Models

Linear Programming

Blending or Mixing Problem

Blending or Mixing Problem

Statement

Another classic problem that can be modeled as a linear program concerns blending or mixing ingredients to obtain a product with certain characteristics or properties. We illustrate this class with the problem of determining the optimum amounts of three ingredients to include in an animal feed mix. The final product must satisfy several nutrient restrictions. The possible ingredients, their nutritive contents (in kilograms of nutrient per kilograms of ingredient) and the unit cost are shown in the following table.

The mixture must meet the following restrictions:

- Calcium — at least 0.8% but not more than 1.2%.
- Protein — at least 22%.
- Fiber — at most 5%.

The problem is to find the composition of the feed mix that satisfies these constraints while minimizing cost.

Nutritive content and price of ingredients				
Ingredient	Calcium (kg/kg)	Protein (kg/kg)	Fiber (kg/kg)	Unit cost (cents/kg)
Limestone	0.38	0.0	0.0	10.0
Corn	0.001	0.09	0.02	30.5
Soybean meal	0.002	0.50	0.08	90.0

Model

Variable Definitions

L, C, S : proportions of limestone, corn, and soybean meal, respectively, in the mixture.

Constraints

The number of hours available on each machine type is 40 times the number of machines. All the constraints are dimensioned in hours. For machine 1, for example, we have 40 hrs/machine ¥ 4 machines = 160 hrs. In writing out the constraints, it is customary to provide a column in the model for each variable.

Minimum calcium:	$0.38L$	$+ 0.001C$	$+ 0.002S$	≥ 0.008
Maximum calcium:	$0.38L$	$+ 0.001C$	$+ 0.002S$	≤ 0.012
Minimum protein:		$+ 0.09C$	$+ 0.50S$	≥ 0.22
Maximum fiber:		$+ 0.02C$	$+ 0.08S$	≤ 0.05
Conservation:	L	$+ C$	$+ S$	$= 1$

Nonnegativity

$$L, C, S \geq 0$$

Objective Function

Because each decision variable is defined as a fraction of a kilogram, the objective is to minimize the cost of providing one kilogram of feed mix.

$$\text{Minimize } Z = 10L + 30.5C + 90S$$

Excel Model and Solution

The model was created by the Math Programming add-in and solved with the Jensen LP/IP Solver. The solution shows the optimum proportion of each component of the feed. The objective value is the cost per kilogram of the minimum cost mix that meets the nutruitive requirements.

◇	A	B	C	D	E	F	G	H	I	J	K
1	Linear Model				Name:	Mix1				Solver:	Jensen LP/IP
2	TRUE				Type:	LP1				Type:	Linear
3	FALSE	Change			Goal:	Min				Sens.:	Yes
4	TRUE				Cost:	49.156				Side:	No
5	FALSE	Solve									
6	FALSE				Variables			1	2	3	
7	100	Vary			Name:			L	C	S	
8	100				Values:			0.0282	0.6486	0.3233	
9	0	Change Relation			Lower Bounds:			0	0	0	
10	60				Upper Bounds:			1	1	1	
11											
12					Linear Obj. Coef.:			10	30.5	90	
13		Constraints									
14		Num.	Name	Value	Rel.	RHS		Linear Constraint Coefficients			
15		1	Low Calc.	0.012	>=	0.008		0.38	0.001	0.002	
16		2	High Calc.	0.012	<=	0.012		0.38	0.001	0.002	
17		3	Protein	0.22	>=	0.22		0	0.09	0.5	
18		4	Fiber	0.0388	<=	0.05		0	0.02	0.08	
19		5	Conserve	1	=	1		1	1	1	

Sensitivity Analysis

The sensitivity analysis created by the Jensen Solver shows that the upper bound of the calcium constraint and the lower bound of the protein constraint are limiting the solution. The shadow prices are the derviatives of the unit cost of the feed with respect to the tight bounds. For example, if one increases the upper bound of the calcium constraint from 0.012 to 0.013 the cost will be reduced by about $0.20.

cost increase = shadow price * bound increase = (-19.62)(0.001) = -0.1952.

In like manner, if the lower bound on protein were increased from 0.22 to 0.23 the cost would be increased by about $1.45.

cost increase = shadow price * bound increase = (145.17)(0.01) = 1.4517.

The fiber constraint is strictly between its bounds, so the shadow price is zero.

◇	A	B	C	D	E	F	G	H
1	Sensitivity Analysis for Worksheet Mix1							
2								
3	Variable Analysis					Objective Value:	49.1563	
4	Num.	Name	Value	Status	Reduced Cost	Objective Coefficient	Range Lower Limit	Range Upper Limit
5	1	L	0.0282	Basic	0.	10.	---	17.439
6	2	C	0.6486	Basic	0.	30.5	24.4	90.2116
7	3	S	0.3233	Basic	0.	90.	30.4459	123.8889
8								
9	Constraint Analysis							
10	Num.	Name	Value	Status	Shadow Price	Constraint Limit	Range Lower Limit	Range Upper Limit
11	1	Low Calc.	0.012	Basic	0.	0.008	---	0.012
12	2	High Calc.	0.012	Upper	-19.6167	0.012	0.008	0.2137
13	3	Protein	0.22	Lower	145.1698	0.22	0.0874	0.2963
14	4	Fiber	0.0388	Basic	0.	0.05	0.0388	---
15	5	Conserve	1.	Equality	17.4543	1.	0.4693	1.2876

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