2\right)\left(c^2 + c\,\text{ymax} - \text{ymax}^2\right)\sqrt{c^2+\text{ymax}^2} - 4\left(c^2 - 2c\,\text{xmax} + \text{xmax}^2 + c\,\text{ymax} - \text{xmax}\,\text{ymax} - \text{ymax}^2\right)\left(c^2 - 2c\,\text{xmax} + \text{xmax}^2 - c\,\text{ymax} + \text{xmax}\,\text{ymax} - \text{ymax}^2\right)\sqrt{c^2 - 2c\,\text{xmax} + \text{xmax}^2 + \text{ymax}^2} - 4\left(c^2 + 2c\,\text{xmax} + \text{xmax}^2 - c\,\text{ymax} - \text{xmax}\,\text{ymax} - \text{ymax}^2\right)\left(c^2 + 2c\,\text{xmax} + \text{xmax}^2 + c\,\text{ymax} + \text{xmax}\,\text{ymax} - \text{ymax}^2\right)\sqrt{c^2 + 2c\,\text{xmax} + \text{xmax}^2 + \text{ymax}^2} + \text{Log}\left[\left(\frac{c}{\text{ymax}+\sqrt{c^2+\text{ymax}^2}}\right)^{20c^4\,\text{ymax}}\left(\frac{\text{ymax}+\sqrt{(c-\text{xmax})^2+\text{ymax}^2}}{\text{Abs}[c-\text{xmax}]}\right)^{10(c-\text{xmax})^4\,\text{ymax}}\left(\frac{\text{ymax}+\sqrt{(c+\text{xmax})^2+\text{ymax}^2}}{c+\text{xmax}}\right)^{10(c+\text{xmax})^4\,\text{ymax}}\left(\frac{c+\text{xmax}+\sqrt{(c+\text{xmax})^2+\text{ymax}^2}}{c+\sqrt{c^2+\text{ymax}^2}}\right)^{10(c+\text{xmax})\,\text{ymax}^4}\left(\frac{c-\text{xmax}+\sqrt{(c-\text{xmax})^2+\text{ymax}^2}}{c+\sqrt{c^2+\text{ymax}^2}}\right)^{10(c-\text{xmax})\,\text{ymax}^4}\right]\right)\right);$$

Finally, we have to add to the solution above for points drawn from within the same patch (soln) with probability 1-f:

In[ ]:=

$$
\begin{aligned}
\text{finalX} = {} & \frac{1}{120}\,\frac{f}{\text{xmax}^2\,\text{ymax}^2} \\
& \Bigl(4\,\bigl((c-\text{xmax})^2\bigr)^{5/2} - 4\,\bigl(c^5 - 5\,c^4\,\text{xmax} - 10\,c^3\,\text{xmax}^2 - 10\,c^2\,\text{xmax}^3 - 5\,c\,\text{xmax}^4 - \text{xmax}^5\bigr) + \\
& \quad 8\,\bigl(c^2 - c\,\text{ymax} - \text{ymax}^2\bigr)\,\bigl(c^2 + c\,\text{ymax} - \text{ymax}^2\bigr)\,\sqrt{c^2+\text{ymax}^2} - \\
& \quad 4\,\bigl(c^2 - 2\,c\,\text{xmax} + \text{xmax}^2 + c\,\text{ymax} - \text{xmax}\,\text{ymax} - \text{ymax}^2\bigr) \\
& \quad\;\bigl(c^2 - 2\,c\,\text{xmax} + \text{xmax}^2 - c\,\text{ymax} + \text{xmax}\,\text{ymax} - \text{ymax}^2\bigr)\,\sqrt{c^2 - 2\,c\,\text{xmax} + \text{xmax}^2 + \text{ymax}^2} - \\
& \quad 4\,\bigl(c^2 + 2\,c\,\text{xmax} + \text{xmax}^2 - c\,\text{ymax} - \text{xmax}\,\text{ymax} - \text{ymax}^2\bigr) \\
& \quad\;\bigl(c^2 + 2\,c\,\text{xmax} + \text{xmax}^2 + c\,\text{ymax} + \text{xmax}\,\text{ymax} - \text{ymax}^2\bigr)\,\sqrt{c^2 + 2\,c\,\text{xmax} + \text{xmax}^2 + \text{ymax}^2} + \\
& \quad \text{Log}\Bigl[\left(\frac{c}{\text{ymax}+\sqrt{c^2+\text{ymax}^2}}\right)^{20\,c^4\,\text{ymax}} \left(\frac{\text{ymax}+\sqrt{(c-\text{xmax})^2+\text{ymax}^2}}{\text{Abs}[c-\text{xmax}]}\right)^{10\,(c-\text{xmax})^4\,\text{ymax}} \\
& \quad\; \left(\frac{\text{ymax}+\sqrt{(c+\text{xmax})^2+\text{ymax}^2}}{c+\text{xmax}}\right)^{10\,(c+\text{xmax})^4\,\text{ymax}} \\
& \quad\; \left(\frac{c+\text{xmax}+\sqrt{(c+\text{xmax})^2+\text{ymax}^2}}{c+\sqrt{c^2+\text{ymax}^2}}\right)^{10\,(c+\text{xmax})\,\text{ymax}^4} \\
& \quad\; \left(\frac{c-\text{xmax}+\sqrt{(c-\text{xmax})^2+\text{ymax}^2}}{c+\sqrt{c^2+\text{ymax}^2}}\right)^{10\,(c-\text{xmax})\,\text{ymax}^4}\Bigr]\Bigr) + \\
& \frac{(1-f)}{30\,\text{xmax}^2\,\text{ymax}^2} \\
& \Bigl(-2\,\sqrt{\text{xmax}^2+\text{ymax}^2}\,\bigl(\text{xmax}^4 - 3\,\text{xmax}^2\,\text{ymax}^2 + \text{ymax}^4\bigr) + 2\,\bigl(\text{xmax}^5+\text{ymax}^5\bigr) + 5\,\text{xmax}\,\text{ymax} \\
& \quad \Bigl(\text{ymax}^3\,\text{Log}\Bigl[\frac{\text{xmax}+\sqrt{\text{xmax}^2+\text{ymax}^2}}{\text{ymax}}\Bigr] + \text{xmax}^3\,\text{Log}\Bigl[\frac{\text{ymax}+\sqrt{\text{xmax}^2+\text{ymax}^2}}{\text{xmax}}\Bigr]\Bigr)\Bigr);
\end{aligned}
$$

We expect that this will equal the non-split patch case if either f=0 or c=0, so working with the difference from soln might be useful:

In[∘]:=
```
Collect[finalX - soln, Log, Simplify[#, fullconditions] &] /.
   Abs[c - xmax] → ((c - xmax)^2)^(1/2);
collectLog[collectLog4[%]];
finaldif =
 Collect[%, Log, Simplify[#, fullconditions] &] /. Abs[c - xmax] → ((c - xmax)^2)^(1/2)
```

Out[∘]=

$$
\begin{aligned}
&\frac{1}{120\,\text{xmax}^2\,\text{ymax}^2}\\
&f\Bigg(4\left((c-\text{xmax})^2\right)^{5/2}+4\left(-c^5+5c^4\,\text{xmax}+10c^3\,\text{xmax}^2+10c^2\,\text{xmax}^3+5c\,\text{xmax}^4+\text{xmax}^5\right)+\\
&\quad 8\left(c^2-c\,\text{ymax}-\text{ymax}^2\right)\left(c^2+c\,\text{ymax}-\text{ymax}^2\right)\sqrt{c^2+\text{ymax}^2}-\\
&\quad 4\left((c-\text{xmax})^2+c\,\text{ymax}-\text{xmax}\,\text{ymax}-\text{ymax}^2\right)\\
&\quad\left((c-\text{xmax})^2-c\,\text{ymax}+\text{xmax}\,\text{ymax}-\text{ymax}^2\right)\sqrt{(c-\text{xmax})^2+\text{ymax}^2}+\\
&\quad 8\sqrt{\text{xmax}^2+\text{ymax}^2}\left(\text{xmax}^4-3\,\text{xmax}^2\,\text{ymax}^2+\text{ymax}^4\right)-8\left(\text{xmax}^5+\text{ymax}^5\right)-\\
&\quad 4\left(c^2+2c\,\text{xmax}+\text{xmax}^2-c\,\text{ymax}-\text{xmax}\,\text{ymax}-\text{ymax}^2\right)\sqrt{c^2+2c\,\text{xmax}+\text{xmax}^2+\text{ymax}^2}\\
&\quad\left(c^2+\text{xmax}^2+\text{xmax}\,\text{ymax}-\text{ymax}^2+c\,(2\,\text{xmax}+\text{ymax})\right)+\text{Log}\Big[\left((c-\text{xmax})^2\right)^{-5(c-\text{xmax})^4\,\text{ymax}}\\
&\quad\left(\frac{c}{\text{ymax}+\sqrt{c^2+\text{ymax}^2}}\right)^{20c^4\,\text{ymax}}\left(\frac{c-\text{xmax}+\sqrt{(c-\text{xmax})^2+\text{ymax}^2}}{c+\sqrt{c^2+\text{ymax}^2}}\right)^{10(c-\text{xmax})\,\text{ymax}^4}\\
&\quad\left(\text{ymax}+\sqrt{(c-\text{xmax})^2+\text{ymax}^2}\right)^{10(c-\text{xmax})^4\,\text{ymax}}\\
&\quad\left(\left(\frac{\text{xmax}+\sqrt{\text{xmax}^2+\text{ymax}^2}}{\text{ymax}}\right)^{\text{ymax}^3}\left(\frac{\text{ymax}+\sqrt{\text{xmax}^2+\text{ymax}^2}}{\text{xmax}}\right)^{\text{xmax}^3}\right)^{-20\,\text{xmax}\,\text{ymax}}\\
&\quad\left(\frac{c+\text{xmax}+\sqrt{(c+\text{xmax})^2+\text{ymax}^2}}{c+\sqrt{c^2+\text{ymax}^2}}\right)^{10(c+\text{xmax})\,\text{ymax}^4}\\
&\quad\left(\frac{\text{ymax}+\sqrt{(c+\text{xmax})^2+\text{ymax}^2}}{c+\text{xmax}}\right)^{10(c+\text{xmax})^4\,\text{ymax}}\Big]\Bigg)
\end{aligned}
$$

Rewriting in terms of the scaled distance between the patches ($c_s = \frac{c}{\text{xmax}}$) and the extent of patch asymmetry ($\alpha = \frac{\text{ymax}}{\text{xmax}}$):

In[∘]:=
$$\text{Collect}\left[\text{Simplify}\left[\frac{\text{finaldif}}{\frac{1}{120\,\alpha^2}\text{f max}}\;/.\;c\to c_s\,\text{xmax}\;/.\;\text{xmax}\to\text{max}\;/.\;\text{ymax}\to\alpha\,\text{max}\right],\right.$$
$$\left\{\sqrt{1+\alpha^2},\ \sqrt{\alpha^2+c_s^2},\ \sqrt{\alpha^2+(-1+c_s)^2},\ \sqrt{1+\alpha^2+2\,c_s+c_s^2}\right\},$$
$$\left.\text{Simplify}[\#,\text{fullconditions}]\ \&\right]\;/.\;\sqrt{1+\alpha^2}\to \text{H}\;/.\;\sqrt{\alpha^2+(-1+c_s)^2}\to\text{H1}\;/.$$
$$\sqrt{1+\alpha^2+2\,c_s+c_s^2}\to\text{H3}\;/.\;\sqrt{\alpha^2+c_s^2}\to\text{H2}\;/.\;(-1+c_s)^2\to\text{H4}^2$$

Out[∘]=
$$4\left(\text{H4}^2\right)^{5/2}+8\,\text{H}\left(1-3\,\alpha^2+\alpha^4\right)-8\left(1+\alpha^5\right)+$$
$$\frac{1}{\text{max}^5}\text{Log}\Big[\left(\text{H4}^2\right)^{-5\,\text{max}^5\,\alpha\,(-1+c_s)^4}\left(\left(\frac{1+\text{H}}{\alpha}\right)^{\text{max}^3\,\alpha^3}\right)^{-20\,\text{max}^2\,\alpha}(\text{H}+\alpha)^{-20\,\text{max}^5\,\alpha}\,(\text{H1}+\alpha)^{10\,\text{max}^5\,\alpha\,(-1+c_s)^4}$$
$$\left(\frac{c_s}{\text{H2}+\alpha}\right)^{20\,\text{max}^5\,\alpha\,c_s^4}\left(\frac{\text{H3}+\alpha}{1+c_s}\right)^{10\,\text{max}^5\,\alpha\,(1+c_s)^4}\left(\frac{-1+\text{H1}+c_s}{\text{H2}+c_s}\right)^{10\,\text{max}^5\,\alpha^4\,(-1+c_s)}$$
$$\left(\frac{1+\text{H3}+c_s}{\text{H2}+c_s}\right)^{10\,\text{max}^5\,\alpha^4\,(1+c_s)}\Big]+8\,\text{H2}\left(\alpha^2-\alpha\,c_s-c_s^2\right)\left(\alpha^2+\alpha\,c_s-c_s^2\right)-$$
$$4\,\text{H1}\left(-1+\alpha+\alpha^2-(-2+\alpha)\,c_s-c_s^2\right)\left(-1-\alpha+\alpha^2+(2+\alpha)\,c_s-c_s^2\right)+$$
$$4\,\text{H3}\left(-1+\alpha+\alpha^2+(-2+\alpha)\,c_s-c_s^2\right)\left(1+\alpha-\alpha^2+(2+\alpha)\,c_s+c_s^2\right)+$$
$$4\left(1+5\,c_s+10\,c_s^2+10\,c_s^3+5\,c_s^4-c_s^5\right)$$

In[∘]:=
$$\text{backtransform2}=\left\{\text{H}\to\sqrt{1+\alpha^2},\ \text{H1}\to\sqrt{\alpha^2+(-1+c_s)^2},\right.$$
$$\left.\text{H2}\to\sqrt{\alpha^2+c_s^2},\ \text{H3}\to\sqrt{\alpha^2+(1+c_s)^2},\ \text{H4}\to\sqrt{(-1+c_s)^2}\right\};$$

Simplifying:

$$4\left(\left((1+c_s)^5-2\,c_s^5-2\left(1+\alpha^5\right)\right)+2\,\text{H}\left(1-3\,\alpha^2+\alpha^4\right)+2\,\text{H2}\left(\alpha^2-\alpha\,c_s-c_s^2\right)\left(\alpha^2+\alpha\,c_s-c_s^2\right)-\right.$$
$$\text{H1}\left(-1+\alpha+\alpha^2-(-2+\alpha)\,c_s-c_s^2\right)\left(-1-\alpha+\alpha^2+(2+\alpha)\,c_s-c_s^2\right)+$$
$$\left.\text{H3}\left(-1+\alpha+\alpha^2+(-2+\alpha)\,c_s-c_s^2\right)\left(1+\alpha-\alpha^2+(2+\alpha)\,c_s+c_s^2\right)+\text{H4}^5\right)+$$
$$5\,\alpha\,\text{Log}\Big[(\text{H}+\alpha)^{-4}\left(\frac{\text{H1}+\alpha}{\text{H4}}\right)^{2\,(-1+c_s)^4}\left(\frac{c_s}{\text{H2}+\alpha}\right)^{4\,c_s^4}\left(\frac{\text{H3}+\alpha}{1+c_s}\right)^{2\,(1+c_s)^4}$$
$$\left(\left(\frac{\alpha}{1+\text{H}}\right)^2\frac{1+\text{H3}+c_s}{-1+\text{H1}+c_s}\right)^{2\,\alpha^3}\left(\frac{(-1+\text{H1}+c_s)\,(1+\text{H3}+c_s)}{(\text{H2}+c_s)^2}\right)^{2\,\alpha^3\,c_s}\Big];$$

Dividing by four:

In[∘]:=
$$\text{distcapt}=$$
$$\left(\left((1+c_s)^5-2\left(1+\alpha^5+c_s^5\right)\right)+2\,\text{H}\left(1-3\,\alpha^2+\alpha^4\right)+2\,\text{H2}\left(\alpha^2-\alpha\,c_s-c_s^2\right)\left(\alpha^2+\alpha\,c_s-c_s^2\right)-\right.$$
$$\text{H1}\left(-1+\alpha+\alpha^2-(-2+\alpha)\,c_s-c_s^2\right)\left(-1-\alpha+\alpha^2+(2+\alpha)\,c_s-c_s^2\right)+$$
$$\left.\text{H3}\left(-1+\alpha+\alpha^2+(-2+\alpha)\,c_s-c_s^2\right)\left(1+\alpha-\alpha^2+(2+\alpha)\,c_s+c_s^2\right)+\text{H4}^5\right)+$$
$$\frac{5}{4}\,\alpha\,\text{Log}\Big[(\text{H}+\alpha)^{-4}\left(\frac{\text{H1}+\alpha}{\text{H4}}\right)^{2\,(-1+c_s)^4}\left(\frac{c_s}{\text{H2}+\alpha}\right)^{4\,c_s^4}\left(\frac{\text{H3}+\alpha}{1+c_s}\right)^{2\,(1+c_s)^4}$$
$$\left(\left(\frac{\alpha}{1+\text{H}}\right)^2\frac{1+\text{H3}+c_s}{-1+\text{H1}+c_s}\right)^{2\,\alpha^3}\left(\frac{(-1+\text{H1}+c_s)\,(1+\text{H3}+c_s)}{(\text{H2}+c_s)^2}\right)^{2\,\alpha^3\,c_s}\Big];$$

Altogether, $E[X] = soln + \frac{f\,max}{30\,\alpha^2}\,distcapt$.

```
solnc = soln + f max/(30 α^2) distcapt;
```

In the symmetric case:

In[∘]:= `distcapt /. α → 1`

Out[∘]=

$$-16 - 8H + 4H4^5 + \mathrm{Log}\left[\frac{\left(\frac{1+H1}{H4}\right)^{10(-1+c_s)^4}\left(\frac{c_s}{1+H2}\right)^{20c_s^4}\left(\frac{1+H3}{1+c_s}\right)^{10(1+c_s)^4}(1+H3+c_s)^{10}\left(\frac{(-1+H1+c_s)(1+H3+c_s)}{(H2+c_s)^2}\right)^{10c_s}}{(1+H)^{40}(-1+H1+c_s)^{10}}\right] +$$
$$8H2\left(1-c_s-c_s^2\right)\left(1+c_s-c_s^2\right) - 4H1\left(1+c_s-c_s^2\right)\left(-1+3c_s-c_s^2\right) +$$
$$4H3\left(1-c_s-c_s^2\right)\left(1+3c_s+c_s^2\right) + 4\left(-2c_s^5+(1+c_s)^5\right)$$

In[∘]:= `backtransform = {H → Sqrt[xmax^2 + ymax^2], H1 → Sqrt[(c - xmax)^2 + ymax^2], H2 → Sqrt[c^2 + ymax^2], H3 → Sqrt[(c + xmax)^2 + ymax^2]};`

Note that while distcapt cannot be directly evaluated when the patch distance is zero, if we evaluate the log and the non-log terms separately they disappear:

In[∘]:= `Simplify[distcapt /. backtransform2 /. Log[x_] → 0 /. cs → 0, fullconditions]`

Out[∘]=

0

In[∘]:= `Log[Simplify[Exp[(distcapt - (distcapt /. Log[x_] → 0))/(5 α/4)] /. H1 → H /. H3 → H /. H2 → α /. H4 → 1 /. H → Sqrt[1 + α^2] /. (cs/α)^(4 cs^4) → 1 /. cs → 0, fullconditions]]`

Out[∘]=

0

assuming that α does not also vanish so that $\left(\frac{c_s}{\alpha}\right)^{4c_s^4}$ can be evaluated in the limit as 1.

Herd split patches: $E[X]^2$ for two patches at a distance c from one another

Repeating the above but for the squared distance:

Integrate[((x2+c)-x1)^2 + (y2 - y1)^2, x2]

We carry out these integrations in steps (it is faster to use the indefinite integrals and simplify when applying the limits of integration), first rewriting (x2-x1)^2+(y2-y1)^2 in terms of the distances dx=(x2+c)-x1 and dy=y2-y1.

We start with the simpler of the integrals, along the y-axis:

Step 1: Integrating over dy

In[∘]:=
```
Integrate[dx^2 + dy^2, dy]
```
Out[∘]=

$$dx^2\, dy + \frac{dy^3}{3}$$

We evaluate the integrals assuming that the top-most point within a patch is (x2,y2), which means that we are ignoring the half of the space where (x2,y2) is bottom most (this halves the area that we are looking over, accounted for below).

In[∘]:=
```
Simplify[(% /. dy → ymax - y1) - (% /. dy → 0), fullconditions]
```
Out[∘]=

$$dx^2\,(-y1 + ymax) + \frac{1}{3}\,(-y1 + ymax)^3$$

Next integrating over y1

In[∘]:=
```
Simplify[Integrate[%, y1]]
```
Out[∘]=

$$-\frac{1}{12}\,y1\,(y1 - 2\,ymax)\,\left(6\,dx^2 + y1^2 - 2\,y1\,ymax + 2\,ymax^2\right)$$

In[∘]:=
```
Simplify[(% /. y1 → ymax) - (% /. y1 → 0), fullconditions]
```
Out[∘]=

$$\frac{1}{12}\,ymax^2\,\left(6\,dx^2 + ymax^2\right)$$

Step 2: Integrating over dx

Because we need to consider $dx^2 = ((x2 + c) - x1)^2$ for both x2<x1 and x2>x1, we proceed with integrating with respect to x2 and x1 directly:

In[∘]:=
```
1/12 ymax^2 (6 dx^2 + ymax^2) /. dx → x2 + c - x1;

Simplify[Integrate[%, x2], fullconditions]
```
Out[∘]=

$$\frac{1}{12}\,ymax^2\,\left(2\,(c - x1 + x2)^3 + x2\,ymax^2\right)$$

In[∘]:=
```
Simplify[(% /. x2 → xmax) - (% /. x2 → 0), fullconditions]
```
Out[∘]=

$$\frac{1}{12}\,ymax^2\,\left(-2\,(c - x1)^3 + 2\,(c - x1 + xmax)^3 + xmax\,ymax^2\right)$$

Next integrating with respect to x1:

In[∘]:=
```
Simplify[Integrate[%, x1], fullconditions]
```
Out[∘]=

$$\frac{1}{24}\,ymax^2\,\left((c - x1)^4 - (c - x1 + xmax)^4 + 2\,x1\,xmax\,ymax^2\right)$$

In[∘]:= `Simplify[(% /. x1 → xmax) - (% /. x1 → 0), fullconditions]`

Out[∘]=

$$\frac{1}{12}\ \text{xmax}^2\ \text{ymax}^2\ \left(6\ \text{c}^2+\text{xmax}^2+\text{ymax}^2\right)$$

This correctly reduces to $\text{solnX2} = \frac{1}{6}\ \left(\text{xmax}^2+\text{ymax}^2\right)$ when c=0.

This has to be multiplied by the probability density, which depends on the patch sizes. Given that f is the probability of drawing two individuals from different patches, we have $g[\text{x1, y1}] \times g[\text{x2, y2}] = \frac{f}{(\text{xmax ymax})^2}$, which accounts for the total probability of choosing the first individual from the left or the right patch. Recalling also that we only looked at half of the y region (y1<y2) and so doubling, we get :

In[∘]:=

$$2\ \frac{\text{f}}{\text{xmax}^2\ \text{ymax}^2}\ \frac{1}{12}\ \text{xmax}^2\ \text{ymax}^2\ \left(6\ \text{c}^2+\text{xmax}^2+\text{ymax}^2\right)$$

Out[∘]=

$$\frac{1}{6}\ \text{f}\ \left(6\ \text{c}^2+\text{xmax}^2+\text{ymax}^2\right)$$

Finally, we have to add the solution above for points drawn from within the same patch (soln) with probability 1-f:

$$\text{solncX2} = \frac{1}{6}\ (1-\text{f})\ \left(\text{xmax}^2+\text{ymax}^2\right)+\frac{1}{6}\ \text{f}\ \left(6\ \text{c}^2+\text{xmax}^2+\text{ymax}^2\right);$$

which can be more simply written as the non-split case (solnX2) plus a departure that depends on c and f:

$$\text{solncX2} = \frac{\text{xmax}^2+\text{ymax}^2}{6}+\text{c}^2\ \text{f};$$

Herd split patches: CV

$$\text{Sqrt}\left[\frac{\frac{\text{xmax}^2+\text{ymax}^2}{6}+\text{c}^2\ \text{f}}{\left(\text{soln}+\frac{\text{f max}}{30\ \alpha^2}\ \text{distcapt}\right)^2}-1\right];$$

Writing $E[X]^2$ and soln in terms of the scaled parameters:

In[∘]:=

$$\text{Factor}\left[\frac{\text{xmax}^2+\text{ymax}^2}{6}+\text{c}^2\ \text{f}\ /.\ \text{c}\to \text{c}_s\ \text{xmax}\ /.\ \text{xmax}\to \text{max}\ /.\ \text{ymax}\to \alpha\ \text{max}\right]$$

Out[∘]=

$$\frac{1}{6}\ \text{max}^2\ \left(1+\alpha^2+6\ \text{f}\ \text{c}_s^2\right)$$

In[∘]:= `Simplify[soln /. c → c_s xmax /. xmax → max /. ymax → α max, fullconditions]`

Out[∘]=

$$\frac{\text{max}\ \left(2+2\ \alpha^5-2\ \sqrt{1+\alpha^2}+6\ \alpha^2\ \sqrt{1+\alpha^2}-2\ \alpha^4\ \sqrt{1+\alpha^2}+5\ \alpha^4\ \text{Log}\left[\frac{1+\sqrt{1+\alpha^2}}{\alpha}\right]+5\ \alpha\ \text{Log}\left[\alpha+\sqrt{1+\alpha^2}\right]\right)}{30\ \alpha^2}$$

In[ ]:= solncapt = Simplify[collectLog[%/ $\frac{\max}{30\ \alpha^2}$ /. $\sqrt{1+\alpha^2}$ → H]]

Out[ ]=

$$2+2\alpha^5-2\,H\left(1-3\alpha^2+\alpha^4\right)+\mathrm{Log}\left[\left(\frac{1+H}{\alpha}\right)^{5\alpha^4}(H+\alpha)^{5\alpha}\right]$$

Altogether, we have:

$$\mathrm{Sqrt}\left[\frac{\frac{1}{6}\max^2\left(1+\alpha^2+6\,f\,c_s^2\right)}{\left(\frac{\max}{30\ \alpha^2}\ \mathrm{solncapt}+\frac{f\max}{30\ \alpha^2}\ \mathrm{distcapt}\right)^2}-1\right];$$

which equals

In[ ]:=
$$\mathrm{CVherdALTc}=\mathrm{Sqrt}\left[\frac{\alpha^4\,150\left(1+\alpha^2+6\,f\,c_s^2\right)}{(\mathrm{solncapt}+f\,\mathrm{distcapt})^2}-1\right];$$

This is scale free (neither solncapt nor distcapt depend on the the size of the patches, just on α and c_s) and reduces correctly to the non-split case, CVherdALT = $\mathrm{Sqrt}\left[\frac{150\,\alpha^4\,(1+\alpha^2)}{\mathrm{solncapt}^2}-1\right]$, when f=0 or when c=0.

E[X_i]: Average pairwise distance from individual i

We now focus on an individual at position {x1,y1} and determine its average pairwise distance to all other individuals measured. This is then used as the basis for calculating the CV among individuals. Again working with dx=x2-x1 and dy=y2-y1:

Step 1: Integrating over dx and dy

In[ ]:=
```
Integrate[Sqrt[dx^2 + dy^2], dx];
Collect[collectLog[% // TrigToExp], Log, Simplify[#, Assumptions → fullconditions] &]
```

Out[ ]=

$$\frac{1}{4}\left(2\,dx\sqrt{dx^2+dy^2}-dy^2\,\mathrm{Log}\left[\frac{-dx+\sqrt{dx^2+dy^2}}{dx+\sqrt{dx^2+dy^2}}\right]\right)$$

In[ ]:=
```
Simplify[TrigToExp[Integrate[%, dy]], fullconditions, Trig → False];
Collect[Nest[collectLog, %, Length[%]], Log, Simplify]
```

Out[ ]=

$$\frac{1}{12}\left(4\,dx\,dy\sqrt{dx^2+dy^2}+\mathrm{Log}\left[\left(1-\frac{dy}{\sqrt{dx^2+dy^2}}\right)^{-dx^3}\left(1+\frac{dy}{\sqrt{dx^2+dy^2}}\right)^{dx^3}\left(\frac{-dx+\sqrt{dx^2+dy^2}}{dx+\sqrt{dx^2+dy^2}}\right)^{-dy^3}\right]\right)$$

Tidying up:

In[ ]:=
$$\frac{1}{12}\left(4\,dx\,dy\sqrt{dx^2+dy^2}+\mathrm{Log}\left[\left(\frac{dy+\sqrt{dx^2+dy^2}}{dx}\right)^{2\,dx^3}\left(\frac{dx+\sqrt{dx^2+dy^2}}{dy}\right)^{2\,dy^3}\right]\right);$$

For an individual at position x1, the integral of x2 ranges from x2 = 0 (dx = -x1) to x2 = xmax (dx = xmax-x1):

In[∘]:=
```
Simplify[(% /. dx → xmax - x1) - (% /. dx → -x1), fullconditions, Trig → False];
Collect[Nest[collectLog, %, Length[%]], Log, Simplify]
```

Out[∘]=

$$\frac{1}{12}\left(4\,dy\,x1\,\sqrt{dy^2+x1^2}+4\,dy\,\sqrt{dy^2+(x1-xmax)^2}\,(-x1+xmax)+\text{Log}\left[\left(-\frac{dy+\sqrt{dy^2+x1^2}}{x1}\right)^{2\,x1^3}\left(\frac{-x1+\sqrt{dy^2+x1^2}}{dy}\right)^{-2\,dy^3}\left(\frac{dy+\sqrt{dy^2+(x1-xmax)^2}}{-x1+xmax}\right)^{2\,(-x1+xmax)^3}\left(\frac{-x1+\sqrt{dy^2+(x1-xmax)^2}+xmax}{dy}\right)^{2\,dy^3}\right]\right)$$

For an individual at position y1, the integral of y2 ranges from y2 = 0 (dy = -y1) to y2 = ymax (dy = ymax-y1):

In[∘]:=
```
FullSimplify[(% /. dy → ymax - y1) - (% /. dy → -y1), fullconditions];
Collect[Nest[collectLog, %, Length[%]], Log, Simplify]
```

Out[∘]=

$$\frac{1}{12}\left(4\,x1\,y1\,\sqrt{x1^2+y1^2}+4\,(-x1+xmax)\,y1\,\sqrt{(x1-xmax)^2+y1^2}+4\,(x1-xmax)\,\sqrt{(x1-xmax)^2+(y1-ymax)^2}\,(y1-ymax)+4\,x1\,\sqrt{x1^2+(y1-ymax)^2}\,(-y1+ymax)+\text{Log}\left[\left(\frac{y1-\sqrt{x1^2+y1^2}}{x1}\right)^{-2\,x1^3}\left(\frac{y1-\sqrt{(x1-xmax)^2+y1^2}}{x1-xmax}\right)^{2\,(x1-xmax)^3}\left(\frac{\left(x1+\sqrt{x1^2+y1^2}\right)\left(-x1+xmax+\sqrt{(x1-xmax)^2+y1^2}\right)}{y1^2}\right)^{2\,y1^3}\left(\frac{-x1+\sqrt{x1^2+(y1-ymax)^2}}{-x1+xmax+\sqrt{(x1-xmax)^2+(y1-ymax)^2}}\right)^{2\,(y1-ymax)^3}\left(-\frac{-y1+\sqrt{x1^2+(y1-ymax)^2}+ymax}{x1}\right)^{2\,x1^3}\left(\frac{-y1+\sqrt{(x1-xmax)^2+(y1-ymax)^2}+ymax}{-x1+xmax}\right)^{2\,(-x1+xmax)^3}\right]\right)$$

After a few rounds of tidying up and multiplying by the probability density function for the position of the second individual $g[x2,\ y2]=\frac{1}{xmax\ ymax}$, we have the individual's average pairwise distance to others:

In[ ]:=
$$\text{intraind} = \frac{1}{3}\frac{1}{\text{xmax ymax}}\left(\text{x1 y1}\sqrt{\text{x1}^2+\text{y1}^2} + (\text{xmax}-\text{x1})\,\text{y1}\sqrt{(\text{xmax}-\text{x1})^2+\text{y1}^2} + (\text{ymax}-\text{y1})\,\text{x1}\sqrt{\text{x1}^2+(\text{ymax}-\text{y1})^2} + (\text{xmax}-\text{x1})\,(\text{ymax}-\text{y1})\sqrt{(\text{xmax}-\text{x1})^2+(\text{ymax}-\text{y1})^2}\right) + \frac{1}{6}\frac{1}{\text{xmax ymax}}\,\text{Log}\Big[\left(\frac{\text{xmax}-\text{x1}+\sqrt{(\text{xmax}-\text{x1})^2+\text{y1}^2}}{\sqrt{\text{x1}^2+\text{y1}^2}-\text{x1}}\right)^{\text{y1}^3}\left(\frac{\text{ymax}-\text{y1}+\sqrt{\text{x1}^2+(\text{ymax}-\text{y1})^2}}{\sqrt{\text{x1}^2+\text{y1}^2}-\text{y1}}\right)^{\text{x1}^3}\left(\frac{\sqrt{(\text{xmax}-\text{x1})^2+\text{y1}^2}-\text{y1}}{\text{ymax}-\text{y1}+\sqrt{(\text{xmax}-\text{x1})^2+(\text{ymax}-\text{y1})^2}}\right)^{(\text{x1}-\text{xmax})^3}\left(\frac{\sqrt{(\text{ymax}-\text{y1})^2+\text{x1}^2}-\text{x1}}{\text{xmax}-\text{x1}+\sqrt{(\text{xmax}-\text{x1})^2+(\text{ymax}-\text{y1})^2}}\right)^{(\text{y1}-\text{ymax})^3}\Big];$$

$\overline{E[X_i]}$: Average of the individual average pairwise distances [same as soln for E[X] above]

Here, we calculate the average of E[X_i] (given by “intraind”) over all positions {x1,y1} where the individual i may be.

We start by breaking intraind into two parts, without and with the log:

In[ ]:=
$$\text{part1} = \frac{1}{3}\frac{1}{\text{xmax ymax}}\left(\text{x1 y1}\sqrt{\text{x1}^2+\text{y1}^2} + (\text{xmax}-\text{x1})\,\text{y1}\sqrt{(\text{xmax}-\text{x1})^2+\text{y1}^2} + (\text{ymax}-\text{y1})\,\text{x1}\sqrt{\text{x1}^2+(\text{ymax}-\text{y1})^2} + (\text{xmax}-\text{x1})\,(\text{ymax}-\text{y1})\sqrt{(\text{xmax}-\text{x1})^2+(\text{ymax}-\text{y1})^2}\right);$$

$$\text{part2} = \frac{1}{6}\frac{1}{\text{xmax ymax}}\,\text{Log}\Big[\left(\frac{\text{xmax}-\text{x1}+\sqrt{(\text{xmax}-\text{x1})^2+\text{y1}^2}}{\sqrt{\text{x1}^2+\text{y1}^2}-\text{x1}}\right)^{\text{y1}^3}\left(\frac{\text{ymax}-\text{y1}+\sqrt{\text{x1}^2+(\text{ymax}-\text{y1})^2}}{\sqrt{\text{x1}^2+\text{y1}^2}-\text{y1}}\right)^{\text{x1}^3}\left(\frac{\sqrt{(\text{xmax}-\text{x1})^2+\text{y1}^2}-\text{y1}}{\text{ymax}-\text{y1}+\sqrt{(\text{xmax}-\text{x1})^2+(\text{ymax}-\text{y1})^2}}\right)^{(\text{x1}-\text{xmax})^3}\left(\frac{\sqrt{(\text{ymax}-\text{y1})^2+\text{x1}^2}-\text{x1}}{\text{xmax}-\text{x1}+\sqrt{(\text{xmax}-\text{x1})^2+(\text{ymax}-\text{y1})^2}}\right)^{(\text{y1}-\text{ymax})^3}\Big];$$

Integrating part1:

In[ ]:=
```
Integrate[part1, x1, y1];
Collect[Factor[%], Sqrt[x1^2 - 2 x1 xmax + xmax^2 + y1^2 - 2 y1 ymax + ymax^2], Simplify]
```

Out[ ]=

$$-\frac{-\left(x1^2+y1^2\right)^{5/2}+\left(x1^2-2\,x1\,xmax+xmax^2+y1^2\right)^{5/2}+\left(x1^2+(y1-ymax)^2\right)^{5/2}}{45\,xmax\,ymax}+\frac{\left(x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2\right)^2\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2-2\,y1\,ymax+ymax^2}}{45\,xmax\,ymax}$$

In[ ]:=
```
FullSimplify[(% /. x1 → xmax) - (% /. x1 → 0), fullconditions];
FullSimplify[(% /. y1 → ymax) - (% /. y1 → 0), fullconditions]
```

Out[ ]=

$$\frac{1}{45\,xmax\,ymax}4\left(-xmax^5+xmax^4\sqrt{xmax^2+ymax^2}+2\,xmax^2\,ymax^2\sqrt{xmax^2+ymax^2}+ymax^4\left(-ymax+\sqrt{xmax^2+ymax^2}\right)\right)$$

Integrating part2 (multiplied by 6xmax ymax for ease of calculation) was more involved.

First integrating with respect to x1 and y1:

In[∘]:=
```
Integrate[6 xmax ymax part2, x1];
Collect[Nest[collectLog, %, Length[%]], Log, Simplify[#, fullconditions] &];
Collect[Nest[collectLog, %, Length[%]], Log, Simplify[#, fullconditions] &];
Collect[Nest[collectLog, %, Length[%]], Log, Simplify[#, fullconditions] &]
```

Out[∘]=

$$\frac{1}{12}\Bigg(y1\left(x1^2-14\,y1^2\right)\sqrt{x1^2+y1^2}-y1\left(x1^2-2\,x1\,xmax+xmax^2-14\,y1^2\right)\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}-\left(x1^2-14\,(y1-ymax)^2\right)\sqrt{x1^2+(y1-ymax)^2}\,(y1-ymax)+\left(x1^2-2\,x1\,xmax+xmax^2-14\,(y1-ymax)^2\right)\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}\,(y1-ymax)+\mathrm{Log}\Bigg[\left(x1-xmax+\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}\right)^{12\,xmax\,y1^3}\left(\frac{-x1+xmax+\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}}{-x1+\sqrt{x1^2+y1^2}}\right)^{12\,x1\,y1^3}\left(\frac{x1-\sqrt{x1^2+(y1-ymax)^2}}{x1-xmax-\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}}\right)^{12\,x1\,(y1-ymax)^3}\left(x1-xmax+\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}\right)^{-12\,xmax\,(y1-ymax)^3}\left(\frac{y1+\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}-ymax}{y1+\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}}\right)^{3\,xmax^4}\left(\frac{-y1+\sqrt{x1^2+(y1-ymax)^2}+ymax}{-y1+\sqrt{x1^2+y1^2}}\right)^{3\,x1^4}\left(\frac{-y1+\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}}{-y1+\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}+ymax}\right)^{3\,x1\,\left(x1^3-4\,x1^2\,xmax+6\,x1\,xmax^2-4\,xmax^3\right)}\Bigg]\Bigg)$$

```
Integrate[%, y1];
Collect[Nest[collectLog, %, Length[%]], Log, Simplify[#, fullconditions] &];
Collect[Nest[collectLog, %, Length[%]], Log, Simplify[#, fullconditions] &];
Collect[Nest[collectLog, %, Length[%]], Log, Simplify[#, fullconditions] &]
```

Out[∘]=

$$\frac{1}{12}\Bigg(-\frac{11}{5}\,x1^4\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}+\frac{44}{5}\,x1^3\,xmax\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}-\frac{11}{5}\,xmax^4\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}+\frac{3}{5}\,xmax^2\,y1^2\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}+\frac{14}{5}\,y1^4\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}+\frac{3}{5}\,x1^2\left(-22\,xmax^2+y1^2\right)\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}+\sqrt{x1^2+y1^2}\left(-5\,x1^4+x1^2\,y1^2\right)+$$

$$\sqrt{x1^2 - 2\,x1\,xmax + xmax^2 + y1^2}\;\left(5\,(x1 - xmax)^4 - (x1 - xmax)^2\,y1^2\right) +$$
$$\frac{2}{5}\,x1\,\sqrt{x1^2 - 2\,x1\,xmax + xmax^2 + y1^2}\;\left(22\,xmax^3 - 3\,xmax\,y1^2\right) +$$
$$\frac{1}{5}\,\sqrt{x1^2 + y1^2}\;\left(11\,x1^4 - 3\,x1^2\,y1^2 - 14\,y1^4\right) + 5\,x1^4\,\sqrt{x1^2 + (y1 - ymax)^2} -$$
$$5\,x1^2\,(x1 - xmax)^2\,\sqrt{x1^2 - 2\,x1\,xmax + xmax^2 + (y1 - ymax)^2} +$$
$$10\,x1\,(x1 - xmax)^2\,xmax\,\sqrt{x1^2 - 2\,x1\,xmax + xmax^2 + (y1 - ymax)^2} -$$
$$5\,(x1 - xmax)^2\,xmax^2\,\sqrt{x1^2 - 2\,x1\,xmax + xmax^2 + (y1 - ymax)^2} +$$
$$\frac{1}{5}\,\left(11\,x1^2 - 22\,x1\,xmax + 11\,xmax^2 - 14\,(y1 - ymax)^2\right)$$
$$\left(x1^2 - 2\,x1\,xmax + xmax^2 + (y1 - ymax)^2\right)^{3/2} + \frac{1}{5}\,\left(x1^2 + (y1 - ymax)^2\right)^{3/2}$$
$$\left(-11\,x1^2 + 14\,(y1 - ymax)^2\right) - x1^2\,\sqrt{x1^2 + (y1 - ymax)^2}\,(y1 - ymax)^2 +$$
$$(x1 - xmax)^2\,\sqrt{x1^2 - 2\,x1\,xmax + xmax^2 + (y1 - ymax)^2}\,(y1 - ymax)^2 - 3\,xmax\,y1^3\,ymax +$$
$$\frac{9}{2}\,xmax\,y1^2\,ymax^2 - 3\,xmax\,y1\,ymax^3 + \mathrm{Log}\Big[\left(x1 - xmax + \sqrt{x1^2 - 2\,x1\,xmax + xmax^2 + y1^2}\right)^{3\,xmax\,y1^4}$$
$$\left(\frac{-x1 + \sqrt{x1^2 + y1^2}}{-x1 + xmax + \sqrt{x1^2 - 2\,x1\,xmax + xmax^2 + y1^2}}\right)^{-3\,x1\,y1^4}\left(x1 + \sqrt{x1^2 + (y1 - ymax)^2}\right)^{-3\,x1\,ymax^4}$$
$$\left(\frac{x1 - \sqrt{x1^2 + (y1 - ymax)^2}}{x1 - xmax - \sqrt{x1^2 - 2\,x1\,xmax + xmax^2 + (y1 - ymax)^2}}\right)^{3\,x1\,y1\,\left(y1^3 - 4\,y1^2\,ymax + 6\,y1\,ymax^2 - 4\,ymax^3\right)}$$
$$\left(x1 - xmax + \sqrt{x1^2 - 2\,x1\,xmax + xmax^2 + (y1 - ymax)^2}\right)^{3\,x1\,ymax^4 + 3\,xmax\,(y1 - ymax)^3\,(3\,y1 + ymax)}$$
$$\left(y1 + \sqrt{x1^2 + (y1 - ymax)^2} - ymax\right)^{3\,x1^4\,ymax}$$
$$\left(y1 + \sqrt{x1^2 - 2\,x1\,xmax + xmax^2 + (y1 - ymax)^2} - ymax\right)^{-3\,(x1 - xmax)^4\,ymax}$$
$$\left(\left(x1 - xmax + \sqrt{x1^2 - 2\,x1\,xmax + xmax^2 + (y1 - ymax)^2}\right)^{-12\,xmax\,(y1 - ymax)^3}\right.$$
$$\left.\left(\frac{y1 + \sqrt{x1^2 - 2\,x1\,xmax + xmax^2 + (y1 - ymax)^2} - ymax}{y1 + \sqrt{x1^2 - 2\,x1\,xmax + xmax^2 + y1^2}}\right)^{3\,xmax^4}\right)^{y1}$$
$$\left(\frac{-y1 + \sqrt{x1^2 + y1^2}}{-y1 + \sqrt{x1^2 + (y1 - ymax)^2} + ymax}\right)^{-3\,x1^4\,y1}$$
$$\left.\left(\frac{-y1 + \sqrt{x1^2 - 2\,x1\,xmax + xmax^2 + y1^2}}{-y1 + \sqrt{x1^2 - 2\,x1\,xmax + xmax^2 + (y1 - ymax)^2} + ymax}\right)^{3\,x1\,\left(x1^3 - 4\,x1^2\,xmax + 6\,x1\,xmax^2 - 4\,xmax^3\right)\,y1}\Big]\right)$$

Breaking the above into two parts (without and with the log):

$$\mathbf{part2a} = \frac{1}{12}\,\left(-\frac{11}{5}\,x1^4\,\sqrt{x1^2 - 2\,x1\,xmax + xmax^2 + y1^2} + \right.$$

$$\frac{44}{5}\,x1^3\,xmax\,\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}-\frac{11}{5}\,xmax^4\,\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}+$$
$$\frac{3}{5}\,xmax^2\,y1^2\,\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}+\frac{14}{5}\,y1^4\,\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}+$$
$$\frac{3}{5}\,x1^2\left(-22\,xmax^2+y1^2\right)\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}+\sqrt{x1^2+y1^2}\left(-5\,x1^4+x1^2\,y1^2\right)+$$
$$\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}\left(5\,(x1-xmax)^4-(x1-xmax)^2\,y1^2\right)+$$
$$\frac{2}{5}\,x1\,\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}\left(22\,xmax^3-3\,xmax\,y1^2\right)+$$
$$\frac{1}{5}\,\sqrt{x1^2+y1^2}\left(11\,x1^4-3\,x1^2\,y1^2-14\,y1^4\right)+5\,x1^4\,\sqrt{x1^2+(y1-ymax)^2}-$$
$$5\,x1^2\,(x1-xmax)^2\,\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}+$$
$$10\,x1\,(x1-xmax)^2\,xmax\,\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}-$$
$$5\,(x1-xmax)^2\,xmax^2\,\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}+$$
$$\frac{1}{5}\left(11\,x1^2-22\,x1\,xmax+11\,xmax^2-14\,(y1-ymax)^2\right)$$
$$\left(x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2\right)^{3/2}+\frac{1}{5}\left(x1^2+(y1-ymax)^2\right)^{3/2}$$
$$\left(-11\,x1^2+14\,(y1-ymax)^2\right)-x1^2\,\sqrt{x1^2+(y1-ymax)^2}\,(y1-ymax)^2+$$
$$(x1-xmax)^2\,\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}\,(y1-ymax)^2-$$
$$\left.3\,xmax\,y1^3\,ymax+\frac{9}{2}\,xmax\,y1^2\,ymax^2-3\,xmax\,y1\,ymax^3\right);$$

$$part2b=\frac{1}{12}\left(\text{Log}\left[\left(x1-xmax+\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}\right)^{3\,xmax\,y1^4}\right.\right.$$
$$\left(\frac{-x1+\sqrt{x1^2+y1^2}}{-x1+xmax+\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}}\right)^{-3\,x1\,y1^4}\left(x1+\sqrt{x1^2+(y1-ymax)^2}\right)^{-3\,x1\,ymax^4}$$
$$\left(\frac{x1-\sqrt{x1^2+(y1-ymax)^2}}{x1-xmax-\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}}\right)^{3\,x1\,y1\,\left(y1^3-4\,y1^2\,ymax+6\,y1\,ymax^2-4\,ymax^3\right)}$$
$$\left(x1-xmax+\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}\right)^{3\,x1\,ymax^4+3\,xmax\,(y1-ymax)^3\,(3\,y1+ymax)}$$
$$\left(y1+\sqrt{x1^2+(y1-ymax)^2}-ymax\right)^{3\,x1^4\,ymax}$$
$$\left(y1+\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}-ymax\right)^{-3\,(x1-xmax)^4\,ymax}$$
$$\left(\left(x1-xmax+\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}\right)^{-12\,xmax\,(y1-ymax)^3}\right.$$
$$\left.\left(\frac{y1+\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}-ymax}{y1+\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}}\right)^{3\,xmax^4}\right)^{y1}$$

$$\left(\frac{-y1+\sqrt{x1^2+y1^2}}{-y1+\sqrt{x1^2+(y1-ymax)^2}+ymax}\right)^{-3\,x1^4\,y1}$$
$$\left(\frac{-y1+\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}}{-y1+\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}+ymax}\right)^{3\,x1\,(x1^3-4\,x1^2\,xmax+6\,x1\,xmax^2-4\,xmax^3)\,y1}\Bigg]\Bigg)$$

;

Evaluating part2a over the limits of integration:

```
part2a;
Simplify[Limit[%, x1 → xmax, Assumptions → fullconditions] -
   Limit[%, x1 → 0, Assumptions → fullconditions], fullconditions];
Simplify[Limit[%, y1 → ymax, Assumptions → fullconditions] -
  Limit[%, y1 → 0, Assumptions → fullconditions], fullconditions]
```

Out[∘]=

$$-\frac{2}{15}\left(-7\,xmax^5+7\,xmax^4\sqrt{xmax^2+ymax^2}-xmax^2\,ymax^2\sqrt{xmax^2+ymax^2}+7\,ymax^4\left(-ymax+\sqrt{xmax^2+ymax^2}\right)\right)$$

Using Simplify[PowerExpand[%]] inside the Log, part2b can be simplified as:

$$part2b=\frac{1}{12}\,Log\Big[\left(-x1+\sqrt{x1^2+y1^2}\right)^{-3\,x1\,y1^4}$$
$$\left(-y1+\sqrt{x1^2+y1^2}\right)^{-3\,x1^4\,y1}\left(x1-xmax+\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}\right)^{3\,xmax\,y1^4}$$
$$\left(-x1+xmax+\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}\right)^{3\,x1\,y1^4}$$
$$\left(-y1+\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}\right)^{3\,x1\,(x1^3-4\,x1^2\,xmax+6\,x1\,xmax^2-4\,xmax^3)\,y1}$$
$$\left(y1+\sqrt{x1^2-2\,x1\,xmax+xmax^2+y1^2}\right)^{-3\,xmax^4\,y1}$$
$$\left(x1-\sqrt{x1^2+(y1-ymax)^2}\right)^{3\,x1\,y1\,(y1^3-4\,y1^2\,ymax+6\,y1\,ymax^2-4\,ymax^3)}$$
$$\left(x1+\sqrt{x1^2+(y1-ymax)^2}\right)^{-3\,x1\,ymax^4}$$
$$\left(x1-xmax-\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}\right)^{-3\,x1\,y1\,(y1^3-4\,y1^2\,ymax+6\,y1\,ymax^2-4\,ymax^3)}$$
$$\left(x1-xmax+\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}\right)^{-3\,xmax\,(y1-ymax)^4+3\,x1\,ymax^4}$$
$$\left(y1+\sqrt{x1^2+(y1-ymax)^2}-ymax\right)^{3\,x1^4\,ymax}$$
$$\left(y1+\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}-ymax\right)^{3\,xmax^4\,y1-3\,(x1-xmax)^4\,ymax}$$
$$\left(-y1+\sqrt{x1^2+(y1-ymax)^2}+ymax\right)^{3\,x1^4\,y1}$$
$$\left(-y1+\sqrt{x1^2-2\,x1\,xmax+xmax^2+(y1-ymax)^2}+ymax\right)^{-3\,x1\,(x1^3-4\,x1^2\,xmax+6\,x1\,xmax^2-4\,xmax^3)\,y1}\Big];$$

Evaluating part2b over the limits of integration:

```
Simplify[Limit[%, x1 → xmax, Assumptions → fullconditions] -
   Limit[%, x1 → 0, Assumptions → fullconditions], fullconditions];
Collect[Nest[collectLog, %, Length[%]], Log, Simplify[#, fullconditions] &]
```

Out[∘]=

$$\frac{1}{12}\,\mathrm{Log}\Big[\left(\frac{y1}{-xmax+\sqrt{xmax^2+y1^2}}\right)^{6\,xmax\,y1^4}\left(-xmax+\sqrt{xmax^2+(y1-ymax)^2}\right)^{3\,xmax\,\left(2\,y1^4-8\,y1^3\,ymax+12\,y1^2\,ymax^2-8\,y1\,ymax^3+ymax^4\right)}\left(xmax+\sqrt{xmax^2+(y1-ymax)^2}\right)^{-3\,xmax\,ymax^4}\left(y1+\sqrt{xmax^2+(y1-ymax)^2}-ymax\right)^{-3\,xmax^4\,(y1-2\,ymax)}(-y1+ymax)^{-6\,xmax\,y1\,\left(y1^3-4\,y1^2\,ymax+6\,y1\,ymax^2-4\,ymax^3\right)}\left(\frac{\left(y1+\sqrt{xmax^2+y1^2}\right)\left(-y1+\sqrt{xmax^2+(y1-ymax)^2}+ymax\right)}{-y1+\sqrt{xmax^2+y1^2}}\right)^{3\,xmax^4\,y1}\Big]$$

Gathering terms and simplifying (using Simplify[PowerExpand[]], multiplying top and bottom by $\left(A+\sqrt{B}\right)$ as beneficial, and gathering parts by hand):

In[∘]:=

$$\frac{1}{12}\,\mathrm{Log}\Big[\left(\frac{\left(xmax+\sqrt{xmax^2+y1^2}\right)\left(-xmax+\sqrt{xmax^2+(y1-ymax)^2}\right)}{y1\,(-y1+ymax)}\right)^{6\,xmax\,y1^4}\left(\frac{\left(y1+\sqrt{xmax^2+y1^2}\right)\left(-y1+\sqrt{xmax^2+(y1-ymax)^2}+ymax\right)}{xmax^2}\right)^{6\,xmax^4\,y1}\left(\frac{ymax-y1}{xmax+\sqrt{xmax^2+(ymax-y1)^2}}\right)^{6\,xmax\,ymax^4}\left(y1+\sqrt{xmax^2+(y1-ymax)^2}-ymax\right)^{6\,xmax^4\,ymax}\left(\frac{-xmax+\sqrt{xmax^2+(y1-ymax)^2}}{-y1+ymax}\right)^{-12\,xmax\,y1\,ymax\,\left(2\,y1^2-3\,y1\,ymax+2\,ymax^2\right)}\Big];$$

In[∘]:=
```
Simplify[Limit[%, y1 → ymax, Assumptions → fullconditions] -
   Limit[%, y1 → 0, Assumptions → fullconditions], fullconditions];
Collect[Nest[collectLog, %, Length[%]], Log, Simplify[#, fullconditions] &]
```

Out[∘]=

$$\frac{1}{12}\,\mathrm{Log}\Big[\left(\frac{ymax}{xmax+\sqrt{xmax^2+ymax^2}}\right)^{-12\,xmax\,ymax^4}\left(\frac{-ymax+\sqrt{xmax^2+ymax^2}}{ymax+\sqrt{xmax^2+ymax^2}}\right)^{-6\,xmax^4\,ymax}\Big]$$

Bringing together all of the above terms [recalling that we need to divide part2 by 6xmax ymax], multiplying the result by the probability density function g[x1, y1] = $\frac{1}{xmax\,ymax}$, the average of an individual's pairwise distances to others is:

In[∘]:=
```
avepairind =
  Collect[1/(xmax ymax) (1/(45 xmax ymax) 4 (-xmax^5 + xmax^4 Sqrt[xmax^2 + ymax^2] + 2 xmax^2 ymax^2
        Sqrt[xmax^2 + ymax^2] + ymax^4 (-ymax + Sqrt[xmax^2 + ymax^2])) +
      1/(6 xmax ymax) (-2/15 (-7 xmax^5 + 7 xmax^4 Sqrt[xmax^2 + ymax^2] - xmax^2 ymax^2 Sqrt[xmax^2 + ymax^2] +
          7 ymax^4 (-ymax + Sqrt[xmax^2 + ymax^2])) + 1/12 Log[(ymax/(xmax + Sqrt[xmax^2 + ymax^2]))^(-12 xmax ymax^4)
          ((-ymax + Sqrt[xmax^2 + ymax^2])/(ymax + Sqrt[xmax^2 + ymax^2]))^(-6 xmax^4 ymax)]]), Log, Simplify[#, fullconditions] &]
```

Out[∘]=

$$\frac{1}{360\, xmax^2\, ymax^2}\left(-24\left(-xmax^5+xmax^4\sqrt{xmax^2+ymax^2}-3\, xmax^2\, ymax^2\sqrt{xmax^2+ymax^2}+ymax^4\left(-ymax+\sqrt{xmax^2+ymax^2}\right)\right)+5\,\mathrm{Log}\left[\left(\frac{ymax}{xmax+\sqrt{xmax^2+ymax^2}}\right)^{-12\, xmax\, ymax^4}\left(\frac{-ymax+\sqrt{xmax^2+ymax^2}}{ymax+\sqrt{xmax^2+ymax^2}}\right)^{-6\, xmax^4\, ymax}\right]\right)$$

In[∘]:=

$$avepairind = \frac{1}{15\, xmax^2\, ymax^2}\left(-\sqrt{xmax^2+ymax^2}\left(xmax^4-3\, xmax^2\, ymax^2+ymax^4\right)+\left(xmax^5+ymax^5\right)\right)+\frac{1}{6\, xmax\, ymax}\,\mathrm{Log}\left[\left(\frac{xmax+\sqrt{xmax^2+ymax^2}}{ymax}\right)^{ymax^3}\left(\frac{ymax+\sqrt{xmax^2+ymax^2}}{xmax}\right)^{xmax^3}\right];$$

Simplifying to the extent possible:

In[∘]:=

$$avepairind = \frac{\sqrt{xmax^2+ymax^2}}{3}+\frac{\left(xmax^5+ymax^5\right)-\left(xmax^2+ymax^2\right)^{5/2}}{15\, xmax^2\, ymax^2}+\frac{1}{6\, xmax\, ymax}\,\mathrm{Log}\left[\left(\frac{xmax+\sqrt{xmax^2+ymax^2}}{ymax}\right)^{ymax^3}\left(\frac{ymax+\sqrt{xmax^2+ymax^2}}{xmax}\right)^{xmax^3}\right];$$

In[∘]:=
```
avepairind /. ymax → 1 /. xmax → {0.2, 1.}
```

Out[∘]=
```
{0.349759, 0.521405}
```

As expected, this is just the solution (soln) that we found for the mean pairwise distance in the first section (ignoring individual identity):

In[∘]:=
$$\text{soln} = \frac{(\text{xmax}^5 + \text{ymax}^5)}{15\,\text{xmax}^2\,\text{ymax}^2} + \frac{1}{15}\,H\left(3 - \frac{\text{xmax}^2}{\text{ymax}^2} - \frac{\text{ymax}^2}{\text{xmax}^2}\right) + \frac{\text{xmax}^3\,\text{Log}\left[\frac{\text{ymax}+H}{\text{xmax}}\right] + \text{ymax}^3\,\text{Log}\left[\frac{\text{xmax}+H}{\text{ymax}}\right]}{6\,\text{xmax}\,\text{ymax}}\ \text{/. } H \to \sqrt{\text{xmax}^2 + \text{ymax}^2}\,;$$

In[∘]:=
```
FullSimplify[PowerExpand[avepairind - soln], fullconditions]
```
Out[∘]=
```
0
```

Below, we assume a square range to calculate the average squared value of intraind, so simplifying the above to this case:

In[∘]:=
```
Simplify[soln /. xmax → max /. ymax → max, max > 0]
```
Out[∘]=
$$\frac{1}{15}\,\text{max}\left(2 + \sqrt{2} + 5\,\text{Log}\left[1 + \sqrt{2}\right]\right)$$

In[∘]:=
$$\text{indsoln} = \frac{1}{15}\,\text{max}\left(2 + \sqrt{2} + 5\,\text{Log}\left[1 + \sqrt{2}\right]\right);$$

Individual CV: Average pairwise distance from individual i [assumes xmax = ymax]

Attempts to integrate the square of E[X_i] (given by “intraind”) over all positions {x1,y1} where the individual i may be were not successful in the general case.

Here we focus on a square range (xmax=ymax=max). We first transform the position of the individual relative to the max, defining X1=x1/max and Y1=y1/max.

In[∘]:=
```
FullSimplify[intraind /. xmax → max /. ymax → max /. x1 → X1 * max /. y1 → Y1 * max,
  {max > 0, 0 < X1 < 1, 0 < Y1 < 1}]
```

Out[∘]=

$$\frac{1}{6}\,\text{max}\,\Bigg(2\,X1\,\sqrt{X1^2+(-1+Y1)^2}-2\,X1\,\sqrt{X1^2+(-1+Y1)^2}\,Y1+2\,\sqrt{2+(-2+X1)\,X1+(-2+Y1)\,Y1}-2\,X1\,\sqrt{2+(-2+X1)\,X1+(-2+Y1)\,Y1}-2\,Y1\,\sqrt{2+(-2+X1)\,X1+(-2+Y1)\,Y1}+2\,X1\,Y1\,\sqrt{2+(-2+X1)\,X1+(-2+Y1)\,Y1}+2\,Y1\,\sqrt{(-1+X1)^2+Y1^2}-2\,X1\,Y1\,\sqrt{(-1+X1)^2+Y1^2}+2\,X1\,Y1\,\sqrt{X1^2+Y1^2}+(-1+Y1)^3\,\text{Log}\left[\frac{X1-\sqrt{X1^2+(-1+Y1)^2}}{-1+X1-\sqrt{2+(-2+X1)\,X1+(-2+Y1)\,Y1}}\right]+(-1+X1)^3\,\text{Log}\left[\frac{-Y1+\sqrt{(-1+X1)^2+Y1^2}}{1-Y1+\sqrt{2+(-2+X1)\,X1+(-2+Y1)\,Y1}}\right]+Y1^3\,\text{Log}\left[\frac{\left(1-X1+\sqrt{(-1+X1)^2+Y1^2}\right)\left(X1+\sqrt{X1^2+Y1^2}\right)}{Y1^2}\right]+X1^3\left(\text{Log}\left[1+\sqrt{X1^2+(-1+Y1)^2}-Y1\right]-\text{Log}\left[-Y1+\sqrt{X1^2+Y1^2}\right]\right)\Bigg)$$

We can numerically calculate the integral of the above squared, by factoring out max^2 from intraind^2 and then recognizing that the change of variables from x1 to X1 requires the substitution of *d*x1 with max *d*X1 and *d*y1 with max *d*Y1, so that overall the expected value of the square for an individuals distance to others is:

```
max² max² NIntegrate[(% / max)², {X1, 0, 1}, {Y1, 0, 1}]
```

Out[∘]=

$0.278925\,\text{max}^4$

Multiplying by the uniform probability density function for the position of this individual $g[x1, y1] = \frac{1}{\text{xmax ymax}}$, we get the expected squared average distance of an individual to all others:

In[∘]:=
```
indsolnsq = 0.27892463475631724` max²;
```

Thus, the individual-level CV for a square range is:

In[∘]:=

$$\text{Sqrt}\left[\frac{\text{indsolnsq}}{\text{indsoln}^2}-1\right]$$

Out[∘]=

0.16116

which is scale-free as expected.

```
CVind = 0.16116021233464142`;
```

1D Analysis - Next we consider the case where the range is very elongated (xmax>>ymax), so much so that ymax approaches 0 and we can calculate a similar result based on the 1D case.

For an individual at position x1, the distribution of distances to other individuals is:

```
Sqrt[(x2 - x1)^2] 1/max
```

Out[ ]=

$$\frac{e^{-\frac{x2^2}{2\,\sigma x^2}}\sqrt{(-x1+x2)^2}}{\sqrt{2\pi}\,\sigma x}$$

where $\frac{1}{max}$ is the probability density function for an individual in a linear range of length max.

We take the focal individual to be at position x1 and calculate its distance to other individuals [it's easier to just take the distance as the positive value, breaking the integral into x2>x1 and x2<x1 (switching the sign in the latter case)]

In[ ]:=
```
Integrate[(x2 - x1) 1/max, x2, Assumptions → {x1 > 0, max > 0}]
```

Out[ ]=

$$\frac{-x1\,x2+\frac{x2^2}{2}}{max}$$

In[ ]:=
```
Simplify[Simplify[Limit[%, x2 → max, Assumptions → {x1 > 0, max > 0}] -
     Limit[%, x2 → x1, Assumptions → {x1 > 0, max > 0}], {x1 > 0, max > 0}] +
   Simplify[Limit[-%, x2 → x1, Assumptions → {x1 > 0, max > 0}] -
     Limit[-%, x2 → 0, Assumptions → {x1 > 0, max > 0}], {x1 > 0, max > 0}]]
```

Out[ ]=

$$\frac{max}{2}-x1+\frac{x1^2}{max}$$

The above gives the average distance of individuals at position x1 to all others. We then calculate the average distance and then the average distance squared among focal individuals, accounting for the uniform probability density of $\frac{1}{max}$ for the focal individual:

In[ ]:=
```
indsoln1 = Integrate[(max/2 - x1 + x1^2/max) 1/max, {x1, 0, max}]
```

Out[ ]=

$$\frac{max}{3}$$

Repeating for the average distance squared:

In[ ]:=
```
indsolnsq1 = Integrate[(max/2 - x1 + x1^2/max)^2 1/max, {x1, 0, max}]
```

Out[ ]=

$$\frac{7\,max^2}{60}$$

In[ ]:=
```
CVind1 = Simplify[Sqrt[(7 max^2/60)/(max/3)^2 - 1]]
```

Out[ ]=

$$\frac{1}{2\sqrt{5}}$$

In[ ]:=
```
% // N
```

Out[ ]=
```
0.223607
```

Given the challenges of integrating the individual-level average distance, attempts to analyse the case of a split herd did not bear fruit. That said, the extreme cases are known. When the two herds overlap completely (c=0), we already have the non-split result above (CVind). At the other extreme, where the two patches are very far apart (c -> ∞), the individual mean μ_{IID} approaches $p_2\,c$ for individuals drawn from patch 1 (with ~0 distance to other individuals within the patch and a distance c when drawn from the other patch) and similarly $p_1\,c$ for individuals drawn from patch 2; altogether, this gives an average distance of $E[\mu_{\mathrm{IID}}] = 2\,p_1\,p_2\,c = f\,c$ and an $E\left[\mu_{\mathrm{IID}}{}^2\right] = p_1\,(p_2\,c)^2 + p_2\,(p_1\,c)^2 = \frac{f}{2}\,c^2$, an individual-level variance of $\mathrm{Var}[\mu_{\mathrm{IID}}] = \frac{f}{2}\,c^2 - (f\,c)^2 = f\left(\frac{1}{2} - f\right)c^2$, and so a CV of $\mathrm{Sqrt}\left[\frac{f\left(\frac{1}{2}-f\right)c^2}{(f\,c)^2}\right] = \sqrt{\frac{\left(\frac{1}{2}-f\right)}{f}}$. When the two patches are equal in size (f=1/2), all individuals have the same average distance to all others (~c/2) and there is no variance (CVind=0), but when the two patches are unequal in size, CVind can rise because individuals in the smaller patch have a much higher average IID to all other individuals, since most other individuals are in the larger patch.

Figure S1 - numerical integrals

Blue: Individual level CV (dots based on simulations, lines based on symmetrical case).
Green: Herd level CV (dots based on simulation, lines based on symmetrical case).
Black: $\mathrm{CV}_{\mathrm{ind}}\,/\,\mathrm{CV}_{\mathrm{herd}}$ (dots based on simulations for $\mathrm{CV}_{\mathrm{ind}}$, lines based on symmetrical case).

Integrations - Previously evaluated [ENTER]

In[ ]:=
```
intplottingherdB =
  {{0, 0.47550492704176501326842085214526889387`19.265782401984602},
   {1/10, 0.48223800269182439424204244018789428309`19.27571708761852},
   {1/5, 0.50356051634595339083427661882324140787`19.305953643770614},
   {3/10, 0.53824858525631198337225006537087354474`19.35147320832805},
   {2/5, 0.58116789766759142365898914989132029409`19.402226285602584},
   {1/2, 0.62485670125279216025118617141741644753`19.448406768590193},
   {3/5, 0.66207157923304649208209225180065497062`19.48394870190656},
   {7/10, 0.68809022450507344486066919543655729458`19.506950021190846},
   {4/5, 0.70199982193047489624491778297599020915`19.51867128656191},
   {9/10, 0.70664163377492790177858078532602223706`19.522497620863668}};
```

In[ ]:=
```
intplottingindB = {{0, 0.16116021233464095`}, {1/10, 0.16312883877526665`},
   {1/5, 0.1693414648365905`}, {3/10, 0.17935572544198097`},
   {2/5, 0.19151852420892146`}, {1/2, 0.20350866146741475`},
   {3/5, 0.21322706435128924`}, {7/10, 0.21955320967881167`},
   {4/5, 0.22262334114521112`}, {9/10, 0.2235285093418294`}};
```

```
In[•]:= intplottingherdC = {{0, 0.47550492704176501326842085214526889`19.265782401984602},
   {1/20, 0.48013013088978453058624972048256633385`19.27262750131087},
   {1/10, 0.49426475599545168060259597054596294437`19.292995007024736},
   {3/20, 0.51242319341691255220116380455905523996`19.317999501648178},
   {1/5, 0.53543242138361696425860387527965538733`19.347936120490846},
   {3/10, 0.61696072853668961648603899077008969569`19.44043441976271},
   {2/5, 0.70756495766796866667079363735718672221`19.523253749693687},
   {1/2, 0.78457452736441301149687179416751554035`19.58094607154419},
   {3/5, 0.84675250188097034359398182722397977721`19.620745261992997},
   {7/10, 0.89689350936346214682246872587932529801`19.649144619400865},
   {4/5, 0.93779194700440521268631533903583857538`19.67018135839725},
   {9/10, 0.97162441294193192884740650788847743111`19.686288512504305}};

In[•]:= intplottingindC = {{0, 0.1611602123346416`}, {1/20, 0.17486833394365656`},
   {1/10, 0.19618094312896772`}, {3/20, 0.19784271022739677`},
   {1/5, 0.1785479266617664`}, {3/10, 0.13038154039884364`},
   {2/5, 0.09334307038687574`}, {1/2, 0.06631141833127747`},
   {3/5, 0.046101508504420165`}, {7/10, 0.030533839376035758`},
   {4/5, 0.01821204461364805`}, {9/10, 0.008233855835082378`}};
```

```
In[*]:= intplottingherdD = {{0, 0.475504927041765013268420852145268894`19.265782401984602},
    {1/20, 0.47751635176037152890545888864425857584`19.268770395810055},
    {1/10, 0.48796289174092435622832295141246646926`19.284015138752537},
    {3/20, 0.51503876573781489517790461553533242126`19.32149836644405},
    {1/5, 0.56640457466872430679594150655614447195`19.38541179435481},
    {3/10, 0.74389621394719582297003504859517360806`19.55174654657169},
    {2/5, 0.96668538449381088452187064820649746148`19.684005918857387},
    {1/2, 1.19339093153619424467596859422845975852`19.769000259670523},
    {3/5, 1.40994580210304239660859311944133692376`19.823031923962073},
    {7/10, 1.61234834952651222049916371834025576462`19.85865540393001},
    {4/5, 1.8000466411358438406099633646634471892`19.88318446161295},
    {9/10, 1.97370151510957192381525891542206852499`19.900766100736888}};

In[*]:= intplottingindD = {{0, 0.1611602123346416`}, {1/20, 0.1670808590417897`},
    {1/10, 0.18676110401421217`}, {3/20, 0.22387952786445203`},
    {1/5, 0.2799178612132464`}, {3/10, 0.4274744039389861`}, {2/5, 0.585175384995669`},
    {1/2, 0.7360269476861879`}, {3/5, 0.876130277741853`}, {7/10, 1.0051394934643687`},
    {4/5, 1.1237204125810092`}, {9/10, 1.2328024772013375`}};
```

```
intplottingherdE =
  {{0, 0.64568779098024611914681035855938937511`19.468702049070505},
   {1/20, 0.65645049086855249420877909092791181455`19.478786401459917},
   {1/10, 0.67851970701684012519565375689391406748`19.498656380829797},
   {3/20, 0.68538894580453075837052381285821178619`19.5046283565488},
   {1/5, 0.68471618007527699205380074758672043231`19.504047799722212},
   {3/10, 0.73435071609659913530787509318968632116`19.544491965376984},
   {2/5, 0.79868188879324703747919508710036911823`19.590462209582196},
   {1/2, 0.85280211204339715680282001472781976014`19.624335941798122},
   {3/5, 0.89588948232536433007109102356288293859`19.648605181226234},
   {7/10, 0.93025995104890403929383458308800273233`19.666440500020705},
   {4/5, 0.95806613866164962180728829142280965978`19.679967122835656},
   {9/10, 0.98092388770242751452407868457771836829`19.69052476635805}};

intplottingindE = {{0, 0.20902207505075407`}, {1/20, 0.24464885437986275`},
   {1/10, 0.28108991782008425`}, {3/20, 0.265716164716257`},
   {1/5, 0.21876595477685665`}, {3/10, 0.14396879796383358`},
   {2/5, 0.09863947779972117`}, {1/2, 0.06843584434446058`}};
```

```
In[•]:= intplottingherdF =
  {{0, 0.64568779098024611914681035855938937511`19.468702049070505},
   {1/10, 0.67261410694071294278601323013056492754`19.49344279159825},
   {1/5, 0.78489435833827004875336586708328911889`19.581165161431162},
   {3/10, 0.98935592579205127551100797642408618 1`19.694297697724526},
   {2/5, 1.21252922644235313541381443054836803116`19.77464754471812},
   {1/2, 1.41820465723962143287284578264601739712`19.824723148534396},
   {3/5, 1.60029138277375863816732787380208986632`19.856834390738165},
   {7/10, 1.76010979994666859597782676496561237426`19.87850899781394},
   {4/5, 1.90053018955267649461055635728052964629`19.893858834109505},
   {9/10, 2.02444183570111969209674863482497199988`19.90517970466281}};

In[•]:= intplottingindF = {{0, 0.20902207505075407`}, {1/10, 0.26716043957826724`},
   {1/5, 0.4057808634969151`}, {3/10, 0.5789387922485483`}, {2/5, 0.7385769098761474`},
   {1/2, 0.8766436401266968`}, {3/5, 0.995233618482544`}, {7/10, 1.0975758181710125`},
   {4/5, 1.1865533957194974`}, {9/10, 1.2645152143794107`}};
```

```
In[∘]:= intplottingherdG =
  {{0, 0.64568779098024611914681035855938936546`19.468702049070505},
   {1/10, 0.6249669682390801206200194188680510117 3`19.44851698986969},
   {1/5, 0.5667721406605546948789891727540838909`19.385838346209535},
   {3/10, 0.5073851200907658822185852483040354723 7`19.31118829930355},
   {2/5, 0.4888278688972020791115917827970255662 9`19.285257194253433},
   {1/2, 0.5272574266623707614713041134757216461 8`19.33751442367978},
   {3/5, 0.6078424280028319965519665652295539334 2`19.431028732475703},
   {7/10, 0.7072676067712154196056196442262782413 3`19.523010422569236},
   {4/5, 0.8100289075250711775986926795863116211 8`19.597902197739398},
   {9/10, 0.9086010837792124168597601324086660310 1`19.65535135806768 7}};

In[∘]:= intplottingindG =
  {{0, 0.20902207505075487`}, {1/10, 0.2042641665810927`}, {1/5, 0.18747332064958802`},
   {3/10, 0.16062093784044443`}, {2/5, 0.1295368593806871`},
   {1/2, 0.09879840490042426`}, {3/5, 0.0711969511023559`}, {7/10, 0.0478075547072212`},
   {4/5, 0.028556820614599104`}, {9/10, 0.012853038153653885`}};
```

In[ ]:=
```
intplottingherdH =
  {{0, 0.64568779098024611914681035855938936546`19.468702049070505},
   {1/10, 0.63821733361904228518045953746304667882`19.461543909556},
   {1/5, 0.61728185316447255139965530005849847612`19.440761741207897},
   {3/10, 0.59790126821397128444458506460041604767`19.420523613392756},
   {2/5, 0.60442618459330801140969221832750656441`19.427448585585985},
   {1/2, 0.66386656617806497664370931479574105168`19.485582365622008},
   {3/5, 0.79581593158189812992553463828967217222`19.588553181318694},
   {7/10, 1.00642530826658600642287130212809893002`19.701742646135234},
   {4/5, 1.29610807956585342183896455195110603335`19.797164470362084},
   {9/10, 1.66849469042825827994206464021472177499`19.86671293054847}};
```

In[ ]:=
```
intplottingindH =
  {{0, 0.20902207505075487`}, {1/10, 0.20752393610013836`}, {1/5, 0.2054810541316196`},
   {3/10, 0.21471127501254128`}, {2/5, 0.25254140536828656`},
   {1/2, 0.329671545841258`}, {3/5, 0.4470304269403403`}, {7/10, 0.603240955625845 2`},
   {4/5, 0.7993585175970265`}, {9/10, 1.039904565627 6136`}};
```

Panel A: Symmetric range contraction [max along x axis, decreasing from left to right]

In[ ]:=
```
SeedRandom[12 241];
```

We seek to examine a range of xmax=ymax=max varying between 0 to 1. [We expect no effect.] In our plots, we'll show the size of the square (max) decreasing along the x-axis varying x from 0 to 1. To do so, we define max = $(1-x)^{V}$, which allows us to vary max from 1 at x=0 to 0 at x=1. V allows us to change the midpoint, which we set to 2, so that a value of x=1/2 corresponds to max=1/4. To convert the x-axis to max, we use $1 - \text{max}^{\frac{1}{V}}$:

In[ ]:=
```
tryV = 2;
```

In[ ]:=
```
Solve[max == (1 - x)^V, x]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[ ]=

$$\left\{\left\{x \to 1 - max^{\frac{1}{V}}\right\}\right\}$$

In[ ]:=
```
tab = Join[Table[i, {i, 0, 9 / 10, 1 / 10}], {{19 / 20}}] // Flatten
```

Out[ ]=

$$\left\{0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{9}{10}, \frac{19}{20}\right\}$$

In[ ]:=
```
numind = 100;
```

In[ ]:=
```
For[t = 1, t ≤ Length[tab], t++,
 table =
  Table[RandomReal[UniformDistribution[{{0, (1 - tab[[t]])^tryV}, {0, (1 - tab[[t]])^tryV}}]],
   {i, 1, numind}];
 disttable = Flatten[
   Table[EuclideanDistance[table[[i]], table[[j]]], {i, 1, numind}, {j, i + 1, numind}]];
 CVherdsim[tab[[t]], tab[[t]]] = StandardDeviation[disttable] / Mean[disttable];
 disttable2 = Table[
   1 / (Length[table] - 1) (Sum[EuclideanDistance[table[[i]], table[[j]]], {j, 1, i - 1}] + Sum[
       EuclideanDistance[table[[i]], table[[j]]], {j, i + 1, numind}]), {i, 1, numind}];
 CVindsim[tab[[t]], tab[[t]]] = StandardDeviation[disttable2] / Mean[disttable2]
]
```

In[ ]:=
```
forplottingherdA = Table[{tab[[t]], CVherdsim[tab[[t]], tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$$\left\{\{0, 0.472711\}, \left\{\frac{1}{10}, 0.474576\right\}, \left\{\frac{1}{5}, 0.47388\right\}, \left\{\frac{3}{10}, 0.475617\right\}, \left\{\frac{2}{5}, 0.478777\right\}, \left\{\frac{1}{2}, 0.475558\right\}, \left\{\frac{3}{5}, 0.479834\right\}, \left\{\frac{7}{10}, 0.485189\right\}, \left\{\frac{4}{5}, 0.470129\right\}, \left\{\frac{9}{10}, 0.480268\right\}, \left\{\frac{19}{20}, 0.484186\right\}\right\}$$

In[ ]:=
```
forplottingindA = Table[{tab⟦t⟧, CVindsim[tab⟦t⟧, tab⟦t⟧]}, {t, 1, Length[tab]}]
```

Out[ ]=

$$\left\{\{0, 0.173186\}, \left\{\frac{1}{10}, 0.169755\right\}, \left\{\frac{1}{5}, 0.166214\right\}, \left\{\frac{3}{10}, 0.173614\right\}, \left\{\frac{2}{5}, 0.18024\right\}, \left\{\frac{1}{2}, 0.165392\right\}, \left\{\frac{3}{5}, 0.174014\right\}, \left\{\frac{7}{10}, 0.189727\right\}, \left\{\frac{4}{5}, 0.156804\right\}, \left\{\frac{9}{10}, 0.171152\right\}, \left\{\frac{19}{20}, 0.177563\right\}\right\}$$

To label the x axis with max values:

In[ ]:=
```
{x, (1 - x)^tryV} /. x → tab // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & \frac{1}{10} & \frac{1}{5} & \frac{3}{10} & \frac{2}{5} & \frac{1}{2} & \frac{3}{5} & \frac{7}{10} & \frac{4}{5} & \frac{9}{10} & \frac{19}{20} \\ 1 & \frac{81}{100} & \frac{16}{25} & \frac{49}{100} & \frac{9}{25} & \frac{1}{4} & \frac{4}{25} & \frac{9}{100} & \frac{1}{25} & \frac{1}{100} & \frac{1}{400} \end{pmatrix}$$

In[ ]:=
```
Select[x /. Solve[(1 - x)^tryV == 1 / 10, x], # < 1 &]
```

Out[ ]=

$$\left\{\frac{1}{10}\left(10 - \sqrt{10}\right)\right\}$$

In[ ]:=
```
xticks = {{0.01, "1"}, {1/2 (2 - √2), "1/2"}, {1 / 2, "1/4"},
   {1/10 (10 - √10), "1/10"}, {4 / 5, "1/25"}, {9 / 10, "1/100"}, {1, "1/∞"}}
```

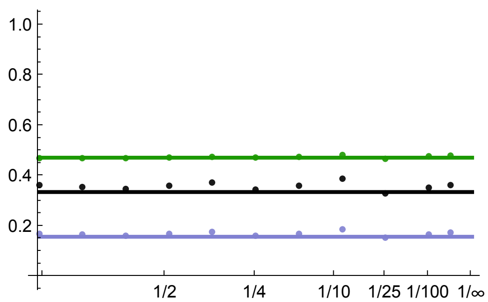
Out[ ]=

$$\left\{\{0.01, 1\}, \left\{\frac{1}{2}\left(2 - \sqrt{2}\right), 1/2\right\}, \left\{\frac{1}{2}, 1/4\right\}, \left\{\frac{1}{10}\left(10 - \sqrt{10}\right), 1/10\right\}, \left\{\frac{4}{5}, 1/25\right\}, \left\{\frac{9}{10}, 1/100\right\}, \{1, 1/\infty\}\right\}$$

In[ ]:=
```
maxc = 1;
```

In[ ]:=
```
plotA = Show[
  Plot[CVherd /. xmax → ymax /. ymax → (1 - x)^tryV,
   {x, 0, 1}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  Plot[CVind, {x, 0, 1}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[(CVind / CVherd /. xmax → ymax /. ymax → (1 - x)^tryV),
   {x, 0, 1}, PlotStyle → Black],
  ListPlot[forplottingherdA, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[forplottingindA, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[Table[{forplottingindA[[i, 1]], forplottingindA[[i, 2]] / forplottingherdA[[i, 2]]},
    {i, 1, Length[forplottingindA]}], PlotStyle → {Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, 1}, {0, 1}},
  Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[ ]=



Panel B: Asymmetric contraction leading to elongated ranges [xmax/ymax along x axis]

In[ ]:=
```
SeedRandom[21 773];
```

We seek to examine a range of ymax from 0 to 1, holding xmax=1. In our plots, we'll show xmax/ymax along the x-axis using values of x from 0 to 1. To do so, we define ymax = $(1-x)^V$, which allows us to vary xmax/ymax from 1 at x=0 to infinity at x=1. V allows us to change the midpoint, which we set to 2, so that a value of x=1/2 corresponds to xmax/ymax=4. To convert the x-axis to ymax, we use $1 - \text{ymax}^{\frac{1}{V}}$:

In[ ]:=
```
tryV = 2;
```

In[ ]:=
```
Solve[ymax == (1 - x)^V, x]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[ ]=

$\left\{\left\{x \to 1 - \text{ymax}^{\frac{1}{V}}\right\}\right\}$

In[•]:=
```
tab = Join[Table[i, {i, 0, 9 / 10, 1 / 10}]] // Flatten
```
Out[•]=

$$\left\{0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{9}{10}\right\}$$

In[•]:=
```
tryxmax = 1;
```
In[•]:=
```
numind = 100;
```
In[•]:=
```
For[t = 1, t ≤ Length[tab], t++,
  table = Table[RandomReal[
     UniformDistribution[{{0, tryxmax}, {0, (1 - tab[[t]])^tryV}}]], {i, 1, numind}];
  disttable = Flatten[
    Table[EuclideanDistance[table[[i]], table[[j]]], {i, 1, numind}, {j, i + 1, numind}]];
  CVherdsim[tryxmax, tab[[t]]] = StandardDeviation[disttable] / Mean[disttable];
  disttable2 = Table[
    1 / (Length[table] - 1) (Sum[EuclideanDistance[table[[i]], table[[j]]], {j, 1, i - 1}] + Sum[
        EuclideanDistance[table[[i]], table[[j]]], {j, i + 1, numind}]), {i, 1, numind}];
  CVindsim[tryxmax, tab[[t]]] = StandardDeviation[disttable2] / Mean[disttable2]
 ]
```
In[•]:=
```
forplottingherdB = Table[{tab[[t]], CVherdsim[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```
Out[•]=

$$\left\{\{0, 0.47049\}, \left\{\frac{1}{10}, 0.491607\right\}, \left\{\frac{1}{5}, 0.507825\right\}, \left\{\frac{3}{10}, 0.537111\right\}, \left\{\frac{2}{5}, 0.568451\right\}, \left\{\frac{1}{2}, 0.62333\right\}, \left\{\frac{3}{5}, 0.663144\right\}, \left\{\frac{7}{10}, 0.685779\right\}, \left\{\frac{4}{5}, 0.694607\right\}, \left\{\frac{9}{10}, 0.701092\right\}\right\}$$

In[•]:=
```
forplottingindB = Table[{tab[[t]], CVindsim[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```
Out[•]=

$$\left\{\{0, 0.157596\}, \left\{\frac{1}{10}, 0.176405\right\}, \left\{\frac{1}{5}, 0.164852\right\}, \left\{\frac{3}{10}, 0.193088\right\}, \left\{\frac{2}{5}, 0.175682\right\}, \left\{\frac{1}{2}, 0.195336\right\}, \left\{\frac{3}{5}, 0.198762\right\}, \left\{\frac{7}{10}, 0.189041\right\}, \left\{\frac{4}{5}, 0.21651\right\}, \left\{\frac{9}{10}, 0.223265\right\}\right\}$$

Below, we'll want the ratio of $\frac{\text{CVind}}{\text{CVherd}}$ in the limit for a very elongated range:

In[•]:=
```
limratio = Limit[CVind1 / CVherd /. xmax → tryxmax, ymax → Infinity]
```
Out[•]=

$$\frac{1}{\sqrt{10}}$$

To label the x axis:

```
In[•]:= {x, tryxmax / (1 - x)^tryV} /. x → tab // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & \frac{1}{10} & \frac{1}{5} & \frac{3}{10} & \frac{2}{5} & \frac{1}{2} & \frac{3}{5} & \frac{7}{10} & \frac{4}{5} & \frac{9}{10} \\ 1 & \frac{100}{81} & \frac{25}{16} & \frac{100}{49} & \frac{25}{9} & 4 & \frac{25}{4} & \frac{100}{9} & 25 & 100 \end{pmatrix}$$

```
In[•]:= Select[x /. Solve[1 / (1 - x)^tryV == 10, x], # < 1 &]
```

Out[•]=

$$\left\{\frac{1}{10}\left(10-\sqrt{10}\right)\right\}$$

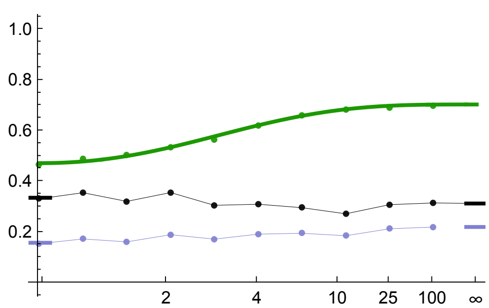
```
In[•]:= xticks = {{0.01, "1"}, {1/2 (2 - √2), "2"}, {1 / 2, "4"},
    {1/10 (10 - √10), "10"}, {4 / 5, "25"}, {9 / 10, "100"}, {1, "∞"}}
```

Out[•]=

$$\left\{\{0.01, 1\}, \left\{\frac{1}{2}\left(2-\sqrt{2}\right), 2\right\}, \left\{\frac{1}{2}, 4\right\}, \left\{\frac{1}{10}\left(10-\sqrt{10}\right), 10\right\}, \left\{\frac{4}{5}, 25\right\}, \left\{\frac{9}{10}, 100\right\}, \{1, \infty\}\right\}$$

In[ ]:=
```
plot1 = Show[
  Plot[CVherd /. xmax → tryxmax /. ymax → (1 - x)^tryV,
   {x, 0, 0.999}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  Plot[CVind, {x, -0.025, 0.025}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVind1, {x, 0.975, 1.025}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[(CVind / CVherd /. xmax → tryxmax /. ymax → tryxmax),
   {x, -0.025, 0.025}, PlotStyle → Black],
  Plot[limratio, {x, 0.975, 1.025}, PlotStyle → Black],
  ListPlot[forplottingherdB, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[forplottingindB, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[forplottingindB,
   PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}, Joined → True],
  ListPlot[Table[{forplottingindB[[i, 1]], forplottingindB[[i, 2]] / forplottingherdB[[i, 2]]},
    {i, 1, Length[forplottingindB]}], PlotStyle → {Black}],
  ListPlot[Join[Table[{forplottingindB[[i, 1]], forplottingindB[[i, 2]] / forplottingherdB[[i, 2]]},
     {i, 1, Length[forplottingindB]}], {{1, limratio}}],
   Joined → True, PlotStyle → {Black, Thin}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, 1}, {0, 1}},
  Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[ ]=



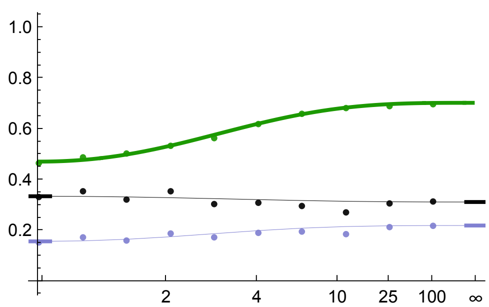
Dividing CVind by CVherd gives very similar answers in the 1D and symmetrical 2D cases (at opposite ends of this "elongation" axis). While the dashed line looks flat, the values are not exactly the same.

In[ ]:=
```
{CVind / (CVherd /. xmax → 1 /. ymax → 1), CVind1 / CVherd1} // N
```

Out[ ]=
```
{0.338924, 0.316228}
```

In[ ]:=
```
plotB = Show[
  Plot[CVherd /. xmax → tryxmax /. ymax → (1 - x)^tryV,
   {x, 0, 0.99}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  Plot[CVind, {x, -0.025, 0.025}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVind1, {x, 0.975, 1.025}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[(CVind / CVherd /. xmax → tryxmax /. ymax → tryxmax),
   {x, -0.025, 0.025}, PlotStyle → Black],
  Plot[limratio, {x, 0.975, 1.025}, PlotStyle → Black],
  ListPlot[forplottingherdB, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[forplottingindB, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[Join[intplottingindB, {{1, CVind1}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}],
  ListPlot[Table[{forplottingindB[[i, 1]], forplottingindB[[i, 2]] / forplottingherdB[[i, 2]]},
    {i, 1, Length[forplottingindB]}], PlotStyle → {Black}],
  ListPlot[Join[Table[
      {intplottingindB[[i, 1]], intplottingindB[[i, 2]] / (CVherd /. xmax → tryxmax /. ymax → (1 - tab[[i]])^tryV)},
      {i, 1, Length[intplottingindB]}], {{1, limratio}}],
   Joined → True, PlotStyle → {Thin, Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, 1}, {0, 1}},
  Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[ ]=



Panel B: Numerical integration [SLOW - Don't enter]

For comparison, we can numerically integrate. Integration was inaccurate and so we first simplify and then take the limit as c approaches its numerical value:

In[ ]:=
```
tryc = 0;
```

In[∘]:=
```
meanTEMP = Simplify[Limit[
   soln + (f xmax)/(30 α^2) ((((1 + c_s)^5 - 2 (1 + α^5 + c_s^5)) + 2 H (1 - 3 α^2 + α^4) + 2 H2 (α^2 - α c_s - c_s^2) (α^2 +
          α c_s - c_s^2) - H1 (-1 + α + α^2 - (-2 + α) c_s - c_s^2) (-1 - α + α^2 + (2 + α) c_s - c_s^2) +
        H3 (-1 + α + α^2 + (-2 + α) c_s - c_s^2) (1 + α - α^2 + (2 + α) c_s + c_s^2) + H4^5) +
       5/4 α Log[(H + α)^-4 ((H1 + α)/H4)^(2 (-1+c_s)^4) (c_s/(H2 + α))^(4 c_s^4) ((H3 + α)/(1 + c_s))^(2 (1+c_s)^4)
         ((α/(1 + H))^2 (1 + H3 + c_s)/(-1 + H1 + c_s))^(2 α^3) ((-1 + H1 + c_s) (1 + H3 + c_s)/(H2 + c_s)^2)^(2 α^3 c_s) ]) /.
     {H → Sqrt[1 + α^2], H1 → Sqrt[α^2 + (-1 + c_s)^2], H2 → Sqrt[α^2 + c_s^2], H3 → Sqrt[α^2 + (1 + c_s)^2],
      H4 → Sqrt[(-1 + c_s)^2]} /. c_s → c / xmax /.
    α → ymax / xmax /. xmax → tryxmax /. f → tryf, c → tryc], ymax ≥ 0]
```

Out[∘]=

$$\frac{1}{30\,\text{ymax}^2}\left(2+2\,\text{ymax}^5-2\sqrt{1+\text{ymax}^2}+6\,\text{ymax}^2\sqrt{1+\text{ymax}^2}-2\,\text{ymax}^4\sqrt{1+\text{ymax}^2}+5\,\text{ymax}^4\,\text{Log}\left[\frac{1+\sqrt{1+\text{ymax}^2}}{\text{ymax}}\right]+5\,\text{ymax}\,\text{Log}\left[\text{ymax}+\sqrt{1+\text{ymax}^2}\right]\right)$$

In[∘]:=
```
For[t = 1, t ≤ Length[tab], t++,
 tryymax = (1 - tab[[t]])^tryV;
 mean = N[Limit[meanTEMP, ymax → tryymax], 20];
 sqH = (xmax^2 + ymax^2)/6 + c^2 f /. xmax → tryxmax /. ymax → tryymax /. f → tryf /. c → tryc;

 CVherdintC[tryxmax, tab[[t]]] = N[Sqrt[sqH - mean^2] / mean, 20];
 CVindintC[tryxmax, tab[[t]]] =
  N[Sqrt[sqI[tryxmax, tryymax, tryc, tryf] - mean^2] / mean, 20];
 Print[{CVherdintC[tryxmax, tab[[t]]], CVindintC[tryxmax, tab[[t]]]}]
]
```

```
{0.4755049270417650133, 0.16116}
{0.4822380026918243942, 0.163129}
{0.5035605163459533908, 0.169341}
{0.5382485852563119834, 0.179356}
{0.5811678976675914237, 0.191519}
{0.6248567012527921603, 0.203509}
{0.6620715792330464921, 0.213227}
{0.6880902245050734449, 0.219553}
{0.7019998219304748962, 0.222623}
{0.7066416337749279018, 0.223529}
```

In[ ]:=
```
intplottingherdB = Table[{tab[[t]], CVherdintC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$$\left\{\{0, 0.4755049270417650133\}, \left\{\frac{1}{10}, 0.4822380026918243942\right\}, \left\{\frac{1}{5}, 0.5035605163459533908\right\}, \left\{\frac{3}{10}, 0.5382485852563119834\right\}, \left\{\frac{2}{5}, 0.5811678976675914237\right\}, \left\{\frac{1}{2}, 0.6248567012527921603\right\}, \left\{\frac{3}{5}, 0.6620715792330464921\right\}, \left\{\frac{7}{10}, 0.6880902245050734449\right\}, \left\{\frac{4}{5}, 0.7019998219304748962\right\}, \left\{\frac{9}{10}, 0.7066416337749279018\right\}\right\}$$

In[ ]:=
```
intplottingindB = Table[{tab[[t]], CVindintC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$$\left\{\{0, 0.16116\}, \left\{\frac{1}{10}, 0.163129\right\}, \left\{\frac{1}{5}, 0.169341\right\}, \left\{\frac{3}{10}, 0.179356\right\}, \left\{\frac{2}{5}, 0.191519\right\}, \left\{\frac{1}{2}, 0.203509\right\}, \left\{\frac{3}{5}, 0.213227\right\}, \left\{\frac{7}{10}, 0.219553\right\}, \left\{\frac{4}{5}, 0.222623\right\}, \left\{\frac{9}{10}, 0.223529\right\}\right\}$$

Panel C: Even herd split [midpoint at c=4]

In[ ]:=
```
SeedRandom[32 912];
```

We seek to examine a range of distances, c, between the peaks. To do so, we use c/(V+c) along the x-axis, which allows us to vary c from 0 at x=0 to infinity at x=1, within a single plot (we use V = 4 so that patches 4 sd apart are half-way along the plot). To convert the x-axis to c, we use:

In[ ]:=
```
tryV = 4;
```

In[ ]:=
```
Solve[c / (tryV + c) == x, c]
```

Out[ ]=

$$\left\{\left\{c \to -\frac{4\,x}{-1 + x}\right\}\right\}$$

```
In[∘]:= tab =
         Join[Table[i, {i, 0/10, 4/20, 1/20}], Table[i, {i, 3/10, 9/10, 1/10}]] // Flatten
```

Out[∘]=

$$\left\{0, \frac{1}{20}, \frac{1}{10}, \frac{3}{20}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{9}{10}\right\}$$

```
In[∘]:= numind1 = 50;
        numind2 = 50;
        numind = numind1 + numind2;

In[∘]:= tryf = 2 (numind2 / numind) (numind1 / numind)
          (*Unequal split with 95% in one patch so f = 0.095*)
```

Out[∘]=

$$\frac{1}{2}$$

```
In[∘]:= trymax = 1;
        tryxmax = trymax;
        tryymax = trymax;

In[∘]:= For[t = 1, t ≤ Length[tab], t++,
         table = Join[Table[RandomReal[UniformDistribution[{{0, tryxmax}, {0, tryymax}}]],
             {i, 1, numind1}], Table[RandomReal[UniformDistribution[
                {{tryV tab[[t]] / (1 - tab[[t]]), tryV tab[[t]] / (1 - tab[[t]]) + tryxmax}, {0, tryymax}}]], {i, 1, numind2}]];
         disttable = Flatten[
           Table[EuclideanDistance[table[[i]], table[[j]]], {i, 1, numind}, {j, i + 1, numind}]];
         CVherdsimC[tryxmax, tab[[t]]] = StandardDeviation[disttable] / Mean[disttable];
         disttable2 = Table[
           1 / (Length[table] - 1) (Sum[EuclideanDistance[table[[i]], table[[j]]], {j, 1, i - 1}] + Sum[
                EuclideanDistance[table[[i]], table[[j]]], {j, i + 1, numind}]), {i, 1, numind}];
         CVindsimC[tryxmax, tab[[t]]] = StandardDeviation[disttable2] / Mean[disttable2]
        ]
```

In[∘]:=
```
forplottingherdC = Table[{tab⟦t⟧, CVherdsimC[tryxmax, tab⟦t⟧]}, {t, 1, Length[tab]}]
```

Out[∘]=

$$\left\{\{0, 0.475331\}, \left\{\frac{1}{20}, 0.487266\right\}, \left\{\frac{1}{10}, 0.495214\right\}, \left\{\frac{3}{20}, 0.524657\right\}, \left\{\frac{1}{5}, 0.53443\right\}, \left\{\frac{3}{10}, 0.626007\right\}, \left\{\frac{2}{5}, 0.69305\right\}, \left\{\frac{1}{2}, 0.78026\right\}, \left\{\frac{3}{5}, 0.838624\right\}, \left\{\frac{7}{10}, 0.89089\right\}, \left\{\frac{4}{5}, 0.929399\right\}, \left\{\frac{9}{10}, 0.961288\right\}\right\}$$

In[∘]:=
```
forplottingindC = Table[{tab⟦t⟧, CVindsimC[tryxmax, tab⟦t⟧]}, {t, 1, Length[tab]}]
```

Out[∘]=

$$\left\{\{0, 0.173211\}, \left\{\frac{1}{20}, 0.182847\right\}, \left\{\frac{1}{10}, 0.203726\right\}, \left\{\frac{3}{20}, 0.227707\right\}, \left\{\frac{1}{5}, 0.188517\right\}, \left\{\frac{3}{10}, 0.125227\right\}, \left\{\frac{2}{5}, 0.100634\right\}, \left\{\frac{1}{2}, 0.0657055\right\}, \left\{\frac{3}{5}, 0.0490179\right\}, \left\{\frac{7}{10}, 0.0290495\right\}, \left\{\frac{4}{5}, 0.0192999\right\}, \left\{\frac{9}{10}, 0.00768273\right\}\right\}$$

In[∘]:=
```
maxc = 1;
```

Below, we'll want the ratio of $\frac{\text{CVind}}{\text{CVherd}}$ in the limit for very distant patches when the patches are even:

In[∘]:=
```
limratio =
  Limit[CVindfarsplit/CVherdC /. xmax → tryxmax /. ymax → tryymax /. f → tryf, c → Infinity]
```

Out[∘]=

0

We also plot CVherd in the limit for very distant patches when the patches are even:

In[∘]:=
```
limratioHERD =
  Limit[CVherdC /. xmax → tryxmax /. ymax → tryymax /. f → tryf, c → Infinity]
```

Out[∘]=

1

Choosing x-axis tick positions:

In[∘]:=

$$\left\{x, \frac{\text{tryV}\,x}{1-x}\right\} \text{ /. x → tab // Transpose}$$

Out[∘]=

$$\left\{\{0, 0\}, \left\{\frac{1}{20}, \frac{4}{19}\right\}, \left\{\frac{1}{10}, \frac{4}{9}\right\}, \left\{\frac{3}{20}, \frac{12}{17}\right\}, \left\{\frac{1}{5}, 1\right\}, \left\{\frac{3}{10}, \frac{12}{7}\right\}, \left\{\frac{2}{5}, \frac{8}{3}\right\}, \left\{\frac{1}{2}, 4\right\}, \left\{\frac{3}{5}, 6\right\}, \left\{\frac{7}{10}, \frac{28}{3}\right\}, \left\{\frac{4}{5}, 16\right\}, \left\{\frac{9}{10}, 36\right\}\right\}$$

In[∘]:=

$$\text{Solve}\left[\frac{\text{tryV}\,x}{1-x} == 2, x\right]$$

Out[∘]=

$$\left\{\left\{x \to \frac{1}{3}\right\}\right\}$$

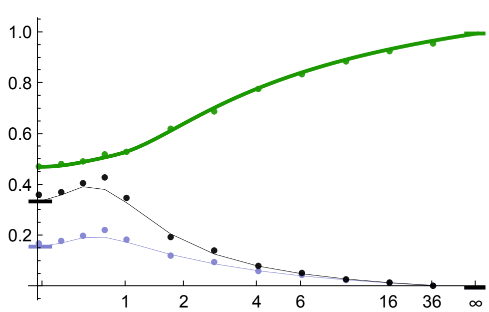
In[ ]:=
```
xticks = {{0.01, "0"}, {1/5, "1"}, {1/3, "2"},
   {1/2, "4"}, {3/5, "6"}, {4/5, "16"}, {9/10, "36"}, {1, "∞"}}
```

Out[ ]=

$$\left\{\{0.01, 0\}, \left\{\frac{1}{5}, 1\right\}, \left\{\frac{1}{3}, 2\right\}, \left\{\frac{1}{2}, 4\right\}, \left\{\frac{3}{5}, 6\right\}, \left\{\frac{4}{5}, 16\right\}, \left\{\frac{9}{10}, 36\right\}, \{1, \infty\}\right\}$$

In[ ]:=
```
plotC = Show[
   Plot[CVherdC /. xmax → trymax /. ymax → trymax /. f → tryf /. c → (tryV x)/(1 - x), {x, 0, maxc},
    WorkingPrecision → 50, PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
   Plot[limratioHERD, {x, 0.975, 1.05},
    PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
   Plot[CVind /. xmax → tryxmax /. ymax → tryymax,
    {x, -0.025, 0.025}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
   Plot[CVindfarsplit /. xmax → tryxmax /. ymax → tryymax /. f → tryf,
    {x, 0.975, 1.05}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
   Plot[CVind/CVherd /. xmax → tryxmax /. ymax → tryymax,
    {x, -0.025, 0.025}, PlotStyle → Black],
   Plot[limratio, {x, 0.975, 1.05}, PlotStyle → Black],
   ListPlot[forplottingherdC, PlotStyle → RGBColor[0.11, 0.6, 0.]],
   ListPlot[forplottingindC, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
   ListPlot[intplottingindC,
    Joined → True, PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}],
   ListPlot[Table[{forplottingherdC[[i, 1]], forplottingindC[[i, 2]]/forplottingherdC[[i, 2]]},
     {i, 1, Length[forplottingindC]}], PlotStyle → {Black}],
   ListPlot[Table[{intplottingindC[[i, 1]], intplottingindC[[i, 2]]/intplottingherdC[[i, 2]]},
     {i, 1, Length[forplottingindC]}], Joined → True, PlotStyle → {Thin, Black}],
   AxesOrigin → {0, 0},
   PlotRange → {{0, maxc}, {0, 1}}, Ticks → {xticks, Automatic}, ImageSize → 250
  ]
```

Out[ ]=



Panel C: Numerical integration [SLOW - Don't enter]

For comparison, we can numerically integrate. Integration was inaccurate and so we first simplify and then take the limit as c approaches its numerical value:

In[ ]:=

$$
\begin{aligned}
&\mathtt{meanTEMP} = \mathtt{Simplify}\Big[\\
&\quad \mathtt{soln} + \frac{\mathtt{f\ xmax}}{30\ \alpha^2}\Bigg(\Big(\Big(\big((1+c_s)^5 - 2\left(1+\alpha^5+c_s^5\right)\big) + 2\,\mathtt{H}\left(1-3\alpha^2+\alpha^4\right) + 2\,\mathtt{H2}\left(\alpha^2-\alpha c_s-c_s^2\right)\left(\alpha^2+\right.\\
&\qquad \left.\alpha c_s - c_s^2\right) - \mathtt{H1}\left(-1+\alpha+\alpha^2-(-2+\alpha)\,c_s-c_s^2\right)\left(-1-\alpha+\alpha^2+(2+\alpha)\,c_s-c_s^2\right) +\\
&\qquad \mathtt{H3}\left(-1+\alpha+\alpha^2+(-2+\alpha)\,c_s-c_s^2\right)\left(1+\alpha-\alpha^2+(2+\alpha)\,c_s+c_s^2\right)+\mathtt{H4}^5\Big)+\\
&\qquad \frac{5}{4}\,\alpha\,\mathtt{Log}\Big[(\mathtt{H}+\alpha)^{-4}\left(\frac{\mathtt{H1}+\alpha}{\mathtt{H4}}\right)^{2\,(-1+c_s)^4}\left(\frac{c_s}{\mathtt{H2}+\alpha}\right)^{4 c_s^4}\left(\frac{\mathtt{H3}+\alpha}{1+c_s}\right)^{2\,(1+c_s)^4}\\
&\qquad \left(\left(\frac{\alpha}{1+\mathtt{H}}\right)^2\frac{1+\mathtt{H3}+c_s}{-1+\mathtt{H1}+c_s}\right)^{2\alpha^3}\left(\frac{(-1+\mathtt{H1}+c_s)\,(1+\mathtt{H3}+c_s)}{(\mathtt{H2}+c_s)^2}\right)^{2\alpha^3 c_s}\Big]\Big)\Bigg)\ /.\\
&\quad \Big\{\mathtt{H}\to\sqrt{1+\alpha^2},\ \mathtt{H1}\to\sqrt{\alpha^2+(-1+c_s)^2},\ \mathtt{H2}\to\sqrt{\alpha^2+c_s^2},\ \mathtt{H3}\to\sqrt{\alpha^2+(1+c_s)^2},\\
&\quad \mathtt{H4}\to\sqrt{(-1+c_s)^2}\Big\}\ /.\ c_s\to \mathtt{c}/\mathtt{xmax}\ /.\ \alpha\to\mathtt{ymax}/\mathtt{xmax}\ /.\\
&\quad \mathtt{xmax}\to\mathtt{tryxmax}\ /.\ \mathtt{ymax}\to\mathtt{tryymax}\ /.\ \mathtt{f}\to\mathtt{tryf},\ \mathtt{c}\ge 0\Big]
\end{aligned}
$$

Out[ ]=

$$
\begin{aligned}
&\frac{1}{60}\Bigg(8+2\sqrt{2}+(1+c)^5-\\
&\left(1-3c+c^2\right)\sqrt{2-2c+c^2}\left(-1-c+c^2\right)+2\sqrt{1+c^2}\left(-1-c+c^2\right)\left(-1+c+c^2\right)-\\
&\left(-1+c+c^2\right)\sqrt{2+2c+c^2}\left(1+3c+c^2\right)-2\left(2+c^5\right)+\mathrm{Abs}[-1+c]^5+20\,\mathrm{Log}\left[1+\sqrt{2}\right]+\\
&\frac{5}{4}\,\mathrm{Log}\Big[\frac{1}{\left(1+\sqrt{2}\right)^8}\left(1+\sqrt{1+(-1+c)^2}\right)^{2\,(-1+c)^4}\left(-1+\sqrt{1+(-1+c)^2}+c\right)^{2\,(-1+c)}\\
&\quad\left(\frac{c}{1+\sqrt{1+c^2}}\right)^{4c^4}\left(c+\sqrt{1+c^2}\right)^{-4c}\left(\frac{1+c}{1+\sqrt{1+(1+c)^2}}\right)^{-2\,(1+c)^4}\\
&\quad\left(1+c+\sqrt{1+(1+c)^2}\right)^{2\,(1+c)}\mathrm{Abs}[-1+c]^{-2\,(-1+c)^4}\Big]\Bigg)
\end{aligned}
$$

In[∘]:=
```
For[t = 1, t ≤ Length[tab], t++,
 tryc = (tryV tab[[t]])/(1 - tab[[t]]);
 mean = N[Limit[meanTEMP, c → tryc], 20];
 sqH = (xmax^2 + ymax^2)/6 + c^2 f /. xmax → tryxmax /. ymax → tryymax /. f → tryf /. c → tryc;

 CVherdintC[tryxmax, tab[[t]]] = N[Sqrt[sqH - mean^2] / mean, 20];
 CVindintC[trymax, tab[[t]]] = N[Sqrt[sqI[trymax, tryc, tryf] - mean^2] / mean, 20];
 Print[{CVherdintC[trymax, tab[[t]]], CVindintC[trymax, tab[[t]]]}]
]
```

```
{0.4755049270417650133, 0.16116}
{0.4801301308897845306, 0.174868}
{0.4942647559954516806, 0.196181}
{0.5124231934169125522, 0.197843}
{0.5354324213836169643, 0.178548}
{0.6169607285366896165, 0.130382}
{0.7075649576679686667, 0.0933431}
{0.7845745273644130115, 0.0663114}
{0.8467525018809703436, 0.0461015}
{0.8968935093634621468, 0.0305338}
{0.9377919470044052127, 0.018212}
{0.9716244129419319288, 0.00823386}
```

In[∘]:=
```
intplottingherdC = Table[{tab[[t]], CVherdintC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[∘]=

$$\left\{\{0, 0.4755049270417650133\}, \left\{\frac{1}{20}, 0.4801301308897845306\right\}, \left\{\frac{1}{10}, 0.4942647559954516806\right\}, \left\{\frac{3}{20}, 0.5124231934169125522\right\}, \left\{\frac{1}{5}, 0.5354324213836169643\right\}, \left\{\frac{3}{10}, 0.6169607285366896165\right\}, \left\{\frac{2}{5}, 0.7075649576679686667\right\}, \left\{\frac{1}{2}, 0.7845745273644130115\right\}, \left\{\frac{3}{5}, 0.8467525018809703436\right\}, \left\{\frac{7}{10}, 0.8968935093634621468\right\}, \left\{\frac{4}{5}, 0.9377919470044052127\right\}, \left\{\frac{9}{10}, 0.9716244129419319288\right\}\right\}$$

In[ ]:=
```
intplottingindC = Table[{tab[[t]], CVindintC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```
Out[ ]=

$$\left\{\{0, 0.16116\}, \left\{\frac{1}{20}, 0.174868\right\}, \left\{\frac{1}{10}, 0.196181\right\}, \left\{\frac{3}{20}, 0.197843\right\}, \left\{\frac{1}{5}, 0.178548\right\}, \left\{\frac{3}{10}, 0.130382\right\}, \left\{\frac{2}{5}, 0.0933431\right\}, \left\{\frac{1}{2}, 0.0663114\right\}, \left\{\frac{3}{5}, 0.0461015\right\}, \left\{\frac{7}{10}, 0.0305338\right\}, \left\{\frac{4}{5}, 0.018212\right\}, \left\{\frac{9}{10}, 0.00823386\right\}\right\}$$

Panel D: Uneven herd split [midpoint at c=4]

In[ ]:=
```
SeedRandom[43 812];
```

We seek to examine a range of distances, c, between the peaks. To do so, we use c/(V+c) along the x-axis, which allows us to vary c from 0 at x=0 to infinity at x=1, within a single plot (we use V = 4 so that patches 4 sd apart are half-way along the plot). To convert the x-axis to c, we use:

In[ ]:=
```
tryV = 4;
```

In[ ]:=
```
Solve[c / (tryV + c) == x, c]
```
Out[ ]=

$$\left\{\left\{c \to -\frac{4\,x}{-1 + x}\right\}\right\}$$

In[ ]:=
```
tab =
  Join[Table[i, {i, 0 / 10, 4 / 20, 1 / 20}], Table[i, {i, 3 / 10, 9 / 10, 1 / 10}]] // Flatten
```
Out[ ]=

$$\left\{0, \frac{1}{20}, \frac{1}{10}, \frac{3}{20}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{9}{10}\right\}$$

In[ ]:=
```
numind1 = 90;
numind2 = 10;
numind = numind1 + numind2;
```

In[ ]:=
```
tryf = 2 (numind2 / numind) (numind1 / numind)
   (*Unequal split with 90% in one patch so f = 0.18*)
```
Out[ ]=

$$\frac{9}{50}$$

In[ ]:=
```
trymax = 1;
tryxmax = trymax;
tryymax = trymax;
```

```
In[•]:= For[t = 1, t ≤ Length[tab], t++,
  table = Join[Table[RandomReal[UniformDistribution[{{0, tryxmax}, {0, tryymax}}]],
     {i, 1, numind1}], Table[RandomReal[UniformDistribution[
        {{tryV tab[[t]]/(1 - tab[[t]]), tryV tab[[t]]/(1 - tab[[t]]) + tryxmax}, {0, tryymax}}]], {i, 1, numind2}]];
  disttable = Flatten[
    Table[EuclideanDistance[table[[i]], table[[j]]], {i, 1, numind}, {j, i + 1, numind}]];
  CVherdsimC[tryxmax, tab[[t]]] = StandardDeviation[disttable]/Mean[disttable];
  disttable2 = Table[
    1/(Length[table] - 1) (Sum[EuclideanDistance[table[[i]], table[[j]]], {j, 1, i - 1}] + Sum[
        EuclideanDistance[table[[i]], table[[j]]], {j, i + 1, numind}]), {i, 1, numind}];
  CVindsimC[tryxmax, tab[[t]]] = StandardDeviation[disttable2]/Mean[disttable2]
  ]

In[•]:= forplottingherdD = Table[{tab[[t]], CVherdsimC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[•]=

$$\left\{\{0, 0.474266\}, \left\{\frac{1}{20}, 0.470027\right\}, \left\{\frac{1}{10}, 0.484829\right\}, \left\{\frac{3}{20}, 0.483136\right\}, \left\{\frac{1}{5}, 0.57089\right\}, \left\{\frac{3}{10}, 0.798745\right\}, \left\{\frac{2}{5}, 0.960725\right\}, \left\{\frac{1}{2}, 1.22076\right\}, \left\{\frac{3}{5}, 1.41601\right\}, \left\{\frac{7}{10}, 1.58171\right\}, \left\{\frac{4}{5}, 1.81007\right\}, \left\{\frac{9}{10}, 1.96378\right\}\right\}$$

```
In[•]:= forplottingindD = Table[{tab[[t]], CVindsimC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[•]=

$$\left\{\{0, 0.16428\}, \left\{\frac{1}{20}, 0.160544\right\}, \left\{\frac{1}{10}, 0.183164\right\}, \left\{\frac{3}{20}, 0.191312\right\}, \left\{\frac{1}{5}, 0.284328\right\}, \left\{\frac{3}{10}, 0.470485\right\}, \left\{\frac{2}{5}, 0.583888\right\}, \left\{\frac{1}{2}, 0.761376\right\}, \left\{\frac{3}{5}, 0.887567\right\}, \left\{\frac{7}{10}, 0.998112\right\}, \left\{\frac{4}{5}, 1.1435\right\}, \left\{\frac{9}{10}, 1.24055\right\}\right\}$$

```
In[•]:= maxc = 1;
```

Below, we'll want the ratio of $\frac{\text{CVind}}{\text{CVherd}}$ in the limit for very distant patches when the patches are uneven:

```
In[•]:= limratio =
  Limit[CVindfarsplit/CVherdC /. xmax → tryxmax /. ymax → tryymax /. f → tryf, c → Infinity] // N
```

Out[•]=

```
0.624695
```

We also plot CVherd in the limit for very distant patches when the patches are uneven:

In[∘]:=
```
limratioHERD =
 Limit[CVherdC /. xmax → tryxmax /. ymax → tryymax /. f → tryf, c → Infinity] // N
```
Out[∘]=

2.13437

Choosing x-axis tick positions:

In[∘]:=
$$\left\{x, \frac{\text{tryV}\,x}{1-x}\right\} \text{ /. x → tab // Transpose}$$

Out[∘]=
$$\left\{\{0, 0\}, \left\{\frac{1}{20}, \frac{4}{19}\right\}, \left\{\frac{1}{10}, \frac{4}{9}\right\}, \left\{\frac{3}{20}, \frac{12}{17}\right\}, \left\{\frac{1}{5}, 1\right\}, \left\{\frac{3}{10}, \frac{12}{7}\right\}, \left\{\frac{2}{5}, \frac{8}{3}\right\}, \left\{\frac{1}{2}, 4\right\}, \left\{\frac{3}{5}, 6\right\}, \left\{\frac{7}{10}, \frac{28}{3}\right\}, \left\{\frac{4}{5}, 16\right\}, \left\{\frac{9}{10}, 36\right\}\right\}$$

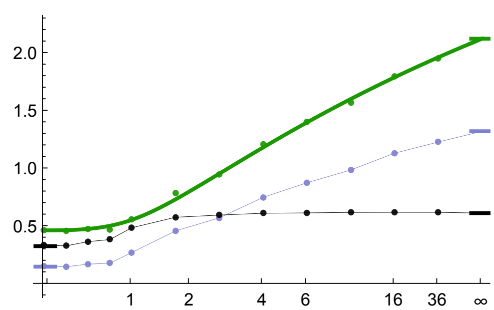
In[∘]:=
$$\text{Solve}\left[\frac{\text{tryV}\,x}{1-x} == 2, x\right]$$

Out[∘]=
$$\left\{\left\{x \to \frac{1}{3}\right\}\right\}$$

In[∘]:=
```
xticks = {{0.01, "0"}, {1 / 5, "1"}, {1 / 3, "2"},
  {1 / 2, "4"}, {3 / 5, "6"}, {4 / 5, "16"}, {9 / 10, "36"}, {1, "∞"}}
```
Out[∘]=
$$\left\{\{0.01, 0\}, \left\{\frac{1}{5}, 1\right\}, \left\{\frac{1}{3}, 2\right\}, \left\{\frac{1}{2}, 4\right\}, \left\{\frac{3}{5}, 6\right\}, \left\{\frac{4}{5}, 16\right\}, \left\{\frac{9}{10}, 36\right\}, \{1, \infty\}\right\}$$

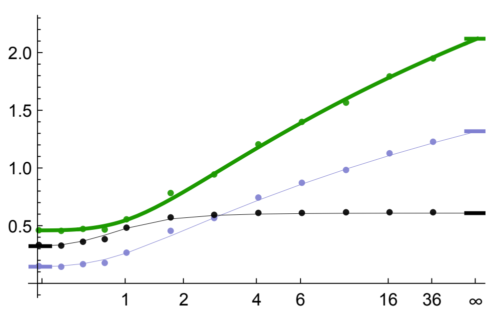
In[ ]:=
```
Show[
 Plot[CVherdC /. xmax → trymax /. ymax → trymax /. f → tryf /. c → (tryV x)/(1 - x), {x, 0, maxc},
  WorkingPrecision → 50, PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
 Plot[limratioHERD, {x, 0.975, 1.05},
  PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
 Plot[CVind /. xmax → tryxmax /. ymax → tryymax,
  {x, -0.025, 0.025}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
 Plot[CVindfarsplit /. f → tryf,
  {x, 0.975, 1.05}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
 Plot[CVind/CVherd /. xmax → tryxmax /. ymax → tryymax,
  {x, -0.025, 0.025}, PlotStyle → Black],
 Plot[limratio, {x, 0.975, 1.05}, PlotStyle → Black],
 ListPlot[forplottingherdD, PlotStyle → RGBColor[0.11, 0.6, 0.]],
 ListPlot[forplottingindD, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
 ListPlot[Join[forplottingindD, {{1, CVindfarsplit /. f → tryf}}],
  Joined → True, PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}],
 ListPlot[Table[{forplottingherdD[[i, 1]], forplottingindD[[i, 2]]/forplottingherdD[[i, 2]]},
   {i, 1, Length[forplottingindD]}], PlotStyle → {Black}],
 ListPlot[Join[Table[{forplottingherdD[[i, 1]], forplottingindD[[i, 2]]/forplottingherdD[[i, 2]]},
    {i, 1, Length[forplottingindD]}], {{1, limratio}}],
  Joined → True, PlotStyle → {Thin, Black}],
 AxesOrigin → {0, 0},
 PlotRange → {{0, maxc}, {0, 2.2}}, Ticks → {xticks, Automatic}, ImageSize → 250
]
```

Out[ ]=



In[ ]:=
```
plotD = Show[
  Plot[CVherdC /. xmax → trymax /. ymax → trymax /. f → tryf /. c → (tryV x)/(1 - x), {x, 0, maxc},
   WorkingPrecision → 50, PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
  Plot[limratioHERD, {x, 0.975, 1.05},
   PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
  Plot[CVind /. xmax → tryxmax /. ymax → tryymax,
   {x, -0.025, 0.025}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVindfarsplit /. f → tryf,
   {x, 0.975, 1.05}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVind/CVherd /. xmax → tryxmax /. ymax → tryymax,
   {x, -0.025, 0.025}, PlotStyle → Black],
  Plot[limratio, {x, 0.975, 1.05}, PlotStyle → Black],
  ListPlot[forplottingherdD, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[forplottingindD, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[Join[intplottingindD, {{1, CVindfarsplit /. f → tryf}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}],
  ListPlot[Table[{forplottingherdD[[i, 1]], forplottingindD[[i, 2]]/forplottingherdD[[i, 2]]},
    {i, 1, Length[forplottingindD]}], PlotStyle → {Black}],
  ListPlot[Join[Table[{intplottingindD[[i, 1]], intplottingindD[[i, 2]]/intplottingherdD[[i, 2]]},
     {i, 1, Length[forplottingindD]}], {{1, limratio}}],
   Joined → True, PlotStyle → {Thin, Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, maxc}, {0, 2.2}}, Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[ ]=



Panel D: Numerical integration [SLOW - Don’t enter]

For comparison, we can numerically integrate. Integration was inaccurate and so we first simplify and then take the limit as c approaches its numerical value:

In[ ]:=
```
meanTEMP = Simplify[
  soln + (f xmax)/(30 α^2) ((((1 + c_s)^5 - 2 (1 + α^5 + c_s^5)) + 2 H (1 - 3 α^2 + α^4) + 2 H2 (α^2 - α c_s - c_s^2) (α^2 +
          α c_s - c_s^2) - H1 (-1 + α + α^2 - (-2 + α) c_s - c_s^2) (-1 - α + α^2 + (2 + α) c_s - c_s^2) +
        H3 (-1 + α + α^2 + (-2 + α) c_s - c_s^2) (1 + α - α^2 + (2 + α) c_s + c_s^2) + H4^5) +
      5/4 α Log[(H + α)^-4 ((H1 + α)/H4)^(2 (-1+c_s)^4) (c_s/(H2 + α))^(4 c_s^4) ((H3 + α)/(1 + c_s))^(2 (1+c_s)^4)
         (((α/(1 + H))^2 (1 + H3 + c_s)/(-1 + H1 + c_s))^(2 α^3) ((-1 + H1 + c_s) (1 + H3 + c_s)/(H2 + c_s)^2)^(2 α^3 c_s)]) /.
    {H → Sqrt[1 + α^2], H1 → Sqrt[α^2 + (-1 + c_s)^2], H2 → Sqrt[α^2 + c_s^2], H3 → Sqrt[α^2 + (1 + c_s)^2],
     H4 → Sqrt[(-1 + c_s)^2]} /. c_s → c / xmax /. α → ymax / xmax /.
   xmax → tryxmax /. ymax → tryymax /. f → tryf, c ≥ 0]
```

Out[ ]=

$$\frac{2}{15}+\frac{\sqrt{2}}{15}+\frac{1}{3}\text{Log}\left[1+\sqrt{2}\right]+$$
$$\frac{3}{500}\Bigg(-2\sqrt{2}+(1+c)^5-\left(1-3c+c^2\right)\sqrt{2-2c+c^2}\left(-1-c+c^2\right)+2\sqrt{1+c^2}\left(-1-c+c^2\right)$$
$$\left(-1+c+c^2\right)-\left(-1+c+c^2\right)\sqrt{2+2c+c^2}\left(1+3c+c^2\right)-2\left(2+c^5\right)+\text{Abs}[-1+c]^5+$$
$$\frac{5}{4}\text{Log}\Bigg[\frac{1}{\left(1+\sqrt{2}\right)^8}\left(1+\sqrt{1+(-1+c)^2}\right)^{2(-1+c)^4}\left(-1+\sqrt{1+(-1+c)^2}+c\right)^{2(-1+c)}$$
$$\left(\frac{c}{1+\sqrt{1+c^2}}\right)^{4c^4}\left(c+\sqrt{1+c^2}\right)^{-4c}\left(\frac{1+c}{1+\sqrt{1+(1+c)^2}}\right)^{-2(1+c)^4}$$
$$\left(1+c+\sqrt{1+(1+c)^2}\right)^{2(1+c)}\text{Abs}[-1+c]^{-2(-1+c)^4}\Bigg]\Bigg)$$

In[ ]:=
```
For[t = 1, t ≤ Length[tab], t++,
 tryc = (tryV tab[[t]])/(1 - tab[[t]]);
 mean = N[Limit[meanTEMP, c → tryc], 20];
 sqH = (xmax^2 + ymax^2)/6 + c^2 f /. xmax → tryxmax /. ymax → tryymax /. f → tryf /. c → tryc;

 CVherdintC[tryxmax, tab[[t]]] = N[Sqrt[sqH - mean^2] / mean, 20];
 CVindintC[tryxmax, tab[[t]]] = N[Sqrt[sqI[tryxmax, tryc, tryf] - mean^2] / mean, 20];
 Print[{CVherdintC[trymax, tab[[t]]], CVindintC[tryxmax, tab[[t]]]}]
]
```

```
{0.4755049270417650133, 0.16116}
{0.4775163517603715289, 0.167081}
{0.4879628917409243562, 0.186761}
{0.5150387657378148952, 0.22388}
{0.5664045746687243068, 0.279918}
{0.7438962139471958230, 0.427474}
{0.9666853844938108845, 0.585175}
{1.1933909315361942447, 0.736027}
{1.4099458021030423966, 0.87613}
{1.6123483495265122205, 1.00514}
{1.8000466411358438406, 1.12372}
{1.9737015151095719238, 1.2328}
```

In[∘]:=
```
intplottingherdD = Table[{tab⟦t⟧, CVherdintC[tryxmax, tab⟦t⟧]}, {t, 1, Length[tab]}]
```

Out[∘]=

$\left\{\{0, 0.4755049270417650133\}, \left\{\frac{1}{20}, 0.4775163517603715289\right\}, \left\{\frac{1}{10}, 0.4879628917409243562\right\}, \left\{\frac{3}{20}, 0.5150387657378148952\right\}, \left\{\frac{1}{5}, 0.5664045746687243068\right\}, \left\{\frac{3}{10}, 0.7438962139471958230\right\}, \left\{\frac{2}{5}, 0.9666853844938108845\right\}, \left\{\frac{1}{2}, 1.1933909315361942447\right\}, \left\{\frac{3}{5}, 1.4099458021030423966\right\}, \left\{\frac{7}{10}, 1.6123483495265122205\right\}, \left\{\frac{4}{5}, 1.8000466411358438406\right\}, \left\{\frac{9}{10}, 1.9737015151095719238\right\}\right\}$

In[∘]:=
```
intplottingindD = Table[{tab⟦t⟧, CVindintC[tryxmax, tab⟦t⟧]}, {t, 1, Length[tab]}]
```

Out[∘]=

$\left\{\{0, 0.16116\}, \left\{\frac{1}{20}, 0.167081\right\}, \left\{\frac{1}{10}, 0.186761\right\}, \left\{\frac{3}{20}, 0.22388\right\}, \left\{\frac{1}{5}, 0.279918\right\}, \left\{\frac{3}{10}, 0.427474\right\}, \left\{\frac{2}{5}, 0.585175\right\}, \left\{\frac{1}{2}, 0.736027\right\}, \left\{\frac{3}{5}, 0.87613\right\}, \left\{\frac{7}{10}, 1.00514\right\}, \left\{\frac{4}{5}, 1.12372\right\}, \left\{\frac{9}{10}, 1.2328\right\}\right\}$

Panel E: Even herd split with elongated patches [ymax short]

In[∘]:=
```
SeedRandom[59 238];
```

We seek to examine a range of distances, c, between the peaks, now allowing each patch to be elongated. To do so, we use c/(V+c) along the x-axis, which allows us to vary c from 0 at x=0 to infinity at x=1, within a single plot (we use V = 4 so that patches 4 sd apart are half-way along the plot). To convert the x-axis to c, we use:

```
In[ ]:= tryV = 4;

In[ ]:= Solve[c / (tryV + c) == x, c]
Out[ ]=
```

$$\left\{\left\{c \to -\frac{4\,x}{-1 + x}\right\}\right\}$$

```
In[ ]:= tab =
         Join[Table[i, {i, 0 / 10, 2 / 10, 1 / 20}], Table[i, {i, 3 / 10, 9 / 10, 1 / 10}]] // Flatten
Out[ ]=
```

$$\left\{0, \frac{1}{20}, \frac{1}{10}, \frac{3}{20}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{9}{10}\right\}$$

```
In[ ]:= numind1 = 50;
        numind2 = 50;
        numind = numind1 + numind2;

In[ ]:= tryf = 2 (numind2 / numind) (numind1 / numind)
          (*Equal split with 50% in one patch so f = 1/2*)
Out[ ]=
```

$$\frac{1}{2}$$

```
In[ ]:= tryxmax = 1;
        tryymax = 1 / 5;

In[ ]:= For[t = 1, t ≤ Length[tab], t++,
         table = Join[Table[RandomReal[UniformDistribution[{{0, tryxmax}, {0, tryymax}}]],
             {i, 1, numind1}], Table[RandomReal[UniformDistribution[
               {{(tryV tab[[t]])/(1 - tab[[t]]), (tryV tab[[t]])/(1 - tab[[t]]) + tryxmax}, {0, tryymax}}]], {i, 1, numind2}]];
         disttable = Flatten[
           Table[EuclideanDistance[table[[i]], table[[j]]], {i, 1, numind}, {j, i + 1, numind}]];
         CVherdsimC[tryxmax, tab[[t]]] = StandardDeviation[disttable] / Mean[disttable];
         disttable2 = Table[
           1/(Length[table] - 1) (Sum[EuclideanDistance[table[[i]], table[[j]]], {j, 1, i - 1}] + Sum[
               EuclideanDistance[table[[i]], table[[j]]], {j, i + 1, numind}]), {i, 1, numind}];
         CVindsimC[tryxmax, tab[[t]]] = StandardDeviation[disttable2] / Mean[disttable2]
        ]
```

In[◦]:=
```
forplottingherdE = Table[{tab⟦t⟧, CVherdsimC[tryxmax, tab⟦t⟧]}, {t, 1, Length[tab]}]
```
Out[◦]=

$$\left\{\{0, 0.65269\}, \left\{\frac{1}{20}, 0.651785\right\}, \left\{\frac{1}{10}, 0.681042\right\}, \left\{\frac{3}{20}, 0.681407\right\}, \left\{\frac{1}{5}, 0.679038\right\}, \left\{\frac{3}{10}, 0.722538\right\}, \left\{\frac{2}{5}, 0.792907\right\}, \left\{\frac{1}{2}, 0.853867\right\}, \left\{\frac{3}{5}, 0.890349\right\}, \left\{\frac{7}{10}, 0.91603\right\}, \left\{\frac{4}{5}, 0.94696\right\}, \left\{\frac{9}{10}, 0.97109\right\}\right\}$$

In[◦]:=
```
forplottingindE = Table[{tab⟦t⟧, CVindsimC[tryxmax, tab⟦t⟧]}, {t, 1, Length[tab]}]
```
Out[◦]=

$$\left\{\{0, 0.215848\}, \left\{\frac{1}{20}, 0.243695\right\}, \left\{\frac{1}{10}, 0.295809\right\}, \left\{\frac{3}{20}, 0.267889\right\}, \left\{\frac{1}{5}, 0.229122\right\}, \left\{\frac{3}{10}, 0.147649\right\}, \left\{\frac{2}{5}, 0.0951801\right\}, \left\{\frac{1}{2}, 0.0614867\right\}, \left\{\frac{3}{5}, 0.0379738\right\}, \left\{\frac{7}{10}, 0.0323404\right\}, \left\{\frac{4}{5}, 0.0175789\right\}, \left\{\frac{9}{10}, 0.00834784\right\}\right\}$$

Below, we’ll want the ratio of $\frac{\text{CVind}}{\text{CVherd}}$ in the limit for very distant patches when the patches are uneven:

In[◦]:=
```
limratio =
  Limit[CVindfarsplit / CVherdC /. xmax → tryxmax /. ymax → tryymax /. f → tryf, c → Infinity]
```
Out[◦]=

0

We also plot CVherd in the limit for very distant patches when the patches are uneven:

In[◦]:=
```
CVherdfar = Limit[CVherdC /. xmax → tryxmax /. ymax → tryymax /. f → tryf, c → Infinity]
```
Out[◦]=

1

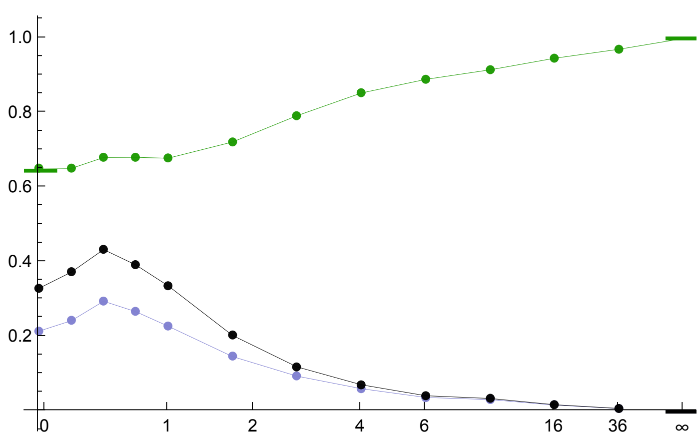
In[◦]:=
```
maxc = 1;
```

In[◦]:=
```
xticks = {{0.01, "0"}, {1/5, "1"}, {1/3, "2"},
   {1/2, "4"}, {3/5, "6"}, {4/5, "16"}, {9/10, "36"}, {1, "∞"}}
```
Out[◦]=

$$\left\{\{0.01, 0\}, \left\{\frac{1}{5}, 1\right\}, \left\{\frac{1}{3}, 2\right\}, \left\{\frac{1}{2}, 4\right\}, \left\{\frac{3}{5}, 6\right\}, \left\{\frac{4}{5}, 16\right\}, \left\{\frac{9}{10}, 36\right\}, \{1, \infty\}\right\}$$

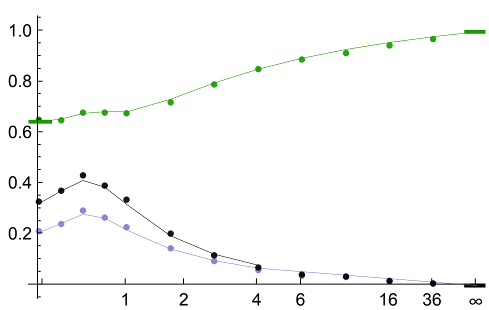
In[ ]:=
```
plotE = Show[
  Plot[CVherdfar, {x, 0.975, 1.05},
   PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
  Plot[CVindfarsplit /. xmax → tryxmax /. ymax → tryymax /. f → tryf,
   {x, 0.975, 1.05}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVherd /. xmax → tryxmax /. ymax → tryymax,
   {x, -0.025, 0.025}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  Plot[limratio, {x, 0.975, 1.05}, PlotStyle → Black],
  ListPlot[forplottingherdE, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[Join[forplottingherdE, {{1, CVherdfar}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.11, 0.6, 0.]}],
  ListPlot[forplottingindE, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[forplottingindE,
   Joined → True, PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}],
  ListPlot[Table[{forplottingherdE[[i, 1]], forplottingindE[[i, 2]] / forplottingherdE[[i, 2]]},
    {i, 1, Length[forplottingindE]}], PlotStyle → {Black}],
  ListPlot[Table[{forplottingherdE[[i, 1]], forplottingindE[[i, 2]] / forplottingherdE[[i, 2]]},
    {i, 1, Length[forplottingindE]}], Joined → True, PlotStyle → {Thin, Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, maxc}, {0, 1}}, Ticks → {xticks, Automatic}
 ]
```

Out[ ]=



In[ ]:=
```
plotE = Show[
  Plot[CVherdfar, {x, 0.975, 1.05},
   PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
  Plot[CVindfarsplit /. xmax → tryxmax /. ymax → tryymax /. f → tryf,
   {x, 0.975, 1.05}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVherd /. xmax → tryxmax /. ymax → tryymax,
   {x, -0.025, 0.025}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  Plot[limratio, {x, 0.975, 1.05}, PlotStyle → Black],
  ListPlot[forplottingherdE, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[Join[intplottingherdE, {{1, CVherdfar}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.11, 0.6, 0.]}],
  ListPlot[forplottingindE, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[Join[intplottingindE, {{1, CVindfarsplit /. f → tryf}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}],
  ListPlot[Table[{forplottingherdE[[i, 1]], forplottingindE[[i, 2]]/forplottingherdE[[i, 2]]},
    {i, 1, Length[forplottingindE]}], PlotStyle → {Black}],
  ListPlot[Table[{intplottingherdE[[i, 1]], intplottingindE[[i, 2]]/intplottingherdE[[i, 2]]},
    {i, 1, Length[intplottingindE]}], Joined → True, PlotStyle → {Thin, Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, maxc}, {0, 1}}, Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[ ]=



Panel E: Numerical integration [SLOW - Don’t enter]

For comparison, we can numerically integrate. Integration was inaccurate and so we first simplify and then take the limit as c approaches its numerical value:

In[ ]:=

$$\text{meanTEMP} = \text{Simplify}\Big[\text{soln} + \frac{\text{f xmax}}{30\ \alpha^2}\Big(\Big(\Big((1+c_s)^5 - 2\left(1+\alpha^5+c_s^5\right)\Big) + 2\,\text{H}\left(1-3\alpha^2+\alpha^4\right) + 2\,\text{H2}\left(\alpha^2-\alpha c_s-c_s^2\right)\left(\alpha^2+\alpha c_s-c_s^2\right) - \text{H1}\left(-1+\alpha+\alpha^2-(-2+\alpha)\,c_s-c_s^2\right)\left(-1-\alpha+\alpha^2+(2+\alpha)\,c_s-c_s^2\right) + \text{H3}\left(-1+\alpha+\alpha^2+(-2+\alpha)\,c_s-c_s^2\right)\left(1+\alpha-\alpha^2+(2+\alpha)\,c_s+c_s^2\right) + \text{H4}^5\Big) + \frac{5}{4}\,\alpha\,\text{Log}\Big[(\text{H}+\alpha)^{-4}\left(\frac{\text{H1}+\alpha}{\text{H4}}\right)^{2\,(-1+c_s)^4}\left(\frac{c_s}{\text{H2}+\alpha}\right)^{4\,c_s^4}\left(\frac{\text{H3}+\alpha}{1+c_s}\right)^{2\,(1+c_s)^4}\left(\left(\frac{\alpha}{1+\text{H}}\right)^2\frac{1+\text{H3}+c_s}{-1+\text{H1}+c_s}\right)^{2\,\alpha^3}\left(\frac{(-1+\text{H1}+c_s)\,(1+\text{H3}+c_s)}{(\text{H2}+c_s)^2}\right)^{2\,\alpha^3\,c_s}\Big]\Big)\ /.\ \left\{\text{H}\to\sqrt{1+\alpha^2},\ \text{H1}\to\sqrt{\alpha^2+(-1+c_s)^2},\ \text{H2}\to\sqrt{\alpha^2+c_s^2},\ \text{H3}\to\sqrt{\alpha^2+(1+c_s)^2},\ \text{H4}\to\sqrt{(-1+c_s)^2}\right\}\ /.\ c_s\to\text{c}/\text{xmax}\ /.\ \alpha\to\text{ymax}/\text{xmax}\ /.\ \text{xmax}\to\text{tryxmax}\ /.\ \text{ymax}\to\text{tryymax}\ /.\ \text{f}\to\text{tryf},\ \text{c}\geq 0\Big]$$

Out[ ]=

$$\frac{1042}{625} - \frac{551\sqrt{26}}{1875} + \frac{1}{150}\left(125\,\text{Log}\Big[\frac{1}{5}\left(1+\sqrt{26}\right)\Big] + \text{Log}\Big[5+\sqrt{26}\Big]\right) + \frac{5}{12}\Bigg(\frac{1102\sqrt{26}}{3125} + (1+c)^5 - \frac{\left(29-55c+25c^2\right)\sqrt{26-50c+25c^2}\left(19-45c+25c^2\right)}{3125} + \frac{2\sqrt{1+25c^2}\left(-1-5c+25c^2\right)\left(-1+5c+25c^2\right)}{3125} - 2\left(\frac{3126}{3125}+c^5\right) + \left(-\frac{19}{25}-\frac{9c}{5}-c^2\right)\left(\frac{29}{25}+\frac{11c}{5}+c^2\right)\sqrt{\frac{1}{25}+(1+c)^2} + \text{Abs}[-1+c]^5 + \frac{1}{4}\,\text{Log}\Big[\Big(5^{-24c^2}\left(\frac{c}{1+\sqrt{1+25c^2}}\right)^{4c^4}\left(5c+\sqrt{1+25c^2}\right)^{-4c/125}\left(1+\sqrt{26-50c+25c^2}\right)^{2\,(-1+c)^4}\left(-5+5c+\sqrt{26-50c+25c^2}\right)^{\frac{2}{125}(-1+c)}\left(\frac{1+c}{1+\sqrt{26+50c+25c^2}}\right)^{-2\,(1+c)^4}\left(5+5c+\sqrt{26+50c+25c^2}\right)^{\frac{2\,(1+c)}{125}}\text{Abs}[-1+c]^{-2\,(-1+c)^4}\Big)\Big/\left(\left(1+\sqrt{26}\right)^4\left(5+\sqrt{26}\right)^{4/125}\right)\Big]\Bigg)$$

In[∘]:=
```
For[t = 1, t ≤ Length[tab], t++,
  tryc = (tryV tab[[t]])/(1 - tab[[t]]);
  mean = N[Limit[meanTEMP, c → tryc], 20];
  sqH = (xmax^2 + ymax^2)/6 + c^2 f /. xmax → tryxmax /. ymax → tryymax /. f → tryf /. c → tryc;

  CVherdintC[tryxmax, tab[[t]]] = N[Sqrt[sqH - mean^2] / mean, 20];
  CVindintC[tryxmax, tab[[t]]] =
   N[Sqrt[sqI[tryxmax, tryymax, tryc, tryf] - mean^2] / mean, 20];
  Print[{CVherdintC[tryxmax, tab[[t]]], CVindintC[tryxmax, tab[[t]]]}]
 ]
```

```
{0.6456877909802461191, 0.209022}
{0.6564504908685524942, 0.244649}
{0.6785197070168401252, 0.28109}
{0.6853889458045307584, 0.265716}
{0.6847161800752769921, 0.218766}
{0.7343507160965991353, 0.143969}
{0.7986818887932470375, 0.0986395}
{0.8528021120433971568, 0.0684358}
{0.8958894823253643301, 0.0468901}
{0.9302599510489040393, 0.0307513}
{0.9580661386616496218, 0.0182129}
{0.9809238877024275145, 0.0081917}
```

In[∘]:=
```
intplottingherdE = Table[{tab[[t]], CVherdintC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[∘]=

$$\left\{\{0, 0.6456877909802461191\}, \left\{\frac{1}{20}, 0.6564504908685524942\right\}, \left\{\frac{1}{10}, 0.6785197070168401252\right\}, \left\{\frac{3}{20}, 0.6853889458045307584\right\}, \left\{\frac{1}{5}, 0.6847161800752769921\right\}, \left\{\frac{3}{10}, 0.7343507160965991353\right\}, \left\{\frac{2}{5}, 0.7986818887932470375\right\}, \left\{\frac{1}{2}, 0.8528021120433971568\right\}, \left\{\frac{3}{5}, 0.8958894823253643301\right\}, \left\{\frac{7}{10}, 0.9302599510489040393\right\}, \left\{\frac{4}{5}, 0.9580661386616496218\right\}, \left\{\frac{9}{10}, 0.9809238877024275145\right\}\right\}$$

In[ ]:=
```
intplottingindE = Table[{tab⟦t⟧, CVindintC[tryxmax, tab⟦t⟧]}, {t, 1, 8}]
```
Out[ ]=

$$\left\{\{0, 0.209022\}, \left\{\frac{1}{20}, 0.244649\right\}, \left\{\frac{1}{10}, 0.28109\right\}, \left\{\frac{3}{20}, 0.265716\right\}, \left\{\frac{1}{5}, 0.218766\right\}, \left\{\frac{3}{10}, 0.143969\right\}, \left\{\frac{2}{5}, 0.0986395\right\}, \left\{\frac{1}{2}, 0.0684358\right\}\right\}$$

Panel F: Uneven herd split with elongated patches [ymax short]

In[ ]:=
```
SeedRandom[66127];
```

We seek to examine a range of distances, c, between the peaks, now allowing each patch to be elongated. To do so, we use c/(V+c) along the x-axis, which allows us to vary c from 0 at x=0 to infinity at x=1, within a single plot (we use V = 4 so that patches 4 sd apart are half-way along the plot). To convert the x-axis to c, we use:

In[ ]:=
```
tryV = 4;
```

In[ ]:=
```
Solve[c / (tryV + c) == x, c]
```
Out[ ]=

$$\left\{\left\{c \to -\frac{4\,x}{-1 + x}\right\}\right\}$$

In[ ]:=
```
tab = Join[Table[i, {i, 0 / 10, 9 / 10, 1 / 10}]] // Flatten
```
Out[ ]=

$$\left\{0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{9}{10}\right\}$$

In[ ]:=
```
numind1 = 90;
numind2 = 10;
numind = numind1 + numind2;
```

In[ ]:=
```
tryf = 2 (numind2 / numind) (numind1 / numind)
  (*Unequal split with 90% in one patch so f = 0.18*)
```
Out[ ]=

$$\frac{9}{50}$$

In[ ]:=
```
tryxmax = 1;
tryymax = 1 / 5;
```

In[ ]:=
```
For[t = 1, t ≤ Length[tab], t++,
 table = Join[Table[RandomReal[UniformDistribution[{{0, tryxmax}, {0, tryymax}}]],
    {i, 1, numind1}], Table[RandomReal[UniformDistribution[
      {{tryV tab[[t]]/(1 - tab[[t]]), tryV tab[[t]]/(1 - tab[[t]]) + tryxmax}, {0, tryymax}}]], {i, 1, numind2}]];
 disttable = Flatten[
   Table[EuclideanDistance[table[[i]], table[[j]]], {i, 1, numind}, {j, i + 1, numind}]];
 CVherdsimC[tryxmax, tab[[t]]] = StandardDeviation[disttable]/Mean[disttable];
 disttable2 = Table[
   1/(Length[table] - 1) (Sum[EuclideanDistance[table[[i]], table[[j]]], {j, 1, i - 1}] + Sum[
       EuclideanDistance[table[[i]], table[[j]]], {j, i + 1, numind}]), {i, 1, numind}];
 CVindsimC[tryxmax, tab[[t]]] = StandardDeviation[disttable2]/Mean[disttable2]
]
```

In[ ]:=
```
forplottingherdF = Table[{tab[[t]], CVherdsimC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$$\left\{\{0, 0.65037\}, \left\{\frac{1}{10}, 0.663809\right\}, \left\{\frac{1}{5}, 0.793686\right\}, \left\{\frac{3}{10}, 1.0085\right\}, \left\{\frac{2}{5}, 1.21359\right\}, \left\{\frac{1}{2}, 1.4339\right\}, \left\{\frac{3}{5}, 1.60634\right\}, \left\{\frac{7}{10}, 1.73474\right\}, \left\{\frac{4}{5}, 1.88707\right\}, \left\{\frac{9}{10}, 2.01019\right\}\right\}$$

In[ ]:=
```
forplottingindF = Table[{tab[[t]], CVindsimC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$$\left\{\{0, 0.219119\}, \left\{\frac{1}{10}, 0.270121\right\}, \left\{\frac{1}{5}, 0.414427\right\}, \left\{\frac{3}{10}, 0.60206\right\}, \left\{\frac{2}{5}, 0.745494\right\}, \left\{\frac{1}{2}, 0.897708\right\}, \left\{\frac{3}{5}, 1.01159\right\}, \left\{\frac{7}{10}, 1.09366\right\}, \left\{\frac{4}{5}, 1.19192\right\}, \left\{\frac{9}{10}, 1.2694\right\}\right\}$$

Below, we'll want the ratio of $\frac{\text{CVind}}{\text{CVherd}}$ in the limit for very distant patches when the patches are uneven:

In[ ]:=
```
limratio =
 Limit[CVindfarsplit/CVherdC /. xmax → tryxmax /. ymax → tryymax /. f → tryf, c → Infinity]
```

Out[ ]=

$$\frac{400}{\sqrt{410000 - 4\,\text{Log}[2]^2 - \text{Log}[4]^2 + \text{Log}[4]\,\text{Log}[16]}}$$

We also plot CVherd in the limit for very distant patches when the patches are uneven:

In[ ]:=
```
Simplify[CVherdC /. xmax → tryxmax /. ymax → tryymax /. f → tryf, c > 1];
CVherdfar = Limit[%, c → Infinity]
```

Out[ ]=

$$\frac{\sqrt{41}}{3}$$

In[ ]:=
```
maxc = 1;
```

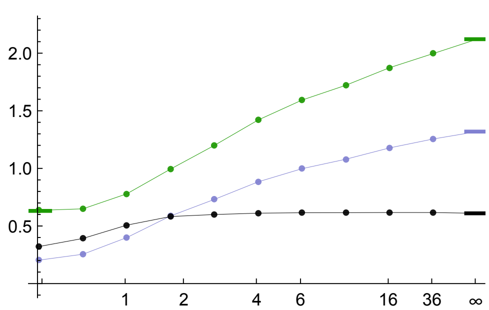
In[ ]:=
```
xticks = {{0.01, "0"}, {1 / 5, "1"}, {1 / 3, "2"},
  {1 / 2, "4"}, {3 / 5, "6"}, {4 / 5, "16"}, {9 / 10, "36"}, {1, "∞"}}
```

Out[ ]=

$$\left\{\{0.01, 0\}, \left\{\frac{1}{5}, 1\right\}, \left\{\frac{1}{3}, 2\right\}, \left\{\frac{1}{2}, 4\right\}, \left\{\frac{3}{5}, 6\right\}, \left\{\frac{4}{5}, 16\right\}, \left\{\frac{9}{10}, 36\right\}, \{1, \infty\}\right\}$$

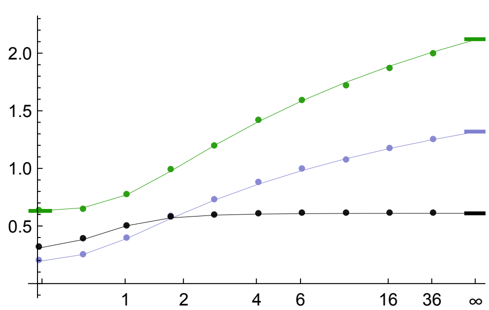
In[ ]:=
```
plotF = Show[
  Plot[CVherdfar, {x, 0.975, 1.05},
   PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
  Plot[CVindfarsplit /. f → tryf,
   {x, 0.975, 1.05}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVherd /. xmax → tryxmax /. ymax → tryymax,
   {x, -0.025, 0.025}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  Plot[limratio, {x, 0.975, 1.05}, PlotStyle → Black],
  ListPlot[forplottingherdF, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[Join[forplottingherdF, {{1, CVherdfar}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.11, 0.6, 0.]}],
  ListPlot[forplottingindF, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[Join[forplottingindF, {{1, CVindfarsplit /. f → tryf}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}],
  ListPlot[Table[{forplottingherdF[[i, 1]], forplottingindF[[i, 2]]/forplottingherdF[[i, 2]]},
    {i, 1, Length[forplottingindF]}], PlotStyle → {Black}],
  ListPlot[Join[Table[{forplottingherdF[[i, 1]], forplottingindF[[i, 2]]/forplottingherdF[[i, 2]]},
     {i, 1, Length[forplottingindF]}], {{1, limratio}}],
   Joined → True, PlotStyle → {Thin, Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, maxc}, {0, 2.2}}, Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[ ]=



In[∘]:=
```
plotF = Show[
  Plot[CVherdfar, {x, 0.975, 1.05},
   PlotStyle → {RGBColor[0.11, 0.6, 0.]}, PlotRange → All],
  Plot[CVindfarsplit /. f → tryf,
   {x, 0.975, 1.05}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVherd /. xmax → tryxmax /. ymax → tryymax,
   {x, -0.025, 0.025}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  Plot[limratio, {x, 0.975, 1.05}, PlotStyle → Black],
  ListPlot[forplottingherdF, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[Join[intplottingherdF, {{1, CVherdfar}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.11, 0.6, 0.]}],
  ListPlot[forplottingindF, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[Join[intplottingindF, {{1, CVindfarsplit /. f → tryf}}], Joined → True,
   PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}, PlotRange → All],
  ListPlot[Table[{forplottingherdF[[i, 1]], forplottingindF[[i, 2]]/forplottingherdF[[i, 2]]},
    {i, 1, Length[forplottingindF]}], PlotStyle → {Black}],
  ListPlot[Join[Table[{intplottingherdF[[i, 1]], intplottingindF[[i, 2]]/intplottingherdF[[i, 2]]},
     {i, 1, Length[intplottingindF]}], {{1, limratio}}],
   Joined → True, PlotStyle → {Thin, Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, maxc}, {0, 2.2}}, Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[∘]=



Panel F: Numerical integration [SLOW - Don’t enter]

For comparison, we can numerically integrate. Integration was inaccurate and so we first simplify and then take the limit as c approaches its numerical value:

In[∘]:=

```
meanTEMP = Simplify[
  soln + f xmax/(30 α^2) ((((1 + cs)^5 - 2 (1 + α^5 + cs^5)) + 2 H (1 - 3 α^2 + α^4) + 2 H2 (α^2 - α cs - cs^2) (α^2 +
           α cs - cs^2) - H1 (-1 + α + α^2 - (-2 + α) cs - cs^2) (-1 - α + α^2 + (2 + α) cs - cs^2) +
         H3 (-1 + α + α^2 + (-2 + α) cs - cs^2) (1 + α - α^2 + (2 + α) cs + cs^2) + H4^5) +
       5/4 α Log[(H + α)^-4 ((H1 + α)/H4)^(2 (-1+cs)^4) (cs/(H2 + α))^(4 cs^4) ((H3 + α)/(1 + cs))^(2 (1+cs)^4)
          ((α/(1 + H))^2 (1 + H3 + cs)/(-1 + H1 + cs))^(2 α^3) ((-1 + H1 + cs) (1 + H3 + cs)/(H2 + cs)^2)^(2 α^3 cs) ]) /.
      {H → Sqrt[1 + α^2], H1 → Sqrt[α^2 + (-1 + cs)^2], H2 → Sqrt[α^2 + cs^2], H3 → Sqrt[α^2 + (1 + cs)^2],
       H4 → Sqrt[(-1 + cs)^2]} /. cs → c / xmax /. α → ymax / xmax /.
    xmax → tryxmax /. ymax → tryymax /. f → tryf, c ≥ 0]
```

Out[∘]=

$$\frac{1042}{625}-\frac{551\sqrt{26}}{1875}+\frac{1}{150}\left(125\,\mathrm{Log}\left[\frac{1}{5}\left(1+\sqrt{26}\right)\right]+\mathrm{Log}\left[5+\sqrt{26}\right]\right)+$$
$$\frac{3}{20}\left(\frac{1102\sqrt{26}}{3125}+(1+c)^5-\frac{\left(29-55c+25c^2\right)\sqrt{26-50c+25c^2}\left(19-45c+25c^2\right)}{3125}+\right.$$
$$\frac{2\sqrt{1+25c^2}\left(-1-5c+25c^2\right)\left(-1+5c+25c^2\right)}{3125}-2\left(\frac{3126}{3125}+c^5\right)+$$
$$\left(-\frac{19}{25}-\frac{9c}{5}-c^2\right)\left(\frac{29}{25}+\frac{11c}{5}+c^2\right)\sqrt{\frac{1}{25}+(1+c)^2}+\mathrm{Abs}[-1+c]^5+$$
$$\frac{1}{4}\,\mathrm{Log}\left[\frac{1}{\left(1+\sqrt{26}\right)^4\left(5+\sqrt{26}\right)^{4/125}}\,5^{-24c^2}\left(\frac{c}{1+\sqrt{1+25c^2}}\right)^{4c^4}\left(5c+\sqrt{1+25c^2}\right)^{-4c/125}\right.$$
$$\left(1+\sqrt{26-50c+25c^2}\right)^{2(-1+c)^4}\left(-5+5c+\sqrt{26-50c+25c^2}\right)^{\frac{2}{125}(-1+c)}$$
$$\left.\left.\left(\frac{1+c}{1+\sqrt{26+50c+25c^2}}\right)^{-2(1+c)^4}\left(5+5c+\sqrt{26+50c+25c^2}\right)^{\frac{2(1+c)}{125}}\mathrm{Abs}[-1+c]^{-2(-1+c)^4}\right]\right)$$

In[ ]:=
```
For[t = 1, t ≤ Length[tab], t++,
 tryc = tryV tab[[t]] / (1 - tab[[t]]);
 mean = N[Limit[meanTEMP, c → tryc], 20];
 sqH = (xmax^2 + ymax^2)/6 + c^2 f /. xmax → tryxmax /. ymax → tryymax /. f → tryf /. c → tryc;

 CVherdintC[tryxmax, tab[[t]]] = N[Sqrt[sqH - mean^2] / mean, 20];
 CVindintC[tryxmax, tab[[t]]] =
  N[Sqrt[sqI[tryxmax, tryymax, tryc, tryf] - mean^2] / mean, 20];
 Print[{CVherdintC[tryxmax, tab[[t]]], CVindintC[tryxmax, tab[[t]]]}]
]
```

```
{0.6456877909802461191, 0.209022}
{0.6726141069407129428, 0.26716}
{0.7848943583382700488, 0.405781}
{0.9893559257920512755, 0.578939}
{1.2125292264423531354, 0.738577}
{1.4182046572396214329, 0.876644}
{1.6002913827737586382, 0.995234}
{1.7601097999466685960, 1.09758}
{1.9005301895526764946, 1.18655}
{2.0244418357011196921, 1.26452}
```

In[ ]:=
```
intplottingherdF = Table[{tab[[t]], CVherdintC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$$\left\{\{0, 0.6456877909802461191\}, \left\{\frac{1}{10}, 0.6726141069407129428\right\}, \left\{\frac{1}{5}, 0.7848943583382700488\right\}, \left\{\frac{3}{10}, 0.9893559257920512755\right\}, \left\{\frac{2}{5}, 1.2125292264423531354\right\}, \left\{\frac{1}{2}, 1.4182046572396214329\right\}, \left\{\frac{3}{5}, 1.6002913827737586382\right\}, \left\{\frac{7}{10}, 1.7601097999466685960\right\}, \left\{\frac{4}{5}, 1.9005301895526764946\right\}, \left\{\frac{9}{10}, 2.0244418357011196921\right\}\right\}$$

In[ ]:=
```
intplottingindF = Table[{tab[[t]], CVindintC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$$\left\{\{0, 0.209022\}, \left\{\frac{1}{10}, 0.26716\right\}, \left\{\frac{1}{5}, 0.405781\right\}, \left\{\frac{3}{10}, 0.578939\right\}, \left\{\frac{2}{5}, 0.738577\right\}, \left\{\frac{1}{2}, 0.876644\right\}, \left\{\frac{3}{5}, 0.995234\right\}, \left\{\frac{7}{10}, 1.09758\right\}, \left\{\frac{4}{5}, 1.18655\right\}, \left\{\frac{9}{10}, 1.26452\right\}\right\}$$

Panel G: Even herd split with elongated patches [ymax long]

In[ ]:=
```
SeedRandom[77212];
```

We seek to examine a range of distances, c, between the peaks, now allowing each patch to be elongated. To do so, we use c/(V+c) along the x-axis, which allows us to vary c from 0 at x=0 to infinity at x=1, within a single plot (we use V = 4 so that patches 4 sd apart are half-way along the plot). To convert the x-axis to c, we use:

In[ ]:=
```
tryV = 4;
```

In[ ]:=
```
Solve[c / (tryV + c) == x, c]
```

Out[ ]=

$$\left\{\left\{c \to -\frac{4\,x}{-1+x}\right\}\right\}$$

In[ ]:=
```
tab = Join[Table[i, {i, 0 / 10, 9 / 10, 1 / 10}]] // Flatten
```

Out[ ]=

$$\left\{0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{9}{10}\right\}$$

In[ ]:=
```
numind1 = 50;
numind2 = 50;
numind = 100;
```

In[ ]:=
```
tryf = 2 (numind2 / numind) (numind1 / numind)
  (*Equal split with 50% in one patch so f = 1/2*)
```

Out[ ]=

$$\frac{1}{2}$$

In[ ]:=
```
tryxmax = 1;
tryymax = 5;
```

In[ ]:=
```
For[t = 1, t ≤ Length[tab], t++,
 table = Join[Table[RandomReal[UniformDistribution[{{0, tryxmax}, {0, tryymax}}]],
    {i, 1, numind1}], Table[RandomReal[UniformDistribution[
      {{tryV tab[[t]]/(1 - tab[[t]]), tryV tab[[t]]/(1 - tab[[t]]) + tryxmax}, {0, tryymax}}]], {i, 1, numind2}]];
 disttable = Flatten[
   Table[EuclideanDistance[table[[i]], table[[j]]], {i, 1, numind}, {j, i + 1, numind}]];
 CVherdsimC[tryxmax, tab[[t]]] = StandardDeviation[disttable]/Mean[disttable];
 disttable2 = Table[
   1/(Length[table] - 1) (Sum[EuclideanDistance[table[[i]], table[[j]]], {j, 1, i - 1}] + Sum[
       EuclideanDistance[table[[i]], table[[j]]], {j, i + 1, numind}]), {i, 1, numind}];
 CVindsimC[tryxmax, tab[[t]]] = StandardDeviation[disttable2]/Mean[disttable2]
]
```

In[ ]:=
```
forplottingherdG = Table[{tab[[t]], CVherdsimC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$\{\{0, 0.63467\}, \{\frac{1}{10}, 0.615477\}, \{\frac{1}{5}, 0.555801\}, \{\frac{3}{10}, 0.502156\}, \{\frac{2}{5}, 0.495109\}, \{\frac{1}{2}, 0.525298\}, \{\frac{3}{5}, 0.602384\}, \{\frac{7}{10}, 0.704638\}, \{\frac{4}{5}, 0.815552\}, \{\frac{9}{10}, 0.900771\}\}$

In[ ]:=
```
forplottingindG = Table[{tab[[t]], CVindsimC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$\{\{0, 0.203677\}, \{\frac{1}{10}, 0.252975\}, \{\frac{1}{5}, 0.179129\}, \{\frac{3}{10}, 0.156466\}, \{\frac{2}{5}, 0.121663\}, \{\frac{1}{2}, 0.100978\}, \{\frac{3}{5}, 0.0688491\}, \{\frac{7}{10}, 0.0445305\}, \{\frac{4}{5}, 0.0329098\}, \{\frac{9}{10}, 0.0134876\}\}$

Below, we’ll want the ratio of $\frac{\text{CVind}}{\text{CVherd}}$ in the limit for very distant patches when the patches are uneven:

In[ ]:=
```
limratio =
 Limit[CVindfarsplit/CVherdC /. xmax → tryxmax /. ymax → tryymax /. f → tryf, c → Infinity]
```

Out[ ]=

0

We also plot CVherd in the limit for very distant patches when the patches are uneven:

In[ ]:=
```
Simplify[CVherdC /. xmax → tryxmax /. ymax → tryymax /. f → tryf, c > 1];
CVherdfar = Limit[%, c → Infinity]
```

Out[ ]=

1

In[∘]:= **maxc = 1;**

In[∘]:= **Solve**$\left[\frac{\text{tryV } x}{1 - x} == 6, x\right]$

Out[∘]=

$\left\{\left\{x \to \frac{3}{5}\right\}\right\}$

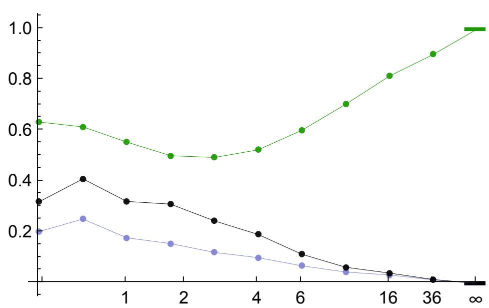
In[∘]:=
```
xticks = {{0.01, "0"}, {1 / 5, "1"}, {1 / 3, "2"},
   {1 / 2, "4"}, {3 / 5, "6"}, {4 / 5, "16"}, {9 / 10, "36"}, {1, "∞"}}
```

Out[∘]=

$\left\{\{0.01, 0\}, \left\{\frac{1}{5}, 1\right\}, \left\{\frac{1}{3}, 2\right\}, \left\{\frac{1}{2}, 4\right\}, \left\{\frac{3}{5}, 6\right\}, \left\{\frac{4}{5}, 16\right\}, \left\{\frac{9}{10}, 36\right\}, \{1, \infty\}\right\}$

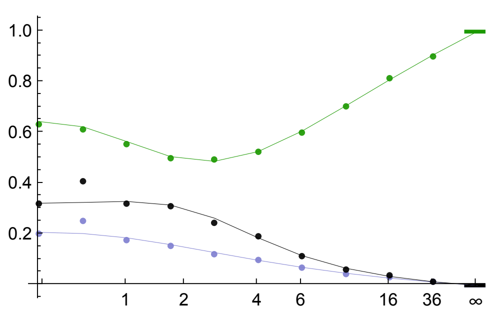
In[ ]:=
```
plotG = Show[
  Plot[CVherdfar, {x, 0.975, 1.05},
   PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
  Plot[CVindfarsplit /. f → tryf,
   {x, 0.975, 1.05}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVherd /. xmax → tryxmax /. ymax → tryymax,
   {x, -0.025, 0.025}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  Plot[limratio, {x, 0.975, 1.05}, PlotStyle → Black],
  ListPlot[forplottingherdG, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[Join[forplottingherdG, {{1, CVherdfar}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.11, 0.6, 0.]}],
  ListPlot[forplottingindG, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[Join[forplottingindG, {{1, CVindfarsplit /. f → tryf}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}],
  ListPlot[Table[{forplottingherdG[[i, 1]], forplottingindG[[i, 2]] / forplottingherdG[[i, 2]]},
    {i, 1, Length[forplottingindG]}], PlotStyle → {Black}],
  ListPlot[Join[Table[{forplottingherdG[[i, 1]], forplottingindG[[i, 2]] / forplottingherdG[[i, 2]]},
     {i, 1, Length[forplottingindG]}], {{1, limratio}}],
   Joined → True, PlotStyle → {Thin, Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, maxc}, {0, 1}}, Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[ ]=



In[∘]:=
```
plotG = Show[
  Plot[CVherdfar, {x, 0.975, 1.05},
   PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
  Plot[CVindfarsplit /. f → tryf,
   {x, 0.975, 1.05}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVherd /. xmax → tryxmax /. ymax → tryymax,
   {x, -0.025, 0.025}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  Plot[limratio, {x, 0.975, 1.05}, PlotStyle → Black],
  ListPlot[forplottingherdG, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[Join[intplottingherdG, {{1, CVherdfar}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.11, 0.6, 0.]}],
  ListPlot[forplottingindG, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[Join[intplottingindG, {{1, CVindfarsplit /. f → tryf}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}],
  ListPlot[Table[{forplottingherdG[[i, 1]], forplottingindG[[i, 2]] / forplottingherdG[[i, 2]]},
    {i, 1, Length[forplottingindG]}], PlotStyle → {Black}],
  ListPlot[Join[Table[{intplottingherdG[[i, 1]], intplottingindG[[i, 2]] / intplottingherdG[[i, 2]]},
     {i, 1, Length[intplottingindG]}], {{1, limratio}}],
   Joined → True, PlotStyle → {Thin, Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, maxc}, {0, 1}}, Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[∘]=



Panel G: Numerical integration [SLOW - Don’t enter]

For comparison, we can numerically integrate. Integration was inaccurate and so we first simplify and then take the limit as c approaches its numerical value:

In[ ]:=

$$\mathbf{meanTEMP} = \mathrm{Simplify}\Big[\mathrm{soln} + \frac{f\,\mathrm{xmax}}{30\,\alpha^2}\Big(\Big(\Big((1+c_s)^5 - 2\left(1+\alpha^5+c_s^5\right)\Big) + 2\,H\left(1-3\,\alpha^2+\alpha^4\right) + 2\,H2\left(\alpha^2-\alpha\,c_s-c_s^2\right)\left(\alpha^2+\alpha\,c_s-c_s^2\right) - H1\left(-1+\alpha+\alpha^2-(-2+\alpha)\,c_s-c_s^2\right)\left(-1-\alpha+\alpha^2+(2+\alpha)\,c_s-c_s^2\right) + H3\left(-1+\alpha+\alpha^2+(-2+\alpha)\,c_s-c_s^2\right)\left(1+\alpha-\alpha^2+(2+\alpha)\,c_s+c_s^2\right) + H4^5\Big) + \frac{5}{4}\,\alpha\,\mathrm{Log}\Big[(H+\alpha)^{-4}\left(\frac{H1+\alpha}{H4}\right)^{2\,(-1+c_s)^4}\left(\frac{c_s}{H2+\alpha}\right)^{4\,c_s^4}\left(\frac{H3+\alpha}{1+c_s}\right)^{2\,(1+c_s)^4}\left(\left(\frac{\alpha}{1+H}\right)^2\frac{1+H3+c_s}{-1+H1+c_s}\right)^{2\,\alpha^3}\left(\frac{(-1+H1+c_s)\,(1+H3+c_s)}{(H2+c_s)^2}\right)^{2\,\alpha^3\,c_s}\Big]\Big)\ /.$$
$$\left\{H\to\sqrt{1+\alpha^2},\ H1\to\sqrt{\alpha^2+(-1+c_s)^2},\ H2\to\sqrt{\alpha^2+c_s^2},\ H3\to\sqrt{\alpha^2+(1+c_s)^2},\ H4\to\sqrt{(-1+c_s)^2}\right\}\ /.\ c_s\to c\,/\,\mathrm{xmax}\ /.\ \alpha\to\mathrm{ymax}\,/\,\mathrm{xmax}\ /.$$
$$\mathrm{xmax}\to\mathrm{tryxmax}\ /.\ \mathrm{ymax}\to\mathrm{tryymax}\ /.\ f\to\mathrm{tryf},\ c\geq 0\Big]$$

Out[ ]=

$$\frac{1}{1500}\Big(12\,504 - 1102\,\sqrt{26} + (1+c)^5 - \left(-19-7\,c+c^2\right)\sqrt{26-2\,c+c^2}\left(-29+3\,c+c^2\right) + 2\,\sqrt{25+c^2}\left(-25-5\,c+c^2\right)\left(-25+5\,c+c^2\right) + \left(29+3\,c-c^2\right)\sqrt{26+2\,c+c^2}\left(-19+7\,c+c^2\right) - 2\left(3126+c^5\right) + \mathrm{Abs}[-1+c]^5 + 50\left(125\,\mathrm{Log}\left[\frac{1}{5}\left(1+\sqrt{26}\right)\right] + \mathrm{Log}\left[5+\sqrt{26}\right]\right) + \frac{25}{4}\,\mathrm{Log}\Big[$$
$$\Big(30\,549\,363\,634\,996\,046\,820\,519\,793\,932\,136\,176\,997\,894\,027\,405\,723\,266\,638\,936\,139\,092\,812\,\backslash$$
$$916\,265\,247\,204\,577\,018\,572\,351\,080\,152\,282\,568\,751\,526\,935\,904\,671\,553\,178\,534\,278\,042\,\backslash$$
$$839\,697\,351\,331\,142\,009\,178\,896\,307\,244\,205\,337\,728\,522\,220\,355\,888\,195\,318\,837\,008\,165\,\backslash$$
$$086\,679\,301\,794\,879\,136\,633\,899\,370\,525\,163\,649\,789\,227\,021\,200\,352\,450\,820\,912\,190\,874\,\backslash$$
$$482\,021\,196\,014\,946\,372\,110\,934\,030\,798\,550\,767\,828\,365\,183\,620\,409\,339\,937\,395\,998\,276\,\backslash$$
$$770\,114\,898\,681\,640\,625\left(\frac{5+\sqrt{25+(-1+c)^2}}{\sqrt{(-1+c)^2}}\right)^{2\,(-1+c)^4}\left(\frac{c}{5+\sqrt{25+c^2}}\right)^{4\,c^4}\left(\frac{5+\sqrt{25+(1+c)^2}}{1+c}\right)^{2\,(1+c)^4}\left(1+c+\sqrt{25+(1+c)^2}\right)^{250}\left(\frac{\left(-1+\sqrt{25+(-1+c)^2}+c\right)\left(1+c+\sqrt{25+(1+c)^2}\right)}{\left(c+\sqrt{25+c^2}\right)^2}\right)^{250\,c}\Big)\Big/$$
$$\left(\left(1+\sqrt{26}\right)^{500}\left(5+\sqrt{26}\right)^4\left(-1+\sqrt{25+(-1+c)^2}+c\right)^{250}\right)\Big]\Big)$$

In[ ]:=
```
For[t = 1, t ≤ Length[tab], t++,
  tryc = (tryV tab[[t]])/(1 - tab[[t]]);
  mean = N[Limit[meanTEMP, c → tryc], 20];
  sqH = (xmax^2 + ymax^2)/6 + c^2 f /. xmax → tryxmax /. ymax → tryymax /. f → tryf /. c → tryc;

  CVherdintC[tryxmax, tab[[t]]] = N[Sqrt[sqH - mean^2] / mean, 20];
  CVindintC[tryxmax, tab[[t]]] =
   N[Sqrt[sqI[tryxmax, tryymax, tryc, tryf] - mean^2] / mean, 20];
  Print[{CVherdintC[tryxmax, tab[[t]]], CVindintC[tryxmax, tab[[t]]]}]
 ]
```

```
{0.6456877909802461191, 0.209022}
{0.6249669682390801206, 0.204264}
{0.5667721406605546949, 0.187473}
{0.5073851200907658822, 0.160621}
{0.4888278688972020791, 0.129537}
{0.5272574266623707615, 0.0987984}
{0.6078424280028319966, 0.071197}
{0.7072676067712154196, 0.0478076}
{0.8100289075250711776, 0.0285568}
{0.9086010837792124169, 0.012853}
```

In[ ]:=
```
intplottingherdG = Table[{tab[[t]], CVherdintC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$$\left\{\{0, 0.6456877909802461191\}, \left\{\frac{1}{10}, 0.6249669682390801206\right\}, \left\{\frac{1}{5}, 0.5667721406605546949\right\}, \left\{\frac{3}{10}, 0.5073851200907658822\right\}, \left\{\frac{2}{5}, 0.4888278688972020791\right\}, \left\{\frac{1}{2}, 0.5272574266623707615\right\}, \left\{\frac{3}{5}, 0.6078424280028319966\right\}, \left\{\frac{7}{10}, 0.7072676067712154196\right\}, \left\{\frac{4}{5}, 0.8100289075250711776\right\}, \left\{\frac{9}{10}, 0.9086010837792124169\right\}\right\}$$

In[ ]:=
```
intplottingindG = Table[{tab[[t]], CVindintC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$$\left\{\{0, 0.209022\}, \left\{\frac{1}{10}, 0.204264\right\}, \left\{\frac{1}{5}, 0.187473\right\}, \left\{\frac{3}{10}, 0.160621\right\}, \left\{\frac{2}{5}, 0.129537\right\}, \left\{\frac{1}{2}, 0.0987984\right\}, \left\{\frac{3}{5}, 0.071197\right\}, \left\{\frac{7}{10}, 0.0478076\right\}, \left\{\frac{4}{5}, 0.0285568\right\}, \left\{\frac{9}{10}, 0.012853\right\}\right\}$$

Panel H: Uneven herd split with elongated patches [ymax long]

In[•]:=
```
SeedRandom[827 523];
```

We seek to examine a range of distances, c, between the peaks, now allowing each patch to be elongated. To do so, we use c/(V+c) along the x-axis, which allows us to vary c from 0 at x=0 to infinity at x=1, within a single plot (we use V = 4 so that patches 4 sd apart are half-way along the plot). To convert the x-axis to c, we use:

In[•]:=
```
tryV = 4;
```

In[•]:=
```
Solve[c / (tryV + c) == x, c]
```

Out[•]=

$$\left\{\left\{c \to -\frac{4\,x}{-1 + x}\right\}\right\}$$

In[•]:=
```
tab = Join[Table[i, {i, 0 / 10, 9 / 10, 1 / 10}]] // Flatten
```

Out[•]=

$$\left\{0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{9}{10}\right\}$$

In[•]:=
```
numind1 = 90;
numind2 = 10;
numind = numind1 + numind2;
```

In[•]:=
```
tryf = 2 (numind2 / numind) (numind1 / numind)
  (*Unequal split with 90% in one patch so f = 0.18*)
```

Out[•]=

$$\frac{9}{50}$$

In[•]:=
```
tryxmax = 1;
tryymax = 5;
```

In[ ]:=
```
For[t = 1, t ≤ Length[tab], t++,
 table = Join[Table[RandomReal[UniformDistribution[{{0, tryxmax}, {0, tryymax}}]],
    {i, 1, numind1}], Table[RandomReal[UniformDistribution[
      {{tryV tab[[t]]/(1 - tab[[t]]), tryV tab[[t]]/(1 - tab[[t]]) + tryxmax}, {0, tryymax}}]], {i, 1, numind2}]];
 disttable = Flatten[
   Table[EuclideanDistance[table[[i]], table[[j]]], {i, 1, numind}, {j, i + 1, numind}]];
 CVherdsimC[tryxmax, tab[[t]]] = StandardDeviation[disttable]/Mean[disttable];
 disttable2 = Table[
   1/(Length[table] - 1) (Sum[EuclideanDistance[table[[i]], table[[j]]], {j, 1, i - 1}] + Sum[
       EuclideanDistance[table[[i]], table[[j]]], {j, i + 1, numind}]), {i, 1, numind}];
 CVindsimC[tryxmax, tab[[t]]] = StandardDeviation[disttable2]/Mean[disttable2]
]
```

In[ ]:=
```
forplottingherdH = Table[{tab[[t]], CVherdsimC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$$\left\{\{0, 0.64555\}, \left\{\frac{1}{10}, 0.647195\right\}, \left\{\frac{1}{5}, 0.624738\right\}, \left\{\frac{3}{10}, 0.595356\right\}, \left\{\frac{2}{5}, 0.599694\right\}, \left\{\frac{1}{2}, 0.650181\right\}, \left\{\frac{3}{5}, 0.821925\right\}, \left\{\frac{7}{10}, 0.973951\right\}, \left\{\frac{4}{5}, 1.28448\right\}, \left\{\frac{9}{10}, 1.68513\right\}\right\}$$

In[ ]:=
```
forplottingindH = Table[{tab[[t]], CVindsimC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$$\left\{\{0, 0.189993\}, \left\{\frac{1}{10}, 0.20105\right\}, \left\{\frac{1}{5}, 0.197965\right\}, \left\{\frac{3}{10}, 0.238172\right\}, \left\{\frac{2}{5}, 0.231567\right\}, \left\{\frac{1}{2}, 0.304318\right\}, \left\{\frac{3}{5}, 0.474573\right\}, \left\{\frac{7}{10}, 0.581909\right\}, \left\{\frac{4}{5}, 0.801142\right\}, \left\{\frac{9}{10}, 1.06177\right\}\right\}$$

Below, we'll want the ratio of $\frac{\text{CVind}}{\text{CVherd}}$ in the limit for very distant patches when the patches are uneven:

In[ ]:=
```
limratio =
 Limit[CVindfarsplit/CVherdC /. xmax → tryxmax /. ymax → tryymax /. f → tryf, c → Infinity]
```

Out[ ]=

$$\frac{4}{\sqrt{41}}$$

We also plot CVherd in the limit for very distant patches when the patches are uneven:

In[ ]:=
```
Simplify[CVherdC /. xmax → tryxmax /. ymax → tryymax /. f → tryf, c > 1];
CVherdfar = Limit[%, c → Infinity]
```

Out[ ]=

$$\frac{\sqrt{41}}{3}$$

In[ ]:=
```
maxc = 1;
```

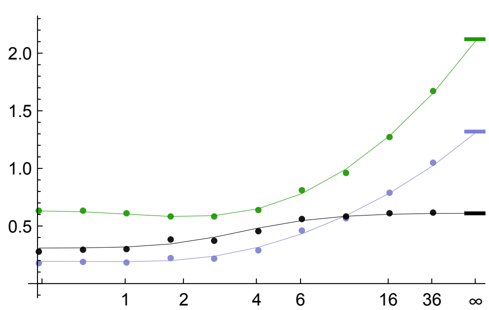
In[ ]:=
```
xticks = {{0.01, "0"}, {1 / 5, "1"}, {1 / 3, "2"},
  {1 / 2, "4"}, {3 / 5, "6"}, {4 / 5, "16"}, {9 / 10, "36"}, {1, "∞"}}
```

Out[ ]=

$$\left\{\{0.01, 0\}, \left\{\frac{1}{5}, 1\right\}, \left\{\frac{1}{3}, 2\right\}, \left\{\frac{1}{2}, 4\right\}, \left\{\frac{3}{5}, 6\right\}, \left\{\frac{4}{5}, 16\right\}, \left\{\frac{9}{10}, 36\right\}, \{1, \infty\}\right\}$$

In[∘]:=
```
plotH = Show[
  Plot[CVherdfar, {x, 0.975, 1.05},
   PlotStyle → RGBColor[0.11, 0.6, 0.], PlotRange → All],
  Plot[CVindfarsplit /. f → tryf,
   {x, 0.975, 1.05}, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  Plot[CVherd /. xmax → tryxmax /. ymax → tryymax,
   {x, -0.025, 0.025}, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  Plot[limratio, {x, 0.975, 1.05}, PlotStyle → Black],
  ListPlot[forplottingherdH, PlotStyle → RGBColor[0.11, 0.6, 0.]],
  ListPlot[Join[intplottingherdH, {{1, CVherdfar}}],
   Joined → True, PlotStyle → {Thin, RGBColor[0.11, 0.6, 0.]}],
  ListPlot[forplottingindH, PlotStyle → RGBColor[0.5, 0.5, 0.82]],
  ListPlot[Join[intplottingindH, {{1, CVindfarsplit /. f → tryf}}], Joined → True,
   PlotStyle → {Thin, RGBColor[0.5, 0.5, 0.82]}, PlotRange → All],
  ListPlot[Table[{forplottingherdH[[i, 1]], forplottingindH[[i, 2]]/forplottingherdH[[i, 2]]},
    {i, 1, Length[forplottingindH]}], PlotStyle → {Black}],
  ListPlot[Join[Table[{intplottingherdH[[i, 1]], intplottingindH[[i, 2]]/intplottingherdH[[i, 2]]},
     {i, 1, Length[intplottingindH]}], {{1, limratio}}],
   Joined → True, PlotStyle → {Thin, Black}],
  AxesOrigin → {0, 0},
  PlotRange → {{0, maxc}, {0, 2.2}}, Ticks → {xticks, Automatic}, ImageSize → 250
 ]
```

Out[∘]=



Panel H: Numerical integration [SLOW - Don’t enter]

For comparison, we can numerically integrate. Integration was inaccurate and so we first simplify and then take the limit as c approaches its numerical value:

In[∘]:=

```
meanTEMP = Simplify[
  soln + (f xmax)/(30 α^2) ((((1 + c_s)^5 - 2 (1 + α^5 + c_s^5)) + 2 H (1 - 3 α^2 + α^4) + 2 H2 (α^2 - α c_s - c_s^2) (α^2 +
        α c_s - c_s^2) - H1 (-1 + α + α^2 - (-2 + α) c_s - c_s^2) (-1 - α + α^2 + (2 + α) c_s - c_s^2) +
      H3 (-1 + α + α^2 + (-2 + α) c_s - c_s^2) (1 + α - α^2 + (2 + α) c_s + c_s^2) + H4^5) +
     5/4 α Log[(H + α)^-4 ((H1 + α)/H4)^(2 (-1+c_s)^4) (c_s/(H2 + α))^(4 c_s^4) ((H3 + α)/(1 + c_s))^(2 (1+c_s)^4)
       (((α/(1 + H))^2 (1 + H3 + c_s)/(-1 + H1 + c_s))^(2 α^3) (((-1 + H1 + c_s) (1 + H3 + c_s))/(H2 + c_s)^2)^(2 α^3 c_s)]) /.
    {H → Sqrt[1 + α^2], H1 → Sqrt[α^2 + (-1 + c_s)^2], H2 → Sqrt[α^2 + c_s^2], H3 → Sqrt[α^2 + (1 + c_s)^2],
     H4 → Sqrt[(-1 + c_s)^2]} /. c_s → c / xmax /. α → ymax / xmax /.
   xmax → tryxmax /. ymax → tryymax /. f → tryf, c ≥ 0]
```

Out[∘]=

$$
\begin{aligned}
&\frac{1}{37\,500}\Bigg(312\,600 - 45\,182\sqrt{26} + 9\,(1+c)^5 - 9\left(-19-7c+c^2\right)\sqrt{26-2c+c^2}\left(-29+3c+c^2\right) + \\
&18\sqrt{25+c^2}\left(-25-5c+c^2\right)\left(-25+5c+c^2\right) - \\
&9\left(-29-3c+c^2\right)\sqrt{26+2c+c^2}\left(-19+7c+c^2\right) - 18\left(3126+c^5\right) + \\
&9\,\mathrm{Abs}[-1+c]^5 + 1250\left(125\,\mathrm{Log}\left[\frac{1}{5}\left(1+\sqrt{26}\right)\right] + \mathrm{Log}\left[5+\sqrt{26}\right]\right) + \frac{225}{4}\,\mathrm{Log}\Bigg[\\
&\Bigg(30\,549\,363\,634\,996\,046\,820\,519\,793\,932\,136\,176\,997\,894\,027\,405\,723\,266\,638\,936\,139\,092\,812\,\backslash \\
&\quad 916\,265\,247\,204\,577\,018\,572\,351\,080\,152\,282\,568\,751\,526\,935\,904\,671\,553\,178\,534\,278\,042\,\backslash \\
&\quad 839\,697\,351\,331\,142\,009\,178\,896\,307\,244\,205\,337\,728\,522\,220\,355\,888\,195\,318\,837\,008\,165\,\backslash \\
&\quad 086\,679\,301\,794\,879\,136\,633\,899\,370\,525\,163\,649\,789\,227\,021\,200\,352\,450\,820\,912\,190\,874\,\backslash \\
&\quad 482\,021\,196\,014\,946\,372\,110\,934\,030\,798\,550\,767\,828\,365\,183\,620\,409\,339\,937\,395\,998\,276\,\backslash \\
&\quad 770\,114\,898\,681\,640\,625\left(\frac{5+\sqrt{25+(-1+c)^2}}{\sqrt{(-1+c)^2}}\right)^{2(-1+c)^4} \\
&\quad \left(\frac{c}{5+\sqrt{25+c^2}}\right)^{4c^4}\left(\frac{5+\sqrt{25+(1+c)^2}}{1+c}\right)^{2(1+c)^4}\left(1+c+\sqrt{25+(1+c)^2}\right)^{250} \\
&\quad \left(\frac{\left(-1+\sqrt{25+(-1+c)^2}+c\right)\left(1+c+\sqrt{25+(1+c)^2}\right)}{\left(c+\sqrt{25+c^2}\right)^2}\right)^{250c}\Bigg)\Bigg/ \\
&\left(\left(1+\sqrt{26}\right)^{500}\left(5+\sqrt{26}\right)^4\left(-1+\sqrt{25+(-1+c)^2}+c\right)^{250}\right)\Bigg]\Bigg)
\end{aligned}
$$

In[ ]:=
```
For[t = 1, t ≤ Length[tab], t++,
  tryc = (tryV tab[[t]])/(1 - tab[[t]]);
  mean = N[Limit[meanTEMP, c → tryc], 20];
  sqH = (xmax^2 + ymax^2)/6 + c^2 f /. xmax → tryxmax /. ymax → tryymax /. f → tryf /. c → tryc;

  CVherdintC[tryxmax, tab[[t]]] = N[Sqrt[sqH - mean^2] / mean, 20];
  CVindintC[tryxmax, tab[[t]]] =
   N[Sqrt[sqI[tryxmax, tryymax, tryc, tryf] - mean^2] / mean, 20];
  Print[{CVherdintC[tryxmax, tab[[t]]], CVindintC[tryxmax, tab[[t]]]}]
 ]
```

```
{0.6456877909802461191, 0.209022}
{0.6382173336190422852, 0.207524}
{0.6172818531644725514, 0.205481}
{0.5979012682139712844, 0.214711}
{0.6044261845933080114, 0.252541}
{0.6638665661780649766, 0.329672}
{0.7958159315818981299, 0.44703}
{1.0064253082665860064, 0.603241}
{1.2961080795658534218, 0.799359}
{1.6684946904282582799, 1.0399}
```

In[ ]:=
```
intplottingherdH = Table[{tab[[t]], CVherdintC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$$\left\{\{0, 0.6456877909802461191\}, \left\{\frac{1}{10}, 0.6382173336190422852\right\}, \left\{\frac{1}{5}, 0.6172818531644725514\right\}, \left\{\frac{3}{10}, 0.5979012682139712844\right\}, \left\{\frac{2}{5}, 0.6044261845933080114\right\}, \left\{\frac{1}{2}, 0.6638665661780649766\right\}, \left\{\frac{3}{5}, 0.7958159315818981299\right\}, \left\{\frac{7}{10}, 1.0064253082665860064\right\}, \left\{\frac{4}{5}, 1.2961080795658534218\right\}, \left\{\frac{9}{10}, 1.6684946904282582799\right\}\right\}$$

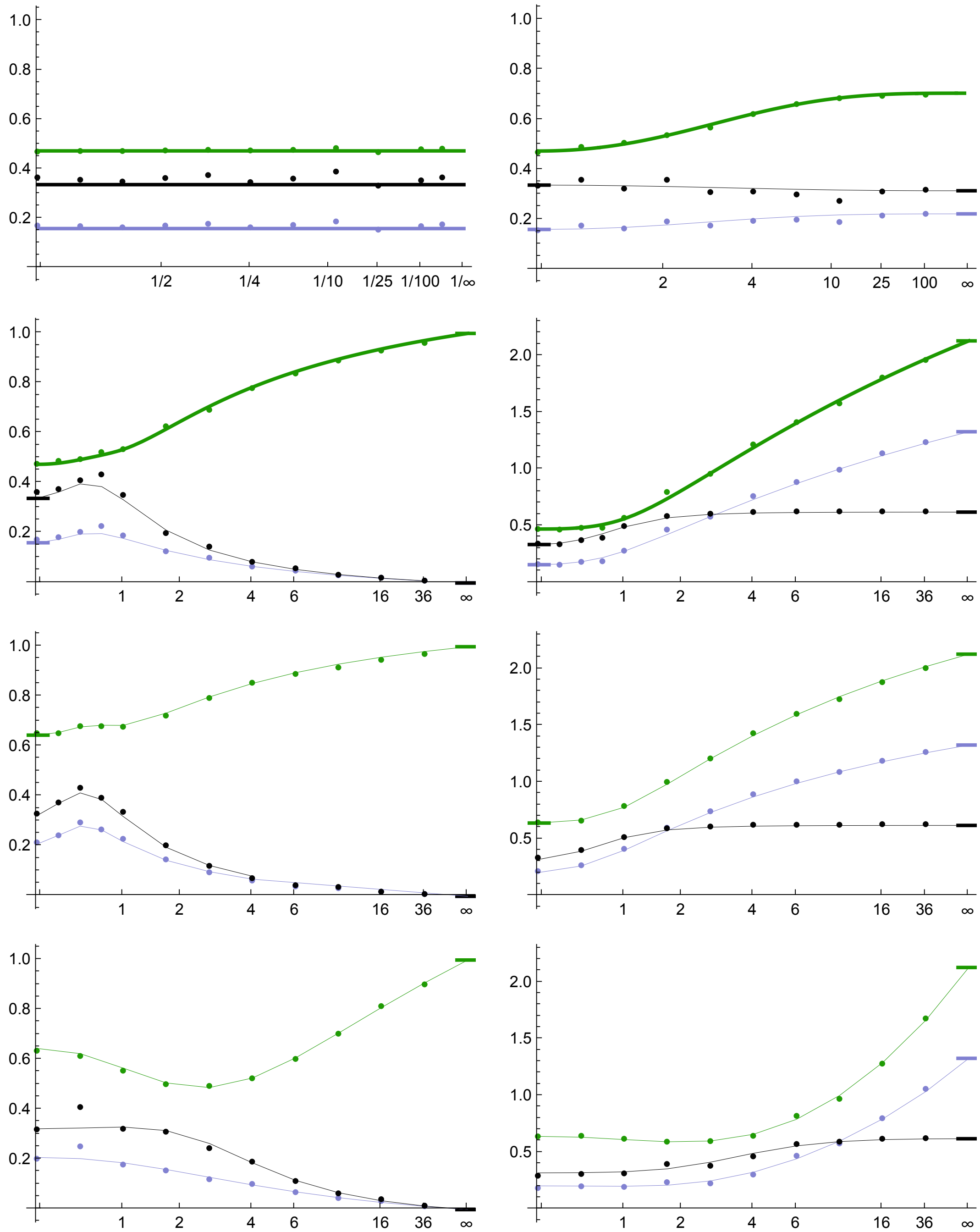
In[ ]:=
```
intplottingindH = Table[{tab[[t]], CVindintC[tryxmax, tab[[t]]]}, {t, 1, Length[tab]}]
```

Out[ ]=

$$\left\{\{0, 0.209022\}, \left\{\frac{1}{10}, 0.207524\right\}, \left\{\frac{1}{5}, 0.205481\right\}, \left\{\frac{3}{10}, 0.214711\right\}, \left\{\frac{2}{5}, 0.252541\right\}, \left\{\frac{1}{2}, 0.329672\right\}, \left\{\frac{3}{5}, 0.44703\right\}, \left\{\frac{7}{10}, 0.603241\right\}, \left\{\frac{4}{5}, 0.799359\right\}, \left\{\frac{9}{10}, 1.0399\right\}\right\}$$

Altogether

In[•]:=
```
GraphicsGrid[{{plotA, plotB}, {plotC, plotD}, {plotE, plotF}, {plotG, plotH}}]
```
Out[•]=



Individuals distributed uniformly across an elliptical range

Simulation only

5000 individuals were simulated from the given population distribution (more individuals to get a more accurate estimate).

Ranges were drawn from a uniform disk, either circular with radius r or elliptical with radii r1 and r2.

Herd with a circular range (CV_{herd} ~ 0.469, CV_{ind} ~ 0.147)

```
In[∘]:= SeedRandom[748 645];
```

Assume a random distribution of individuals (blue):

```
In[∘]:= tryr = 1;
numind = 5000;
```

```
In[∘]:= table = RandomPoint[Disk[{0, 0}, tryr], numind];
```

Calculating the mean Euclidean distance for all pairwise distances (blue)

This table includes all of the pairwise distances:

```
In[∘]:= disttable = Flatten[
   Table[EuclideanDistance[table⟦i⟧, table⟦j⟧], {i, 1, numind}, {j, i + 1, numind}]];
```

The mean and SD:

```
In[∘]:= { Mean[disttable], StandardDeviation[disttable]}
Out[∘]=
{0.908522, 0.425707}
```

The coefficient of variation across the herd (CV_{herd}):

```
In[∘]:= StandardDeviation[disttable]
        ----------------------------
              Mean[disttable]
Out[∘]=
0.468571
```

Looking instead at how far, on average, each individual is to all others, then calculating the CV in this value among individuals (CV_{ind}):

```
In[∘]:= disttable2 = Table[
          1
   ----------------- (Sum[EuclideanDistance[table⟦i⟧, table⟦j⟧], {j, 1, i - 1}] + Sum[
   Length[table] - 1
      EuclideanDistance[table⟦i⟧, table⟦j⟧], {j, i + 1, numind}]), {i, 1, numind}];
```

The mean and SD:

In[ ]:=
```
{Mean[disttable2], StandardDeviation[disttable2]}
```
Out[ ]=
```
{0.908522, 0.133448}
```

Because of the averaging that occurs for each individual’s average pairwise distance, we get a lower CV_{ind}:

In[ ]:=
$$\frac{\texttt{StandardDeviation[disttable2]}}{\texttt{Mean[disttable2]}}$$
Out[ ]=
```
0.146885
```

Herd with a very elongated range (CV_{herd} ~ 0.710, CV_{ind} ~ 0.256)

In[ ]:=
```
SeedRandom[748 645];
```

Assume a random distribution of individuals (blue):

In[ ]:=
```
tryr1 = 1;
tryr2 = 10^6;
numind = 5000;
```

In[ ]:=
```
table = RandomPoint[Disk[{0, 0}, {tryr1, tryr2}], numind];
```

Calculating the mean Euclidean distance for all pairwise distances (blue)

This table includes all of the pairwise distances:

In[ ]:=
```
disttable = Flatten[
   Table[EuclideanDistance[table[[i]], table[[j]]], {i, 1, numind}, {j, i + 1, numind}]];
```

The mean and SD:

In[ ]:=
```
{ Mean[disttable], StandardDeviation[disttable]}
```
Out[ ]=
```
{580 988., 412 674.}
```

The coefficient of variation across the herd (CV_{herd}):

In[ ]:=
$$\frac{\texttt{StandardDeviation[disttable]}}{\texttt{Mean[disttable]}}$$
Out[ ]=
```
0.710298
```

Looking instead at how far, on average, each individual is to all others, then calculating the CV in this value among individuals (CV_{ind}):

In[ ]:=
```
disttable2 = Table[
   1/(Length[table] - 1) (Sum[EuclideanDistance[table[[i]], table[[j]]], {j, 1, i - 1}] + Sum[
      EuclideanDistance[table[[i]], table[[j]]], {j, i + 1, numind}]), {i, 1, numind}];
```

The mean and SD:

In[∘]:=
```
{Mean[disttable2], StandardDeviation[disttable2]}
```
Out[∘]=
```
{580 988., 148 566.}
```

Because of the averaging that occurs for each individual’s average pairwise distance, we get a lower CV_{ind}:

In[∘]:=
$$\frac{\texttt{StandardDeviation[disttable2]}}{\texttt{Mean[disttable2]}}$$

Out[∘]=
```
0.255713
```