Targeting in Multi-Criteria Decision Making

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Abstract

In this work, we introduce the notion of targeting for multi-criteria decision making. The problem involves selecting the best alternatives related to one particular alternative, called the target. We use an axiomatic approach to this problem by establishing properties that any targeting method should satisfy. We present a representation theorem and show that satisfying the main properties of targeting requires aggregating the evaluations of the alternatives related to the target. We propose various candidate targeting methods and examine the properties satisfied by each method.

Introduction

In this work, we tackle the following problem: suppose you have a set of objects or alternatives, each evaluated according to multiple criteria. Traditional multi-criteria decision-making approaches (**??**) typically aim to identify the best options among all alternatives. However, our focus is different. Instead of seeking the overall "best" alternatives, we aim to identify the best alternatives relative to a specific, targeted alternative, which we simply call the *target*. This target has a distinctive evaluation across the criteria, and our goal is to find the best alternatives most relevant to it. We term this problem *targeting*.

For example, imagine Agathe, a user, browsing through a selection of laptops and choosing one to use as a reference or target based on her personal preferences. This target laptop might perform well in terms of battery life (c_1), average on connectivity (c_2), but poorly in terms of processing speed (c_3). Agathe may not be explicitly aware of these two criteria, but she selects the laptop because she values its long battery life, which aligns with her specific needs (e.g., frequent travel). She is not looking for the "best" laptop overall but she is rather asking Brian, the shop staff, to recommend alternatives that are similar to her chosen laptop, particularly those with strong battery performance. Agathe's target laptop serves as a point of reference, and Brian's goal is to find the best alternatives in the same direction, i.e., laptops with improved battery life, as selecting laptops based on high processing speed would be irrelevant to Agathe. Our aim is to approach this problem systematically by providing a set of principles to guide the targeting process.

Previous work related to the selection of some solutions has explored selecting subsets of nondominated solutions, often to avoid overwhelming users with too many options, which can impede effective decision-making. One common approach is to select k diverse solutions, i.e., a set of k nondominated solutions that are as far apart as possible in the solution space, maximizing pairwise diversity (**???**). Another approach focuses on selecting a representative subset of the solution space (**???**). These studies primarily aim to reduce the output size or computational effort but do not consider methods tailored to a specific target. More closely related to targeting is the work by Schwind *et al.* (**?**), which introduced solution concepts aimed at identifying the best point in the direction of a given ideal point, that is, any arbitrarily chosen point that dominates the utopia point. However, their approach applies specifically to multiobjective constraint optimization problems, with the ideal point always lying outside the solution space. In contrast, our approach considers a target within the solution space itself. This is particularly practical when a user wishes to improve upon a specific alternative, especially when the set of objectives is not fully understood. Our contribution is also novel in that we formalize the targeting process through a set of properties, providing a principled framework for selecting alternatives. This allows to better understand the behaviour of the different methods and to compare them more easily. This also allows us to show that the methods that satisfy the set of main properties we proposed have to use an aggregation function on the evaluations of the alternative.

In the following, we begin by introducing necessary preliminaries before defining a set of principles and properties relevant to the targeting problem. We then characterize these principles in terms of preference orderings over alternatives, showing that functions satisfying these properties must perform an aggregation of the evaluations across the criteria. Building on this, we propose specific choice functions, analyze their properties, and illustrate their behavior through an example. [1]

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[1]Proofs of the results are provided in the supplementary material.

Preliminaries

Let us introduce the basic formal concepts used throughout the paper.

We are given a finite set of objects (also called alternatives) $\Omega = \{o_1, \ldots, o_n\}$, which we will refer to by the letters p, q, r, and $\mathbf{a}$ is a specific object from Ω called *target*. We are also given a finite set of criteria $\mathcal{C} = \{c_1, \ldots, c_m\}$. In this work, each criterion c_i is a function $c_i : \Omega \longrightarrow \mathbb{R}^+$ that assigns a non-negative value to each object with respect to the i-th criterion. The higher the value of a criterion for an object, the better the object is considered for that criterion. We will denote by $\gamma(p)$ the vector of evaluations of object $p \in \Omega$ with respect to the set of criteria $\mathcal{C}$, i.e., $\gamma(p) = (c_1(p), \ldots, c_m(p))$.

A subset S of $\mathcal{C}$ is called a *support*. Intuitively, a support is a set of criteria relevant for evaluating and comparing objects in a given context. We denote by $\overline{\mathsf{S}}$ the complement of S in $\mathcal{C}$, i.e., $\overline{\mathsf{S}} = \mathcal{C} \setminus \mathsf{S}$. Let $\mathsf{S} \subseteq \mathcal{C}$ and $p, q \in \Omega$. We say that p *Pareto dominates* q on S, denoted by $p \succeq_\star^\mathsf{S} q$, whenever $c_i(q) \leq c_i(p)$ for each criterion $c_i \in \mathsf{S}$. In other words, p Pareto dominates q on S if p is "at least as good as" q on each criterion in the support S. Note that $\succeq_\star^\mathsf{S}$ defines a partial ordering on Ω, and we denote by $\succ_\star^\mathsf{S}$ the strict part of $\succeq_\star^\mathsf{S}$ (i.e., $p \succ_\star^\mathsf{S} q$ iff $p \succeq_\star^\mathsf{S} q$ and $q \not\succeq_\star^\mathsf{S} p$), and by $\simeq_\star^\mathsf{S}$ its symmetric part (i.e., $p \simeq_\star^\mathsf{S} q$ iff $p \succeq_\star^\mathsf{S} q$ and $q \succeq_\star^\mathsf{S} p$). Given a subset of objects $O \subseteq \Omega$, the *Pareto front of O restricted to S* is the set $PF_\mathsf{S}(O) = \max(O, \succeq_\star^\mathsf{S}) = \{p \in O \mid \forall q \in O, q \not\succ_\star^\mathsf{S} p\}$, i.e., $PF_\mathsf{S}(O)$ consists of the objects in O that are not strictly Pareto dominated on S by any other object in O.

Given an object $\mathbf{a} \in \Omega$, the *support of $\mathbf{a}$*, denoted by $\mathsf{S}(\mathbf{a})$, is the set of criteria on which $\mathbf{a}$ has a strictly positive evaluation[2], i.e., $\mathsf{S}(\mathbf{a}) = \{c_i \in \mathcal{C} \mid c_i(\mathbf{a}) > 0\}$. Given a support $\mathsf{S} \subseteq \mathcal{C}$ and an object p, let $\gamma_\mathsf{S}(p)$ be the evaluation of p restricted to S, i.e., $\gamma_\mathsf{S}(p) = (c_{i_1}(p), \ldots, c_{i_k}(p))$ where $\{i_1, \ldots, i_k\} = \mathsf{S}$ and $i_j < i_{j'}$ iff $j < j'$. In particular, note that $\gamma_\mathcal{C}(p) = \gamma(p)$, and $p \simeq_\star^\mathsf{S} q$ iff $\gamma_\mathsf{S}(p) = \gamma_\mathsf{S}(q)$.

A *targeting method* is a mapping $\beta : 2^\Omega \times \Omega \to 2^\Omega$ that associates every subset of objects $O \subseteq \Omega$ and a *target* $\mathbf{a} \in \Omega$ with a subset of objects, denoted by $\beta_\mathbf{a}(O)$, such that $\beta_\mathbf{a}(O) \subseteq O$. Our goal is to specify a set of properties that an appropriate targeting method should satisfy. This will be the focus of the next section.

Properties on Targeting Methods

Let us introduce a set of properties that a targeting method β should satisfy. We divide these properties into two groups: the first group consists of the main properties that we expect every targeting method to satisfy, while the second group contains optional properties that may be relevant for specific subclasses of targeting methods.

For simplicity in presenting the properties, in the remainder of this section, we assume the following variables to be universally quantified: $\mathbf{a}$, $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ are any objects from Ω; O and O' are any subsets of Ω; p, q are any objects from O; S is any support with $\mathsf{S} \subseteq \mathcal{C}$; c_i, c_j are any criteria from $\mathcal{C}$;

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[2]Note that it is also possible to choose another value than 0 as threshold if one wants to be more selective on the relevant criteria.

x is any number in $\mathbb{R}^+$; μ is any number in $\mathbb{Q}^*$; and π is any permutation of $\{1, \ldots, m\}$.

Main Properties

We begin by introducing the key properties for targeting methods.

The first requirement is that the method always returns a result when the input set is non-empty:

S1 If $O \neq \emptyset$ then $\beta_\mathbf{a}(O) \neq \emptyset$ Non Vacuity

The second requirement is that the method must guarantee that two targets with the same evaluations lead to the same best elements. This property ensures that there is no other information than the evaluations of the target that is taken into account for selecting the results.

S2 If $\gamma(\mathbf{a}_1) = \gamma(\mathbf{a}_2)$, then $\beta_{\mathbf{a}_1}(O) = \beta_{\mathbf{a}_2}(O)$ Neutrality

Next, the property of (Contribution) requires that only the evaluation of objects is used to determine the best elements, with only the relevant criteria — the support of the target — being considered:

S3 If $p \simeq_\star^{\mathsf{S}(\mathbf{a})} q$ and $p \in \beta_\mathbf{a}(O)$, then $q \in \beta_\mathbf{a}(O)$
       Contribution

Note that (Contribution) implies a property of "anonymity," stating that two objects with identical evaluation vectors are treated equivalently:

If $\gamma(p) = \gamma(q)$ and $p \in \beta_\mathbf{a}(O)$, then $q \in \beta_\mathbf{a}(O)$
       Anonymity

Indeed, $p \simeq_\star^{\mathsf{S}(\mathbf{a})} q$ always holds whenever $\gamma(p) = \gamma(q)$ holds, irrespective of the target's support $\mathsf{S}(\mathbf{a})$.

The next property ensures that all criteria that are relevant according to the target are considered during evaluation. Specifically, if one object strictly outperforms another on those criteria, the underperforming object cannot be among the best:

S4 If $p \succ_\star^{\mathsf{S}(\mathbf{a})} q$, then $q \notin \beta_\mathbf{a}(O)$ Responsiveness

This implies that the best objects selected by β are always within the Pareto front of O restricted to the target's support. Formally, if β satisfies (Responsiveness), then $\beta_\mathbf{a}(O) \subseteq PF_{\mathsf{S}(\mathbf{a})}(O)$.

The next property ensures consistency when restricting the evaluation to a subset of objects. If we consider a subset of O that includes some of the best objects from the original set, these objects should remain the best in the subset:

S5 If $O' \subseteq O$ and $\beta_\mathbf{a}(O) \cap O' \neq \emptyset$, then $\beta_\mathbf{a}(O') = \beta_\mathbf{a}(O) \cap O'$
       Contraction

This guarantees that the selection process is robust to focusing on smaller subsets.

The following property ensures that criteria on which the target performs identically have exactly the same impact on the evaluation of the objects. Specifically, if switching an object's evaluation on two such criteria produces an evaluation that matches another object, the two objects should be considered equivalently good. This property guarantees

that some criteria are not intrinsically (arbitrarily) more important than others. This also emphasizes the fact that we consider a commensurability between the criteria. If for an application this is not the case from the scales used for the different criteria, then a first homogenization/normalization step will be required to make the scales commensurate.

S6 If $c_i(\mathbf{a}) = c_j(\mathbf{a})$, $p \simeq_\star^{\mathsf{S}(\mathbf{a})\setminus\{c_i,c_j\}} q$, $c_i(p) = c_j(q)$, $c_j(p) = c_i(q)$ and $p \in \beta_\mathbf{a}(O)$, then $q \in \beta_\mathbf{a}(O)$
<div align="right">Equivalence</div>

So, this property ensures that the evaluation respects a form of symmetry when criteria are evaluated identically by the target.

The next property captures the intuition that criteria where the target performs better should have a stronger influence on object evaluation. If one criterion is more important for the target, and objects differ only on these criteria, the object with better performance on the more important criterion should be preferred:

S7 If $c_i(\mathbf{a}) > c_j(\mathbf{a})$, $p \simeq_\star^{\mathsf{S}(\mathbf{a})\setminus\{c_i,c_j\}} q$, $c_i(p) > c_j(p)$, $c_j(p) = c_i(q)$ and $c_i(p) = c_j(q)$, then $q \notin \beta_\mathbf{a}(O)$
<div align="right">Relevance</div>

The next property reflects the idea that if an object is the best for two disjoint sets of criteria, it should remain the best when these criteria are combined. Formally:

S8 If $\mathsf{S}(\mathbf{a}_1) \cap \mathsf{S}(\mathbf{a}_2) = \emptyset$ and $\gamma(\mathbf{a}_3) = \gamma(\mathbf{a}_1) + \gamma(\mathbf{a}_2)$, then $\beta_{\mathbf{a}_1}(O) \cap \beta_{\mathbf{a}_2}(O) \subseteq \beta_{\mathbf{a}_3}(O)$
<div align="right">Composition</div>

This property ensures that the targeting method respects the independence of unrelated sets of criteria.

Finally, the (Decomposition) property complements (Composition) by ensuring that the best objects for a union of independent criteria sets are precisely those that are best for both individual sets:

S9 If $\mathsf{S}(\mathbf{a}_1) \cap \mathsf{S}(\mathbf{a}_2) = \emptyset$, $\gamma(\mathbf{a}_3) = \gamma(\mathbf{a}_1) + \gamma(\mathbf{a}_2)$, and $\beta_{\mathbf{a}_1}(O) \cap \beta_{\mathbf{a}_2}(O) \neq \emptyset$, then $\beta_{\mathbf{a}_3}(O) \subseteq \beta_{\mathbf{a}_1}(O) \cap \beta_{\mathbf{a}_2}(O)$
<div align="right">Decomposition</div>

So we are seeking for targeting methods that satisfy this set of nine properties.

Optional Properties

Let us now introduce additional properties that may define sensible subclasses of targeting methods. While not mandatory, these properties offer desirable behavior in specific contexts, depending on the goals and constraints of the targeting method.

The first property, (Dominance), ensures that the selected best objects always Pareto-dominate the target.

O1 If $p \in \beta_\mathbf{a}(O)$ and $\mathbf{a} \in O$, then $p \succeq_\star^{\mathsf{S}(\mathbf{a})} \mathbf{a}$ Dominance

This property is particularly relevant when the goal is to ensure that the best objects are never worse than the target. It is useful in applications where maintaining or exceeding the target's performance across all relevant criteria is essential.

Next, the (Compensation) property assumes that the evaluation scales of all criteria are the same. This means that if two criteria are equally valued by the target, increasing one while decreasing the other by the same amount results in an equally good object:

O2 If $p \simeq_\star^{\mathsf{S}(\mathbf{a})\setminus\{c_i,c_j\}} q$, $c_i(\mathbf{a}) = c_j(\mathbf{a})$, $c_i(q) = c_i(p) - x$, $c_j(q) = c_j(p) + x$ and $p \in \beta_\mathbf{a}(O)$, then $q \in \beta_\mathbf{a}(O)$
<div align="right">Compensation</div>

This property is valuable when trade-offs between criteria are meaningful and the criteria are expressed on comparable scales. It is particularly suitable for scenarios where flexibility in balancing trade-offs is necessary.

Note that this property is a strengthening of the (Equivalence) property:

Proposition 1 *If β satisfies* **O2***, then it satisfies* **S6***.*

We can also show that the (Dominance) and (Compensation) properties are fundamentally incompatible when combined with the required (Non-Vacuity) property (**S1**). This incompatibility arises because (Dominance) disallows trade-offs that (Compensation) requires[3]:

Proposition 2 *If β satisfies* **S1***, then it cannot satisfy both* **O1** *and* **O2***.*

This highlights the need to carefully choose between these properties depending on whether guaranteeing that the best objects are at least as good as the target or allowing trade-off flexibility is more important.

The following (Optimality) property requires that if the target belongs to the Pareto front of the object set (restricted to its support), it must also be selected as one of the best objects:

O3 If $\mathbf{a} \in PF_{\mathsf{S}(\mathbf{a})}(O)$, then $\mathbf{a} \in \beta_\mathbf{a}(O)$ Optimality

This property is most relevant when the target is not Pareto-dominated within its support, ensuring that it is recognized as a reasonable candidate for selection. It applies in contexts where preserving a balance among criteria, as represented by the target, is essential.

There is a direct relationship between this (Optimality) and (Dominance):

Proposition 3 *If β satisfies* **S1***,* **S3** *and* **O1***, then it satisfies* **O3***.*

A direct consequence of proposition 3 is that for targeting methods satisfying (Dominance), since it implies (Optimality) in the presence of (Non-Vacuity) and (Contribution), it follows that when the target is in the Pareto front according to its own support, the best objects are precisely those that share the same evaluation as the target on its support:

Corollary 1 *If β satisfies* **S1***,* **S3** *and* **O1***, and $\mathbf{a} \in PF_{\mathsf{S}(\mathbf{a})}(O)$, then for each $p \in O$, we have that $p \in \beta_\mathbf{a}(O)$ if and only if $p \simeq_\star^{\mathsf{S}(\mathbf{a})} \mathbf{a}$.*

The next property ensures that the result of a targeting method depends only on the relative importance of the criteria in the target's support. It remains unaffected by linear transformations of the target's evaluation:

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[3]All the proofs are given in the supplementary materials.

O4 If $\gamma(\mathbf{a}_1) = \mu \times \gamma(\mathbf{a}_2)$, then $\beta_{\mathbf{a}_1}(O) = \beta_{\mathbf{a}_2}(O)$
Proportionality

This property is interesting when relative importance matters more than absolute values, such as in normalized or scale-invariant evaluations.

Last, the (Symmetry) property states that the result should remain consistent under permutations of the criteria. If both the target and the evaluations of objects are permuted identically, the result should remain unchanged. Let us denote $\gamma(O) = \{\gamma(p) \mid p \in O\}$, and for a set of m-vectors V, $\pi(V) = \{\pi(v) \mid v \in V\}$:

O5 If $\gamma(\mathbf{a}_2) = \pi(\gamma(\mathbf{a}_1))$, $\gamma(O') = \pi(\gamma(O))$, $p' \in O'$, $\gamma(p') = \pi(\gamma(p))$, and $p \in \beta_{\mathbf{a}_1}(O)$, then $p' \in \beta_{\mathbf{a}_2}(O')$
Symmetry

This property is essential in scenarios where criteria are interchangeable, meaning their order or labeling does not affect the evaluation. It ensures fairness in contexts where the criteria are inherently equivalent. Since not satisfying it allows for instance to have some criteria that are inherently (i.e. independently of the target) more important than others. This can make sense in some situations.

Characterization

This section characterizes targeting methods that satisfy the main and optional properties by associating them with a construction involving preference orderings over alternatives.

Specifically, we can show that every function $\beta_{\mathbf{a}}$ can be characterized by a total preorder $\succeq_{\mathbf{a}}$ on alternatives Ω, and then computing $\beta_{\mathbf{a}}(O)$ consists in selecting the alternatives from O that are maximal w.r.t $\succeq_{\mathbf{a}}$. This is formalized through the concept of "targeting assignment":

Definition 1 *A* targeting assignment *is a mapping associating every* $\mathbf{a} \in \Omega$ *with a total preorder* $\succeq_{\mathbf{a}}$ *over elements of* Ω.[4]

Proposition 4 β *satisfies* **S1** *and* **S5** *iff there is a targeting assignment* $\mathbf{a} \mapsto \succeq_{\mathbf{a}}$ *such that for each* $\mathbf{a} \in \Omega$ *and each* $O \subseteq \Omega$, $\beta_{\mathbf{a}}(O) = \max(O, \succeq_{\mathbf{a}})$.

The (if) part of the proof of Proposition 4 consists in defining a targeting method β from a targeting assignment $\mathbf{a} \mapsto \succeq_{\mathbf{a}}$ as $\beta_{\mathbf{a}}(O) = \max(O, \succeq_{\mathbf{a}})$, for each $\mathbf{a} \in \Omega$ and each $O \subseteq \Omega$, and verify that β satisfies **S1** and **S5**. The (only if) part of the proof involves the construction of a targeting assignment $\mathbf{a} \mapsto \succeq_{\mathbf{a}}$ from β defined for all $\mathbf{a}, p, q \in \Omega$ as $p \succeq_{\mathbf{a}} q$ iff $p \in \beta_{\mathbf{a}}(\{p, q\})$. We say that such a targeting assignment *corresponds to* β, and we can show how properties of a targeting method β correspond to properties of its corresponding targeting assignment:

Proposition 5 *Let* β *satisfies* **S1** *and* **S5** *and* $\mathbf{a} \mapsto \succeq_{\mathbf{a}}$ *be its corresponding targeting assignment. Then for each* $i \in \{2, 3, 4, 6, 7, 8, 9\}$, $j \in \{1, 2, 3, 4, 5\}$, β *satisfies* **Si** *(resp.* **Oj**) *iff* $\mathbf{a} \mapsto \succeq_{\mathbf{a}}$ *satisfies the following condition (SRi) (resp. (ORj)):*

(SR2) *If* $\gamma(\mathbf{a}_1) = \gamma(\mathbf{a}_2)$, *then* $\succeq_{\mathbf{a}_1} = \succeq_{\mathbf{a}_2}$

(SR3) *If* $p \simeq_{\star}^{\mathsf{S}(\mathbf{a})} q$, *then* $p \simeq_{\mathbf{a}} q$

(SR4) *If* $p \succ_{\star}^{\mathsf{S}(\mathbf{a})} q$, *then* $p \succ_{\mathbf{a}} q$

(SR6) *If* $c_i(\mathbf{a}) = c_j(\mathbf{a})$, $p \simeq_{\star}^{\mathsf{S}(\mathbf{a}) \setminus \{c_i, c_j\}} q$, $c_i(p) = c_j(q)$ *and* $c_j(p) = c_i(q)$, *then* $p \simeq_{\mathbf{a}} q$

(SR7) *If* $c_i(\mathbf{a}) > c_j(\mathbf{a})$, $p \simeq_{\star}^{\mathsf{S}(\mathbf{a}) \setminus \{c_i, c_j\}} q$, $c_i(p) > c_j(q)$, $c_j(p) = c_i(q)$ *and* $c_i(p) = c_j(q)$, *then* $p \succ_{\mathbf{a}} q$

(SR8) *If* $\mathsf{S}(\mathbf{a}_1) \cap \mathsf{S}(\mathbf{a}_2) = \emptyset$, $\gamma(\mathbf{a}_3) = \gamma(\mathbf{a}_1) + \gamma(\mathbf{a}_2)$, $p \succeq_{\mathbf{a}_1} q$ *and* $p \succeq_{\mathbf{a}_2} q$, *then* $p \succeq_{\mathbf{a}_3} q$

(SR9) *If* $\mathsf{S}(\mathbf{a}_1) \cap \mathsf{S}(\mathbf{a}_2) = \emptyset$, $\gamma(\mathbf{a}_3) = \gamma(\mathbf{a}_1) + \gamma(\mathbf{a}_2)$, $p \succeq_{\mathbf{a}_1} q$ *and* $p \succ_{\mathbf{a}_2} q$, *then* $p \succ_{\mathbf{a}_3} q$

(OR1) *If* $p \not\succeq_{\star}^{\mathsf{S}(\mathbf{a})} \mathbf{a}$, *then* $\mathbf{a} \succ_{\mathbf{a}} p$

(OR2) *If* $p \simeq_{\star}^{\mathsf{S}(\mathbf{a}) \setminus \{c_i, c_j\}} q$, $c_i(\mathbf{a}) = c_j(\mathbf{a})$, $c_i(q) = c_i(p) - x$ *and* $c_j(q) = c_j(p) + x$, *then* $p \simeq_{\mathbf{a}} q$

(OR3) *If* $p \not\succ_{\star}^{\mathsf{S}(\mathbf{a})} \mathbf{a}$, *then* $\mathbf{a} \succeq_{\mathbf{a}} p$

(OR4) *If* $\gamma(\mathbf{a}_1) = \mu \times \gamma(\mathbf{a}_2)$, *then* $\succeq_{\mathbf{a}_1} = \succeq_{\mathbf{a}_2}$

(OR5) *If* $\gamma(\mathbf{a}_2) = \pi(\gamma(\mathbf{a}_1))$, $\gamma(p') = \pi(\gamma(p))$, $\gamma(q') = \pi(\gamma(q))$ *and* $p \succeq_{\mathbf{a}_1} q$, *then* $p' \succeq_{\mathbf{a}_2} q'$

(SR2) states that two targets with the same evaluation will generate exactly the same preference order on objects. *(SR3)* states that two objects that are equivalent on the support of the target will be indifferent for the preference order. *(SR4)* states that if an object performs better than (Pareto dominates) another one, then this object will be strictly preferred. *(SR6)* states that if two criteria are identical for the target, then two objects that are identical on all criteria except on these two ones, where their evaluation is swapped, have to be indifferent for the preference order. *(SR7)* states that if a criterion is more important for the target than another one, then two objects that are identical on all criteria except on these two ones, where their evaluation is swapped, are ranked so that the preferred object is the one that performs best on the more important criterion. *(SR8)* states that if an object is at least as good than another one for two sets of disjoints criteria, then it will also be better for (a target whose support is) the union of the criteria. *(SR9)* states that if an object is at least as good than another one for a set of criteria and strictly better for a disjoint set of criteria, then it will be strictly better for (a target whose support is) the union of the criteria.

(OR1) states that if an object does not Pareto-dominate the target on its support, then the target will be strictly better. *(OR2)* states that if two criteria are identical for the target, then two objects that are identical on all criteria except on these two ones, where their evaluation is reversed but the difference identical, then they have to be indifferent for the preference order. *(OR3)* states that if an object does not strictly Pareto-dominate the target on its support, then it can not be strictly better than the target. *(OR4)* states that if a target has a vector of evaluations that is proportional to the vector of another target, then they will generate exactly the same preference order on objects. *(OR5)* states that if a target vector of evaluations is a permutation of the vector of another target, and if two objects related to the first target have a vector of evaluations that are the permutations of two objects related to the second target, then the relation between

the two objects of the first target is the same than the relation between the two objects of the second target.

Note that this kind of link between choice functions and preferences (preorder) can be related to results that exist for social choice (**?**) or belief change (**??**). But the interesting part is the properties on the preorder that allows to better characterize and understand the operators. The relationship established by Proposition 5 provides a different rationale for the properties of targeting methods (**S2-S9**) and (**O1-O5**), as these properties naturally arise from a preference ordering over alternatives. For example, the way the pairs of properties **S3/S4**, **S6/S7**, and **S8/S9** complement each other becomes evident from their counterparts *(SR3)/(SR4)*, *(SR6)/(SR7)*, and *(SR8)/(SR9)*, within this ordering structure.

In particular, Propositions 4 and 5 imply that for targeting methods satisfying the first five conditions of **S1-S5**, selecting the best alternatives according to a given target is equivalent to aggregating the evaluations of the alternatives:

Corollary 2 β *satisfies* **S1-S5** *iff there exists a set of strictly monotone aggregation functions*[5] $\{f_\beta^{\gamma_{S(\mathbf{a})}(\mathbf{a})} \mid \mathbf{a} \in \Omega\}$ *such that for each* $O \subseteq \Omega$

$$\beta_{\mathbf{a}}(O) = \arg\max_{p \in O}\{f_\beta^{\gamma_{S(\mathbf{a})}(\mathbf{a})}(\gamma_{S(\mathbf{a})}(p))\}$$

This result provides an interesting representation to targeting functions, since it shows that (on a finite set of objects) targeting functions satisfying conditions **S1-S5** correspond to targeting functions defined from an aggregation of the criteria that are relevant for the target.

Targeting Functions

Let us now consider some functions that seem to be valuable candidates as targeting methods. These functions map the evaluation of any given target $\mathbf{a}$ such that $S(\mathbf{a}) \neq \emptyset$ into a "weight vector," $w(\mathbf{a}) = (w_1(\mathbf{a}), \ldots, w_{|\mathcal{C}|}(\mathbf{a}))$, which is defined for each $c_i \in \mathcal{C}$ as:

$$w_i(\mathbf{a}) = c_i(\mathbf{a}) / \sum_{x \in S(\mathbf{a})} c_x(\mathbf{a})$$

Basically this function defines the weight of a criterion for a target $\mathbf{a}$ as the importance of this criterion for the target relatively to the (sum of the) other ones (Note that other functions can also be used to define this weight, leading to variations of the proposed methods below — this possibility is not further investigated in this paper).

In the rest of this section, O denotes any set of objects $O \subseteq \Omega$, and $\mathbf{a}$ any target $\mathbf{a} \in \Omega$ such that $S(\mathbf{a}) \neq \emptyset$.

The first function simply computes the sum of the evaluation of the criteria that are weighted by their relative importance for the target.

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[5]A strictly monotone aggregation function is a mapping f associating a vector of values $(v_1, \ldots, v_k) \in \mathbb{R}^+ \times \ldots \times \mathbb{R}^+$ with a single value $v \in \mathbb{R}^+$ such that if $\forall i, v_i \geq v_i'$ then $f((v_1, \ldots, v_k)) \geq f((v_1', \ldots, v_k'))$, and if additionally $\exists j : v_j > v_j'$, then $f((v_1, \ldots, v_k)) > f((v_1', \ldots, v_k'))$.

Definition 2 (Weighted Sum)

$$\beta_{\mathbf{a}}^{ws}(O) = \arg\max_{p \in O} \mathsf{BWS}_{\mathbf{a}}(p),$$

where $\mathsf{BWS}_{\mathbf{a}}(p) = \sum_i w_i(\mathbf{a}) \times c_i(p)$.

This method focuses on the numerical evaluation on each criteria, which assumes some kind of commensurability between the criteria. That is not always the case in all multi-criteria applications. Then, one possibility is not to focus on the evaluation of each element, but on its rank with respect to each criterion. Given any object $p \in \Omega$ and any criterion $c_i \in \mathcal{C}$, let $r_{c_i}(p)$ denote the *rank* of p w.r.t. c_i, defined as:

- $p \geq_{c_i} q$ iff $c_i(p) \geq c_i(q)$
- best elements b_c for a criteria $\geq_c$ have first rank : $r_{c_i}(b_c) = 1$.
- for any element p, its rank $r_c(p)$ is one plus the length of the shortest path $b_c >_c \ldots >_c p$.

Then the Ordinal Weighted Sum selection function chooses the objects that have the best (weighted) rank :

Definition 3 (Ordinal Weighted Sum)

$$\beta_{\mathbf{a}}^{ows}(O) = \arg\min_{p \in O} \mathsf{OWS}_{\mathbf{a}}(p),$$

where $\mathsf{OWS}_{\mathbf{a}}(p) = \sum_i w_i(\mathbf{a}) \times r_{c_i}(p)$

For this function, we turn to an ordinal evaluation of the elements, but for the weights we kept the evaluation of the target. Another possibility is to also use an ordinal weight of the criteria:

Definition 4 (Purely Ordinal Weighted Sum) *Define the rank of each criterion c with respect to $\mathbf{a}$:*

- $c_i \geq_{\mathbf{a}} c_j$ *iff* $c_i(\mathbf{a}) \geq c_j(\mathbf{a})$
- *best criteria $c_{\mathbf{a}}$ for $\geq_{\mathbf{a}}$ have first rank :* $r_{\mathbf{a}}(c_{\mathbf{a}}) = 1$.
- *for any criterion c s.t. $c(\mathbf{a}) \neq 0$ its rank $r_{\mathbf{a}}(c)$ is one plus the length of the shortest path $c_{\mathbf{a}} > \ldots > c$.*
- *all criteria c s.t. $c(\mathbf{a}) = 0$ are not ranked*

Then we will use a lexicographical comparison with respect to this criteria ranking: $\vec{\mathcal{C}^{\mathbf{a}}} = \langle\{c \text{ s.t. } r_{\mathbf{a}}(c) = 1\}, \{c \text{ s.t. } r_{\mathbf{a}}(c) = 2\}, \ldots, \{c \text{ s.t. } r_{\mathbf{a}}(c) = m\}\rangle$.

$$\beta_{\mathbf{a}}^{pws}(O) = \arg\min_{p \in O}(\mathsf{PWS}_{\mathbf{a}}(p), \geq_{lex}),$$

where $\mathsf{PWS}_{\mathbf{a}}(p) = \langle\sum_{c \in \vec{\mathcal{C}_1^{\mathbf{a}}}} r_c(p), \ldots, \sum_{c \in \vec{\mathcal{C}_m^{\mathbf{a}}}} r_c(p)\rangle$, *and $\geq_{lex}$ is defined for two vectors $x = \langle x_1, \ldots, x_n \rangle$ and $y = \langle y_1, \ldots, y_n \rangle$ as $x >_{lex} y$ if $\exists i$ s.t. $\forall j < i$ $x_j = y_j$ and $x_i > y_j$, and $x \simeq y$ iff $\forall i$ $x_i = y_i$.*

One function that usually allows to obtain results that are more balanced with respect to criteria is the product (just as the Nash welfare function (**???**)). Here, as we consider weights, we will use the power function to weight each criterion evaluation.

Definition 5 (Weighted Geometric Mean)

$\beta_{\mathbf{a}}^{wgm}(O) = \arg\max_{p \in O} \mathsf{WGM}_{\mathbf{a}}(p),$
where $\mathsf{WGM}_{\mathbf{a}}(p) = \prod_{c_i \in \mathcal{C}}[1 + c_i(p)]^{w_i(\mathbf{a})}$

We have to shift every evaluation of criteria by 1 to avoid the problem of the absorbing element 0.

All functions given so far start from the origin point and try to find the best elements "towards" the target. One alternative is not to focus on the origin (the worst potential element), but to start from the best potential element, that we call the ideal element. This ideal element is the (hypothetical) element which evaluation on each element is equal to the maximal evaluation among all elements in Ω. Most of the time this element does not belong to O, so we look at the closest element of O from this point "towards" the target (i.e., by using a distance that is parametrized by the target element). This idea comes from a method that is used in cooperative game theory for the bargaining problem called the Kalai-Smorodinsky solution (**??**).

More formally let us define the coordinates of the ideal point:

Definition 6 (Ideal Point Distance) *The ideal point b^O is defined as $b^O = \langle \max_{p \in O} c_1(p), \ldots, \max_{p \in O} c_n(p) \rangle$.*
$$\beta_{\mathbf{a}}^{ipd}(O) = \arg\min_{p \in O} \mathsf{IPD}_o(p),$$
where $\mathsf{IPD}_{\mathbf{a}}(p) = \sqrt{\sum_{c_i \in \mathcal{C}} [(b_i^O - c_i(p)) * w_i(\mathbf{a})]^2}$

The next method, Weighted LexiMin, ensures the evaluation of the best solutions to be "proportional" to the one of the target. The idea is similar to the one introduced in (**?**) in the context of multi-objective constraint optimization, in which it is called "weighted egalitarian method."

Given a vector of numbers v, v^i denotes the ith component of v. The vector $v_\leq$ (resp. $v_\geq$) denotes the vector composed of each element of v rearranged in a non-decreasing order (resp. non-increasing order). For instance, if $v = (3,2,5,3)$, then $v_\leq = (2,3,3,5)$ and $v_\geq = (5,3,3,2)$. Let v, w two vectors of numbers of the same size, i.e., $v, w \in \mathbb{R}^k$ for some $k \geq 0$. We say that v lexically succeeds w when there exists $i \in \{1, \ldots, m\}$ such that for each $j \in \{1, \ldots, i-1\}$, $v^j = w^j$ and $v^i \geq w^i$. Additionally, when $w^i > 0$ for each $i \in \{1, \ldots, k\}$, we denote by (v/w) the vector $(v/w) = (v^1/w^1, \ldots, v^k/w^k)$, and we denote by $(v * w)$ the vector $(v * w) = (v^1 * w^1, \ldots, v^k * w^k)$. Let us also denote by $w_{\mathsf{S}(\mathbf{a})}(\mathbf{a})$ the vector $w_{\mathsf{S}(\mathbf{a})}(\mathbf{a}) = (w_{i_1}(\mathbf{a}), \ldots, w_{i_{|\mathsf{S}(\mathbf{a})|}}(\mathbf{a}))$ where $i_j < i_{j'}$ iff $j < j'$.

Given $p, \mathbf{a} \in \Omega$, we denote by $\mathsf{WLMin}_{\mathbf{a}}(p)$ the vector $\mathsf{WLMin}_{\mathbf{a}}(p) = (\gamma_{\mathsf{S}(\mathbf{a})}(p)/w_{\mathsf{S}(\mathbf{a})}(\mathbf{a}))_\leq$. Then let $\geq_{leximin}^{\mathbf{a}}$ be the relation over Ω defined for all $p, q \in \Omega$ as $p \geq_{leximin}^{\mathbf{a}} q$ iff $\mathsf{WLMin}_{\mathbf{a}}(p)$ lexically succeeds $\mathsf{WLMin}_{\mathbf{a}}(q)$. It is easy to see that $\geq_{leximin}^{\mathbf{a}}$ is a total ordering. Then:

Definition 7 (Weighted LexiMin)
$$\beta_{\mathbf{a}}^{wlmin}(O) = \arg\max(O, \geq_{leximin}^{\mathbf{a}})$$

The last method, called Weighted LexiMax, gives more priority to the criteria having a high value for both the candidates and the target. It is defined similarly to the Weighted LexiMin as follows. Given $p, \mathbf{a} \in \Omega$, we denote by $\mathsf{WLMax}_{\mathbf{a}}(p)$ the vector $\mathsf{WLMax}_{\mathbf{a}}(p) = (\gamma_{\mathsf{S}(\mathbf{a})}(p) * w(\mathbf{a}))_\geq$. Let $\geq_{leximax}^{\mathbf{a}}$ be the relation over Ω defined for all $p, q \in \Omega$ as $p \geq_{leximax}^{\mathbf{a}} q$ iff $\mathsf{WLMax}_{\mathbf{a}}(p)$ lexically succeeds $\mathsf{WLMax}_{\mathbf{a}}(q)$. Similarly, $\geq_{leximax}^{\mathbf{a}}$ is a strict total ordering,
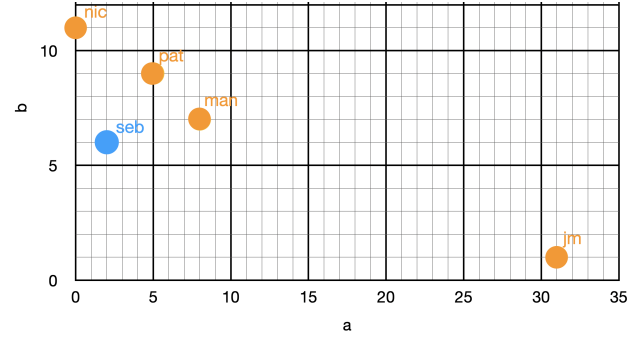

Figure 1: Representation of the alternatives on criteria a and b

so let us denote by $\geq_{leximax}^{\mathbf{a}}$ the corresponding total ordering. Then:

Definition 8 (Weighted LexiMax)
$$\beta_{\mathbf{a}}^{wlmax}(O) = \arg\max(O, \geq_{leximax}^{\mathbf{a}})$$

Let us now consider the following example to illustrate the behaviour of all the methods we defined in this section: $\mathcal{C} = \{a, b, c, d\}$ and $\Omega = O = \{seb, pat, jm, man, nic\}$. The evaluation of the criteria for the participants are the following ones: $\gamma(seb) = \{2, 6, 0, 0\}$; $\gamma(jm) = \{31, 1, 7, 1\}$; $\gamma(pat) = \{5, 9, 0, 11\}$; $\gamma(man) = \{8, 7, 2, 1\}$; $\gamma(nic) = \{0, 11, 12, 8\}$.

We focus on the target seb. As the support of seb has only two criteria we can represent all the alternatives projected on this support, as done in Fig. 1.

Let us see what the proposed methods give as result on this example on Figure 2 (the result for each method is in blue).

β^{ws}		BWS$_{seb}(seb) = 5$
	BWS$_{seb}(jm) = 8.5$	BWS$_{seb}(pat) = 8$
	BWS$_{seb}(man) = 7.25$	BWS$_{seb}(nic) = 8.25$
β^{ows}		OWS$_{seb}(seb) = 4$
	OWS$_{seb}(jm) = 4$	OWS$_{seb}(pat) = 2.25$
	OWS$_{seb}(man) = 2.75$	OWS$_{seb}(nic) = 2$
β^{pws}		PWS$_{seb}(seb) = (8)$
	PWS$_{seb}(jm) = (6)$	PWS$_{seb}(pat) = (5)$
	PWS$_{seb}(man) = (5)$	PWS$_{seb}(nic) = (6)$
β^{wgm}		WGM$_{seb}(seb) = 5.66$
	WGM$_{seb}(jm) = 4$	WGM$_{seb}(pat) = 8.8$
	WGM$_{seb}(man) = 8.24$	WGM$_{seb}(nic) = 6.45$
β^{ipd}		IPD$_{seb}(seb) = 8.16$
	IPD$_{seb}(jm) = 7.5$	IPD$_{seb}(pat) = 6.67$
	IPD$_{seb}(man) = 6.49$	IPD$_{seb}(nic) = 7.75$
β^{wlmin}		WLMin$_{seb}(seb) = (8,8)$
	WLMin$_{seb}(jm) = (1.33, 124)$	WLMin$_{seb}(pat) = (12, 20)$
	WLMin$_{seb}(man) = (9.3, 32)$	WLMin$_{seb}(nic) = (0, 14.6)$
β^{wlmax}		WLMax$_{seb}(seb) = (4.5, 0.5)$
	WLMax$_{seb}(jm) = (7.75, 0.75)$	WLMax$_{seb}(pat) = (6.75, 1.25)$
	WLMax$_{seb}(man) = (5.25, 2)$	WLMax$_{seb}(nic) = (8.25, 0)$

Figure 2: Results of the targeting methods

	S1	S2	S3	S4	S5	S6	S7	S8	S9	O1	O2	O3	O4	O5
Weighted Sum	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✓	✗	✓	✓
Ordinal Weighted Sum	✓	✓	✓	✓	✗	✗	✗	✓	✓	✗	✗	✗	✓	✓
Purely Ordinal Weighted Sum	✓	✓	✓	✓	✗	✗	✗	✓	✓	✗	✗	✗	✓	✓
Weigthed Geometric Mean	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗	✓	✓
Ideal Point Distance	✓	✓	✓	✓	✗	✗	✗	✓	✓	✗	✗	✗	✓	✓
Weighted LexiMin	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✓	✓	✓
Weighted LexiMax	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗	✓	✓

Table 1: Properties of targeting functions

We can see on this example that the results are quite different when considering the different methods. β^{ws} selects jm because he has a very good evaluation on the a criterion. β^{ows} selects nic since he has the best rank on the important criterion b. β^{pws} selects pat, man and nic because the relative ranks of the two criteria a and b are the same in O, and pat, man and nic are well ranked on these two criteria. β^{wgm} selects pat since he has a good average evaluation on each criterion. β^{ipd} selects man because he is the closest from the ideal candidate with respect to the target seb. β^{wlmin} selects pat since his evaluation is proportional to the one of seb. And β^{wlmax} selects nic since both seb and nic have a good evaluation on the criterion a.

Properties of the targeting functions

In the last section we saw that the behaviour of these functions are quite different on an illustrative example. Let us now study more deeply their behaviour by looking at which are the properties satisfied by the different methods.

Proposition 6 *The Weighted Sum method β^{ws} satisfies* **S1, S2, S3, S4, S5, S6, S7, S8, S9, O2, O4, O5**. *It does not satisfy* **O1** *and* **O3**.

Proposition 7 *The Ordinal Weighted Sum method β^{ows} satisfies* **S1, S2, S3, S4, S8, S9, O4, O5**. *It does not satisfy* **S5, S6, S7, O1, O2, O3**.

Proposition 8 *The Purely Ordinal Weighted Sum method β^{pws} satisfies* **S1, S2, S3, S4, S8, S9, O4, O5**. *It does not satisfy* **S5, S6, S7, O1, O2** *and* **O3**.

Proposition 9 *The Weighted Geometric Mean method β^{wgm} satisfies* **S1, S2, S3, S4, S5, S6, S7, S8, S9, O4, O5**. *It does not satisfy* **O1, O2** *and* **O3**.

Proposition 10 *The Ideal Point Distance method β^{ipd} satisfies* **S1, S2, S3, S4, S8, S9, O4, O5**. *It does not satisfied* **S5, S6, S7, O1, O2** *and* **O3**.

Proposition 11 *The Weighted LexiMin method β^{wlmin} satisfies* **S1, S2, S3, S4, S5, S6, S7, S8, S9, O1, O3, O4, O5**. *It does not satisfy* **O2**.

Proposition 12 *The Weighted LexiMax method β^{wlmax} satisfies* **S1, S2, S3, S4, S5, S6, S7, S8, S9, O4, O5**. *It does not satisfy* **O1, O2, O3**.

These results are summarized in Table 1 (See Appendix for the proofs).

The Weighted Sum, Weighted Geometric Mean, Weighted LexiMin and Weighted LexiMax methods satisfy all main properties. The more "ordinal" methods (Ordinal Weighted Sum and Pure Ordinal methods) have to give up some of the main properties. The Ideal Point Distance method, that works quite differently by referring to an hypothetical best element, fails also to satisfy some of the main properties. Notably, Weighted LexiMin is the only method satisfying the optional properties **O1** and **O3**.

Note that **O4** is satisfied by all the proposed targeting functions. Nevertheless, we do not put this property as a main one because the satisfaction of this property is due to the normalization of the weights in all our functions. Some sensible targeting functions without normalization may exist, this is why we let **O4** as an optional property.

Conclusion

In this paper, we introduced a novel multi-criteria decision-making problem called targeting, where the objective is to identify the best elements of a set given a target element. We proposed a set of fundamental axioms that all sensible targeting methods should satisfy, along with additional axioms that define specific subclasses of these methods. We then presented several candidate methods for targeting and analyzed their properties.

To be interesting targeting as we characterized it here should be used when the criteria are quite independent, since it should make sense to focus on the support of the target and to ignore some (irrelevant) criteria. But there are a lot of applications, like the laptop configuration example in the introduction, where this is the case.

We identified four functions that satisfy all mandatory properties: Weighted Sum, Weighted Geometric Mean, Weighted LexiMin and Weighted LexiMax.

Note that our attempts to be more ordinal, i.e. to rely less on exact quantitative values and to focus on ranks instead, failed since the ordinal functions do not satisfy three of our mandatory postulates. Trying to achieve an ordinal variation of targeting looks like an interesting issue.

We implemented these methods in the *Profile Expert Finder* tool (https://coscinus.org/pef/) of coscinus, designed to identify experts corresponding to a given computer science author.

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