Payment Rules through Discriminant-Based Classifiers

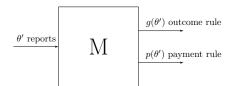
by Paul Dütting, Felix Fischer, Pichayut Jirapinyo, John K. Lai, Benjamin Lubin and David C. Parkes

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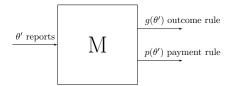
May 19, 2017

Classical Approach



Classical Approach

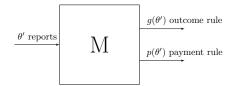
• Impose incentive compatibility (IC) constraint (DSIC, BNIC)



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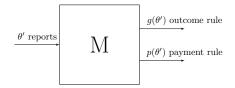
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- Impose incentive compatibility (IC) constraint (DSIC, BNIC)
- Design outcome- and payment rule subject to IC constraint



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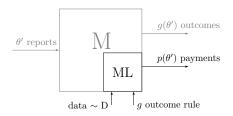


Challenges

- Analytical Complexity
- Exclusion of Mechanisms
- Computational Complexity

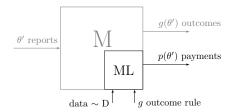


New Approach



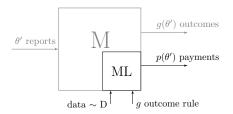
New Approach

lacktriangledown Define an outcome rule g (e.g. optimal outcome rule)



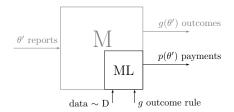
New Approach

- **1** Define an outcome rule g (e.g. optimal outcome rule)
- Sample data from a type distribution D



New Approach

- **1** Define an outcome rule g (e.g. optimal outcome rule)
- Sample data from a type distribution D
- Use Machine Learning (ML) to find a payment rule p that minimizes ex-post regret



ex-post regret an agent has for truthfully reporting in a given instance is the amount by which its utility could be increased through a misreport.

$$rgt_{i}(\theta_{i}, \theta'_{-i}) = \max_{\theta'_{i} \in \Theta_{i}} \underbrace{u_{i}((\theta'_{i}, \theta'_{-i}), \theta_{i})}_{\text{utility misreport}} - \underbrace{u_{i}((\theta_{i}, \theta'_{-i}), \theta_{i})}_{\text{utility truthful report}}$$

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- ightarrow no direct implications

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- $\mathbb{E}(gain) < cost(strategic behaviour)$

- ightarrow strategyproof
- ightarrow no direct implications
- → agents are assumed to report truthfully



Properties of a truthful Mechanism

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Agent-Independent Price:
$$p_i(\theta) = t_i(\theta_{-i}, g_i(\theta))$$

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$$g_i(\theta) \in \arg\max_{o_i \in \Omega_i} v_i(\theta_i, o_i) - t_i(\theta_{-i}, o_i) \quad \forall i \in \Omega_i$$

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Connection

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Connection

Discriminant-Based Classifier:
$$h_w(\theta) \in \underset{o_i \in \Omega_i}{\text{arg max}} \ w_i v_i(\theta_i, o_i) + w_{-i}^T \psi(\theta_{-i}, o_i)$$

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Agent-Independent Price: $p_1(\theta) = t_1(\theta_{-1}, g_1(\theta))$ Agent-Optimizing Outcome: $g_1(\theta) \in \underset{o_1 \in \Omega_1}{\operatorname{arg max}} v_1(\theta_1, o_i) - t_1(\theta_{-1}, o_1)$

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$$t_w(\theta_{-1}, o_1) = -\frac{1}{w_1} w_{-1}^T \psi(\theta_{-1}, o_1)$$

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Payment Rule

$$p_{w}(\theta) = (t_{w}(\theta_{-1}, g_{1}(\theta)), t_{w}(\theta_{-2}, g_{2}(\theta)), ..., t_{w}(\theta_{-n}, g_{n}(\theta)))$$



Theorem (3.2)

Let g be an agent symmetric outcome rule, h_w an admissible classifier, and p_w the payment rule corresponding to h_w . If h_w is a perfect classifier for the partial outcome rule g_1 , then the mechanism (g, p_w) is strategyproof.

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But what happens when h_w is not a perfect classifier? minimizing generalization error of the classifier \Rightarrow minimizing expected ex-post regret of the mechanism.

Discriminant-Based Classifier: $h_w(\theta) \in \arg\max_{o_1 \in \Omega_1} w_1 v_1(\theta_1, o_1) + w_{-1}^T \psi(\theta_{-1}, o_1)$

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the outcome matching the outcome rule of the Single Item Auction

$$g_1(heta) = egin{cases} ext{allocate} & ext{if} & ext{agent 1 has the highest bid} \ ext{not allocate} & ext{else} \end{cases}$$

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$$w_1 = 1$$
 and $w_{-1}^T \psi(\theta_{-1}, o_1) = \begin{cases} -max(\theta_{-1}) & \text{if } o_1 = 1 \\ 0 & \text{if } o_1 = 0 \end{cases}$

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plugging this into the associated price function ightarrow second price payment rule

$$t_w(\theta_{-1}, o_1) = -\frac{1}{w_1} w_{-1}^T \psi(\theta_{-1}, o_1) = max(\theta_{-1})$$



Example: Single Item Auction Instance

$$h_w(\theta) = \underset{o_i}{\operatorname{arg max}} \underbrace{1 * v_i(\theta_i, o_i)}_{\text{left part}} + \underbrace{\begin{cases} -max(\theta_{-1}) & \text{if } o_i = 1\\ 0 & \text{if } o_i = 0 \end{cases}}_{\text{right part}}$$

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	Outcome	Left Part	Right Part	Classifier	Optimal
Agent 1	$egin{aligned} o_1 &= 1 \ o_1 &= 0 \end{aligned}$				
Agent 2	$o_2 = 1$ $o_2 = 0$				

$$h_w(\theta) = \underset{o_i}{\operatorname{arg max}} \underbrace{1 * v_i(\theta_i, o_i)}_{\text{left part}} + \underbrace{\begin{cases} -max(\theta_{-1}) & \text{if } o_i = 1\\ 0 & \text{if } o_i = 0 \end{cases}}_{\text{right part}}$$

	Outcome	Left Part	Right Part	Classifier	Optimal
Agent 1	$o_1 = 1$	$v_1(\theta_1,1)=6$			
	$o_1 = 0$	$v_1(\theta_1,0)=0$			
Agent 2	$o_2 = 1$	$v_2(\theta_2,1)=8$			
	$o_2 = 0$	$v_2(\theta_2,0)=0$			

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	Outcome	Left Part	Right Part	Classifier	Optimal
Agent 1	$o_1 = 1$	$v_1(\theta_1,1)=6$	8	6 - 8 = -2	
	$o_1 = 0$	$v_1(\theta_1,0)=0$	0	0 - 0 = 0	
Agent 2	$o_2 = 1$	$v_2(\theta_2,1)=8$	6	8 - 6 = 2	
	$o_2 = 0$	$v_2(\theta_2,0)=0$	0	0 - 0 = 0	

◆□ → ◆□ → ◆□ → □ → ○○○

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Agent 2	$o_2 = 1$	$v_2(\theta_2,1)=8$	6	8 - 6 = 2	Х
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Structural Support Vector Machine (SSVM)

SSVM

Learn weight vector w by training SSVM using:

training data:
$$\{(\theta^1, o_1^1), (\theta^2, o_1^2), ..., (\theta^\ell, o_1^\ell)\}$$

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SSVM Optimization Problem

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SSVM Optimization Problem

$$\begin{split} & \underset{w,\xi}{\text{minimize}} & & \frac{1}{2}\|w\|^2 + \frac{C}{\ell} \sum_{k=1}^{\ell} \xi^k \\ & \text{subject to} & & (w_1 v_1(\theta_1^k, o_1^k) + w_{-1}^T \psi'(\theta_{-1}^k, o_1^k)) - (w_1 v_1(\theta_1^k, o_1) + w_{-1}^T \psi'(\theta_{-1}^k, o_1)) \\ & & \geq \mathcal{L}(o_1^k, o_1) - \xi^k \text{ , } \forall k = 1, ..., \ell \text{ , } o_1 \in \Omega_1 \\ & & \xi^k \geq 0 \text{ , } \forall k = 1, ..., \ell \end{split}$$

Remarks

Valid Price Function if $w_1 > 0$

Flexible Payment Structure with ψ



Problems of the computed Payment Rule

Negative Payments

computed payments p_w can be negative o normalize payments

Violation of Individual Rationality

truthful report leads to $utility < 0 \longrightarrow$

- introduce payment offsets
- adjust the loss function ${\cal L}$
- introduce deallocation

Multi-Minded Combinatorial Auctions

Multi-Minded CAs

- *r* items {*A*, *B*, ...}
- *n* agents {1, 2, ..., *n*}
- express valuation for bundles
- each agent interested in at most b bundles

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In Terms of the Framework

- type $\theta_i = (v_i(\emptyset), v_i(A), v_i(B), v_i(C), v_i(AB), v_i(AC), v_i(BC), v_i(ABC))$
- type profile $\theta = (\theta_1, \theta_2, ..., \theta_n)$
- outcome $o_1 = 101$
- type distribution D with parameter to control correlation and complementarity between items



Correlation between Accuracy and Regret / IR Violation

Negative Correlation between Accuracy and Regret



Negative Correlation between Accuracy and IR Violation



Degree of Complementarity



low degree of complementarity: $v(AB) \approx v(A) + v(B)$ high degree of complementarity: $v(AB) \gg v(A) + v(B)$



Choice of Outcome Rule and Attribute Map





Training Set Size and IR Fixes

Training Set Size: more training data leads to better results

IR Fixes:

Conclusion

Challenges of Classical Mechanism Design

- Analytical Complexity
- Exclusion of Mechanisms
- Computational Complexity

Conclusion

Challenges of Classical Mechanism Design

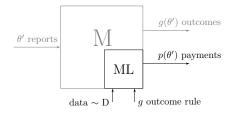
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Conclusion

- introduce new paradigm for computational mechanism design
- shown encouraging experimental results
- further directions of interest that have to be investigated in the future

Discussion - Overview

New Approach



Remarks

- train a classifier for the outcome
- use special structure of the classifier to extract a payment rule
- the better the classifier for the given outcome rule, the less incentive an agent has for not reporting truthfully

