From the expression

$$\begin{bmatrix} f_m \\ \dot{x}_m \end{bmatrix} = \begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix} \cdot \begin{bmatrix} \dot{x}_s \\ -f_s \end{bmatrix}$$

it is easy to compute each elements of the hybrid matrix (Lawrence) as a function of the controllers ($C_m, C_s, C_1, \dots, C_4$) and the master/slave robot impedance (Z_m, Z_s)

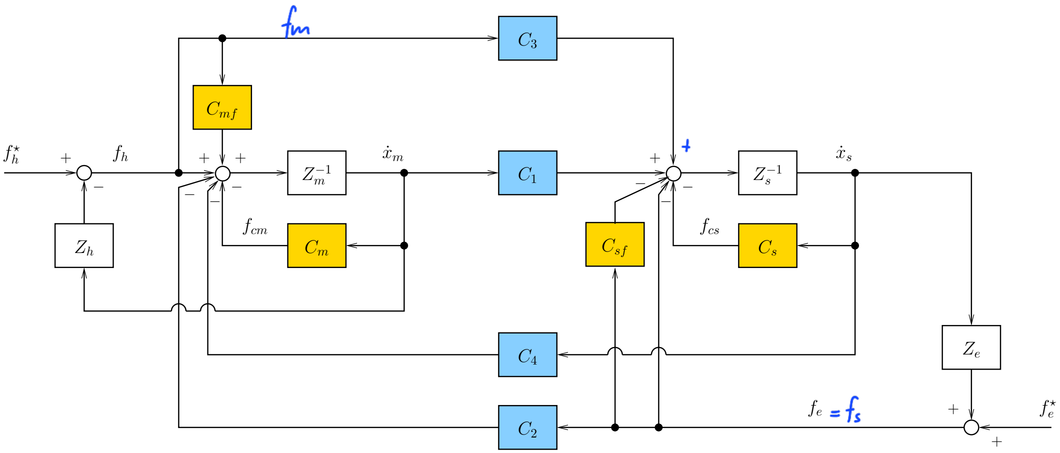
$$\begin{aligned}
H_{11} &:= \left.\frac{f_m}{\dot{x}_s}\right|_{f_s=0} = (Z_m + C_m)D(Z_s + C_s - C_3 C_4) + C_4 \\
H_{12} &:= -\left.\frac{f_m}{f_s}\right|_{\dot{x}_s=0} = -(Z_m + C_m)D(I - C_3 C_2) - C_2 \\
H_{21} &:= \left.\frac{\dot{x}_m}{\dot{x}_s}\right|_{f_s=0} = D(Z_s + C_s - C_3 C_4) \\
H_{22} &:= -\left.\frac{\dot{x}_m}{f_s}\right|_{\dot{x}_s=0} = -D(I - C_3 C_2)
\end{aligned}$$

$Z_{cm} \triangleq Z_m + C_m$

$Z_{cs} \triangleq Z_s + C_s$

where $D = (C_1 + C_3 Z_m + C_3 C_m)^{-1}$.

Four-Channel with local force loops



Hashtrudi-Zaad, Salcudean 2001

$$\begin{bmatrix} f_m \\ \dot{x}_m \end{bmatrix} = \begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix} \begin{bmatrix} \dot{x}_s \\ -f_s \end{bmatrix}$$

$$Z_{cm} = Z_m + C_m$$
$$Z_{cs} = Z_s + C_s$$

① $f_m = H_{11}(s)\,\dot{x}_s$ with $f_s = 0$, $H_{11}(s) = \dfrac{f_m}{\dot{x}_s}$

$$\frac{f_m(1 + C_m f) - C_4 \dot{x}_s - \dot{x}_m C_m - C_2 f_s}{Z_m} = \dot{x}_m$$

$$f_m(1 + C_m f) - C_4 \dot{x}_s - \dot{x}_m C_m - C_2 f_s = \dot{x}_m Z$$

$$\frac{f_m(1 + C_m f) - C_4 \dot{x}_s - C_2 f_s}{C_m + Z_m} = \dot{x}_m$$

Computation for retrieving $\dot{x}_m$ to simplify the above equation

$$\dot{x}_s = \frac{C_3 f_m + C_1 \dot{x}_m - f_e(1 + C_s f) - C_s \dot{x}_s}{Z_s}$$

$$\dot{x}_s Z_s + \dot{x}_s C_s = C_3 f_m + C_1 \dot{x}_m - f_e(1 + C_s f)$$

$$\dot{x}_s(Z_s + C_s) = C_3 f_m + C_1 \dot{x}_m - f_e(1 + C_s f)$$

$$\dot{x}_m = \frac{\dot{x}_s(Z_s + C_s) - C_3 f_m + f_e(1 + C_s f)}{C_1} , \qquad \dot{x}_s = \frac{C_3 f_m + C_1 \dot{x}_m - f_e(1 + C_s f)}{Z_{cs}}$$

$\longrightarrow$ Substituting in eq $*$ we get $\longrightarrow H_{11}(s) = \frac{f_m}{\dot{x}_s}$

$$\frac{f_m(1+C_{mf}) - C_4\dot{x}_s - C_2 f_s}{C_m + Z_m} = \frac{\dot{x}_s(Z_s + C_s) - C_3 f_m - f_e(1+C_{sf})}{C_1}$$

$$C_1 f_m(1+C_{mf}) - C_1(C_4\dot{x}_s \cdot C_2 f_s) = \dot{x}_s(C_m+Z_m)(Z_s+C_s) - (C_m+Z_m)\left[(C_3 f_m) - f_e(1+C_{sf})\right]$$

$$C_1 f_m(1+C_{mf}) - C_1(C_4\dot{x}_s \cdot C_2 f_s) = \dot{x}_s Z_{cm} Z_{cs} - Z_{cm} C_3 f_m + Z_{cm} f_e(1+C_{sf})$$

$$f_m\left[C_1(1+C_{mf}) + Z_{cm}C_3\right] = \dot{x}_s(Z_{cm}Z_{cs} + C_1C_4) - f_s(C_1C_2 + Z_{cm}(1+C_{sf}))$$

$$\frac{f_m}{\dot{x}_s} = \frac{Z_{cm}Z_{cs} + C_1C_4 - f_s(C_1C_2 + Z_{cm}(1+C_{sf}))}{C_1(1+C_{mf}) + C_3 Z_{cm}}$$

$$\left.\frac{f_m}{\dot{x}_s}\right|_{f_s=0} = \frac{Z_{cm}Z_{cs} + C_1C_4}{C_1(1+C_{mf}) + C_3 Z_{cm}} = H_{11} \quad \checkmark$$

$\longrightarrow$ Confronting with H_{11} of the hybrid matrix $\hat{H}$ without local feedback

$$\hat{H}_{11} = \frac{(Z_m+C_m)(Z_s+C_s - C_3C_4)}{C_1 + C_3(Z_m+C_m)} + C_4 =$$

$$= \frac{Z_{cm}Z_{cs} \cancel{- Z_{cm}C_3C_4} + C_1C_4 \cancel{+ C_3C_4Z_{cm}}}{C_1 + C_3 Z_{cm}}$$

$$= \frac{Z_{cm}Z_{cs} + C_1C_4}{C_1 + C_3 Z_{cm}}$$

② $H_{12} = -\dfrac{f_m}{f_s}$ $\qquad \dot{x}_s = 0$

$$C_1 f_m (1 + C_{mf}) - C_1 (\cancel{C_4 \dot{x}_s} - C_2 f_s) = \cancel{\dot{x}_s ((C_m + Z_m)(Z_s + C_s)}) - (C_m + Z_m)[(C_3 f_m) - f_e (1 + C_{sf})]$$

$$C_1 f_m (1 + C_{mf}) - C_1 C_2 f_s = - Z_{cm} C_3 f_m + Z_{cm} f_s (1 + C_{sf})$$

$$f_m [C_1 (1 + C_{mf}) + Z_{cm} C_3] = f_s [C_1 C_2 + Z_{cm} (1 + C_{sf})]$$

$$-\frac{f_m}{f_s}\bigg|_{\dot{x}_s = 0} = -\frac{C_1 C_2 + Z_{cm} (1 + C_{sf})}{C_1 (1 + C_{mf}) + Z_{cm} C_3} = H_{12} \quad ✓$$

→ Confronting with H_{12} of the hybrid matrix $\hat{H}$ without local feedback

$$H_{12} = -\frac{Z_{cm}(1 - C_3 C_2)}{C_1 + C_3 Z_{cm}} - C_2 = \frac{-Z_{cm}(1 - C_3 C_2) - C_2 (C_1 + C_3 Z_{cm})}{C_1 + C_3 Z_{cm}}$$

$$= \frac{-Z_{cm} + \cancel{Z_{cm} C_3 C_2} - C_1 C_2 - \cancel{C_2 C_3 Z_{cm}}}{C_1 + C_3 Z_{cm}}$$

$$= -\frac{C_1 C_2 + Z_{cm}}{C_1 + C_3 Z_{cm}}$$

③ $H_{21} = \left.\dfrac{\dot{x}_m}{\dot{x}_s}\right|_{f_s=0}$

$$\dot{x}_m = \frac{\dot{x}_s(Z_s + C_s) - C_3 f_m + \cancel{f_e(1 + C_s f)}}{C_1}$$

$$\dot{x}_m = \frac{\dot{x}_s Z_{cs}}{C_1} - \frac{C_3}{C_1} f_m$$

Using: $\dfrac{f_m}{\dot{x}_s} = \dfrac{Z_{cm} Z_{cs} + C_1 C_4 - \cancel{f_s(C_1 C_2 + Z_{cm}(1 + C_s f))}}{C_1(1 + C_{mf}) + C_3 Z_{cm}}$

$$\left.\frac{\dot{x}_m}{\dot{x}_s}\right|_{f_s=0} = \frac{Z_{cs}}{C_1} - \frac{C_3}{C_1}\left(\frac{Z_{cm} Z_{cs} + C_1 C_4}{C_1(1 + C_{mf}) + C_3 Z_{cm}}\right)$$

$$= \frac{Z_{cs}\cancel{C_1}(1 + C_{mf}) + \cancel{Z_{cs} C_3 Z_{cm}} - \cancel{C_3 Z_{cm} Z_{cs}} - \cancel{C_1} C_3 C_4}{\cancel{C_1}\left(C_1(1 + C_{mf}) + C_3 Z_{cm}\right)}$$

$$= \frac{Z_{cs}(1 + C_{mf}) - C_3 C_4}{C_1(1 + C_{mf}) + C_3 Z_{cm}} = H_{21} \checkmark$$

→ Confronting with H_{21} of the hybrid matrix $\hat{H}$ without local feedback

$$H_{21} = \frac{Z_{cs} - C_3 C_4}{C_1 + C_3 Z_{cm}}$$

(4) $H_{22} = -\dfrac{\dot{x}_m}{f_s}\Bigg|_{\dot{x}_s = 0}$ $\qquad\qquad \hat{H}_{22} = \dfrac{1 - C_3 C_2}{C_1 + C_3 Z_{cm}}$

$$\dot{x}_m = \frac{\cancel{\dot{x}_s (Z_s + C_s)} - C_3 f_m + f_s (1 + C_s f)}{C_1}$$

Using $\dfrac{f_m}{f_s}\Bigg|_{\dot{x}_s = 0} = \dfrac{C_1 C_2 + Z_{cm}(1 + C_{sf})}{C_1 (1 + C_{mf}) + Z_{cm} C_3}$ we get

$$\frac{\dot{x}_m}{f_s}\Bigg|_{\dot{x}_s = 0} = -\frac{C_3}{C_1}\left(\frac{C_1 C_2 + Z_{cm}(1 + C_{sf})}{C_1 (1 + C_{mf}) + Z_{cm} C_3}\right) + \frac{1 + C_s f}{C_1}$$

$$= \frac{-C_1 C_2 C_3 - Z_{cm} C_3 - Z_{cm} C_3 C_{sf} + C_1 (1 + C_{mf})(1 + C_{sf}) + Z_{cm} C_3 (1 + C_{sf})}{C_1 \left(C_1 (1 + C_{mf}) + Z_{cm} C_3 \right)}$$

$$= \frac{-\cancel{C_1} C_2 C_3 \cancel{- Z_{cm} C_3} \cancel{- Z_{cm} C_3 C_{sf}} + \cancel{C_1} + \cancel{C_1} C_{sf} + \cancel{C_1} C_{mf} + \cancel{C_1} C_{mf} C_{sf} \cancel{+ Z_{cm} C_3} \cancel{+ Z_{cm} C_3 C_{sf}}}{\cancel{C_1} \left(C_1 (1 + C_{mf}) + Z_{cm} C_3 \right)}$$

$$= \frac{1 - C_2 C_3 + C_{sf} + C_{mf} + C_{mf} C_{sf}}{C_1 (1 + C_{mf}) + Z_{cm} C_3} = H_{22} \quad ✓$$

$\rightarrow$ Confronting with H_{22} of the hybrid matrix $\hat{H}$ without local feedback

$$\hat{H}_{22} = \frac{1 - C_3 C_2}{C_1 + C_3 Z_{cm}}$$

• Final Recap

$$Z_{cm} \triangleq Z_m + C_m$$
$$Z_{cs} \triangleq Z_s + C_s$$

$$H_{11} = \frac{Z_{cm} Z_{cs} + C_1 C_4}{C_1 (1 + C_{mf}) + C_3 Z_{cm}}$$

$$H_{12} = - \frac{C_1 C_2 + Z_{cm} (1 + C_{sf})}{C_1 (1 + C_{mf}) + Z_{cm} C_3}$$

$$H_{21} = \frac{Z_{cs} (1 + C_{mf}) - C_3 C_4}{C_1 (1 + C_{mf}) + C_3 Z_{cm}}$$

$$H_{22} = \frac{1 - C_2 C_3 + C_{sf} + C_{mf} + C_{mf} C_{sf}}{C_1 (1 + C_{mf}) + Z_{cm} C_3}$$