浙江大学爱丁堡大学联合学院

ZJU-UoE Institute

Lecture 2 - Geometric image transformations

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A brief recap

Last time we learnt how to open and display images using Python.

matplotlib

General plotting library

scikit-image
image processing in python

Specific functions for image manipulation

NumPy

Library for matrix operations

Reading images as Numpy arrays

matplotlib

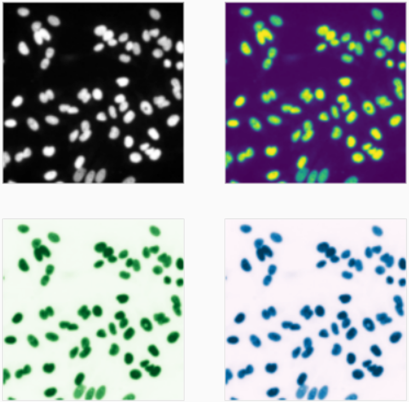
```
import matplotlib.pyplot as plt
img = plt.imread("cells.jpg")
plt.imshow(img)
plt.show()
```

scikit-image
image processing in python

```
from skimage.io import imread
img = io.imread("cells.jpg")
plt.imshow(img)
plt.show()
```

Colour mapping

```python
fig, ax = plt.subplots(ncols=2, nrows=2)
ax[0, 0].imshow(img, cmap="gray")
ax[0, 1].imshow(img, cmap="viridis")
ax[1, 0].imshow(img, cmap="Greens")
ax[1, 1].imshow(img, cmap="PuBu")
plt.show()
```

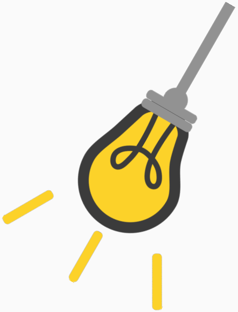


Check out the Matplotlib website for a list of colourmaps!

`https://matplotlib.org/stable/tutorials/colors/colormaps.html`

Learning objectives

- Describe use cases for geometric transformations of images
- Use Python to crop images (in 2 or more dimensions)
- Explain the theory of affine transformations
- Implement basic transformations in Python (translation, scaling, rotation)



Geometric image transformations

Geometric image transformations

Geometric image transformations are operations that change the image geometry without altering its pixel values (mostly…).

Examples are cropping, translating, scaling rotating an image.

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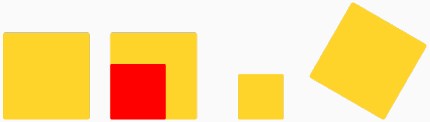
Example use cases:

- Analysing only part of an image (cropping)
- Making sure all input images for a pipeline are the same size (scaling, cropping)
- Aligning multiple images (e.g. in video stabilization) (rotating, translating)
- ...

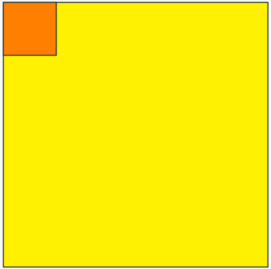


Geometric image transformations

Cropping

Cropping is as easy as subsetting the image matrix.

```
# img contains a 512 x 512 image
# Take the top-left 100 x 100 pixels
img1 = img[0:100, 0:100] # shape (100, 100)
```



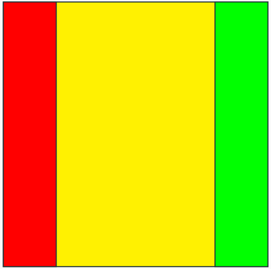
img[0:100, 0:100]

Cropping

Cropping is as easy as subsetting the image matrix.

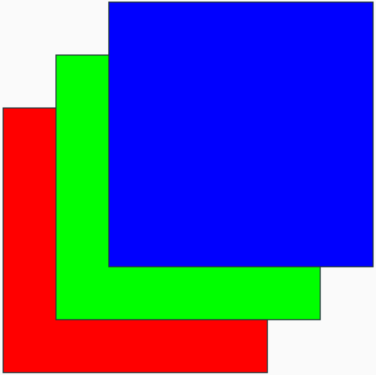
```
# img contains a 512 x 512 image
# Take the top-left 100 x 100 pixels
img1 = img[0:100, 0:100] # shape (100, 100)

# A 100 pixel wide strip on the left...
img2 = img[:, 0:100] # shape (512, 100)
# ...or on the right!
img3 = img[:, -100:]  # shape (512, 100)
```



Since images are just tensors, we can crop them in any dimension.

```
 # img is a 512 x 512 RGB image
# img.shape is (3, 512, 512)
# Extract the green channel
img_green = img[1] # Shape (512, 512)
```

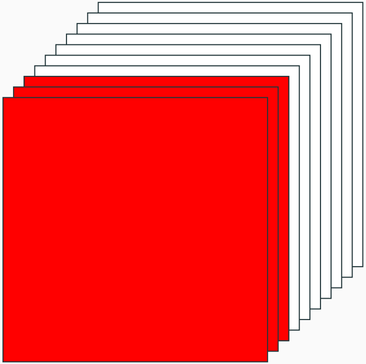


Cropping in more than two dimensions

Since images are just tensors, we can crop them in any dimension.

```
# img is a 512 x 512 RGB image
# img.shape is (3, 512, 512)
# Extract the green channel
img_green = img[1] # Shape (512, 512)

# video is a 512 x 512 video of 300 frames
# video.shape is (300, 512, 512)
# Take the first 50 frames
video_crop = video[0:50]
```



Question time

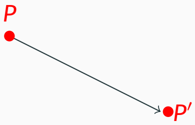
We have a 100 frames video of a 512 x 512 z-stack with 60 planes loaded in `zstack`.
`zstack.shape` is `(100, 60, 512, 512)`

What does this code give you?

```
result = stack[50:70, :, 100:300]
```

Affine transformations

In Euclidean geometry, an affine transformation is a geometric transformation that preserves lines and parallelism but not necessarily distances and angles. [Wikipedia]



We want to transform $P\ (x; y)$ into $P'\ (x'; y')$.

$$P' : \begin{cases} x' = f(x, y) \\ y' = g(x, y) \end{cases}$$

Where f and g are linear functions.

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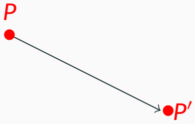

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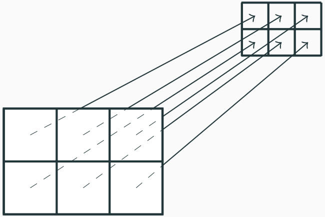
Where f and g are linear functions.

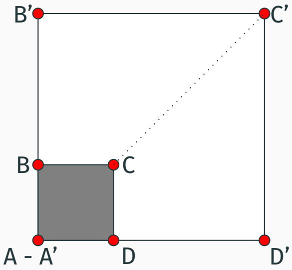
We can generalise this to an image, by applying the transformation to every pixel.



Geometric image transformations

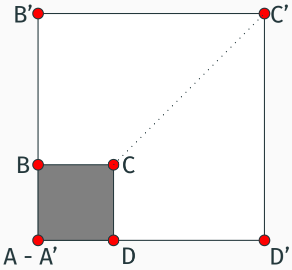
Scaling

Scaling



Scaling transforms the coordinates as:

$$\begin{cases} x' = s_x * x \\ y' = s_y * y \end{cases}$$



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$$\begin{cases} x' = s_x * x \\ y' = s_y * y \end{cases}$$

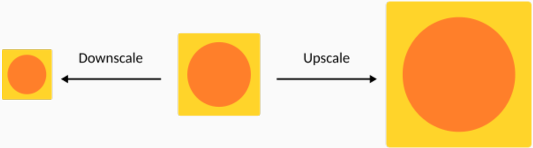
We can write this in matrix form*:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

This is called the scaling **transformation matrix**.

* Don't remember how matrix multiplication works? Check out Wikipedia!

Scaling

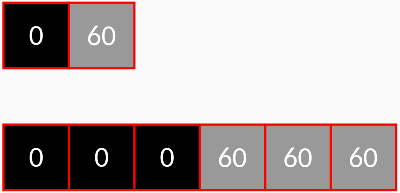


Two problems:
When **upscaling** we need to generate new information.
When **downscaling** we need to decide "how to lose" information.

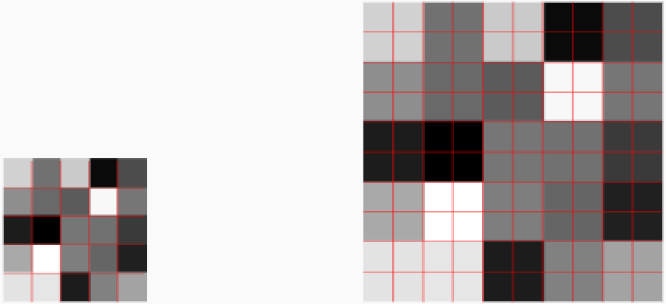
Interpolation is the solution!

Nearest-neighbour interpolation

The simplest way to resize an image is to use nearest-neighbour interpolation. Each pixel of the scaled image will have the colour of the nearest pixel in the original image.
In this "1D" example, we resize a 1x2 image to 1x6

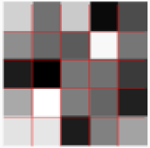


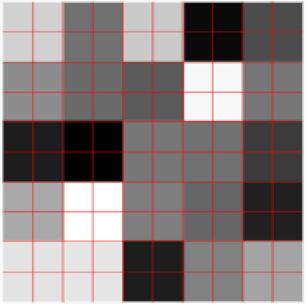
Nearest-neighbour interpolation



Upscaling of a 5x5 image by a factor of 2, to get a 10x10 image, with nearest neighbour interpolation

Nearest-neighbour interpolation



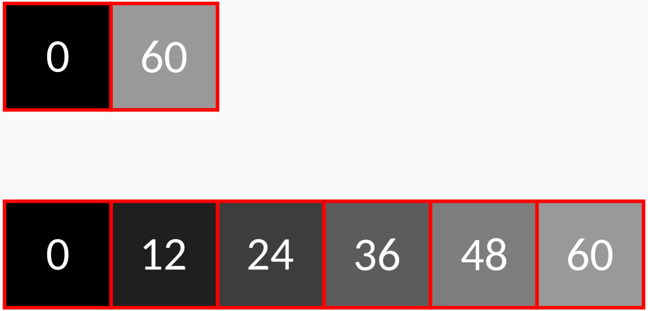


Upscaling of a 5x5 image by a factor of 2, to get a 10x10 image, with nearest neighbour interpolation

```
from skimage.transform import rescale
img_scaled = rescale(img, 2, order=0)
```

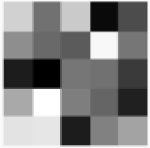
Better ways to scale an image involve changing the pixel values of the rescaled image based on their neighbourhood.

For example we could use linear interpolation



Linear interpolation

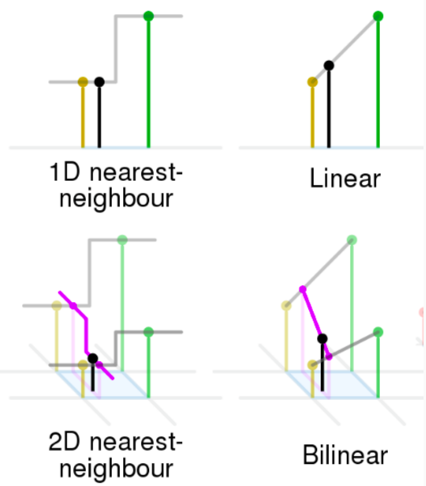
The same applies in 2D, although we need to take into accounts the values of both horizontal and vertical neighbours (**bilinear interpolation**).



Upscaling of a 5x5 image by a factor of 2, to get a 10x10 image, with bi-linear interpolation

```
from skimage.transform import rescale
img_scaled = rescale(img, 2, order=1)
```

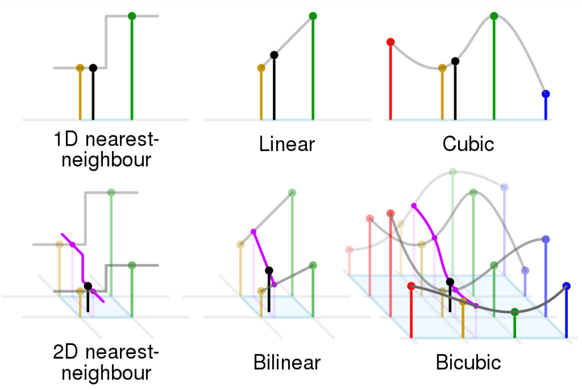
Nearest neighbour vs linear interpolation



Comparison of nearest neighbour and linear interpolation - Source: Wikipedia

Higher orders of interpolation

We can use higher orders of interpolation to produce smoother results.



Comparison of nearest neighbour and linear interpolation - Source: Wikipedia

Scikit Image supports values from 0 to 5 in the `order` parameter of the `rescale` function. 0 is nearest neighbour, 1 is bi-linear, 2 is bi-quadratic and so on.

Scaling to a target size

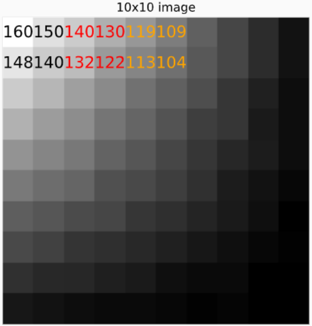
To scale to a target size, rather than by a specific factor, we can use `resize` instead of `rescale`.

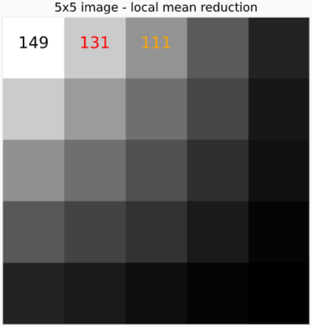
```
import matplotlib.pyplot as plt
from skimage.transform import resize, rescale

img = plt.imread("cells.jpg")
img_scaled1 = rescale(img, 2)
img_scaled2 = resize(img, (150, 150))
```

Local mean downscaling

Another simple solution for downscaling is calculating the local mean of each pixel





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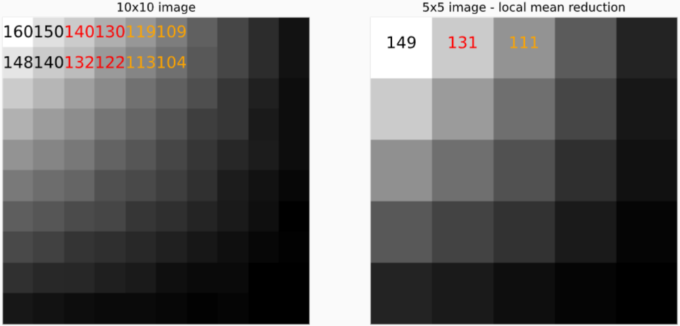



```
from skimage.transform import downscale_local_mean
img_small = downscale_local_mean(img, (2,2))
```

Image needs to be padded if the size is not a multiple of the downscaling factor.
This method is fast but not good at keeping fine details.

Scaling summary

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

`skimage.transform.rescale` $\rightarrow$ scales an image by a specific factor (>1 upscaling; <1 downscaling.). Can specify a different scaling factor for each dimension of the image.

`skimage.transform.resize` $\rightarrow$ scales an image to a target size.

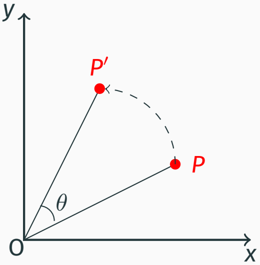
`skimage.transform.downscale_local_mean` $\rightarrow$ downscales the image by a specific factor (>1), using the local mean of each pixel.

Geometric image transformations

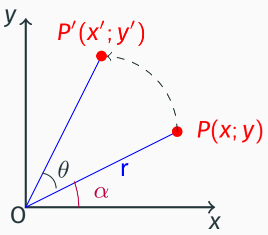
Rotations

Rotating a point around the origin

We want to rotate a point $P(x, y)$ around the origin by an angle θ to get $P'(x', y')$.

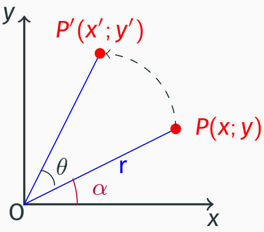


Rotating a point around the origin



$$P : \begin{cases} x = r\cos(\alpha) \\ y = r\sin(\alpha) \end{cases} \quad P' : \begin{cases} x' = r\cos(\alpha+\theta) \\ y' = r\sin(\alpha+\theta) \end{cases}$$

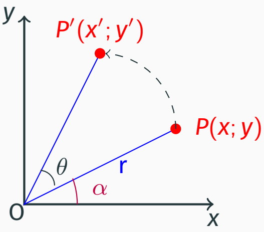
Rotating a point around the origin



$$P:\begin{cases} x = r\cos(\alpha) \\ y = r\sin(\alpha) \end{cases} \qquad P':\begin{cases} x' = r\cos(\alpha+\theta) = r\cos(\alpha)\cos(\theta) - r\sin(\alpha)\sin(\theta) \\ y' = r\sin(\alpha+\theta) = r\sin(\alpha)\cos(\theta) + r\sin(\theta)\cos(\alpha) \end{cases}$$

Why? Wikipedia to the rescue!

Rotating a point around the origin



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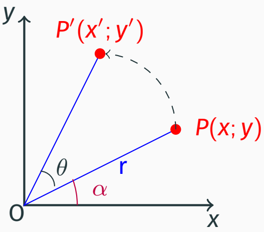
Thus:

$$P' : \begin{cases} x' = x\cos(\theta) - y\sin(\theta) \\ y' = y\cos(\theta) + x\sin(\theta) \end{cases}$$

Rotating a point around the origin



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$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotation transformation matrix

Rotating in Scikit Image

To rotate an image we will:

- Offset our image so that it is centered on the origin
- Generate a rotation matrix
- Multiply the coordinates of each pixel by the rotation matrix
- Shift back the image to its original position

To rotate an image we will:

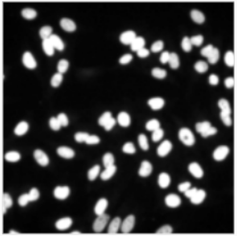
- Offset our image so that it is centered on the origin
- Generate a rotation matrix
- Multiply the coordinates of each pixel by the rotation matrix
- Shift back the image to its original position

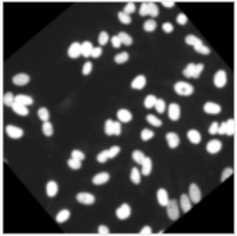
Luckily, Scikit Image has a function for that, `skimage.transform.rotate`.

```
import matplotlib.pyplot as plt
from skimage.transform import rotate

img = plt.imread("cells.jpg")
img_rotated = rotate(img, 20)

plt.imshow(img_rotated, cmap="gray")
plt.show()
```





Note: we lost part of the image and we "gained" black pixels around it.

Geometric image transformations

Translation

Translation

To translate we offset the points by (t_x, t_y): $\begin{cases} x' = x + t_x \\ y' = y + t_y \end{cases}$

The transformation matrix is different from the ones we saw for before:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

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We can generate a transformation matrix using `SimilarityTransform` and apply it using `warp`.

```
from skimage.transform import SimilarityTransform, warp
# Create a 10x10 white image
img = np.ones(shape=(10, 10))
# Make the central pixels black
img[5:7, 5:7] = 0

m = SimilarityTransform(translation = (2, 5))
img_translated = warp(img, m)
```

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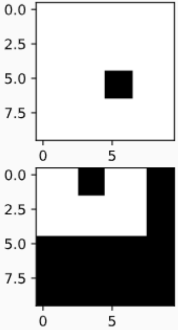

Transformation matrices - summary

Translation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Rotation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scaling

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

The affine transformation matrix

Sometimes we want to combine rotation, scaling and translation into a single operation.
We can rewrite the matrices as such (these are called homogeneous coordinates):

Translation

(now also as matrix multiplication!)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scaling

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Combining matrices

We can now easily combine these 3x3 matrices by multiplying them together.

- Remember that the order is important!
  $A * B$ is **not** the same as $B * A$!
- Use `skimage.transform.SimilarityTransform` to create transformation matrices
- Combine them using the + operator (Under the hood this performs matrix multiplication!).
- Use `skimage.transform.warp` to apply these transformation matrices to an image.

This week's challenge!

It might be interesting to try to code image rotation by yourself.

Hints:

- Generate the transformation matrices using `skimage.transform.SimilarityTransform`
- You will need three matrices: one translation matrix to offset your image by (-xcenter, -ycenter); a rotation matrix to rotate your image around (0, 0); and finally another translation matrix to translate your image back to its original position.
- You can combine the matrices as `m1+m2+m3`
- You can use `skimage.transform.warp` to apply the combined transformation matrix to your image.

If you are stuck, try to look at the source code for `rotate`!
Discuss your results and/or doubts on Slack!

Summary

- Affine transformations are simple yet powerful way to modify an image.
- Scikit Image allows generation of any transformation matrix…
- …but also provides several pre-made functions for rotating, scaling, etc
- **Workshop 1** will allow you to practice what learned so far!