Blending or Mixing Problem

Statement

Another classic problem that can be modeled as a linear program concerns blending or mixing ingredients to obtain a product with certain characteristics or properties. We illustrate this class with the problem of determining the optimum amounts of three ingredients to include in an animal feed mix. The final product must satisfy several nutrient restrictions. The possible ingredients, their nutritive contents (in kilograms of nutrient per kilograms of ingredient) and the unit cost are shown in the following table.

The mixture must meet the following restrictions:

- Calcium — at least 0.8% but not more than 1.2%.
- Protein — at least 22%.
- Fiber — at most 5%.

The problem is to find the composition of the feed mix that satisfies these constraints while minimizing cost.

Nutritive content and price of ingredients

Ingredient	Calcium (kg/kg)	Protein (kg/kg)	Fiber (kg/kg)	Unit cost (cents/kg)
Limestone	0.38	0.0	0.0	10.0
Corn	0.001	0.09	0.02	30.5
Soybean meal	0.002	0.50	0.08	90.0

Model

Variable Definitions

L, C, S : proportions of limestone, corn, and soybean meal, respectively, in the mixture.

Constraints

The number of hours available on each machine type is 40 times the number of machines. All the constraints are dimensioned in hours. For machine 1, for example, we have 40 hrs/machine ¥ 4 machines = 160 hrs. In writing out the constraints, it is customary to provide a column in the model for each variable.

Minimum calcium: $0.38L + 0.001C + 0.002S \geq 0.008$

Maximum calcium: $0.38L + 0.001C + 0.002S \leq 0.012$

Minimum protein: $+ 0.09C + 0.50S \geq 0.22$

Maximum fiber: $+ 0.02C + 0.08S \leq 0.05$

Conservation: $L + C + S = 1$

Nonnegativity

$$L, C, S \geq 0$$

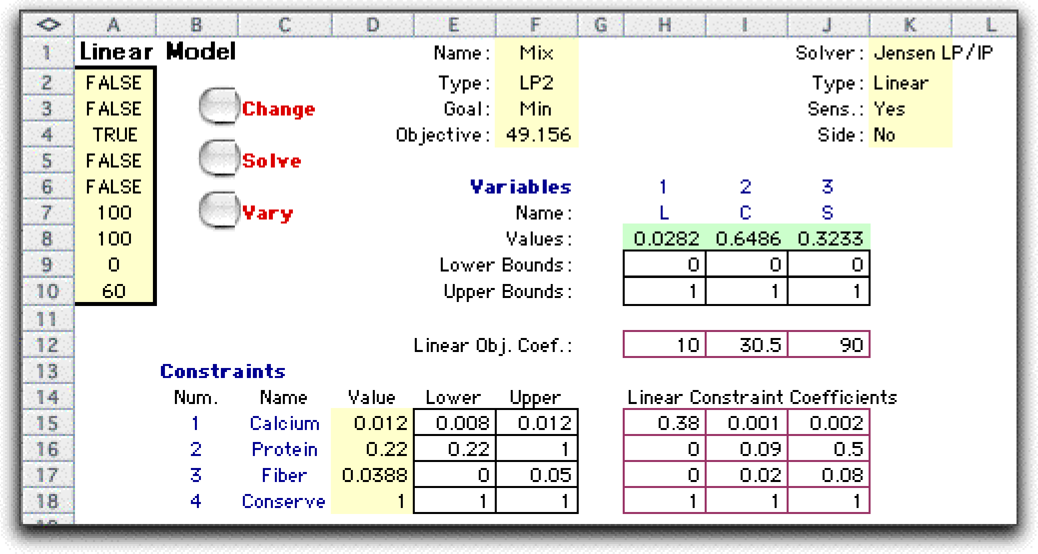
Objective Function

Because each decision variable is defined as a fraction of a kilogram, the objective is to minimize the cost of providing one kilogram of feed mix.

$$\text{Minimize } Z = 10L + 30.5C + 90S$$

Excel Model and Solution

The model was created by the Math Programming add-in and solved with the Jensen LP/IP Solver. The solution shows the optimum proportion of each component of the feed. The objective value is the cost per kilogram of the minimum cost mix that meets the nutruitive requirements. The example illustrates the two-sided constraint option. Each constraint value has both an upper and a lower bound.



◇	A	B	C	D	E	F	G	H	I	J	K	L
1	Linear Model				Name:	Mix				Solver:	Jensen LP/IP	
2	FALSE				Type:	LP2				Type:	Linear	
3	FALSE	○ Change			Goal:	Min				Sens.:	Yes	
4	TRUE				Objective:	49.156				Side:	No	
5	FALSE	○ Solve										
6	FALSE				Variables			1	2	3		
7	100	○ Vary			Name:			L	C	S		
8	100				Values:			0.0282	0.6486	0.3233		
9	0				Lower Bounds:			0	0	0		
10	60				Upper Bounds:			1	1	1		
11												
12					Linear Obj. Coef.:			10	30.5	90		
13		Constraints										
14		Num.	Name	Value	Lower	Upper		Linear Constraint Coefficients				
15		1	Calcium	0.012	0.008	0.012		0.38	0.001	0.002		
16		2	Protein	0.22	0.22	1		0	0.09	0.5		
17		3	Fiber	0.0388	0	0.05		0	0.02	0.08		
18		4	Conserve	1	1	1		1	1	1		

Sensitivity Analysis

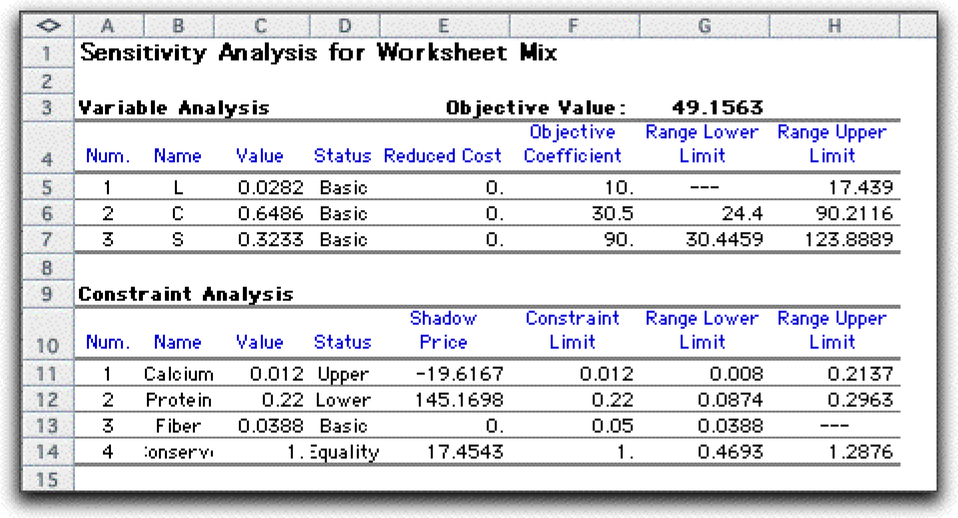
The sensitivity analysis created by the Jensen Solver shows that the upper bound of the calcium constraint and the lower bound of the protein constraint are limiting the solution. The shadow prices are the derviatives of the unit cost of the feed with respect to the tight bounds. For example, if one increases the upper bound of the calcium constraint from 0.012 to 0.013 the cost will be reduced by about $0.20.

cost increase = shadow price * bound increase = (-19.62)(0.001) = -0.1952.

In like manner, if the lower bound on protein were increased from 0.22 to 0.23 the cost would be increased by about $1.45.

cost increase = shadow price * bound increase = (145.17)(0.01) = 1.4517.

The fiber constraint is strictly between its bounds, so the shadow price is zero.



◇	A	B	C	D	E	F	G	H
1	Sensitivity Analysis for Worksheet Mix							
2								
3	Variable Analysis				Objective Value:		49.1563	
4	Num.	Name	Value	Status	Reduced Cost	Objective Coefficient	Range Lower Limit	Range Upper Limit
5	1	L	0.0282	Basic	0.	10.	---	17.439
6	2	C	0.6486	Basic	0.	30.5	24.4	90.2116
7	3	S	0.3233	Basic	0.	90.	30.4459	123.8889
8								
9	Constraint Analysis							
10	Num.	Name	Value	Status	Shadow Price	Constraint Limit	Range Lower Limit	Range Upper Limit
11	1	Calcium	0.012	Upper	-19.6167	0.012	0.008	0.2137
12	2	Protein	0.22	Lower	145.1698	0.22	0.0874	0.2963
13	3	Fiber	0.0388	Basic	0.	0.05	0.0388	---
14	4	onserv	1.	quality	17.4543	1.	0.4693	1.2876
15								