浙江大学爱丁堡大学联合学院
ZJU-UoE Institute

Filters

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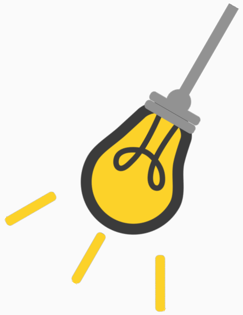
Learning objectives

- Define rank and convolutional filters
- Explain their use in image analysis
- Implement basic filters in Python



Types of pixel operations

Operations for manipulating pixel intensities

Two types of operations:

- Point operations - Change pixel intensity based only on its value - $I'_{(x,y)} = f(I_{x,y})$ (see Lecture 3)
- **Neighbourhood operations** - Change pixel intensity based on the intensity of the pixel and its neighbours.

Filters

Neighbourhood operations, often called **filters** allow to modify an image in a way that is not possible with point operations.

- Detect simple structures such as edges, corners, lines, etc.
- Perform operations such as smoothing, sharpening, etc.
- Noise reduction

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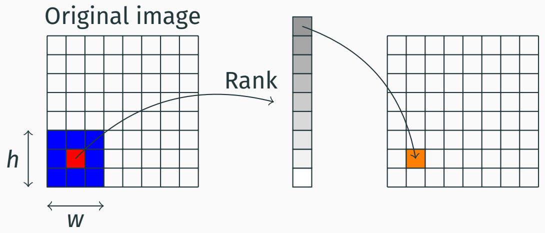
Today we will look at:

- **Rank filters** - the new pixel value is a function of the rank of the pixel values of the neighbourhood
- **Convolutional filters** - the new pixel value is a weighted sum of the pixel values of the neighbourhood

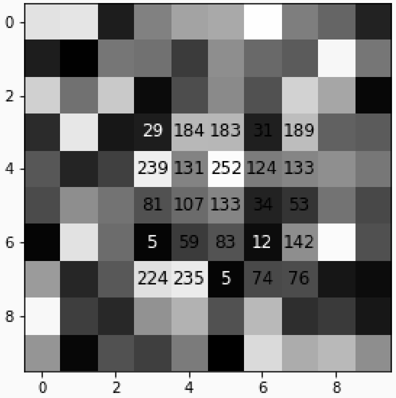
Outline

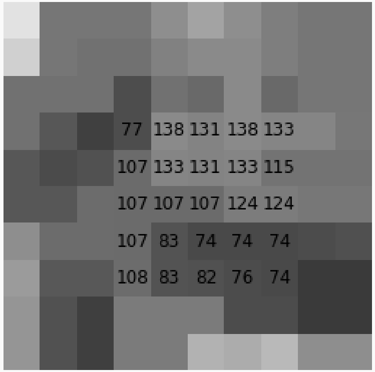
Rank filters



- We decide on a window size $w \times h$ (other non-rectangular shapes are possible)
- We traverse each pixel in the image and take its $w \times h$ neighbourhood
- We rank the intensity of each pixel in the neighbourhood
- We take a specific value (e.g. minimum, maximum, median) and set the output value to this value

Example - median filter





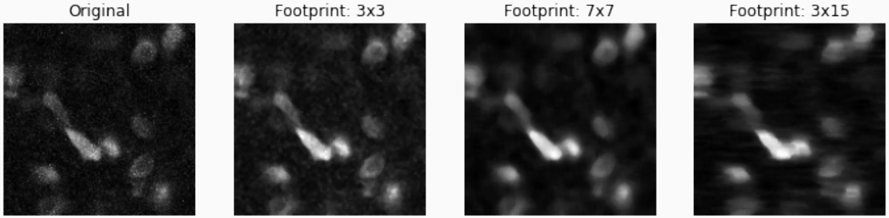
Rank filters in Scikit Image

Rank filters are implemented in Scikit Image in the ‘skimage.rank‘ module.
These filters require a *footprint* of the pixel neighbourhood, as a matrix of 0 and 1.

For example

```
from skimage.rank import median

# 3x3 neighbourhood
footprint = np.ones(3, 3)
img_median = median(img, selem=footprint)
```



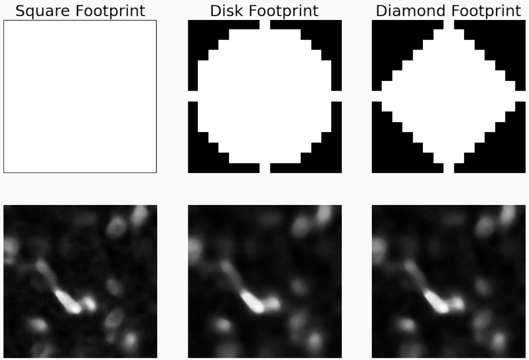
Non-rectangular footprints

The footprint can be a *non-rectangular* matrix. For example, you can generate a **circular** shape using the `skimage.morphology.disk` function or a **diamond** shape using the `skimage.morphology.diamond` function.

```
from skimage.morphology import disk,
diamond

dsk = disk(7) # A disk, radius 7
dia = diamond(7) # A diamond, radius 7

fig, ax = plt.subplots(1, 2)
ax[0].imshow(dsk, cmap="gray")
ax[1].imshow(dia, cmap="gray")
```



Rank filters - use cases

Rank filters are very simple, but have useful applications.
The **median filter** is used to remove noise and to smooth images.

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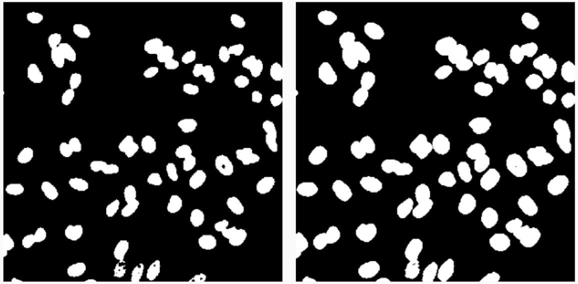
The **maximum filter** can be used in binary images to remove small "holes". It is also a very common filter used in modern neural networks for image analysis.

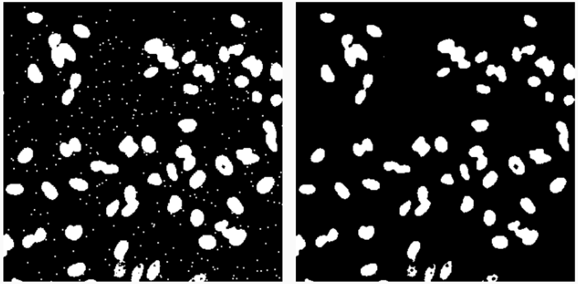


Rank filters - use cases

Rank filters are very simple, but have useful applications.
The **median filter** is used to remove noise and to smooth images.

The **maximum filter** can be used in binary images to remove small "holes". It is also a very common filter used in modern neural networks for image analysis.

The **minimum filter** can be used to remove small bright spots.

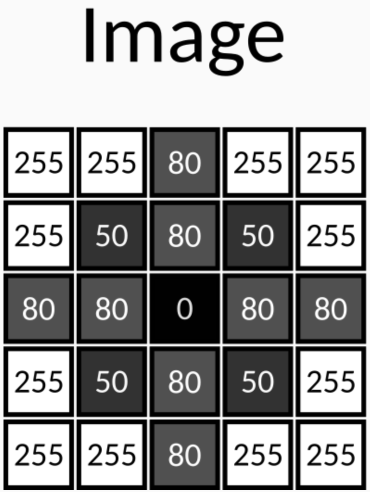


Outline

Convolutional filters

A **convolutional filter** consists of a small matrix, called a **kernel**, that is used to process an image. Convolution takes each pixel of the image, together with its neighbours, and adds them together, weighting each neighbour by the value of a kernel of the same size of the neighbourhood.

Example of a convolutional filter



Image

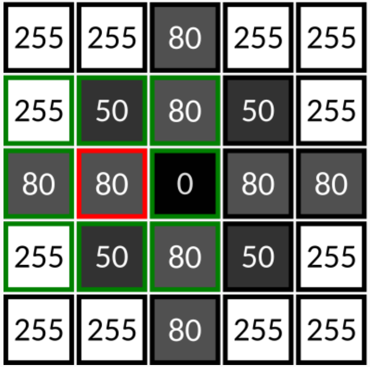
255	255	80	255	255
255	50	80	50	255
80	80	0	80	80
255	50	80	50	255
255	255	80	255	255

Kernel

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Example of a convolutional filter

Image



255	255	80	255	255
255	50	80	50	255
80	80	0	80	80
255	50	80	50	255
255	255	80	255	255

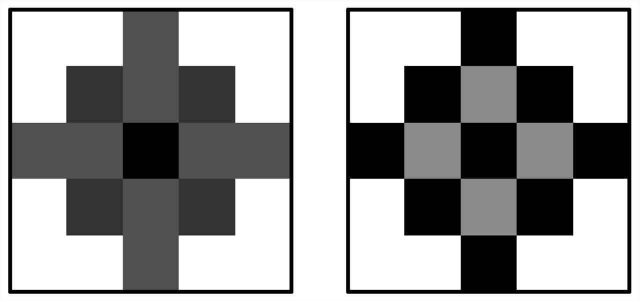
Kernel

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

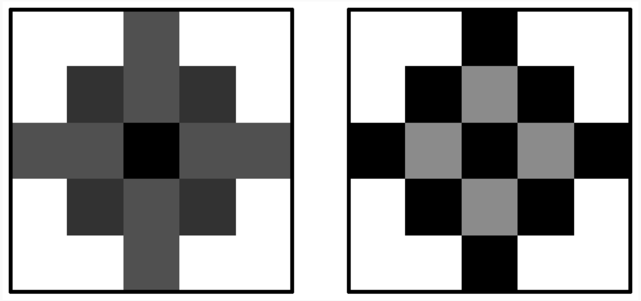
The convolved pixel value will be

$$255 * 0 + 50 * (-1) + 80 * 0 + 80 * (-1) + 80 * 5 + 0 * (-1) + 255 * 0 + 50 * (-1) + 80 * 0 = \mathbf{220}$$

Example of a convolutional filter - result


Our image, after applying the convolutional filter.

Example of a convolutional filter - result


Our image, after applying the convolutional filter.

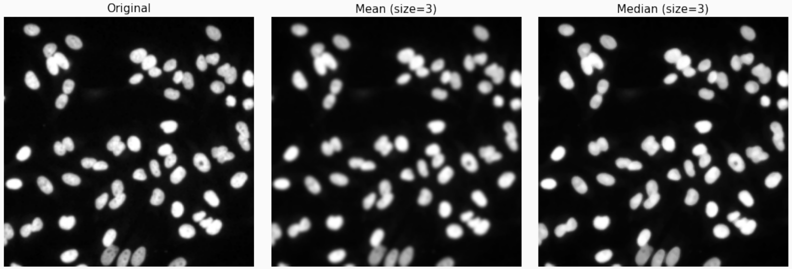
Convolutional filters are **shift invariant**, meaning the result of the convolution is the same regardless of the position of the pixel in the original image (as long as it has the same neighborhood).

Common convolutional filters - averaginig filter

The **averaging filter** or **box blur** is a simple filter that is used to reduce noise in an image.
It simply takes the average of the pixel values in the neighbourhood.

Example 3x3 kernel:

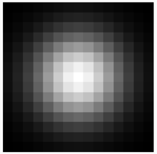
$$\begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$



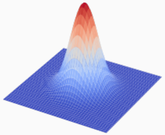
Not as good as the **median filter**, as it is sensitive to outliers and does not preserve edges as well (can you get an intuition as to why?).

Common convolutional filters - Gaussian filter

The **Gaussian filter** is a filter that is used to smooth images.
It uses a **Gaussian** function to weight the pixel values in the neighbourhood.



$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

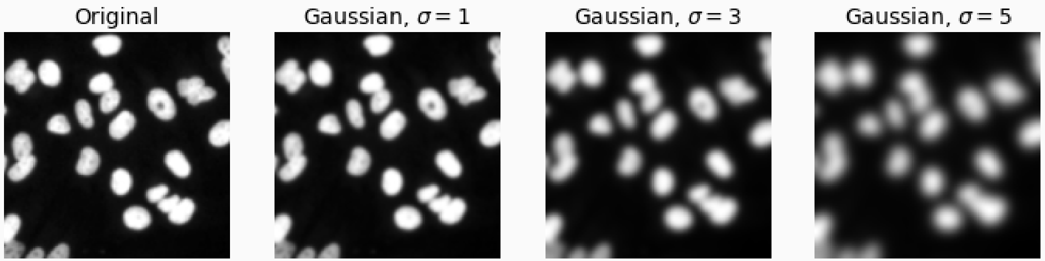


Example of gaussian kernels 3x3 and 5x5 (approx.):

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \qquad \frac{1}{256}\begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

Common convolutional filters - Gaussian filter - result

Example of gaussian filters with increasing sigma values:

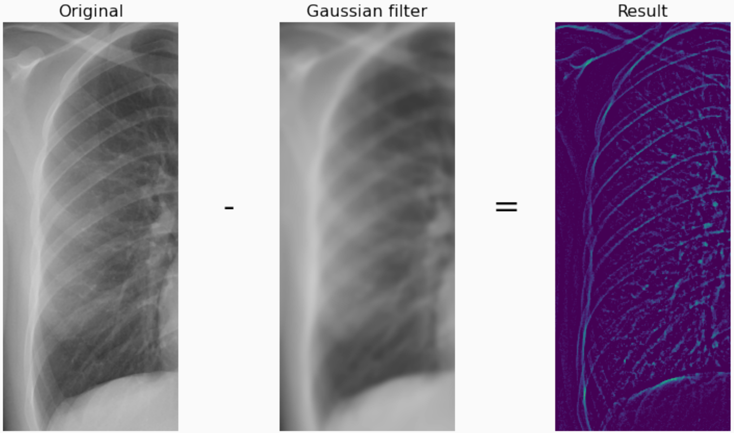


Gaussian filters result in a blurring effect, which can be useful for removing noise, or in general removing the finer detail of the image (low-pass filtering).

```
from skimage.filters import gaussian
img_blurred = gaussian(image, sigma=1)
```

Sharpening filters

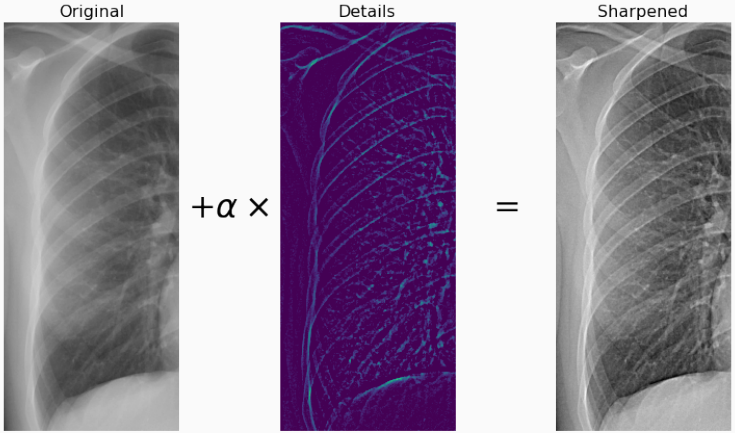
Sharpening filters are used to increase the detail of an image.



If we apply a Gaussian filter to the image we will remove detail, so if we subtract the result from the original image we will get the "details" of the image.

Sharpening filters

Sharpening filters are used to increase the detail of an image.



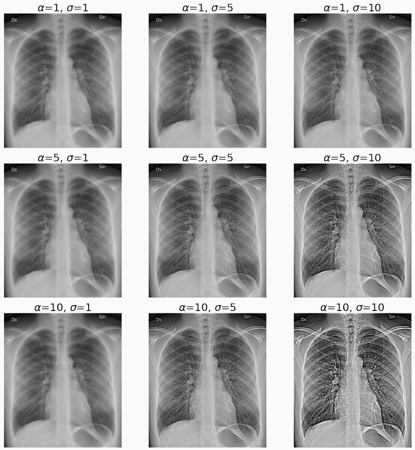
We can now add the details to the original image, thus sharpening it!

This process is called **"unsharp masking"**.

Unsharp masking in Scikit Image

You can use the `skimage.filters.unsharp_mask` function to apply unsharp masking to images.

```
from skimage.filters import unsharp_mask
img_sharpened = unsharp_mask(img, radius=15, amount=2)
```



Exploring the parameter space of unsharp masking.

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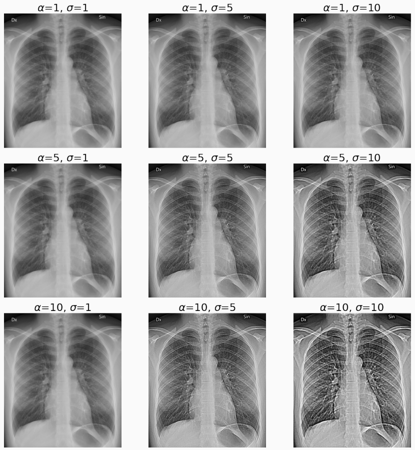
```

Exercise: try writing your own unsharp masking function.
It should accept an image, a radius (the σ of the gaussian blur) and an amount for sharpening and return the sharpened image.



Exploring the parameter space of unsharp masking.

What happens at the edges?

Convolutional filters traverse the image pixel by pixel and apply a function that takes into account the pixel's neighbourhood.

What happens at the edges, where the neighbourhood is not complete?
There are several ways to deal with this.

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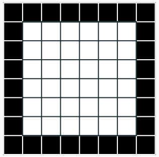
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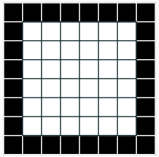
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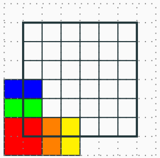
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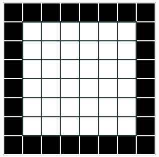
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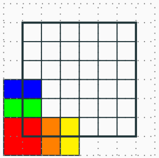
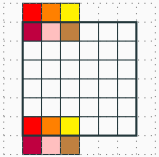
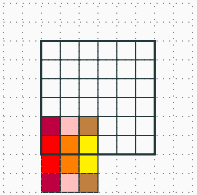

Extend


Wrap


Mirror

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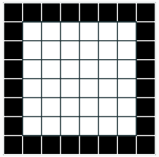
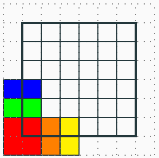
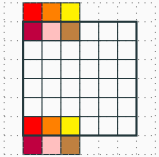
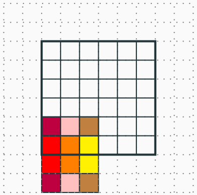
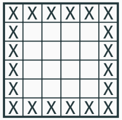
- **Mirror the image**: extra pixels are created by mirroring the image.


Zero-pad

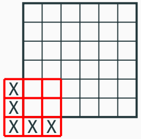

Extend


Wrap


Mirror



Crop the image



Crop the kernel

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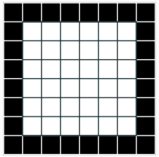
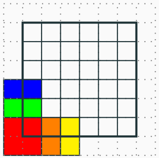
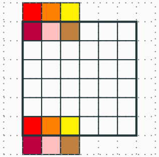
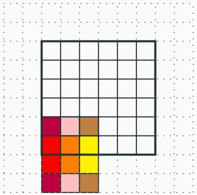
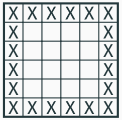
- **Extend the image**: edge pixels are copied, corner pixels are repeated as *wedges*.
- **Wrap the image**: extra pixels are taken from the opposite side of the image.
- **Mirror the image**: extra pixels are created by mirroring the image.
- **Crop the image**: we ignore the edge pixels. This will result in a smaller output image.
- **Crop the kernel**: we only use the kernel values corresponding to a pixel in the image. Kernel weights are adjusted to account for this.

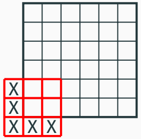

Zero-pad


Extend


Wrap


Mirror


Crop the image


Crop the kernel

Edge behaviour in Scikit Image

Filters in Scikit Image allow you to choose what to do with the edge pixels using the `mode` parameter.

For example

```
from skimage.filters import gaussian
image_smoothed = gaussian(image, sigma=1, mode="mirror")
```

Allowed values for `mode` are

- `constant` - padding with the value specified by `cval`
- `nearest` - extend
- `mirror` and `reflect` - both of these mirror the image, but `reflect` does not duplicate the edge pixel
- `wrap`

Want to try out more?

You can design your own convolutional filter and apply it to an image using the `skimage.filters.edges.convolve` function.

What happens using this kernel?

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

What about this?

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

You can see convolutional filters in action at this website.

Summary

- Image filters are a simple yet powerful way of manipulating images.
- Filters can be used to remove noise, smooth images, sharpen them, and more.
- Rank filters such as median, maximum and minimum work by choosing a specific value of the pixels in the neighbourhood, depending on their rank.
- Convolutional filters use a kernel to apply a linear transformation to the image.
- In the next lecture we will look at other methods for noise removal and filters for edge detection.