Gram-Schmidt

$$A = \{ \underbrace{(0,0,1,1)}_{v_1} \underbrace{(0,1,1,0)}_{v_2} \underbrace{(1,1,0,0)}_{v_3} \}$$

$$A' = \{ u_1 , u_2 , u_3 \}$$

- $u_1 = \dfrac{w_1}{\|w_1\|} \rightarrow \quad w_1 = v_1 = (0,0,1,1)$

$$= \frac{(0,0,1,1)}{\sqrt{2}}$$

$$= \left(0,0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$$

- $u_2 = \dfrac{w_2}{\|w_2\|} \rightarrow \quad u_2 = \dfrac{\left(0,1,\frac{1}{2},-\frac{1}{2}\right)}{\left\|\left(0,1,\frac{1}{2},-\frac{1}{2}\right)\right\|} = \dfrac{\left(0,1,\frac{1}{2},-\frac{1}{2}\right)}{\sqrt{\frac{3}{2}}} = \left(0,\sqrt{\frac{2}{3}},\frac{1}{\sqrt{6}},\frac{-1}{\sqrt{6}}\right)$

$$w_2 = v_2 - (v_2 \cdot u_1)\, u_1$$

$$= (0,1,1,0) - \left[(0,1,1,0)\cdot\left(0,0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)\right]\left(0,0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$$

$$= (0,1,1,0) - \frac{1}{\sqrt{2}}\left(0,0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$$

$$= (0,1,1,0) - \left(0,0,\frac{1}{2},\frac{1}{2}\right)$$

$$= \left(0,1,\frac{1}{2},-\frac{1}{2}\right)$$

- $u_3 = \dfrac{w_3}{\|w_3\|} \rightarrow \quad u_3 = \dfrac{\left(1,0,-\frac{1}{2},\frac{1}{2}\right)}{\left\|\left(1,0,-\frac{1}{2},\frac{1}{2}\right)\right\|} = \dfrac{\left(1,0,-\frac{1}{2},\frac{1}{2}\right)}{\sqrt{\frac{3}{2}}} = \left(\sqrt{\frac{2}{3}},0,-\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}}\right)$

$$w_3 = v_3 - (v_3 \cdot u_1)\, u_1 - (v_3 \cdot u_2)\, u_2$$

$$= (1,1,0,0) - \left[(1,1,0,0)\cdot\left(0,0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)\right]\left(0,0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right) - \left[(1,1,0,0)\cdot\left(0,1,\frac{1}{2},-\frac{1}{2}\right)\right]\left(0,1,\frac{1}{2},-\frac{1}{2}\right)$$

$$= (1,1,0,0) - 0\left(0,0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right) - 1\left(0,1,\frac{1}{2},-\frac{1}{2}\right)$$

$$= (1,1,0,0) - (0,0,0,0) - \left(0,1,\frac{1}{2},-\frac{1}{2}\right)$$

$$= (1,1,0,0) - \left(0,1,\frac{1}{2},-\frac{1}{2}\right)$$

$$= \left(1,0,-\frac{1}{2},\frac{1}{2}\right)$$

Sea $A = (v_1, v_2, \dots, v_n)$

$\Rightarrow A' = (u_1, u_2, \dots, u_n) \leftrightarrow$

$$w_1 = v_1 \Rightarrow u_1 = \frac{w_1}{\|w_1\|}$$

$$w_n = v_n - \sum_{i=1}^{n-1} (v_n \cdot u_i)\, u_i \Rightarrow u_n = \frac{w_n}{\|w_n\|} \quad \text{con } n = 2,3,\dots,n$$