Gram-Schmidt

$A = \{(0,0,1,1)(0,1,1,0)(1,1,0,0)\}$

$v_1 \quad v_2 \quad v_3$

$A' = \{u_1, u_2, u_3\}$

- $u_1 = \frac{w_1}{\|w_1\|} \rightarrow w_1 = v_1 = (0,0,1,1)$

$= \frac{(0,0,1,1)}{\sqrt{2}}$

$= \left(0,0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

- $u_2 = \frac{w_2}{\|w_2\|} \rightarrow u_2 = \frac{\left(0,1,\frac{1}{2},-\frac{1}{2}\right)}{\left\|\left(0,1,\frac{1}{2},-\frac{1}{2}\right)\right\|} = \frac{\left(0,1,\frac{1}{2},-\frac{1}{2}\right)}{\sqrt{\frac{3}{2}}} = \left(0,\sqrt{\frac{2}{3}},\frac{1}{\sqrt{6}},-\frac{1}{\sqrt{6}}\right)$

$w_2 = v_2 - (v_2 \cdot u_1)\, u_1$

$= (0,1,1,0) - \left[(0,1,1,0) \cdot \left(0,0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)\right]\left(0,0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

$= (0,1,1,0) - \frac{1}{\sqrt{2}}\left(0,0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

$= (0,1,1,0) - \left(0,0,\frac{1}{2},\frac{1}{2}\right)$

$= \left(0,1,\frac{1}{2},-\frac{1}{2}\right)$

- $u_3 = \frac{w_3}{\|w_3\|} \rightarrow u_3 = \frac{\left(1,0,-\frac{1}{2},\frac{1}{2}\right)}{\left\|\left(1,0,-\frac{1}{2},\frac{1}{2}\right)\right\|} = \frac{\left(1,0,-\frac{1}{2},\frac{1}{2}\right)}{\sqrt{\frac{3}{2}}} = \left(\sqrt{\frac{2}{3}},0,-\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}}\right)$

$w_3 = v_3 - (v_3 \cdot u_1)\, u_1 - (v_3 \cdot u_2)\, u_2$

$= (1,1,0,0) - \left[(1,1,0,0) \cdot \left(0,0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)\right]\left(0,0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right) - \left[(1,1,0,0) \cdot \left(0,1,\frac{1}{2},-\frac{1}{2}\right)\right]\left(0,1,\frac{1}{2},-\frac{1}{2}\right)$

$= (1,1,0,0) - 0\left(0,0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right) - 1\left(0,1,\frac{1}{2},-\frac{1}{2}\right)$

$= (1,1,0,0) - (0,0,0,0) - \left(0,1,\frac{1}{2},-\frac{1}{2}\right)$

$= (1,1,0,0) - \left(0,1,\frac{1}{2},-\frac{1}{2}\right)$

$= \left(1,0,-\frac{1}{2},\frac{1}{2}\right)$

Sea $A = (v_1, v_2, \dots, v_n)$

$\Rightarrow A' = (w_1, w_2, \dots, w_n) \leftrightarrow w_1 = v_1$

$$w_n = \frac{v_n - \sum_{i=1}^{n-1} (v_n \cdot u_i)\, u_i}{\left\| v_n - \sum_{i=1}^{n-1} (v_n \cdot u_i)\, u_i \right\|} \quad \text{con } n = 2,3,\dots,n$$