浙江大学爱丁堡大学联合学院
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# Geometric image transformations

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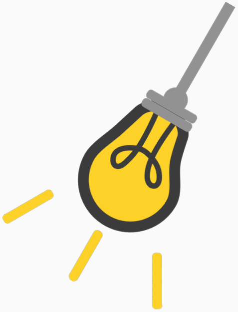
# Learning objectives

- Describe use cases for geometric transformations of images
- Use Python to crop images (in 2 or more dimensions)
- Explain the theory of affine transformations
- Implement basic transformations in Python (translation, scaling, rotation)



A brief recap

Geometric image transformations

- Cropping
- Translation
- Rotations
- Scaling

# Python tools for image processing

Last time we learnt how to open and display images using Python.

matplotlib

General plotting library

scikit-image
image processing in python

Specific functions for image manipulation

NumPy

Library for matrix operations

## Reading images as Numpy arrays

matplotlib

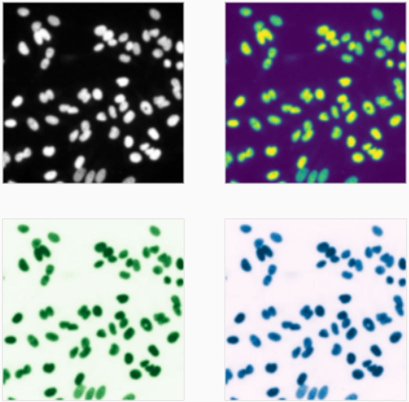
```
import matplotlib.pyplot as plt
img = plt.imread("cells.jpg")
plt.imshow(img)
plt.show()
```

scikit-image
image processing in python

```
from skimage import io
img = io.imread("cells.jpg")
io.imshow(img)
io.show()
```

# Colour mapping

```
fig, ax = plt.subplots(ncols=2, nrows=2)
ax[0, 0].imshow(img, "gray")
ax[0, 1].imshow(img, "viridis")
ax[1, 0].imshow(img, "Greens")
ax[1, 1].imshow(img, "PuBu")
```



Check out the Matplotlib website for a list of colourmaps!

`https://matplotlib.org/stable/tutorials/colors/colormaps.html`

# Geometric image transformations

**Geometric image transformations** are operations that change the image geometry without altering its pixel values (mostly…).

Examples are cropping, translating, scaling rotating an image.

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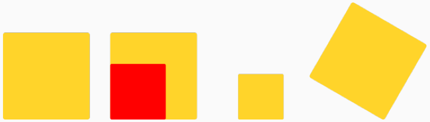
Examples are cropping, translating, scaling rotating an image.
Example use cases:

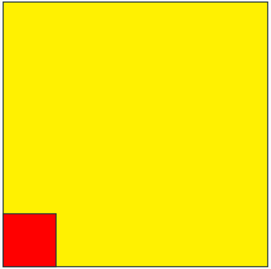
- Analysing only part of an image (cropping)
- Making sure all input images for a pipeline are the same size (scaling, cropping)
- Aligning multiple images (e.g. in video stabilization) (rotating, translating)
- ...



# Cropping

Cropping is as easy as subsetting the image matrix.

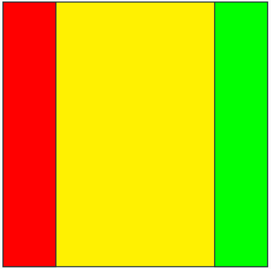
```
# img contains a 512 x 512 image
# Take the top-left 100 x 100 pixels
img1 = img[0:100, 0:100] # shape (100, 100)
```



# Cropping

Cropping is as easy as subsetting the image matrix.

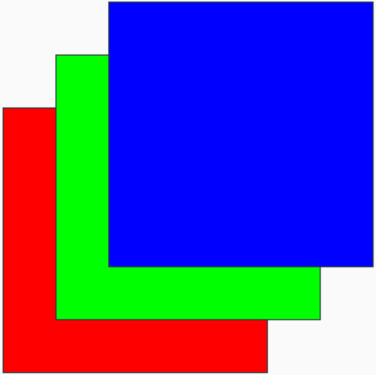
```
# img contains a 512 x 512 image
# Take the top-left 100 x 100 pixels
img1 = img[0:100, 0:100] # shape (100, 100)

# A 100 pixel wide strip on the left...
img2 = img[:, 0:100] # shape (512, 100)
# ...or on the right!
img3 = img[:, -100:]  # shape (512, 100)
```



# Cropping in more than two dimensions

Since images are just tensors, we can crop them in any dimension.

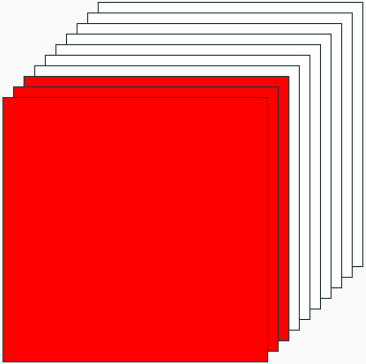
```
# img is a 512 x 512 RGB image
# img.shape is (3, 512, 512)
# Extract the green channel
img_green = img[1] # Shape (512, 512)
```



## Cropping in more than two dimensions

Since images are just tensors, we can crop them in any dimension.

```
# img is a 512 x 512 RGB image
# img.shape is (3, 512, 512)
# Extract the green channel
img_green = img[1] # Shape (512, 512)

# video is a 512 x 512 video of 300 frames
# video.shape is (300, 512, 512)
# Take the first 50 frames
video_crop = video[0:50]
```



# Question time

We have a 100 frames video of a 512 x 512 z-stack with 60 planes loaded in `zstack`.

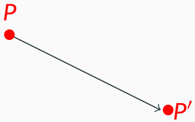
`zstack.shape` is `(100, 60, 512, 512)`

**What does this code give you?**

```
result = stack[50:70, :, 100:300]
```

# Affine transformations

In Euclidean geometry, an affine transformation is a geometric transformation that preserves lines and parallelism but not necessarily distances and angles. [Wikipedia]




We want to transform $P\ (x; y)$ into $P'\ (x'; y')$.

$$P' : \begin{cases} x' = f(x, y) + a \\ y' = f(x, y) + b \end{cases}$$

where $f$ is a linear function

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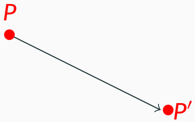


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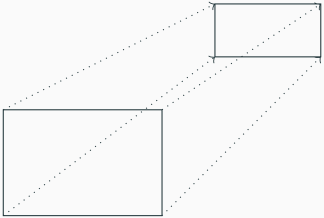
We can generalise this to an image, by applying the transformation to every pixel.



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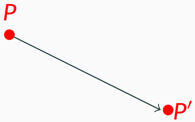


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$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{A} \begin{bmatrix} x \\ y \end{bmatrix} + \mathbf{B} \qquad \mathbf{A} = \begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix} ; \mathbf{B} = \begin{bmatrix} b_{00} \\ b_{10} \end{bmatrix}$$

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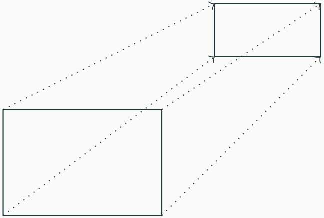

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This can be combined into*

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{00} & a_{01} & b_{00} \\ a_{10} & a_{11} & b_{10} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

This is the **transformation matrix**.

* Don't remember how matrix multiplication works? Check out Wikipedia!

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This is the **transformation matrix**.

Transformation matrices are easily created using the `skimage.transform.SimilarityTransform` function.

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# Translation

To translate we offset the points by $(t_x, t_y)$: $\begin{cases} x' = x + t_x \\ y' = y + t_y \end{cases}$

The translation matrix is: $\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$

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```
from skimage.transform import SimilarityTransform, warp
m = SimilarityTransform(translate = (2, 5))
img = np.ones(shape=(10, 10))
img[5:7, 5:7] = 0
img_translated = warp(img, m)
```

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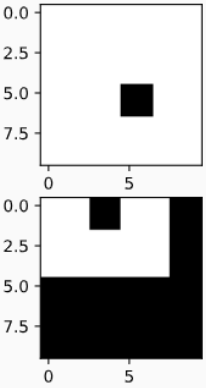
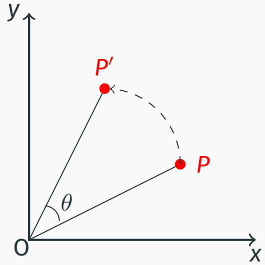


# Outline

# Rotating a point around the origin

We want to rotate a point $P(x, y)$ around the origin by an angle $\theta$ to get $P'(x', y')$.



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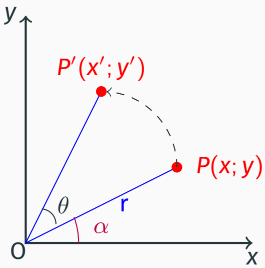


$$P : \begin{cases} x = r\cos(\alpha) \\ y = r\sin(\alpha) \end{cases} \quad P' : \begin{cases} x' = r\cos(\alpha+\theta) \\ y' = r\sin(\alpha+\theta) \end{cases}$$

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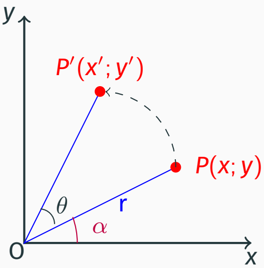


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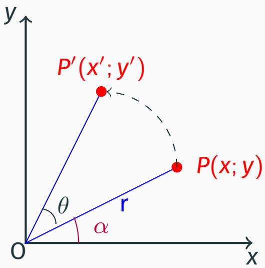
$$P' : \begin{cases} x' = r\cos(\alpha)\cos(\theta) - r\sin(\alpha)\sin(\theta) \\ y' = r\sin(\alpha)\cos(\theta) + r\sin(\theta)\cos(\alpha) \end{cases}$$

Why? Wikipedia to the rescue!

# Rotating a point around the origin

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Thus:

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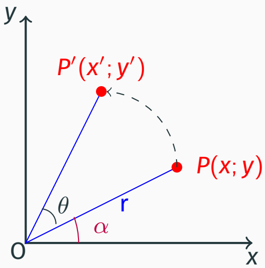


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Thus:

$$P': \begin{cases} x' = x\cos(\theta) - y\sin(\theta) \\ y' = y\cos(\theta) + x\sin(\theta) \end{cases}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Transformation (rotation) matrix

## Rotating in Scikit Image

To rotate an image we will:

- Offset our image so that it is centered on the origin
- Generate a rotation matrix
- Multiply the coordinates of each pixel by the rotation matrix

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- Offset our image so that it is centered on the origin
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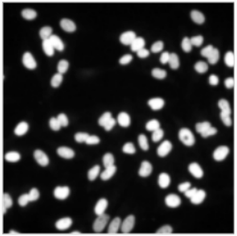
- Multiply the coordinates of each pixel by the rotation matrix

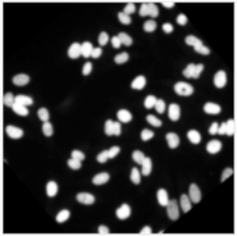
Luckily, Scikit Image has a ready function for that, `skimage.transform.rotate`.

```
import matplotlib.pyplot as plt
from skimage.transform import rotate

img = plt.imread("cells.jpg")
img_rotated = rotate(img, 20)

plt.imshow(img_rotated, cmap="gray")
plt.show()
```





Note: we lost part of the image and we "gained" black pixels around it.

# Optional exercise - want to try by yourself?

It might be interesting to try to code image rotation by yourself.

Hints:

- Generate the transformation matrices using `skimage.transform.SimilarityTransform`
- You will need three matrices: one translation matrix to offset your image by (-xcenter, -ycenter); a rotation matrix to rotate your image around (0, 0); and finally another translation matrix to translate your image back to its original position.
- You can combine the matrices as `m1+m2+m3`
- You can use `skimage.transform.warp` to apply the combined transformation matrix to your image.

**Are there any differences between your result and the results from `rotate`?**

*If you are stuck, try to look at the source code for `rotate`!*

A brief recap

Geometric image transformations

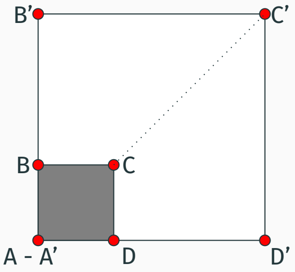
Cropping

Translation

Rotations

Scaling

# Scaling

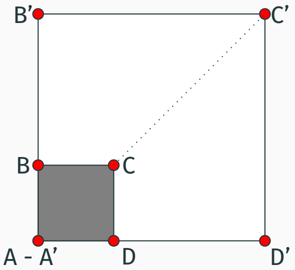



Scaling transforms the coordinates as:

$$\begin{cases} x' = s_x * x \\ y' = s_y * y \end{cases}$$

Transformation matrix = $\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

# Scaling

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Transformation matrix = $\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

```
from skimage.transform import SimilarityTransform
m = SimilarityTransform(scale = (2, 3))
```

# Scaling

Two problems:

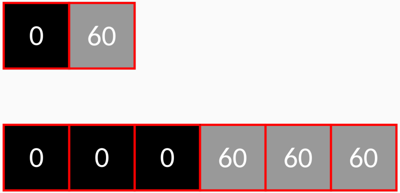
When **upscaling** we need to generate new information.

When **downscaling** we need to decide "how to lose" information.

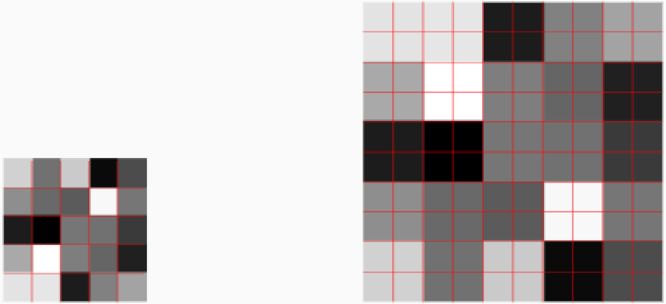
**Interpolation** is the solution!

# Nearest-neighbour interpolation

The simplest way to resize an image is to use nearest-neighbour interpolation. Each pixel of the scaled image will have the colour of the nearest pixel in the original image.
In this "1D" example, we resize a 1x2 image to 1x6

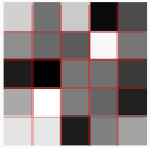


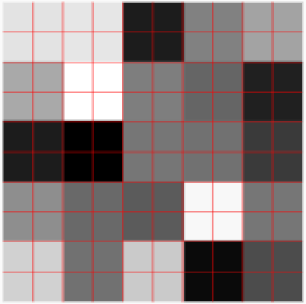
# Nearest-neighbour interpolation



Upscaling of a 5x5 image by a factor of 2, to get a 10x10 image, with nearest neighbour interpolation

# Nearest-neighbour interpolation





Upscaling of a 5x5 image by a factor of 2, to get a 10x10 image, with nearest neighbour interpolation

```
from skimage.transform import rescale
img_scaled = rescale(img, 2, order=0)
```

# Scaling with interpolation

Better ways to scale an image involve changing the pixel values of the rescaled image based on their neighbourhood.

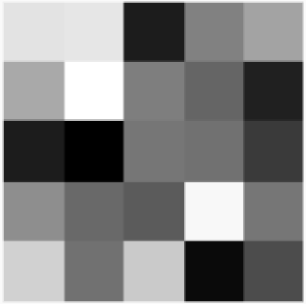
For example we could use linear interpolation

| | |
|---|---|
| 0 | 60 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 12 | 24 | 36 | 48 | 60 |

# Linear interpolation

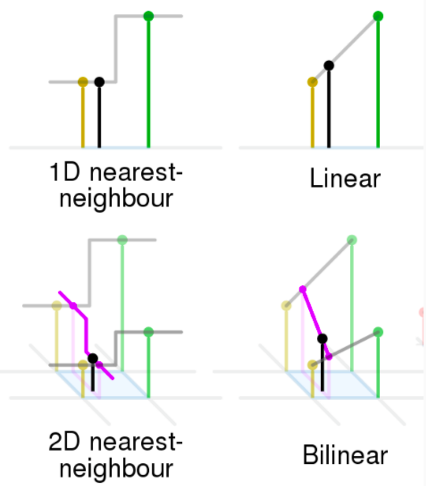
The same applies in 2D, although we need to take into accounts the values of both horizontal and vertical neighbours (**bilinear interpolation**).



Upscaling of a 5x5 image by a factor of 2, to get a 10x10 image, with bi-linear interpolation

```
from skimage.transform import rescale
img_scaled = rescale(img, 2, order=1)
```

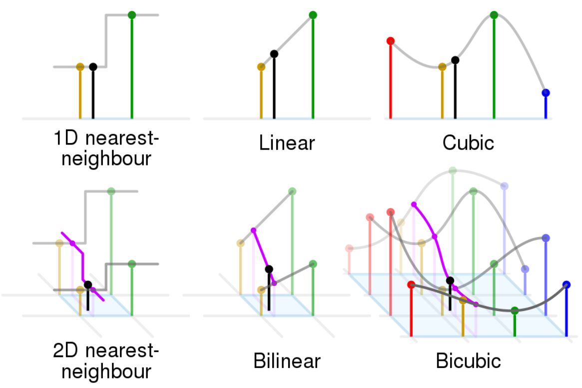
# Nearest neighbour vs linear interpolation




Comparison of nearest neighbour and linear interpolation - Source: Wikipedia

# Higher orders of interpolation

We can use higher orders of interpolation to produce smoother results.




Comparison of nearest neighbour and linear interpolation - Source: Wikipedia

Scikit Image supports values from 0 to 5 in the `order` parameter of the `rescale` function. 0 is nearest neighbour, 1 is bi-linear, 2 is bi-quadratic and so on.

# Scaling to a target size

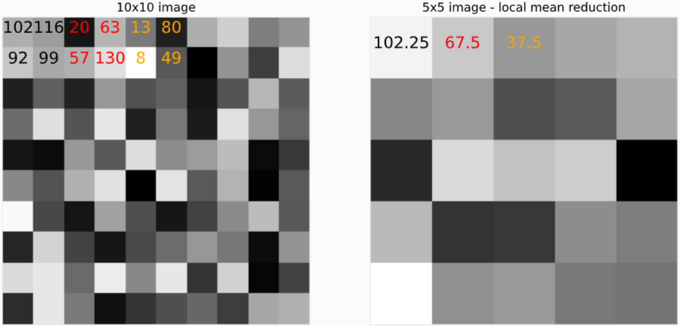
To scale to a target size, rather than by a specific factor, we can use `resize` instead of `rescale`.

```
import matplotlib.pyplot as plt
from skimage.transform import resize, rescale

img = plt.imread("cells.jpg")
img_scaled1 = rescale(img, 2)
img_scaled2 = resize(img, (150, 150))
```

# Local mean downscaling

Another simple solution for downscaling is calculating the local mean of each pixel



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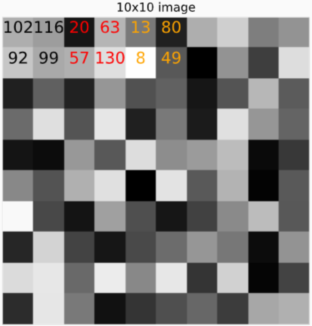


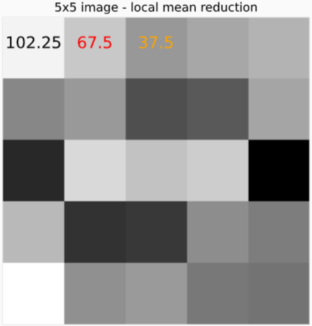


```
from skimage.transform import downscale_local_mean
img_small = downscale_local_mean(img, (2,2))
```

Image needs to be padded if the size is not a multiple of the downscaling factor. This method is fast but not good at keeping fine details.

# Scaling summary

`skimage.transform.rescale` $\rightarrow$ scales an image by a specific factor (>1 upscaling; <1 downscaling.). Can specify a different scaling factor for each dimension of the image.

`skimage.transform.resize` $\rightarrow$ scales an image to a target size.

`skimage.transform.downscale_local_mean` $\rightarrow$ downscales the image by a specific factor (>1), using the local mean of each pixel.

## Summary

- Affine transformations are simple yet powerful way to modify an image.
- Scikit Image allows generation of any transformation matrix…
- …but also provides several pre-made functions for rotating, scaling, etc
- **Workshop 1** will allow you to practice what learned so far!