


浙江大学爱丁堡大学联合学院

ZJU-UoE Institute

Lecture 3 - Image histograms

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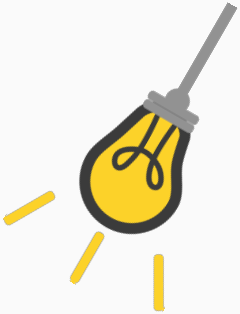
Learning objectives

- Define and produce an image histogram
- Use histograms to interpret image quality
- Apply point operations to manipulate image histograms



Histograms

What is a histogram?

"A histogram is an approximate representation of the distribution of numerical data." (Wikipedia)

Given a variable x, in the interval $[x_{min}; x_{max}]$, we divide this interval (or a suitably larger one) into non-overlapping **bins** and count the number of values of x falling into each bin.
Generally, we choose bins to be of equal size.

Histogram example

Given $x = 5, 7, 10, 22, 35, 88, 26, 74, 22, 95$

We can choose the interval $[0; 99]$, and divide it into bins of size 10 $[0 : 9][10; 19] \ldots [90 : 99]$

Histogram example

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We now count the occurrences of x in each bin.

$$[0 : 9] \to 2$$

$$[10 : 19] \to 1$$

...and so on

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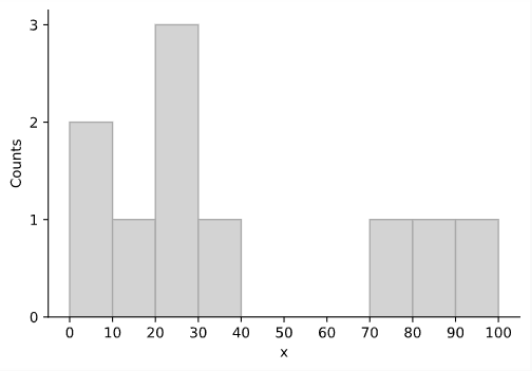


x =	5	7	10	22	35	88	26	74	22	95

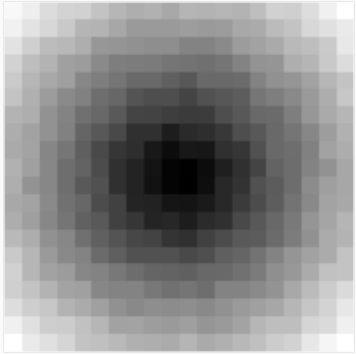
Histogram of an image

Images are just matrices, so we can count the occurrences of each pixel value in the image.

Example

An 8-bit image, 20 x 20

```
> print(img.shape)
(20, 20)
>print(np.min(img))
13
>print(np.max(img))
177
```



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```
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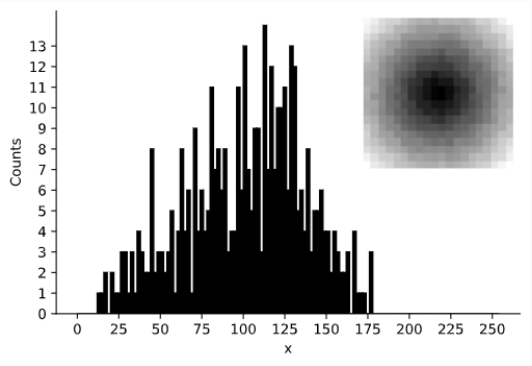
>print(np.min(img))
13
>print(np.max(img))
177
```

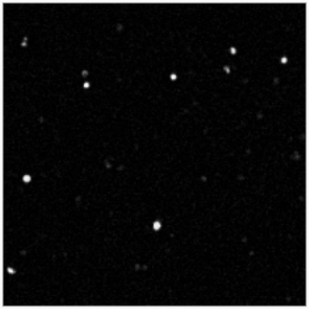
Maximum dynamic range [0, 255]
We count the occurrences of each pixel with intensity between 0 and 255.

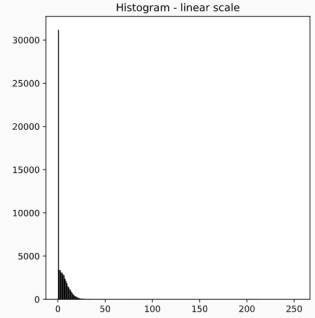


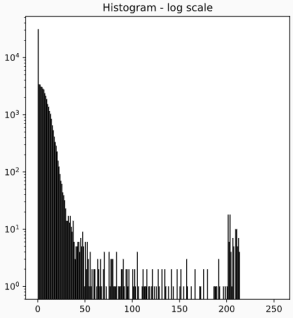
What can you tell about the image from looking at the histogram?

Logarithmic scale

We can plot the histogram on a logarithmic scale. Useful when images contain many pixels of a specific intensity (e.g. cells on a black background).

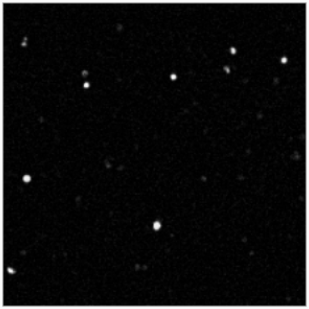


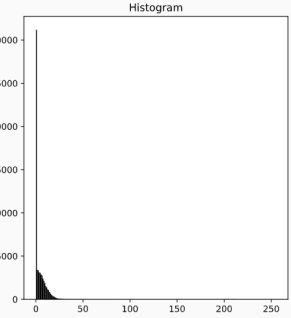


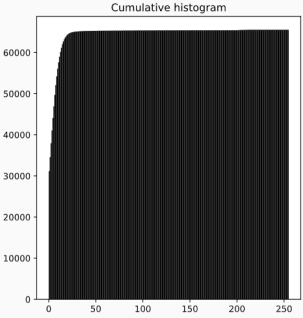


Cumulative histogram

Alternatively, we can plot a cumulative histogram (see later for uses). Each bar is the count of pixels with intensity between 0 and the corresponding bin.

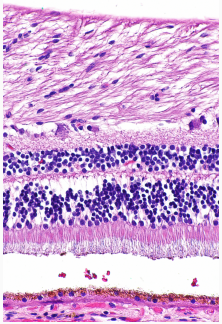




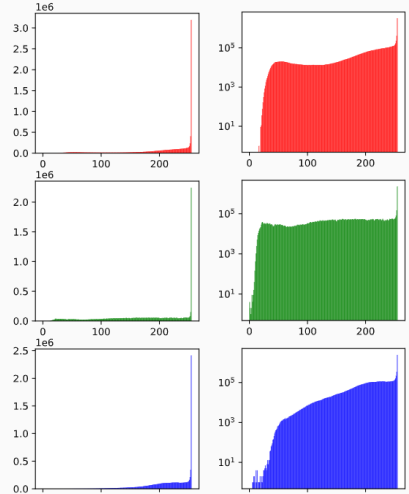


Histograms of RGB images

For RGB (or multi-channel) images, it makes sense to have a single histogram per channel



H&E staining of retina (cell nuclei stained blue-purple and extracellular material stained pink)
Librepath - CC BY-SA 3.0



RGB histogram - left linear, right logarithmic.

Histograms are destructive

It is important to remember that producing a histogram is a many-to-one operation! We can generate a histogram from an image, but cannot recreate the image from the histogram.

These three images have the same histogram!

Drawing a histogram using Matplotlib

The Matplotlib `hist` function makes it very easy to plot a histogram

```
import matplotlib.pyplot as plt
img = io.imread("cells.tif")

fig, ax = plt.subplots(1, 2, figsize=(12, 6))
# Linear
ax[0].hist(img.ravel(), bins=range(255), color="black")
ax[0].set_title("Histogram - linear scale")
```

The `bins` parameter accepts either a sequence defining the edges of the bins, or a single value with the number of desired bins.

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# Linear
ax[0].hist(img.ravel(), bins=range(255), color="black")
ax[0].set_title("Histogram - linear scale")
# Log
ax[1].hist(img.ravel(), bins=range(255), color="black", log=True)
ax[1].set_title("Histogram - log scale")
plt.show()
```

The `bins` parameter accepts either a sequence defining the edges of the bins, or a single value with the number of desired bins.

Brightness and contrast

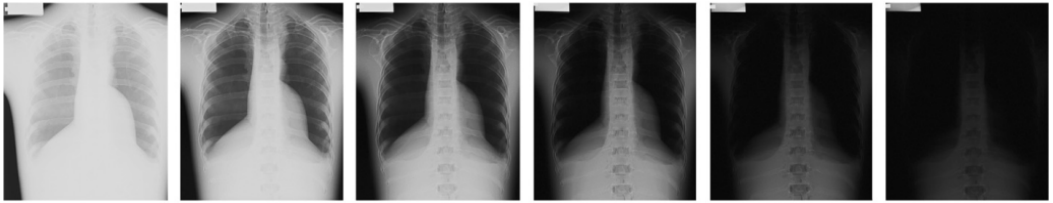
Brightness

The **brightness** of an image is the average intensity of all its pixels

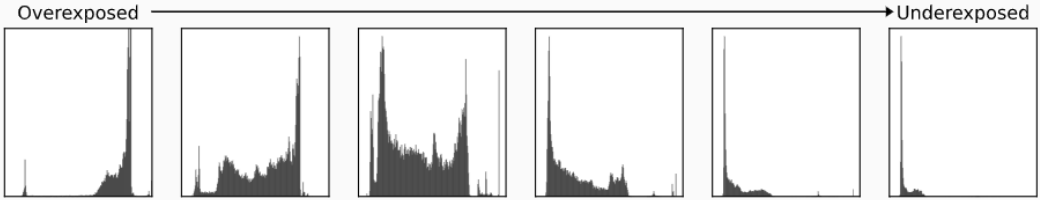
$$B(I) = \frac{1}{w * h} \sum_{r=1}^{h} \sum_{c=1}^{w} I(r, c)$$

Where w and h are the width and height of the image, and $I(r, c)$ is the intensity of the pixel at row r and column c.

Brightness example

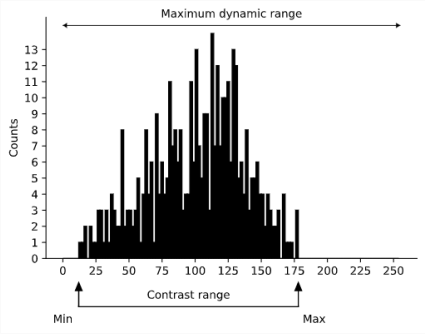


Same X-ray, decreasing brightness - Image from Veldkamp et al., 2009

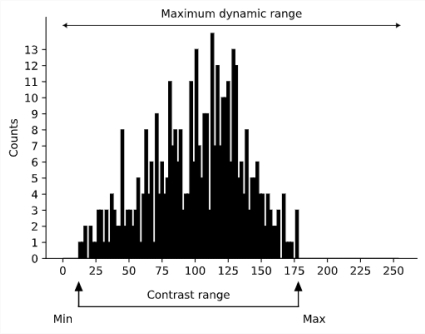


Contrast

Contrast is *the difference in luminance or colour that makes an object distinguishable* (Wikipedia). It measures the relationship between light and dark pixels.



Contrast



Contrast is *the difference in luminance or colour that makes an object distinguishable* (Wikipedia). It measures the relationship between light and dark pixels. Many different definitions in the literature (see Peli, *J. Opt. Soc. Am. A*, 1990)

Weber contrast (range $0 \rightarrow \infty$)

$$\frac{I_{obj} - I_{bg}}{I_{bg}}$$

Michelson contrast (range $0 \rightarrow 1$)

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

RMS contrast

$$\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

where x is the normalised intensity $0 \leq x \leq 1$

Point operations

Point operations and neighborhood operations

Manipulating pixel intensities allows enhancement of images.

Two types of operations:

- Point operations - Change pixel intensity based only on its value
- Neighborhood operations - Change pixel intensity based on the intensity of the pixel and its neighbours.

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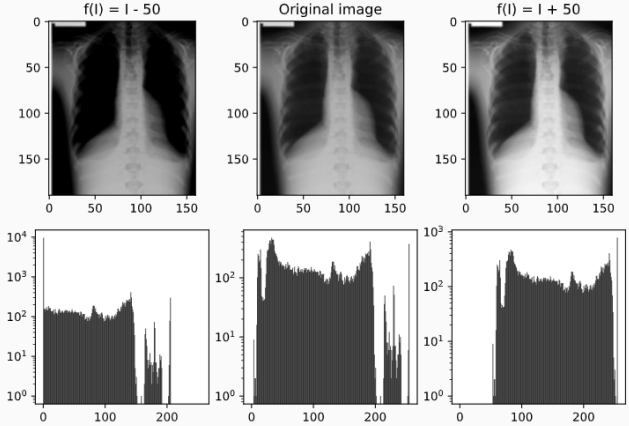
So, the resulting intensity I' of a point operation is $I'_{(x,y)} = f(I_{x,y})$.

Manipulating brightness

We can easily change an image brightness by using a point operation $f(I_{x,y}) = I_{x,y} + \mathit{offset}$, which shifts the image histogram to the left or to the right.



What is happening to the histograms?

Example code for brightness manipulation

```
from skimage import img_as_float, img_as_ubyte

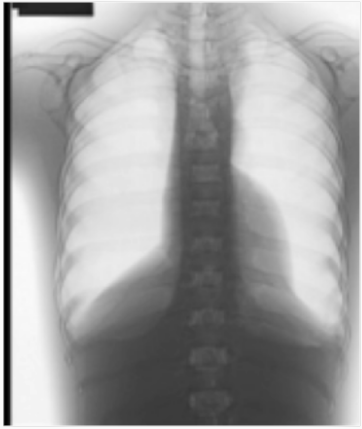
def change_brightness(img, offset):
    """
    Changes the brightness of an image
    img:  the image
    offset:  the brightness offset to apply
    Returns:  the modified image
    """

    img2 = img_as_float(img)
    img2 += offset/255
    img2 = np.clip(img2, 0, 1)
    return (img_as_ubyte(img2))

img_2 = change_brightness(img, 10)
```

Your turn!

Can you think of a point operation to invert an image? What happens to the histogram? Write your own Python function to do this!



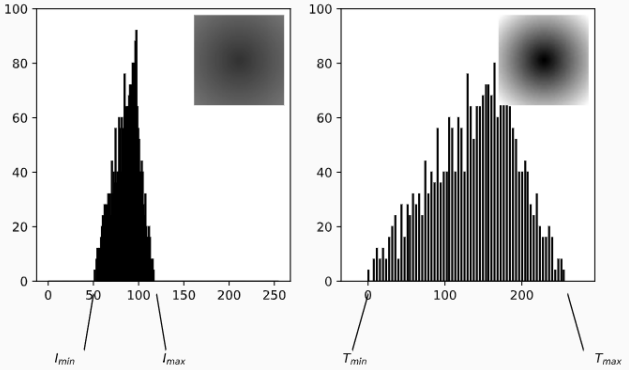
Manipulating contrast - histogram stretching

A simple way of increasing the contrast of an image is to stretch its histogram.

Simplest form:

$$I' = (I - I_{min})\frac{T_{max} - T_{min}}{I_{max} - I_{min}} + T_{min}$$

Where I_{min} and I_{max} are the minimum and maximum intensity of the image, and T_{min} and T_{max} are the minimum and maximum intensity of the target histogram.



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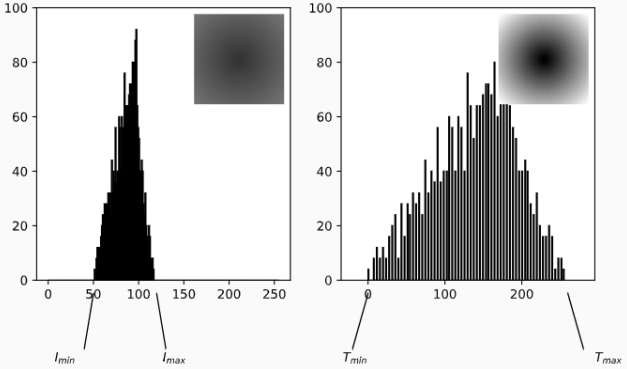


This calculation can become meaningless if we have even a single very light or dark pixel, so we can use the 2^{nd} and 98^{th} percentiles instead of I_{min} and I_{max} values.

Histogram stretching in Python

```
import numpy as np
from skimage.exposure import rescale_intensity

img_rescale = rescale_intensity(img, in_range=(0, 100), out_range=(0, 255))
```

Histogram stretching in Python

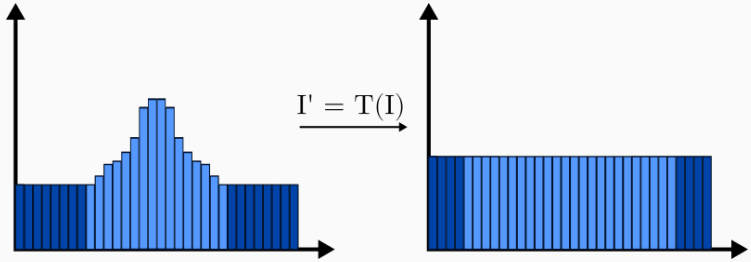
```
import numpy as np
from skimage.exposure import rescale_intensity

img_rescale = rescale_intensity(img, in_range=(0, 100), out_range=(0, 255))
# Or, use the percentile version:
# Calculate the 2nd and 98th percentiles
p2, p98 = np.percentile(img, (2, 98))
img_rescale = rescale_intensity(img, in_range=(p2, p98), out_range=(0, 255))
```

Histogram equalization

Histogram equalization increases contrast by manipulating pixel intensities.

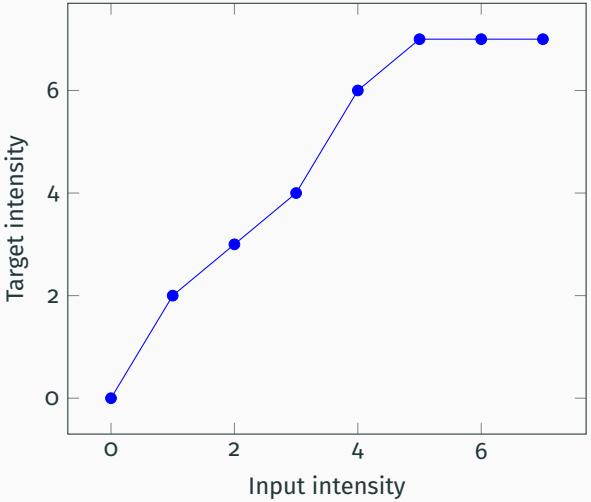
The idea is to generate a **histogram transfer function** $T(I)$ that will map the image intensities to a target, uniform histogram.



Choosing $T(I)$

How do we choose $T(I)$?

$T(I)$ should be monotonically (not strictly) increasing, to preserve the intensity rank of the pixels.



In this example we would change any pixel of the original image with intensity 3 to intensity 4, any with intensity 6 to intensity 7 and so on.

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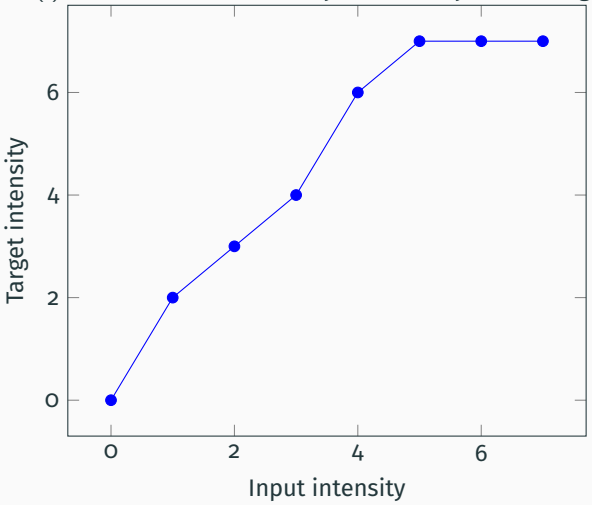
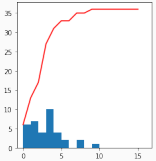


In this example we would change any pixel of the original image with intensity 3 to intensity 4, any with intensity 6 to intensity 7 and so on.
It turns out that using the CDF (or the cumulative histogram, for the discrete case) is the solution!
Look here if you want a mathematical proof!

Histogram equalization - an example

Assume you have a 4-bit image (dynamic range 0-15) with the following histogram

Pixel value	Number of pixels	Cumulative
0	6	6
1	7	13
2	4	17
3	10	27
4	4	31
5	2	33
6	0	33
7	2	35
8	0	35
9	1	36
10	0	36
11	0	36
12	0	36
13	0	36
14	0	36
15	0	36

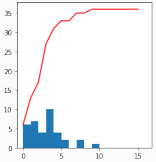


7	0	1	4	0	0
1	0	1	4	3	2
2	3	4	3	9	1
1	3	0	3	2	5
3	1	3	7	1	2
3	3	0	4	3	5

Histogram equalization - an example

Assume you have a 4-bit image (dynamic range 0-15) with the following histogram

Pixel value	Number of pixels	Cumulative	Cum. normalized (h_n)
0	6	6	0.17
1	7	13	0.36
2	4	17	0.47
3	10	27	0.75
4	4	31	0.86
5	2	33	0.92
6	0	33	0.92
7	2	35	0.97
8	0	35	0.97
9	1	36	1.00
10	0	36	1.00
11	0	36	1.00
12	0	36	1.00
13	0	36	1.00
14	0	36	1.00
15	0	36	1.00

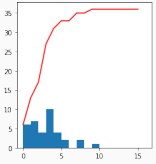


7	0	1	4	0	0
1	0	1	4	3	2
2	3	4	3	9	1
1	3	0	3	2	5
3	1	3	7	1	2
3	3	0	4	3	5

Histogram equalization - an example

Assume you have a 4-bit image (dynamic range 0-15) with the following histogram

Pixel value	Number of pixels	Cumulative	Cum. normalized (h_n)	New pixel value $\lfloor (h_n * l_{max}) \rfloor$
0	6	6	0.17	2
1	7	13	0.36	5
2	4	17	0.47	7
3	10	27	0.75	11
4	4	31	0.86	12
5	2	33	0.92	13
6	0	33	0.92	13
7	2	35	0.97	14
8	0	35	0.97	14
9	1	36	1.00	15
10	0	36	1.00	15
11	0	36	1.00	15
12	0	36	1.00	15
13	0	36	1.00	15
14	0	36	1.00	15
15	0	36	1.00	15



7	0	1	4	0	0
1	0	1	4	3	2
2	3	4	3	9	1
1	3	0	3	2	5
3	1	3	7	1	2
3	3	0	4	3	5

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Assume you have a 4-bit image (dynamic range 0-15) with the following histogram

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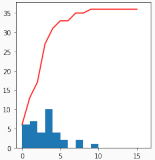
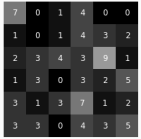
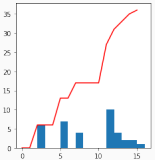
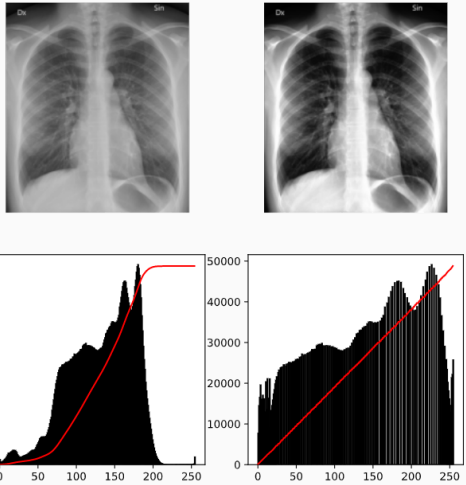
0	6	6	0.17	2
1	7	13	0.36	5
2	4	17	0.47	7
3	10	27	0.75	11
4	4	31	0.86	12
5	2	33	0.92	13
6	0	33	0.92	13
7	2	35	0.97	14
8	0	35	0.97	14
9	1	36	1.00	15
10	0	36	1.00	15
11	0	36	1.00	15
12	0	36	1.00	15
13	0	36	1.00	15
14	0	36	1.00	15
15	0	36	1.00	15







On a real image...



Note how, in practice, we often do not have a uniform histogram.

Beyond histogram equalization...

Histogram equalization can result in increased noise and "unrealistic" looking images. Useful e.g. for X-rays, but other strategies exist

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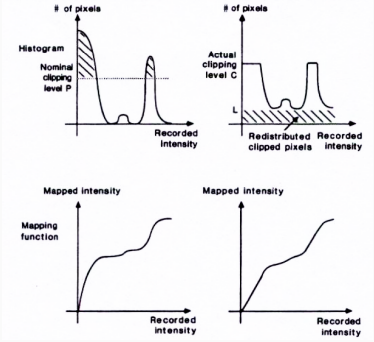
Adaptive histogram equalization (AHE) computes several histograms of different regions of the image to improve local contrast. Drawback: increased noise in areas with low contrast.

Contrast Limited Adaptive Equalization (CLAHE) solves this by reducing the amount of contrast enhancement. This is done by clipping the histogram to a maximum before calculating the cumulative histogram.

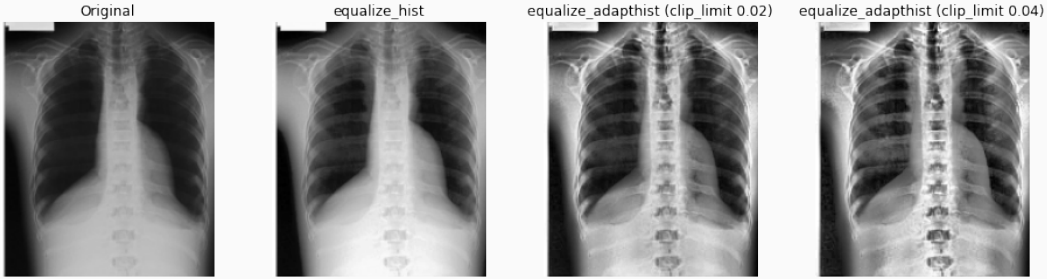
This is implemented in the `skimage.exposure.equalize_adapthist` function.



Histogram equalization - Scikit image

```
from skimage.exposure import equalize_hist, equalize_adapthist

img_equalized = equalize_hist(img)
# Or, with CLAHE
img_equalized_CLAHE = equalize_adapthist(img, clip_limit=0.03)
```



Summary

- Histograms are an important tool for inspecting image quality
- Optimal brightness and contrast can help reveal specific image features
- Manipulation of image histograms e.g. through point operations is one important tool for image enhancement