


浙江大学爱丁堡大学联合学院

ZJU-UoE Institute

Edge detection and noise reduction methods

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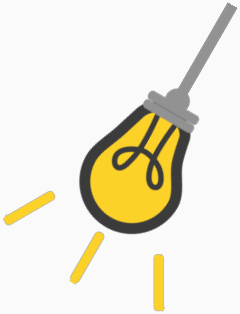
Learning objectives

- Explain the use of image gradients for edge detection.
- Describe various edge detection algorithms and how they work.
- Describe different noise reduction algorithms and how they work.
- Use Scikit Image to implement these techniques.



Convolution

Reminder of what is a convolution

Edge detection

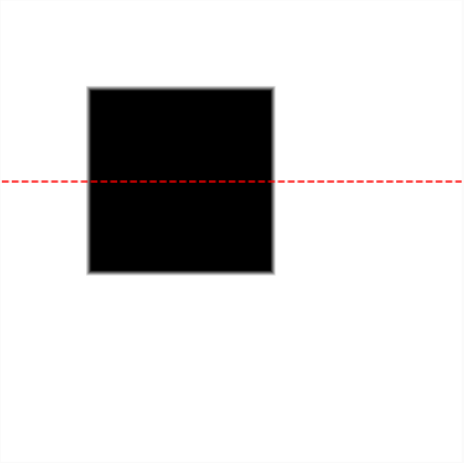
Edge detection problem

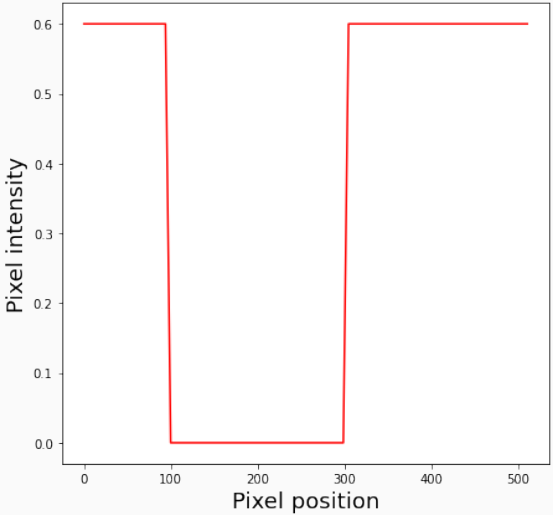
An edge is an area where the brightness of an image changes more or less gradually.

Detecting edges is useful e.g. to find objects in a scene, determine which pixels belong to which objects, and measure their properties.

Monarch butterfly - CC-BY-SA 2.0 Ted @ Flickr

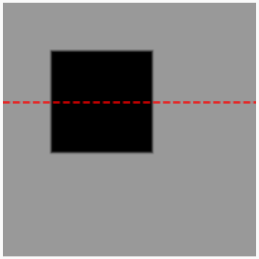
How to detect an edge?

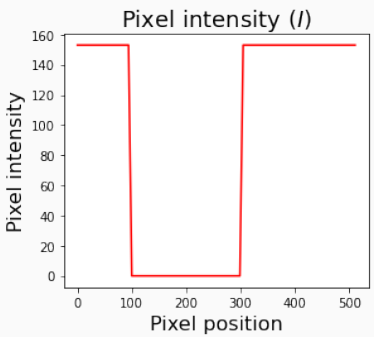


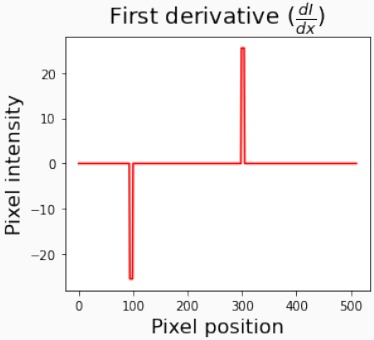


Consider the vertical edges. We can see a change in the intensity of pixels as we move from black to white and vice versa. **Can you think of a way to detect these edges?**

We can use derivatives!







Edges will correspond to minima and maxima of the derivative of the intensity!

Image derivatives

With a 2D image, we can find the x and y derivatives of the image intensity.

The vector of derivatives is called the **gradient** and is given by:

$$\nabla I = \left[\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right]$$

The gradient is a vector with:

- Direction perpendicular to the edge

$$\theta = \arctan\left(\frac{\partial I}{\partial x} \Big/ \frac{\partial I}{\partial y} \right)$$

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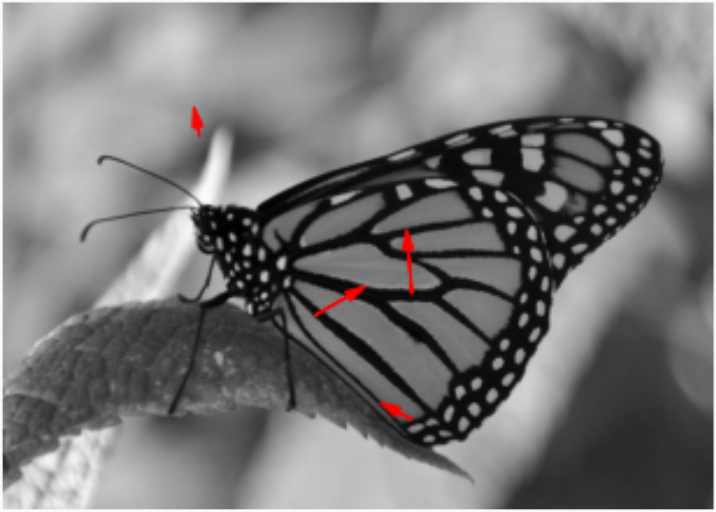
$$\theta = \arctan\left(\frac{\partial I}{\partial x} \Big/ \frac{\partial I}{\partial y} \right)$$

- Length (**gradient magnitude**) proportional to the intensity change

$$||\nabla I|| = \sqrt{\left(\frac{\partial I}{\partial x} \right)^2 + \left(\frac{\partial I}{\partial y} \right)^2}$$

Example of gradient vectors



Calculating a discrete derivative

How to calculate a discrete derivative?

Remember the definition of a derivative for a continuous function $f(x)$:

$$f'(x) = \frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x}$$

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Our image is not continuous, as it is made up of discrete pixels so the minimum Δx value is 1 (a single pixel).

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Our image is not continuous, as it is made up of discrete pixels so the minimum Δx value is 1 (a single pixel).

The discrete derivative is given by:

$$\frac{dI}{dx} = \frac{I(x) - I(x-1)}{1} = I(x) - I(x-1)$$

Calculating the discrete derivative - variations

We can choose to calculate the discrete derivative in three different ways

- **Forward difference**: $I(x) - I(x+1)$
- **Backward difference**: $I(x) - I(x-1)$
- **Central difference**: $I(x+1) - I(x-1)$

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- **Forward difference**: $I(x) - I(x+1)$
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These can be easily calculated using **convolution**!

- **Forward difference**: $\begin{bmatrix} 1 & -1 \end{bmatrix}$
- **Backward difference**: $\begin{bmatrix} -1 & 1 \end{bmatrix}$
- **Central difference**: $\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$

Discrete derivative - example

Let’s calculate the discrete derivative of this 1D image using central difference

10	16	22	36	40	11	17	23	37	41

Discrete derivative - example

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10	16	22	36	40	11	17	23	37	41

Convolving with the kernel

-1	0	1

We obtain the following

0	12	20	18	-25	-23	12	20	18	0

Derivative of an image

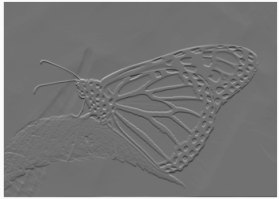
We can apply these convolution kernels to an image as well.

$$K_x = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \qquad K_y = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Original

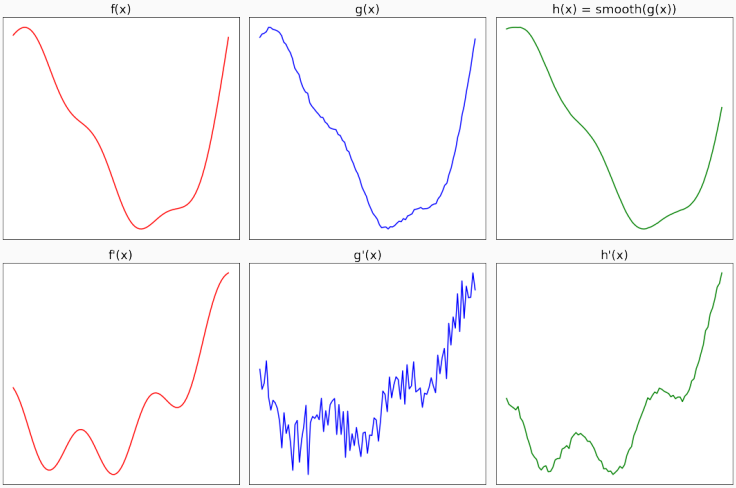
Convolution with K_x

Convolution with K_y



The problem with noise...

Derivatives are very sensitive to noise. Smoothing the function beforehand helps



2D derivative kernels (Prewitt edge detectors)

Expanding the derivative kernels to 2D we can directly smooth our intensity function.

These are called **Prewitt** kernels.

$$K_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \qquad K_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Alternatively the **Sobel** kernels can be used:

$$K_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad K_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Your turn!

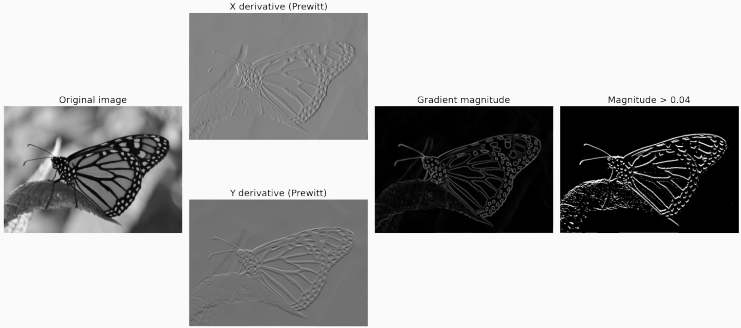
Consider the following image. **What do you expect to obtain and why** after convolution with the Prewitt kernels?
Apply the convolution (you can easily do that by hand); **do the results match** your prediction?

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 50 & 50 & 50 & 50 & 50 & 50 & 50 & 50 \\ 100 & 100 & 100 & 100 & 100 & 100 & 100 & 100 \\ 100 & 100 & 100 & 100 & 100 & 100 & 100 & 100 \\ 50 & 50 & 50 & 50 & 50 & 50 & 50 & 50 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Edge detection - Prewitt and Sobel

Having applied either the Prewitt or Sobel kernels to the image we can now detect edges.

Simply threshold the gradient magnitude of the image to define which pixels are edges.



Edge detection in Scikit Image

```
from skimage.filters import prewitt, sobel
from skimage.io import imread

img = imread("butterfly.jpg")
im_prewitt = prewitt(img)
im_sobel = sobel(img)
```

Prewitt

Sobel

Edge detection in Scikit Image

```
from skimage.filters import prewitt, sobel
from skimage.io import imread

img = imread("butterfly.jpg")
im_prewitt = prewitt(img)
im_sobel = sobel(img)

fig, ax = plt.subplots(2, 1, figsize=(5, 10))
ax[0].imshow(im_prewitt > 0.08, cmap="gray")
ax[0].set_title("Prewitt", fontsize=25)
ax[1].imshow(im_sobel > 0.08, cmap="gray")
ax[1].set_title("Sobel", fontsize=25)
for a in ax:
a.axis("off")
plt.show()
```

Prewitt

Sobel

Problem with noise

Smoothing to remove noise first

Laplacian of Gaussian