Morphology

- The word *Morphology* denotes a branch of biology which deals with form and structure of animals and plants
- Mathematical Morphology is a tool for extracting image components that are useful in the representation and description of region shape
- Morphological techniques can be used to find boundaries, skeletons, convex hulls and also for filtering thinning and pruning.
- Morphological techniques are well-developed for binary images, but many methods can be successfully extended to grayscale.

Dilation and Erosion

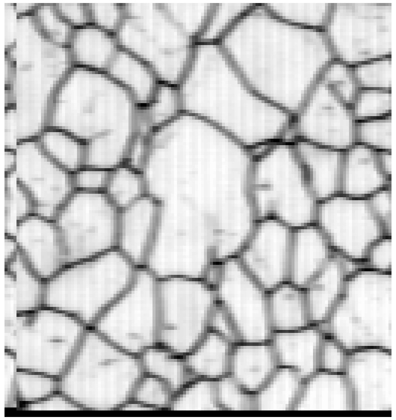
- The basic operations of dilation and erosion form the basis of many more sophisticated techniques

Matlab

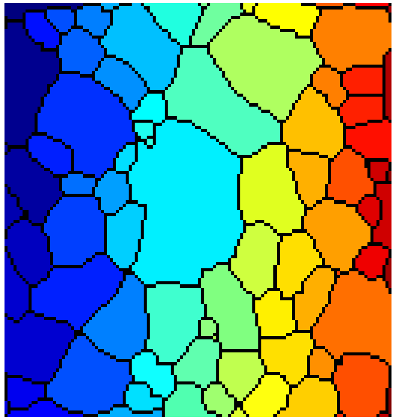
Morphology

Demo

Original image



Labelled steel grains



Definitions

- Let A and B be sets in Z^2, with components a=(a1,a2) and b =(b1,b2)
- The *translation* of A by x=(x1,x2), denoted $(A)_x$ is defined by

$$(A)_x = \{c \mid c = a + x, \text{for } a \in A\}$$

- The *reflection* of B, denoted $\hat{B}$, is defined by

$$\hat{B} = \{x \mid x = -b, \text{ for } b \in B\}$$

More Definitions

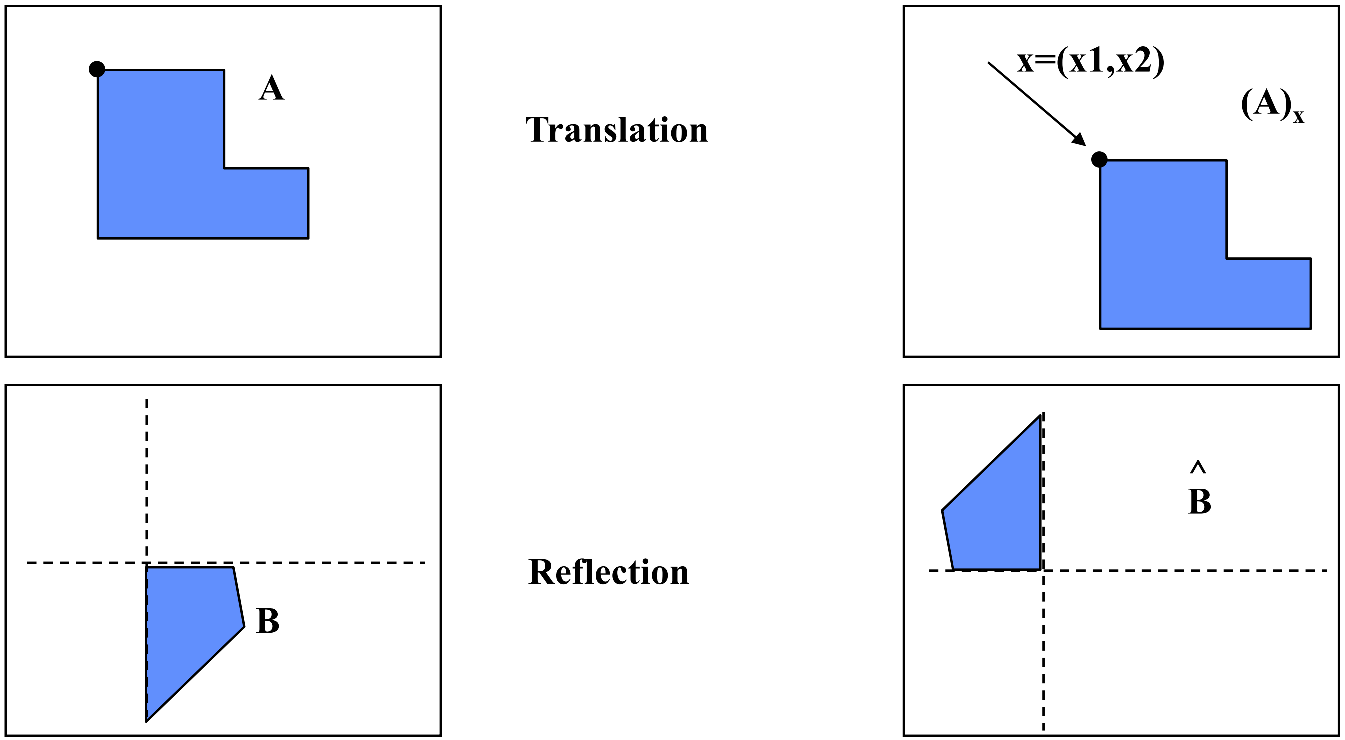
- The *complement* of set A is

$$A^c = \{x \mid x \notin A\}$$

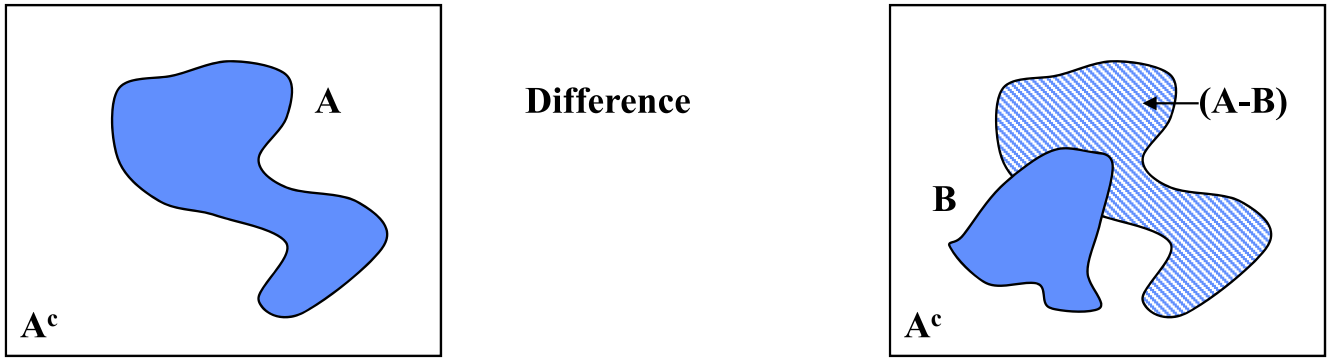
- The *difference* of sets A and B, denoted A-B, is defined by

$$A - B = \{x \mid x \in A, x \notin B\} = A \cap B^c$$

Examples



More Examples



$$A - B = \{x \mid x \in A, x \notin B\} = A \cap B^c$$

Dilation

- with A and B as sets in Z^2 and $\varnothing$ denoting the empty set, the dilation of A by B, denoted $A \oplus B$ is defined by

reflected

$$A \oplus B = \{x \mid (\hat{B})_x \cap A \neq \varnothing\}$$

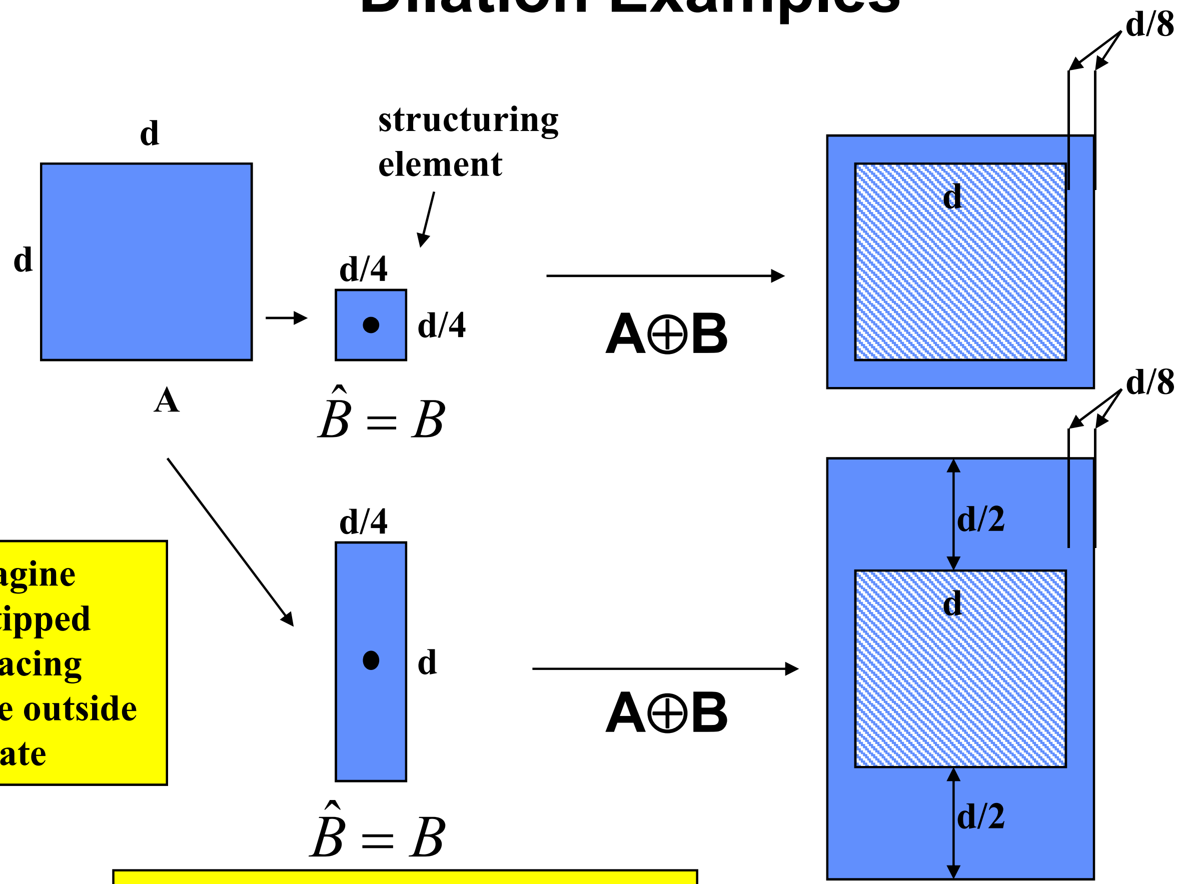
translated

- Thus the dilation process consists of obtaining the refection of $\hat{B}$ about its origin and then shifting this reflection by x. The dilation of A by B then is the set of all x displacements such that B and A overlap by at least one nonzero element.

Dilation (cont)

- Set B is commonly referred to as the *structuring element* in dilation , as well as in other morphological operations.
- Often the set B is viewed as a convolution mask
- The basic process of “flipping” B about its origin and then successively displacing it so that it slides over set (image) A is analogous to the convolution process.

Dilation Examples



Hint – imagine A square tipped felt pen tracing around the outside of a template

Symmetric structuring elements

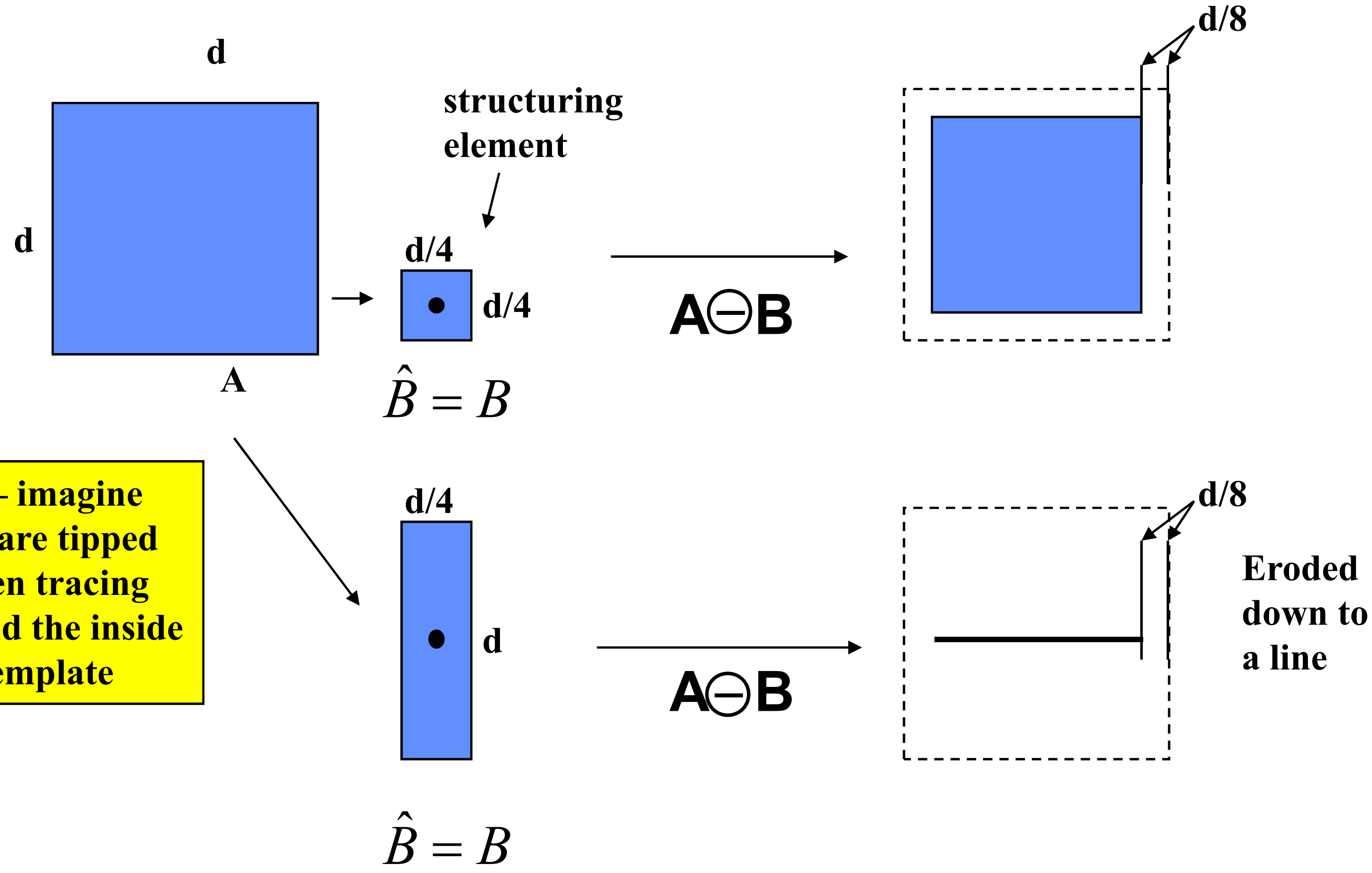
Erosion

- For sets A and B in Z^2 the erosion of A by B denoted $A \ominus B$ is defined by

$$A \ominus B = \{x \mid (\hat{B})_x \subseteq A\}$$

- In other words, the erosion of A by B is the set of all points x such that B, translated by x, is completely contained in A

Erosion Examples



Hint – imagine A square tipped felt pen tracing around the inside of a template

Comment

- Dilation and erosion are duals of each other with respect to set complementation and reflection. That is

complement

$$(A \ominus B)^{c} = A^{c} \oplus B$$

erosion dilation

Opening and Closing

- Dilation expands an image and erosion shrinks it
- Opening generally smoothes the contour of an image, breaks narrow isthmuses, and eliminates thin protrusions
- Closing also tends to smooth sections of contours, but generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour.
- The *Opening* of set A by structuring element B is

$$A \circ B = (A \ominus B) \oplus B$$

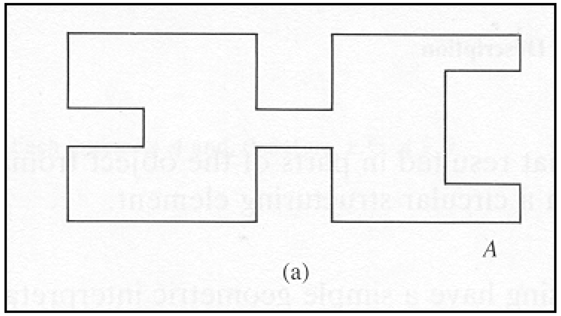
- The *Closing* of set A by structuring element B is

$$A \bullet B = (A \oplus B) \ominus B$$

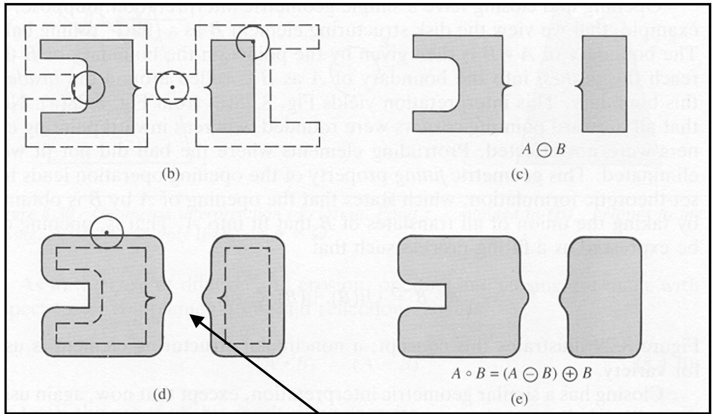
Comments

- In other words
  - Opening is simply the erosion of A by B, followed by the dilation by B
  - Closing is simply the dilation of A by B, followed by the erosion by B
- The operations are extensively used to preprocess images after thresholding operations to “clean up” the images prior to further processing

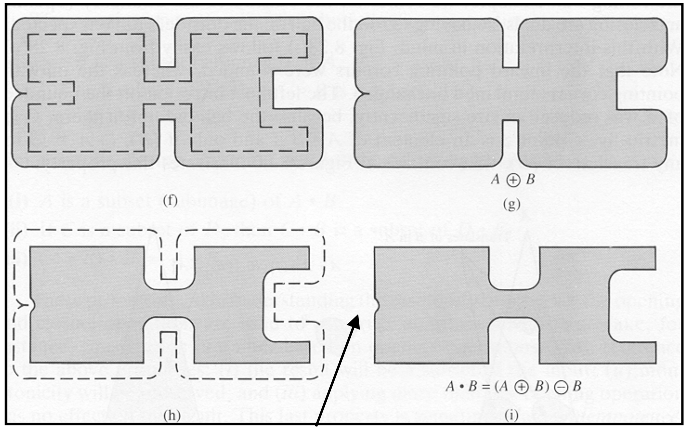
Examples of Opening and Closing



Opening



Closing



Elimination of bridge

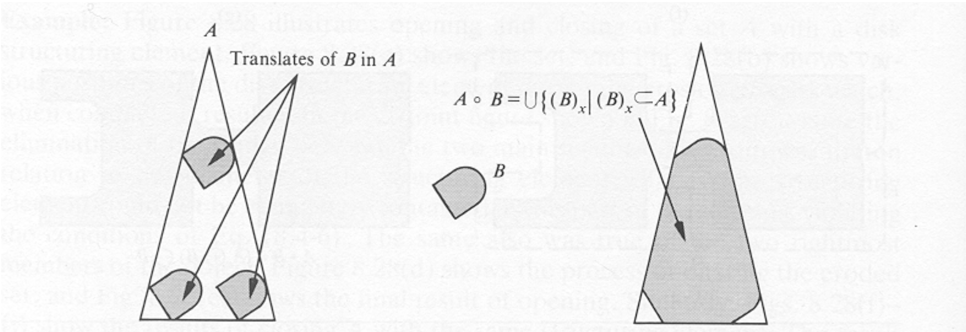
Elimination of notch

Interpretation

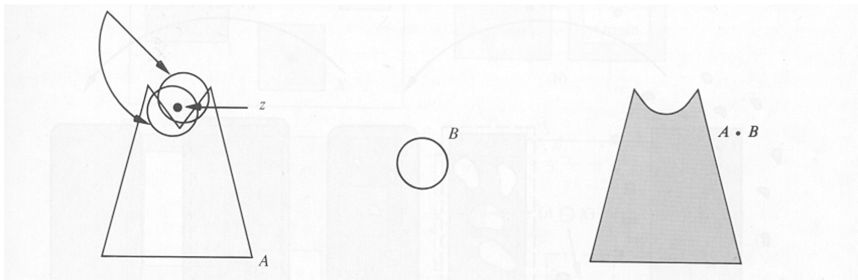
- Opening and closing have simple geometrical interpretations, if we view a disk structuring element as a rolling ball
  - The boundary of the opening is the points on the boundary of the ball (structuring element) that are closest to the boundary of A as B is rolled on the inside of this boundary
  - The boundary of the closing is the points on the boundary of the ball (structuring element) that are closest to the boundary of A as B is rolled on the outside of this boundary

Note: structuring elements do not roll (rotate) in practice

Illustration



"Fitting" Characterization of Opening



"Fitting" Characterization of Closing

Properties of Openings

- Opening operation satisfies:
  1. $A \circ B$ is a subset (subimage) of A
  2. If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$
  3. $(A \circ B) \circ B = A \circ B$
- In other words:
  1. The opening is a subset of the input
  2. Monotonicity is preserved
  3. Applying more than one opening has no effect on the result

Properties of Closings

- Similarly, closing operation satisfies:
  1. A is a subset (subimage) of A • B
  2. If C is a subset of D, then C • B is a subset of D • B
  3. (A • B) • B = A • B
- In other words:
  1. The input is a subset of the closing
  2. Monotonicity is preserved
  3. Applying more than one opening has no effect on the result

Example of Opening/Closing

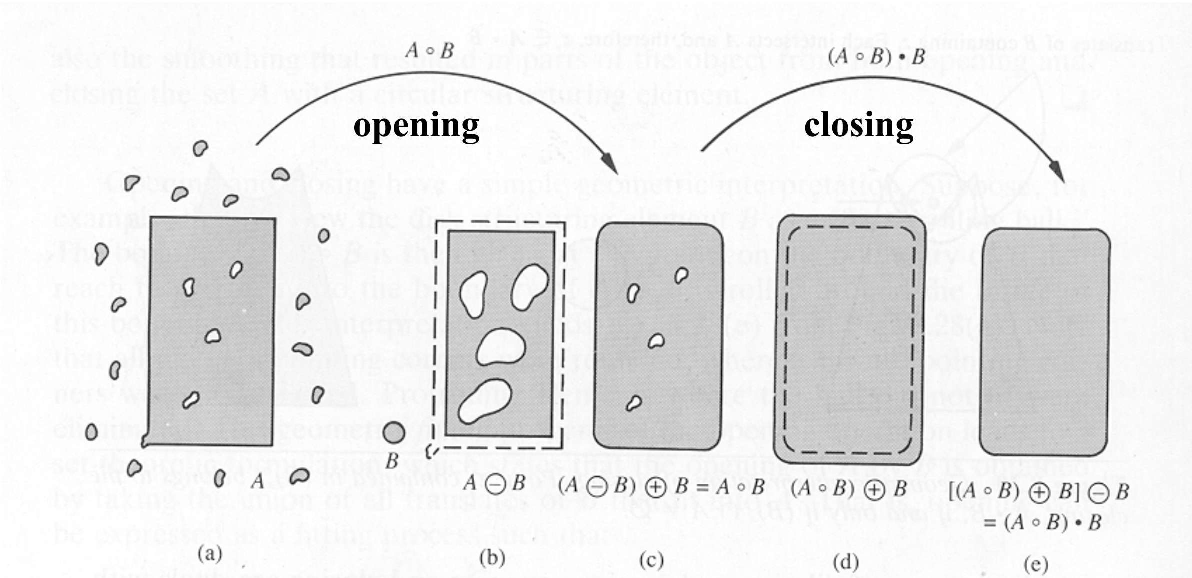


Image consists of a thresholded object with noise both within and without the object boundary. Opening using a structuring element larger than the noise artefacts removes noise outside the object. Closing fills in the voids.

Hit or Miss Transform

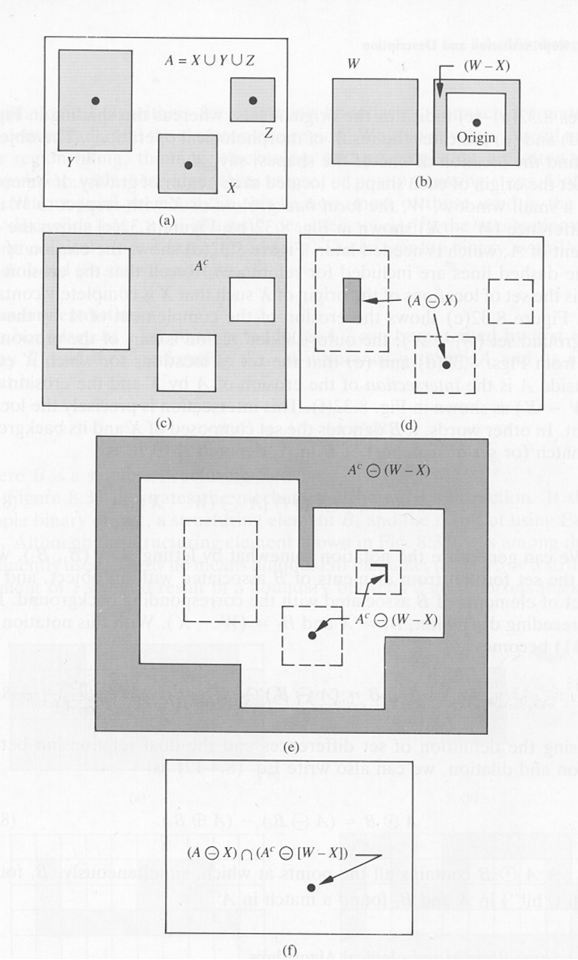
Morphological Template Matching

- This can be used as a basic tool for shape detection
- As an example we will use a set A consisting of three shapes: X, Y, and Z (see next slide)
- If we enclose X by a small window, W, the local background of X wrt W is the set difference (W-X)
- Recall that the erosion of Z by X is the set of locations of the origin of X such that X is completely contained in A
- Now the set of locations for which X exactly fits inside A is the intersection of the erosion of A by X and the erosion of A^c by (W-X)
- This intersection is precisely the location sought.

Hit or Miss Example

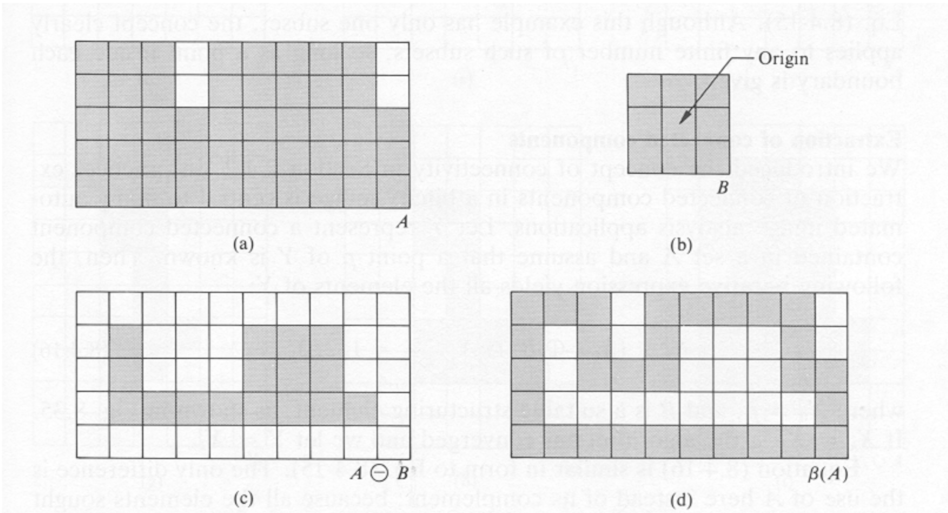
We can generalize the notation somewhat by letting $B=(B_1,B_2)$ where B1 represents the object and B2 represents the local background.

$$A \circledast B = (A \ominus B_1) \bigcap (A^c \ominus B_2)$$



Practical Uses

- **Boundary Extraction**
  - Erode A by B and then perform set difference with A
    - boundary(A)=A - (A $\ominus$ B)



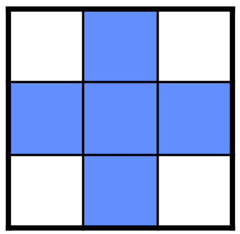
Region Filling

- Conditional Dilation
  - Given the 8-connected boundary of a region, select a point p inside the boundary and conditionally dilate with structuring element B until region is filled using

    $X_k=(X_{k-1}\oplus B)\cap A^c, \quad k=1,2,3,\ldots$

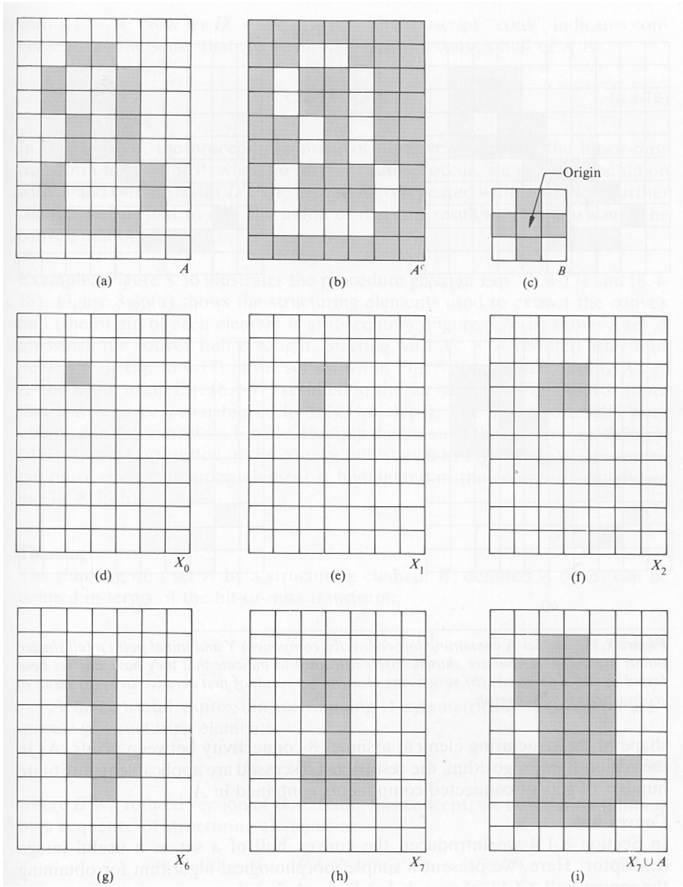
    read as AND

  - Continue until $X_k=X_{k-1}$
  - B must be of the form



Example

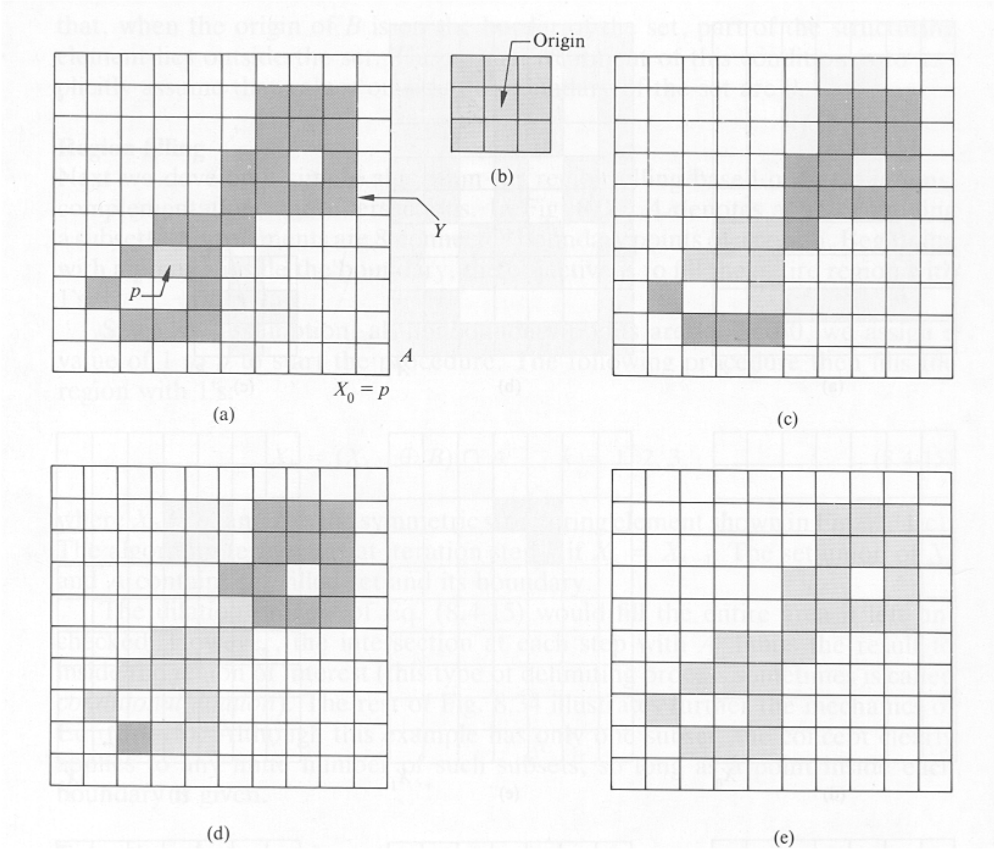
Morphological flood filling using conditional dilation



Extraction of Connected Components

- This process is central to many image processing applications
- Let Y represent a connected component and assume that a point p of Y is known
- Setting X_0 to p, this conditional dilation will yield all points in Y
  - $X_k=(X_{k-1}\oplus B)\cap A$, $k=1,2,3,\ldots$
  - Continue until $X_k=X_{k-1}$
  - Structuring element B should be say a 3x3 pixel group

Example



Convex Hull

- Let B^i, i=1,2,3,4, represent 4 structuring elements.
- The procedure consists of implementing the equation

$$X_k^i = (X \circledast B^i) \cup A \quad i = 1,2,3,4 \text{ and } k = 1,2,3,4.$$

with $X^i_0 = A$

- When each of these has converged, the union is the convex hull.

Example

Convex Hull of Set A

Note: X means "don't care"