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Physics 2CL LAB 6

Worksheet Questions:

Section 3 -

1) For the underdamped, critically damped, and overdamped circuits there is activity a few moments after the initial activity and the battery turning off that looks like a “negative” version of the initial excitement V(t) curve (reflected across the x-axis). This is most likely caused by the capacitor discharging, which makes sense in terms of our previous experiments (RC circuit Charge vs Discharge curves look identical, except one is reflected across the x-axis relative to the other.)

2) The moment the battery is turned on and our circuit begins to charge, we can determine the voltages and current easily from Ohm’s Law and by recalling that a capacitor has effectively no resistance when first starting to charge, so we can therefore ignore its effect on the circuit. We know from the lab manual that the battery is providing $0.5V$ and we can use the measured values of our resistor and inductor’s resistances ($R_R = 1.5,\ R_L = 6.5$) to find the total resistance of our circuit, which allows us to solve for the current via:

$$0.5 = (1.5 + 6.5)I$$

$$I = \frac{0.5}{8} = 0.0625 \approx 0.063\ Amps$$

Now, we can use this current and the resistances of each of our circuit elements to find the voltages across every element:

$$V_{Cap} = (0.063)(R_{cap})$$
$$V_{Cap} = (0.063)(0) = 0\ Volts$$

$$V_R = (0.063)(R_R)$$

$$V_R = (0.063)(1.5) = 0.09375\ Volts$$

$$V_L = (0.063)(R_L)$$

$$V_L = (0.063)(6.5) = 0.4095\ Volts$$

We can check to see if this is reasonable by summing all of the voltages across each element to see if they add up to the total voltage drop across the circuit:

$$0 + 0.4095 + 0.09375 = 0.50325V \approx 0.5V$$

Because all of these values have an uncertainty that I have been ignoring because this is a worksheet question, this slight deviation is within reasonable limits.

Lab Analysis
Section 3.1 - 3.2 -

For the measured inductor values we have:

$$R_L = 6.5 \pm 0.2\,\Omega$$

$$L = 8.22 \pm 0.01\,mH$$

The measured capacitance for our capacitor is:

$$C_{measured} = 10.18 \pm 0.01\,\mu F$$

The measured decade box resistance is as follows:

$$R_D = 1.5 \pm 0.1\,\Omega$$

We can now solve for the total resistance of our circuit, R_{Tot}:

$$R_{Tot} = R_D + R_L$$

$$R_{Tot} = 1.5 + 6.5 = 8\Omega$$

The uncertainty here is:

$$\Delta R_{Tot} = \sqrt{(\Delta R_D)^2(1)^2 + (\Delta R_L)^2(1)^2}$$

$$\Delta R_{Tot} = \sqrt{(0.1)^2(1)^2 + (0.2)^2(1)^2} \rightarrow \sqrt{0.05} = 0.2236$$

$$R_{Tot} = 8.0 \pm 0.2\,\Omega$$

We can now solve for our expected τ:

$$\tau = \frac{L}{R} \rightarrow \frac{0.00822}{8} = 0.0010275\,s$$

The uncertainty here is:

$$\Delta\tau = \sqrt{(\Delta L)^2 (\frac{1}{R_{Tot}})^2 + (\Delta R_{Tot})^2 (-\frac{L}{R_{Tot}^{\ 2}})^2}$$

$$\Delta\tau = \sqrt{(0.00001)^2 (\frac{1}{8})^2 + (0.2)^2 (-\frac{0.00822}{8^2})^2}$$

$$\Delta\tau = \sqrt{(0.0000000000015625) + (0.00000000065985)} = 0.00003571789 \rightarrow \Delta\tau = 0.00004$$

$$\tau = (1.03 \pm 0.04) \cdot 10^{-3} seconds$$

We can similarly solve for ω_D:

$$\omega_D = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \rightarrow \sqrt{\frac{1}{(0.00822)(0.00001018)} - \frac{(8)^2}{4(0.00822)^2}} = \sqrt{(11950343.93) - (236797.0827)}$$

$$\omega_D = 3422.505931\,rad/s$$

The uncertainty is given by:

$$\Delta\omega_D = \sqrt{(\Delta R_{Tot})^2 (-\frac{R}{4L^2\sqrt{\frac{1}{CL}-\frac{R^2}{4L^2}}})^2 + (\Delta C)^2 (-\frac{1}{2C^2L\sqrt{\frac{1}{CL}-\frac{R^2}{4L^2}}})^2 + (\Delta L)^2 (-\frac{\frac{1}{CL^2}-\frac{R^2}{4L^3}}{2\sqrt{\frac{1}{CL}-\frac{R^2}{4L^2}}})^2}$$

$$\Delta\omega_D = = \sqrt{(0.2)^2 (-8.6485)^2 + (0.00000001)^2 (-171497747.9352)^2 + (0.00001)^2 (-203973.0969)^2}$$

$$\Delta\omega_D = \sqrt{(2.99188) + (2.941147754) + (4.16050243)} = \sqrt{10.093531031034967} = 3.177$$

$$\omega_D = 3423 \pm 3\, rad/sec$$

The table of circuit peak voltage-time pairs is as follows:

Time (seconds)	Corresponding Peak Voltage	Pair Number
1.8047	0.125246	1
1.8056	-0.08145	2
1.8066	0.051457	3
1.8074	-0.03603	4

The table of voltage-time pairs, and experimental τandω_D is as follows:

(Note: $\tau$is calculated via $\tau = \frac{t_2 - t_1}{2(ln|V_1| - ln|V_2|)}$ and $\omega_D$is found via $\omega_D = \frac{2\pi}{T}$)

Sample calculation for $\tau_{1-2}$is as follows:

$$\tau = \frac{1.8056 - 1.8047}{2(ln(0.125246) - ln(0.08145))} = \frac{0.009}{0.8605809366} = 0.0010458$$

To find the period, T for any given pair of peaks, I first noted that moving from one peak to another adjacent peak results in $\frac{T}{2}$I then extrapolated out some other relationships to write the period of a pair in terms of a time difference between the two values:

Example for pair 1-2:

$$T = 2(t_2 - t_1) \rightarrow 2(0.0009) = 0.0018$$

Example for pair 1-4:

$$1.5T = (t_4 - t_1) \rightarrow \frac{(0.0027)}{1.5} = 0.0018$$

Sample calculation for $\omega_{D_{1-2}}$ is as follows:

$$\omega = \frac{2\pi}{0.0018} = 3490.6585$$

Pair Numbers	τ(seconds)	Period (seconds)	ω_D (rad/sec)
1-2	0.0010458	0.0018	3490.6585
1-3	0.0010679	0.0019	3306.9396
1-4	0.0010835	0.0018	3490.6585
2-3	0.0010887	0.002	3141.5926
2-4	0.0011034	0.0018	3490.6585
3-4	0.0011223	0.0016	3926.9908

Average $\tau$was calculated via the formula:

$$\frac{\Sigma\tau_n}{n} \rightarrow \frac{(0.0010458+0.0010679+0.0010835+0.0010887+0.0011034+0.0011223)}{6} = \bar{\tau} = 0.0010852667$$

The uncertainty here is:

$$\sqrt{\frac{\Sigma(\tau_n - \bar{\tau})^2}{n-1}}$$

Average τ with uncertainty is:

$$(1.09 \pm 0.03) \cdot 10^{-3}\,\tau$$

Average ω_D was calculated via the formula:

$$\frac{\Sigma\omega_n}{n} \rightarrow \frac{(3490.6585+3490.6585+3490.6585+3306.9396+3141.5926+3926.9908)}{6} = \overline{\omega_D} = 3474.583083$$

The uncertainty here is:

$$\sqrt{\frac{\Sigma(\omega_n - \overline{\omega})^2}{n-1}}$$

Average ω_D with uncertainty is:

$$3500 \pm 300\, rad/sec$$

By comparing the expected and experimental values of τ we have a $t - score$ given by:

$$t_\tau = \frac{|0.00109 - 0.00103|}{\sqrt{0.00003^2 + 0.00004^2}} = 1.2$$

Because $t_\tau > 1$, we know that our expected and experimental values don't necessarily match completely well. However, our values are within six-hundred-thousandths of each other, so I am more inclined to believe that the relatively large t-score here is more caused by the underestimation of error than by incongruent data. Both our experimentally and analytically-derived uncertainties have uncertainties in the hundred-thousandths place, which makes our t-score calculation extremely unforgiving.

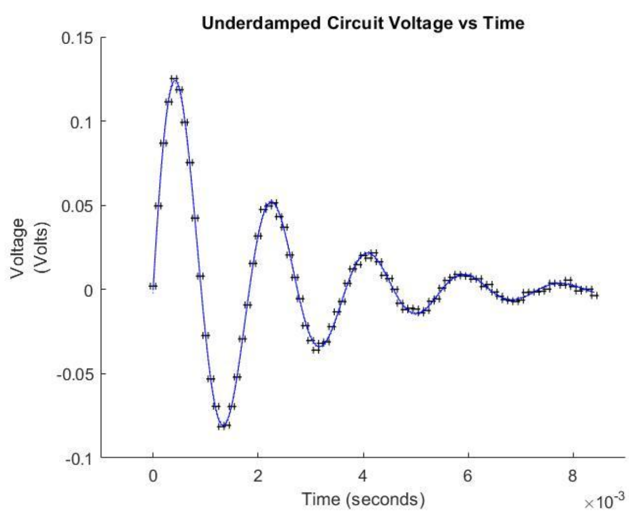
By comparing the expected and experimental values of ω_D we have a $t - score$ given by:

$$t_\omega = \frac{|3500 - 3423|}{\sqrt{300^2 + 3^2}} = 0.257$$

Because $t_\omega < 1$, we know that our expected and experimental values match each other fairly well. Although our expected and experimental values are indeed relatively close, it seems as if the main reason for this low of a t-score is the overestimation of error in our experimentally-derived value of ω_D.

In fitting the underdamped circuit dataset, I chose the sinexpfit function, as this most closely resembles the equation for the voltage as a function of time for an underdamped circuit (as both are sin functions multiplied by e)

The graph of this fit is below:



The final fit parameters are as follows:

$$V_{max} = 0.153 \pm 0.001\,V$$

$$\tau = (1.05 \pm 0.001) \cdot 10^{-3} sec$$

$$\omega_D = 3429 \pm 4\, rad/sec$$

$$\phi = (-1.5 \pm 6) \cdot 10^{-3} rad$$

$$Vertical\,Shift = (-1.8 \pm 1.8) \cdot 10^{-4} V$$

For our second set of $t - scores$ we have:

By comparing the expected and experimental values of τ we have a $t - score$ given by:

$$t_\tau = \frac{|0.00105 - 0.00103|}{\sqrt{0.00001^2 + 0.00004^2}} = 0.485$$

Because $t_\tau < 1$, it is fair to say that our two values of τmatch each other fairly well. This seems like a particularly valid piece of data, especially when considering how small the values of our uncertainties for both the experimentally and analytically derived $\tau's$ are.

By comparing the expected and experimental values of ω_D we have a $t - score$ given by:

$$t_\omega = \frac{|3429-3423|}{\sqrt{4^2+3^2}} = 1.2$$

Because $t_\omega > 1$, it would seem that our experimentally and analytically-derived values don't necessarily match very well. Although our two values of $\omega_D$are within only a few digits of each other, the small size of their relative errors makes this t-score calculation rather unforgiving.

Section 3.3-

The new inductor measurements are as follows:

$L = 144.3 \pm 0.1\, mH$

$R_L = 160.6 \pm 0.1\, \Omega$

The measured resistance of our decade box is:

$R_D = 151.1 \pm 0.1\, \Omega$

We can now solve for the total resistance of our circuit, R_{Tot}:

$R_{Tot} = R_D + R_L$

$R_{Tot} = 151.1 + 160.6 = 310.7\Omega$

The uncertainty here is:

$$\Delta R_{Tot} = \sqrt{(\Delta R_D)^2(1)^2 + (\Delta R_L)^2(1)^2}$$

$$\Delta R_{Tot} = \sqrt{(0.1)^2(1)^2 + (0.1)^2(1)^2} \rightarrow \sqrt{0.02} = 0.14142$$

$$R_{Tot} = 310.7 \pm 0.1\,\Omega$$

We can now solve for our expected τ:

$$\tau = \frac{L}{R} \rightarrow \frac{0.1443}{310.7} = 0.000464435\, s$$

The uncertainty here is:

$$\Delta\tau = \sqrt{(\Delta L\ )^2(\frac{1}{R_{Tot}})^2 + (\Delta R_{Tot})^2(-\frac{L}{{R_{Tot}}^2})^2}$$

$$\Delta\tau = \sqrt{(0.0001)^2(\frac{1}{310.7})^2 + (0.1)^2(-\frac{0.1443}{310.7^2})^2}$$

$$\Delta\tau = \sqrt{(0.0000000000001035899) + (0.000000000000022344346)} = 0.00000035487 \rightarrow \Delta\tau = 0.0000004$$

$$\tau = (4.644 \pm 0.004) \cdot 10^{-4} seconds$$

We can similarly solve for ω_D:

$$\omega_D = \sqrt{-\frac{1}{LC} + \frac{R^2}{4L^2}} \rightarrow \sqrt{-\frac{1}{(0.1443)(0.00001018)} + \frac{(310.7)^2}{4(0.1443)^2}} = \sqrt{(-680747.2426) + (1159017.125)}$$

$$\omega_D = 691.5705912\, rad/s$$

The uncertainty is given by:

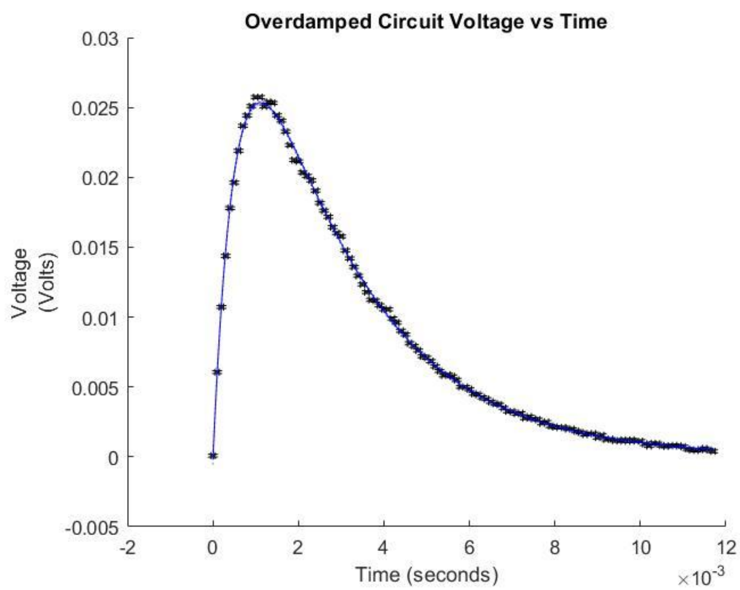
$$\Delta\omega_D = \sqrt{(\Delta R_{Tot})^2(\frac{R}{4L^2\sqrt{\frac{R^2}{4L^2}-\frac{1}{CL}}})^2 + (\Delta C)^2(\frac{1}{2C^2L\sqrt{\frac{R^2}{4L^2}-\frac{1}{CL}}})^2 + (\Delta L)^2(\frac{\frac{1}{CL^2}-\frac{R^2}{4L^3}}{2\sqrt{\frac{R^2}{4L^2}-\frac{1}{CL}}})^2}$$

$$\Delta\omega_D = = \sqrt{(0.1)^2(5.394014169)^2 + (0.00000001)^2(48347230.4181)^2 + (0.0001)^2(-8203.363803)^2}$$

$$\Delta\omega_D = \sqrt{(0.29095388) + (0.2337454689) + (0.67295177)} = \sqrt{1.197651134458448} = 1.0943$$

$$\omega_D = 692 \pm 1\, rad/sec$$

In fitting the overdamped circuit dataset, I chose the sinhexpfit function, as this most closely resembles the equation for the voltage as a function of time for an overdamped circuit (as both are hyperbolic sin functions multiplied by e to some exponent)



The final fit parameters are as follows:

$$V_{max} = 0.102 \pm 0.001\, V$$

$$\tau = (4.70 \pm 0.05) \cdot 10^{-4} sec$$

$$\omega_D = 670 \pm 10\, rad/sec$$

$$\phi = (-2 \pm 2) \cdot 10^{-3} rad$$

$$Vertical\ Shift = (-3 \pm 6) \cdot 10^{-5} V$$

By comparing the expected and experimental values of τ we have a $t - score$ given by:

$$t_\tau = \frac{|0.000470 - 0.0004644|}{\sqrt{0.0000004^2 + 0.000005^2}} = 1.12$$

Because $t_\tau > 1$, we know that our expected and experimental values don't necessarily match well. However, our values are within six millionths of each other, so just as before, I am more inclined to believe that the relatively large t-score here is more caused by the underestimation of error then by incongruent data. Both our experimentally and analytically-derived values of tau have uncertainties in either the millionths or ten-millionths place, which makes our t-score calculation extremely unforgiving.

By comparing the expected and experimental values of ω_D we have a $t - score$ given by:

$$t_\omega = \frac{|692-670|}{\sqrt{1^2+10^2}} = 2.19$$

Because $t_\omega > 1$, we know that our expected and experimental values are a poor match for each other. Although our t-scores for ω_D were fairly small in sections 3.1 and 3.2, we have a much larger score here despite the fact that relative error is larger for our experimentally-derived value. This means that our data must be farther off (by ratio) than it was in the earlier sections.

Section 3.4 -

The expected critical resistance of our circuit is given by:

$$R_{Crit} = 2\sqrt{\frac{L}{C}} \rightarrow 2\sqrt{\frac{0.00822}{0.00001018}} = 56.8319\,\Omega$$

The uncertainty is represented by:

$$\Delta R_{Crit} = \sqrt{(\Delta L)^2\left(\frac{1}{C\sqrt{\frac{L}{C}}}\right)^2 + (\Delta C)^2\left(\frac{L}{C^2\sqrt{\frac{L}{C}}}\right)^2}$$

$$\Delta R_{Crit} = \sqrt{(0.00001)^2\left(\frac{1}{0.00001018\sqrt{\frac{0.00822}{0.00001018}}}\right)^2 + (0.00000001)^2\left(\frac{0.00822}{0.00001018^2\sqrt{\frac{0.00822}{0.00001018}}}\right)^2}$$

$$\Delta R_{Crit} = \sqrt{(0.001195034393090) + (0.00077916329)} = 0.0444$$

$$R_{Crit} = 56.83 \pm 0.04\,\Omega$$

The upper critical bound for the decade box is measured as:

$$R_{DUp} = 76.3\Omega$$

The lower critical bound for the decade box is measured as:

$$R_{DLo} = 30.6\Omega$$

Because $R_{Tot} = R_D + R_L$, we can find:

$$R_{CritUp} = R_{DUp} + R_L$$

$$R_{CritUp} = 76.3 + 6.5 = 82.8\Omega$$

$$R_{CritLo} = R_{DLo} + R_L$$

$$R_{CritLo} = 30.6 + 6.5 = 37.1\Omega$$

Using these values and the bounds treatment, we can solve for R_{Crit} experimentally:

$$R_{Crit} = \frac{R_{CritUp} + R_{CritLo}}{2} \rightarrow \frac{82.8 + 37.1}{2} = 59.95\Omega$$

The uncertainty here is given by:

$$\Delta R_{Crit} = \frac{R_{CritUp} - R_{CritLo}}{2} \rightarrow \frac{82.8 - 37.1}{2} = 22.85\Omega$$

$$R_{Crit} = 60 \pm 20\,\Omega$$

By comparing the expected and experimental values of R_{Crit} we have a $t - score$ given by:

$$t_{Crit} = \frac{|60-56.83|}{\sqrt{20^2 + 0.04^2}} = 0.158$$

Because $t_\tau < 1$, it is fair to say that our two values of τmatch each other fairly well.Regardless, this seems like a particularly strange piece of data, as I am sure that the small t-score is a product of the massive uncertainty associated with our experimentally derived critical resistance using the bounds treatment.

Section 4 -

For the measured inductor values we have:

$$R_L = 6.5 \pm 0.2\,\Omega$$

$$L = 8.22 \pm 0.01\,mH$$

The measured capacitance for our capacitor is:

$$C_{measured} = 10.18 \pm 0.01\,\mu F$$

The measured decade box resistance is as follows:

$$R_D = 10.7 \pm 0.1\,\Omega$$

We can now solve for the total resistance of our circuit, R_{Tot}:

$$R_{Tot} = R_D + R_L$$

$$R_{Tot} = 10.7 + 6.5 = 17.2\Omega$$

The uncertainty here is:

$$\Delta R_{Tot} = \sqrt{(\Delta R_D)^2(1)^2 + (\Delta R_L)^2(1)^2}$$

$$\Delta R_{Tot} = \sqrt{(0.1)^2(1)^2 + (0.2)^2(1)^2} \rightarrow \sqrt{0.05} = 0.2236$$

$$R_{Tot} = 17.2 \pm 0.2\,\Omega$$

We can now solve for our expected τ:

$$\tau = \frac{L}{R} \rightarrow \frac{0.00822}{17.2} = 0.000477906\,s$$

The uncertainty here is:

$$\Delta\tau = \sqrt{(\Delta L\ )^2(\frac{1}{R_{Tot}})^2 + (\Delta R_{Tot})^2(-\frac{L}{R_{Tot}^{\ 2}})^2}$$

$$\Delta\tau = \sqrt{(0.00001)^2(\frac{1}{17.2})^2 + (0.2)^2(-\frac{0.00822}{17.2^2})^2}$$

$$\Delta\tau = \sqrt{(0.00000000000033802) + (0.0000000000308808)} = 0.0000055873 \rightarrow \Delta\tau = 0.000006$$

$$\tau = (4.78 \pm 0.06) \cdot 10^{-4} seconds$$

Expected ω_o is:

$$\omega_o = \frac{1}{\sqrt{LC}} \rightarrow \frac{1}{\sqrt{(0.00822)(0.00001018)}} = 3456.92694\, rad/sec$$

The uncertainty can be represented by:

$$\Delta\omega_0 = \sqrt{(\Delta L)^2(-\frac{C}{2\sqrt{(LC)^3}})^2 + (\Delta C)^2(-\frac{L}{2\sqrt{(LC)^3}})^2}$$

$$\Delta\omega_0 = \sqrt{(0.00001)^2(-0.0000210275)^2 + (0.0000001)^2(-0.00000001697901)^2}$$

$$\Delta\omega_0 = \sqrt{(4.421572780655726) + (2.882868661465510)} = 2.7026$$

$$\omega_0 = 3457 \pm 3\, rad/sec$$

Expected Q can be found as:

$$Q = \sqrt{\frac{L}{C}}\frac{1}{R}$$

$$Q = \sqrt{\frac{0.00822}{0.00001018}}\frac{1}{17.2} = 1.65209$$

We can find the uncertainty as:

$$\Delta Q = \sqrt{(\Delta R_{Tot})^2(\frac{\sqrt{\frac{L}{C}}}{R^2})^2 + (\Delta C)^2(\frac{L}{2C^2R\sqrt{\frac{L}{C}}})^2 + (\Delta L)^2(\frac{1}{2CR\sqrt{\frac{L}{C}}})^2}$$

$$\Delta Q = \sqrt{(0.2)^2(-0.09605171552524)^2 + (0.0000001)^2(81143.88)^2 + (0.00001)^2(100.492062471)^2}$$

$$\Delta Q = \sqrt{0.0003707055} = 0.0192$$

$$Q = 1.65 \pm 0.02$$

For the measured absolute f values we have:

$$f_0 = 591 \pm 1\,Hz$$

$$f_+ = 889 \pm 1\,Hz$$

$$f_- = 424 \pm 1\,Hz$$

Converting these to angular frequencies we have:

$$\omega = 2\pi f$$

With the uncertainty being:

$$\Delta\omega = 2\pi\Delta f$$

So,

$$\omega_0 = 3713 \pm 6\,Hz$$

$$\omega_+ = 5586 \pm 6\,Hz$$

$$\omega_- = 2664 \pm 6\,Hz$$

We can find experimental Q via:

$$Q = \frac{\omega_0}{\omega_+ - \omega_-}$$

$$Q = \frac{3713}{5586-2664} = 1.270705$$

With uncertainty given by:

$$\Delta Q = \sqrt{(\Delta\omega)^2 \cdot \left(\left(\frac{1}{\omega_+ - \omega_-}\right)^2 + \left(\frac{\omega_0}{(\omega_+ - \omega_-)^2}\right)^2 + \left(-\frac{\omega_0}{(\omega_+ - \omega_-)^2}\right)^2\right)}$$

$$\Delta Q = \sqrt{36 \cdot ((0.000000117122) + (0.0000003782326))} = 0.00422$$

$$Q = 1.271 \pm 0.004$$

By comparing the expected and experimental values of Q we have a $t - score$ given by:

$$t_Q = \frac{|1.271 - 1.65|}{\sqrt{0.004^2 + 0.02^2}} = 18.58$$

Because $t > 1$, by a fairly large amount, it is apparent that our experimental and analytical values don't agree with each other very well. I think that this is a result of the underestimation of error for our experimentally derived value of Q.

By comparing the expected and experimental values of ω_0 we have a $t - score$ given by:

$$t_\omega = \frac{|3457 - 3713|}{\sqrt{3^2 + 6^2}} = 35.47$$

Because $t_\omega > 1$, it would seem that our experimentally and analytically-derived values don't necessarily match very well. Although our two values of ω_D are fairly close to each other, the small size of their relative errors makes this t-score calculation rather unforgiving. I think that there is most likely much more error associated with the experimentally derived angular frequency that we are ignoring, which is why the t-score is so high.

Important information for the table of values below:

To get frequency uncertainty, I just used 1 Hz for all values.

To get all values of ω, I computed

$$\omega = 2\pi f$$

To get $\Delta\omega$, I computed:

$$\Delta\omega = 2\pi\Delta f$$

To get ΔA, I used the standard deviation of the measured voltage noise from CAPSTONE.

To derive the time difference uncertainty, I just used:

$$\frac{1}{Sampling\ Rate} \rightarrow \frac{1}{100000} = 0.00001\ s$$

For all values.

To get the phase shift, Δϕ, I used:

$$\Delta\phi = \omega \Delta t$$

Phase shift uncertainty is therefore given by:

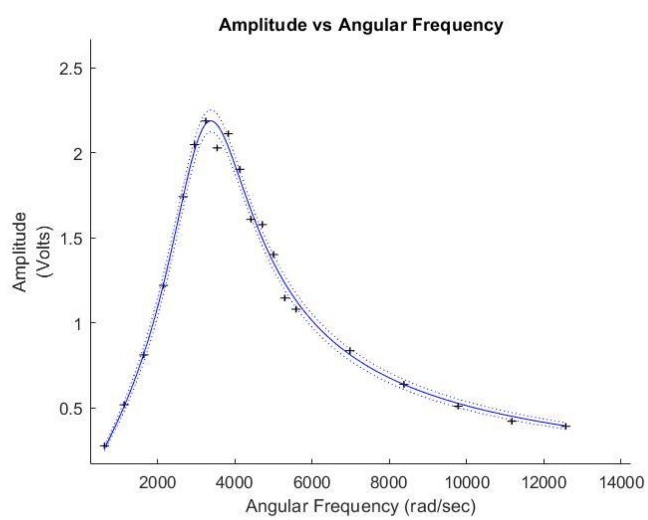
$$\delta\phi = \sqrt{(\delta\omega)^2 (\Delta t)^2 + (\delta t)^2 (\omega)^2}$$

For all ϕ

Freq. (Hz)	Freq. Uncert	ω (rad/s)	ω Uncert.	Amp. (V)	Amp. Uncert.	Δt (s)	Δt Uncert.	Samp. Rate	Δϕ	Δϕ Uncert.
100	1	628	6	0.27794	0.00006	-0.00223	0.00001	100000	-1.4	0.01
181	1	1137	6	0.51915	0.00006	-0.00122	0.00001	100000	-1.38	0.01
262	1	1646	6	0.81254	0.00006	-0.00076	0.00001	100000	-1.25	0.02
343	1	2155	6	1.22023	0.00006	-0.00044	0.00001	100000	-0.94	0.02
424	1	2664	6	1.74118	0.00006	-0.00027	0.00001	100000	-0.72	0.03
471	1	2959	6	2.04708	0.00006	-0.00016	0.00001	100000	-0.46	0.03
517	1	3248	6	2.18682	0.00006	-0.00005	0.00001	100000	-0.17	0.03
564	1	3544	6	2.02795	0.00006	0.00003	0.00001	100000	0.11	0.04
610	1	3833	6	2.1118	0.00006	0.00008	0.00001	100000	0.32	0.04
657	1	4128	6	1.90243	0.00006	0.00014	0.00001	100000	0.56	0.04
703	1	4417	6	1.60831	0.00006	0.00015	0.00001	100000	0.66	0.04
750	1	4712	6	1.57783	0.00006	0.00016	0.00001	100000	0.76	0.05
796	1	5001	6	1.40196	0.00006	0.00019	0.00001	100000	0.94	0.05
843	1	5297	6	1.14631	0.00006	0.00019	0.00001	100000	0.99	0.05
889	1	5586	6	1.08074	0.00006	0.00017	0.00001	100000	0.97	0.06
1111	1	6981	6	0.83735	0.00006	0.00018	0.00001	100000	1.24	0.07
1333	1	8375	6	0.63917	0.00006	0.00016	0.00001	100000	1.33	0.08
1556	1	9777	6	0.51106	0.00006	0.00014	0.00001	100000	1.4	0.1
1778	1	11172	6	0.42247	0.00006	0.00013	0.00001	100000	1.4	0.1
2000	1	12566	6	0.39259	0.00006	0.00011	0.00001	100000	1.4	0.1

In fitting the angular frequency vs. amplitude dataset, I chose the sqrtfit function, as this most closely resembles the equation for the relationship between driving frequency and response amplitude.

The graph is pictured below:



The final fit parameters are as follows:

$$V_{max} = 2.19 \pm 0.03\,V$$

$$Q = 1.58 \pm 0.05$$

$$\omega_0 = 3380 \pm 30\ (rad/sec)$$

By comparing the expected and experimental values of Q we have a $t - score$ given by:

$$t_Q = \frac{|1.58 - 1.65|}{\sqrt{0.05^2 + 0.02^2}} = 1.29$$

Because $t > 1$, it is apparent that our experimental and analytical values don't agree with each other very well. I think that because this is the second time our analytically derived Q gave us a poor t-score, that perhaps this is a result of the underestimation of error for our analytically derived value of Q.

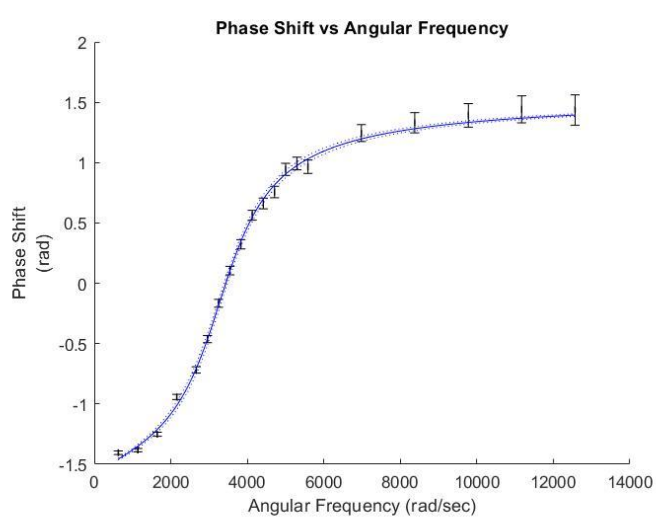
By comparing the expected and experimental values of ω_0 we have a $t - score$ given by:

$$t_\omega = \frac{|3457 - 3380|}{\sqrt{3^2 + 30^2}} = 2.55$$

Because $t_\omega > 1$, it would seem that our experimentally and analytically-derived values don't necessarily match very well. Although our two values of ω_0 have a fair amount of uncertainty to work within, it seems as though we have an accuracy problem, so the t-score remains high. I think that there is most likely much more error associated with the experimentally derived angular frequency that we are ignoring, which is why the t-score is so high.

In fitting the angular frequency vs. phase shift dataset, I chose the arctanfit function, as this most closely resembles the equation for the relationship between driving frequency and phase shift.

The graph is pictured below:



The final fit parameters are as follows:

$$\tau = (4.8 \pm 0.1) \cdot 10^{-4} s$$

$$\omega_0 = 3440 \pm 20 \, (rad/sec)$$

By comparing the expected and experimental values of ω_0 we have a $t - score$ given by:

$$t_\omega = \frac{|3457-3440|}{\sqrt{3^2 + 20^2}} = 0.841$$

Because $t_\omega < 1$, it would seem that our experimentally and analytically-derived values agree with each other. As I predicted earlier, it seems as if the major problem with the last resonant frequency fit was that it wasn't accurate enough to our predicted value and analytically derived value. Now that our experimental and analytic values are closer together, they produce a much smaller t-score despite the fact that there was significantly less uncertainty to work within.

By comparing the expected and experimental values of τ we have a $t - score$ given by:

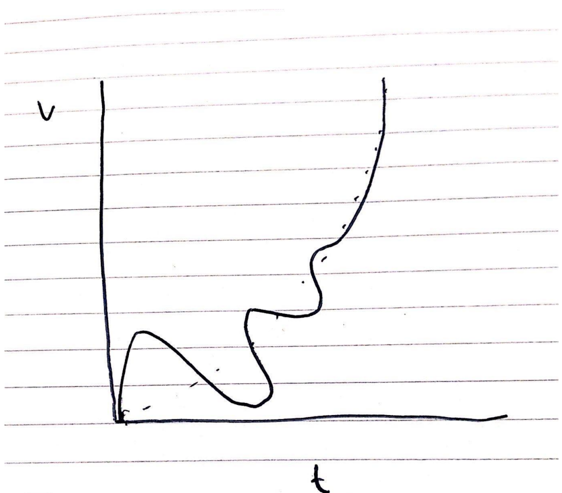
$$t_\omega = \frac{|0.000478-0.00048|}{\sqrt{0.000006^2 + 0.00001^2}} = 0.1715$$

Because $t_\tau < 1$, it would seem that our experimentally and analytically-derived values agree with each other. This data seems to be especially valid, as the t-scores for nearly all of the time constants in this lab have been fairly accurate, with this one having an extremely low t-score. This indicates that our data is both reasonably accurate and precise.

Discussion Question:

Clearly it's hard to visually identify a critically damped circuit. In lab, you distinguished between underdamped and critically damped system with the presence of oscillations, but you identified oscillations by eye. Given a set of (x, y) data points(not a function) how can you mathematically and computationally detect oscillations in general? You should outline the end-to-end data treatment procedure in detail and list any assumptions or limitations in your method. Hint: One way is to consider the second derivative. .

To begin, I would like to first address how we could begin to work with data if it were organized as a function so that I can approach the question of actually determining the presence of oscillations before moving on to the data processing question. Immediately, it can be seen that when a function oscillates, the concavity of the function must be switching signs. While it would be easy to just look for where the second derivative is zero, that only works to find oscillations in systems that damp down to a 'flat' solution holding a constant value. For instance, there might be an oscillation that occurs for a system that would "flatten out" to an accelerating pattern:



In that case, by just looking at whether or not the second-derivative is zero, our model might assume that this system oscillates forever (as concavity will stay positive). Therefore, we should focus on the third

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derivative of our dataset to determine whether or not any oscillations have stopped. If we know that our function/data does approach some non-accelerating value, then we could just look for where the second-derivative is zero, however the third- derivative method isn't that much more difficult , and could be a better solution, as it is useful in more situations. In practice, we would just need to differentiate our function one more time and solve for zeros the same way we would if we were to use the second-derivative test to determine whether or not oscillations have stopped. This is where the difficulty is however, as moving from a set of data points to something differentiable could be fairly difficult regardless of what derivative test we are using. I can see three solutions for this:

1) The data could be processed very similarly to how we have been doing so in these labs, where we input our data into some theoretical model that will fit our data into a function that it should generally follow. From this, we could extract the function for the line of best fit used to fit our data, and then take the third derivative of that function, finding where it is equal to zero. This could have the potential of being the simplest adaptation, but would have some shortcomings, as noisy data might have some issues with generating an accurate enough line of best fit, and we could only use this method on data that we already have a working theoretical model for.
2) The data could be put through a program in something like MATLAB that would just use numerical derivation to take the third derivative of our dataset. This could be something as simple as dividing the data into vectors by variable (but making sure pairs have matching indices) then subtracting neighboring values and then dividing the subtracted variable vectors by each other to generate a vector of slopes at any given point. (something like having vectors <Y> and <X>, then running derivative(n) = (diff(Y(n))./diff(X(n))) in a for loop for the length of the vectors). Once this has been done it could be repeated for the second and third derivatives fairly easily by just naming a new 'dx' vector for step sizes. This method would have the benefits of being fairly easy to implement, not necessitating the need of an already known model, and not having issues with data that might have problems generating a clean or accurate line of best fit. This

method would also be fairly easy to generalize into higher dimensional problems as well. In addition, this method could be made fairly accurate by using very small step sizes in numerical differentiation, allowing the user to control the balance of accuracy versus processing power. This method still has a few problems however. Firstly, if you are trying to process an extremely large or detailed set of data. A numerical method might take up too much processing power and time for sufficiently accurate results. In addition, this still isn't an analytic method, so there could possibly be accuracy problems there too.

3) Finally, the solution that may or may not be possible: use a computer to generate an ultra high degree polynomial to fit the data no matter what. This would have the drawback of not producing a theoretically/conceptually useful line of best fit and creating a fairly difficult derivation depending on the degree of the polynomial though. However, this method could be considered analytic, and would most likely work in a vast majority of cases, as nearly all functions can be written as a polynomial, and all polynomials are continuous and differentiable on the set of all real numbers, so the function should be differentiable so long as there is a computer to deal with the length of all of the terms.