ASSIGNMENT 2

30/09

given
- u_*
- Q_0
- direction at 60 m
- θ_0
- V_{10}, V_{40}, V_{60}, V_{80}, V_{100}

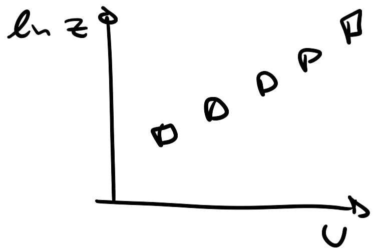
① compute $1/L$

$$\frac{1}{L} = \left(\frac{k g}{\theta_0}\right) \cdot \frac{Q_0}{u_*^3}$$

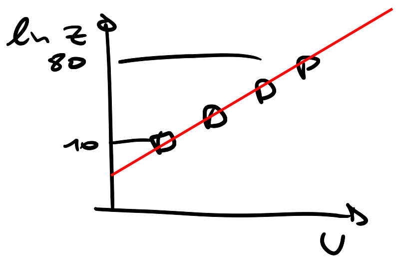
build pdf

② for wind direction $\in [60°, 120°]$
select for neutral condition $\Rightarrow L^{-1} < 0{,}0008$

ⓐ compute average V at each height + error bars



ⓑ fit line



$$\ln z = aU + b$$
$$y = ax + b$$

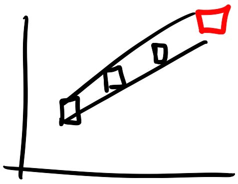
ⓒ compute u_*, z_0

$$U(z) = \frac{u_*}{k} \ln \frac{z}{z_0} = \frac{u_*}{k}(\ln z - \ln z_0) \Rightarrow \ln z = \frac{Uk}{u_*} + \ln z_0$$

$$\Rightarrow \ln z = \underbrace{\frac{k}{u_*}}_{a} U + \underbrace{\ln z_0}_{b}$$

$z_0 = b$ look in the table to compare
$u_* = \frac{k}{a}$

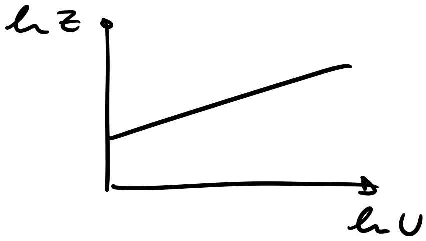
(d) compute U_{N100}



compute % difference from U_{100}, estimated and U_{100} real

(e) compute α

$$\alpha = \frac{d \ln U}{d \ln z} = \frac{1}{\ln \frac{z}{z_0}}$$



$$\ln z = a \ln U + b$$

$$a = \frac{dy}{dx} = \frac{d \ln z}{d \ln U} \Rightarrow \alpha = \frac{1}{a}$$

IMPLIED z_0 ?

(f) α at $z=60$, $z=80$

$$\alpha = \frac{\ln (U_2/U_1)}{\ln (z_2/z_1)}$$

(g) predict U_{100}

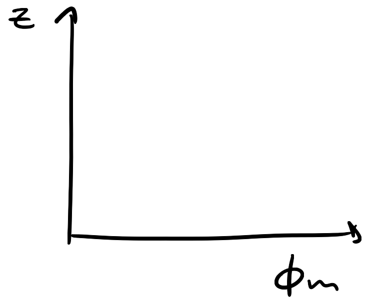
$$U(z) = U_{ref} \left(\frac{z}{z_{ref}} \right)^{\alpha}$$ with $z_{ref} = 80$ m

(h) comparison → plot everything

(3)

?

(4) $\phi_m\left(\frac{z}{L}\right) = \dfrac{\frac{dU}{dz}}{\frac{u_*}{\kappa z}}$



$\phi = 1 + 4.8\,\frac{z}{L}$ for stable

$\phi = \left(1 - 19.3\,\frac{z}{L}\right)^{-1/4}$

NOT SURE IF IT IS THAT SIMPLE

if $\phi_m = 1 \Rightarrow$ neutral log law

(5) $U(z) = \frac{u_*}{\kappa}\left[\log\left(\frac{z}{z_0}\right) - \Psi_m\left(\frac{z}{L}\right)\right]$

$$\phi_m = \frac{\frac{dU}{dz}}{\frac{u_*}{\kappa z}} \Rightarrow \frac{dU}{dz} = \frac{u_*}{\kappa z}\phi_m \Rightarrow \int_0^U dU = \frac{u_*}{\kappa}\int_{z_0}^{z}\frac{1}{z}\phi_m\,dz$$

stable: $\phi = 1 + 4.8\,\frac{z}{L}$

$$U = \frac{u_*}{\kappa}\int_{z_0}^{z}\frac{1}{z}\left(1 + 4.8\,\frac{z}{L}\right)dz = \frac{u_*}{\kappa}\int_{z_0}^{z}\frac{1}{z} + \frac{4.8}{L}\,dz$$

$$= \frac{u_*}{\kappa}\left[\ln\frac{z}{z_0} + \frac{4.8}{L}(z - z_0)\right]$$

$$\Rightarrow \Psi_m = -\frac{4.8}{L}(z - z_0)$$

unstable: $\phi = \left(1 - 19.3\,\frac{z}{L}\right)^{-\frac{1}{4}}$

$$U = \frac{u_*}{\kappa}\int_{z_0}^{z}\frac{1}{z}\left(1 - 19.3\,\frac{z}{L}\right)^{-1/4}dz$$

$$= \frac{u_*}{\kappa}\int_{z_0}^{z}\frac{1}{z\left(1 - \frac{19.3}{L}\,z\right)^{1/4}}\,dz$$

$$\Rightarrow \int_{z_0}^{z} \frac{1}{z\,(1-\beta z)^{1/4}}\, dz$$

substitution $\quad u = (1-\beta z)^{1/4}$

$$du = \frac{1}{4}(1-\beta z)^{-\frac{3}{4}}(-\beta)\, dz$$

$$dz = \frac{-4\, du}{(1-\beta z)^{-\frac{3}{4}}\,\beta} = -\frac{4}{\beta}(1-\beta z)^{\frac{3}{4}}\, du = -\frac{4}{\beta}\, u^3\, du$$

$$u^4 = 1-\beta z \Rightarrow z = \frac{1-u^4}{\beta}$$

$$\Rightarrow \int_{z_0}^{z} \frac{1}{z(1-\beta z)^{1/4}}\, dz = \int \frac{\cancel{\beta}}{(1-u^4)\,u} \cdot -\frac{4}{\cancel{\beta}}\, u^3 du = +4\int \frac{u^2}{u^4-1}\, du$$

$$= 4\int \frac{u^2}{(u^2+1)(u+1)(u-1)}\, du =$$

$$= 4\int \left(\frac{1}{2(u^2+1)} - \frac{1}{4(u+1)} + \frac{1}{4(u-1)}\right) du$$

$$= \underbrace{\int \frac{2}{u^2+1}\, du}_{1} - \underbrace{\int \frac{1}{u+1}\, du}_{2} + \underbrace{\int \frac{1}{u+1}\, du}_{3}$$

1 $\quad \int \frac{2}{u^2+1}\, du = 2\, tg^{-1} u$

2 $\quad -\int \frac{1}{u+1}\, du = -\ln(u+1)$ $\qquad\qquad u = (1-\beta z)^{1/4}$

3 $\quad \int \frac{1}{u-1}\, du = \ln(u-1)$

$$\Rightarrow 4\int \frac{u^2}{u^4-1}\, du = 2\, tg^{-1} u - \ln(u+1) + \ln(u-1)$$

$$\int_{z_0}^{z} \frac{1}{z(1-\beta z)^{1/4}} = 2\, tg^{-1}(1-\beta z)^{1/4} - \ln\left[(1-\beta z)^{1/4}+1\right] + \ln\left[(1-\beta z)^{1/4}-1\right]\Big|_{z_0}^{z}$$

$$\Rightarrow U = \frac{u_*}{\kappa}\int_{z_0}^{z} \frac{1}{z(1-\beta z)^{1/4}} = \frac{u_*}{\kappa}\Big(\underbrace{2\, tg^{-1}(1-\beta z)^{1/4} - \ln\left[(1-\beta z)^{1/4}+1\right] + \ln\left[(1-\beta z)^{1/4}-1\right]}_{\alpha}\Big|_{z_0}^{z}$$

$$\Rightarrow \varphi = \log\frac{z}{z_0} - \alpha$$