Thinning

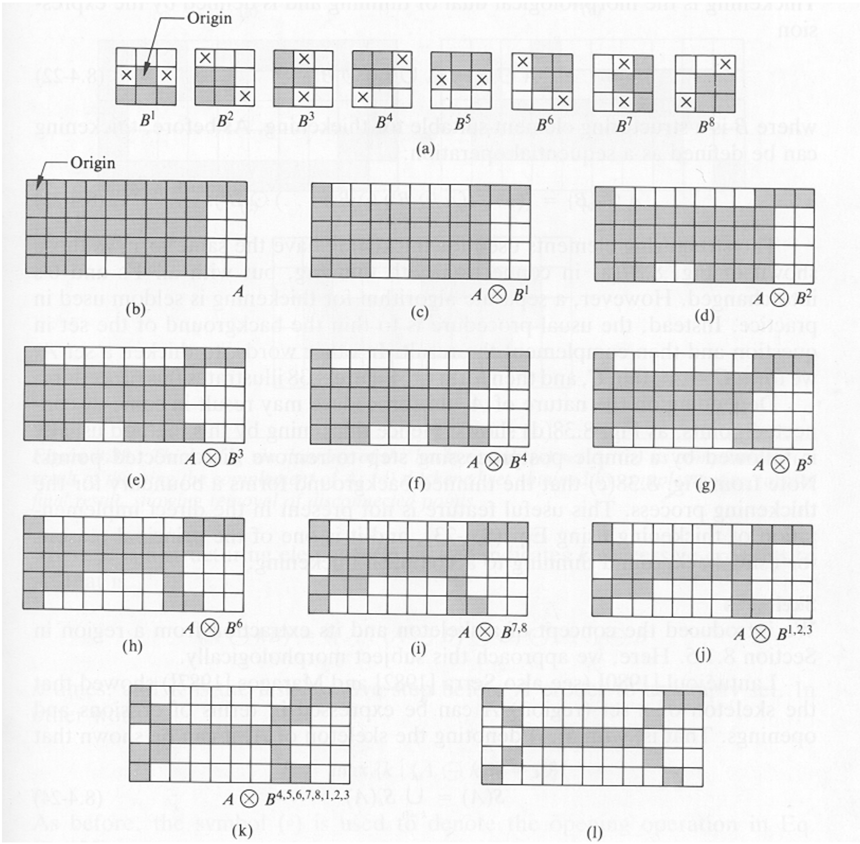
- The thinning of a set A by structuring element B, denoted $A \otimes B$, can be defined in terms of the hit-or-miss transform

$$A \otimes B = A - (A \circledast B)$$

$$= A \cap (A \circledast B)^c$$

- The usual process is to thin A using a sequence of structuring elements $B^1, \ldots B^n$
- In other words, A is thinned by successive passes of structuring elements B^1, B^2,…
- The entire process is repeated until no further change occurs

Thinning Example



Other Morphological Operations

- We also have definitions for the following operations
  - Thickening
    - make lines thicker
  - Skeletonization
    - extract morphological skeleton
  - Pruning
    - extract parasitic components after skeletonization
- See Gonzalez and Woods, “Digital Image Processing,” pp 518-545 for details

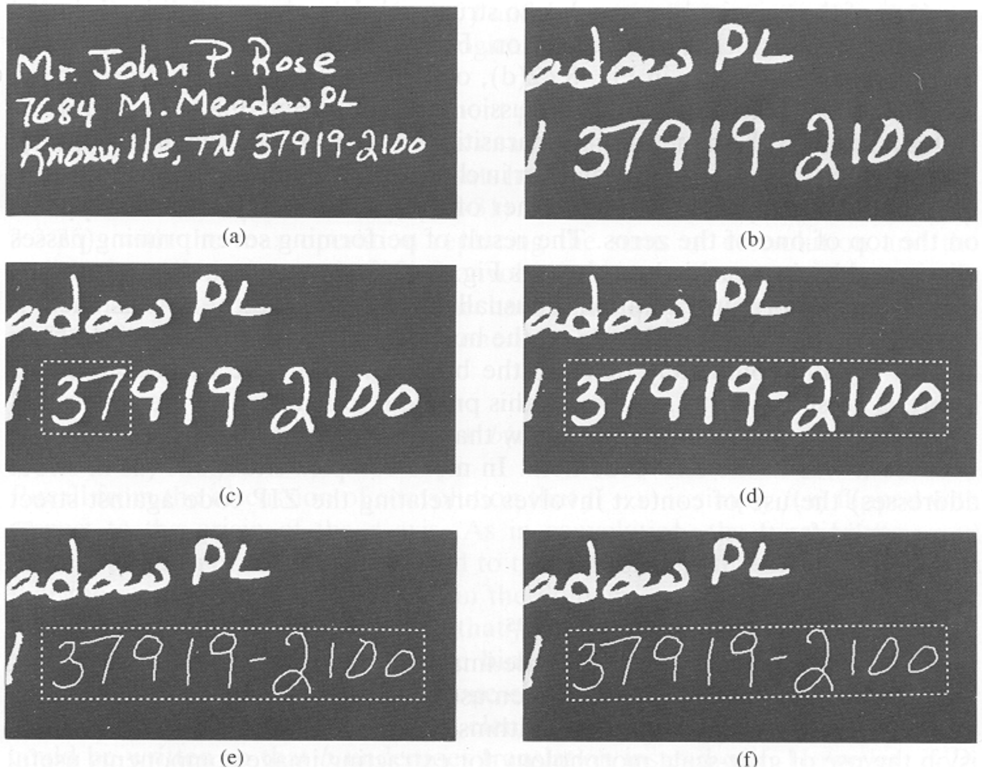
Application

Postcode processing

dilation to bridge breaks in numbers

Erosion to separate numbers

skeleton



pruning

Grayscale Morphology

- Many binary morphological operations are simply extended to grayscale images
- Here we regard the grayscale image as a surface that is eroded and dilated
- Often it is simpler to illustrate the process with 1-D functions, since the extension to 2-D is trivial.

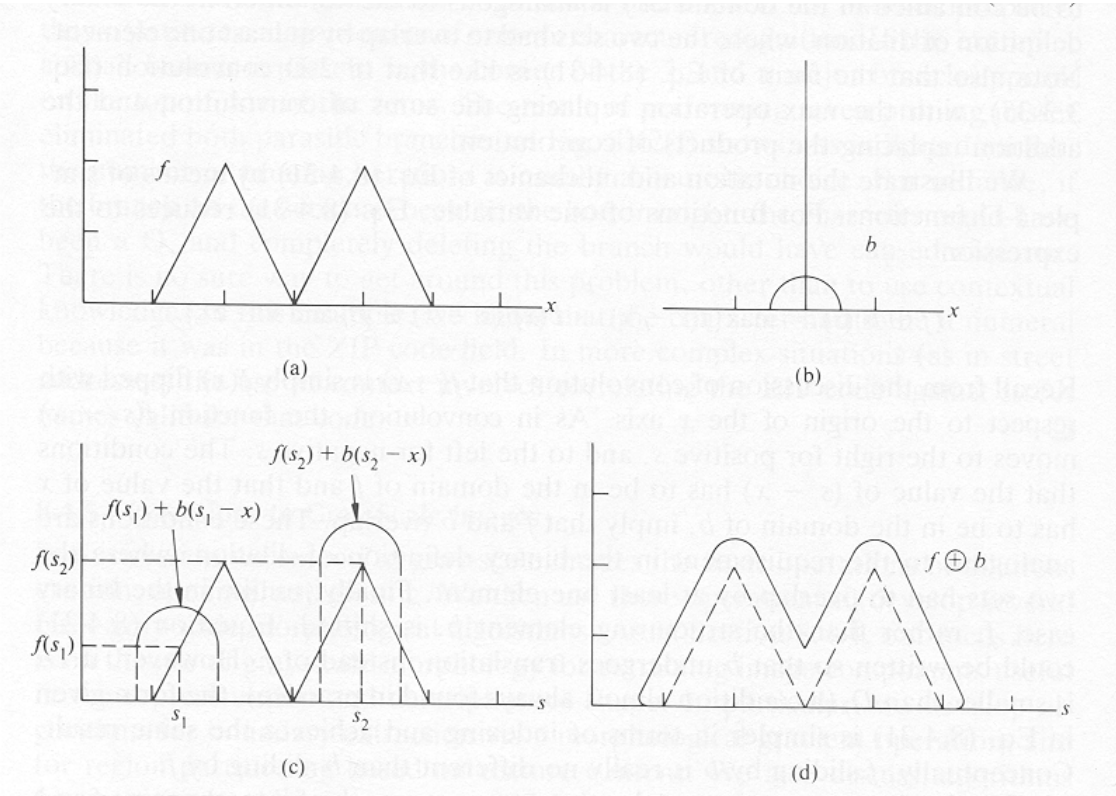
Grayscale Dilation

- Grayscale dilation of f by b, denoted f⊕b, is defined by

$$(f \oplus b)(s,t) =$$
$$\max\{f(s-x,t-y)+b(x,y) \mid (s-x),(t-y) \in D_f;(x,y) \in D_b\}$$

- In other words, we find the maximum of the function f+b in a neighborhood defined by the structuring element b as we slide b over f.
- This is illustrated graphically on the next slide for a 1-D function

Dilation Example



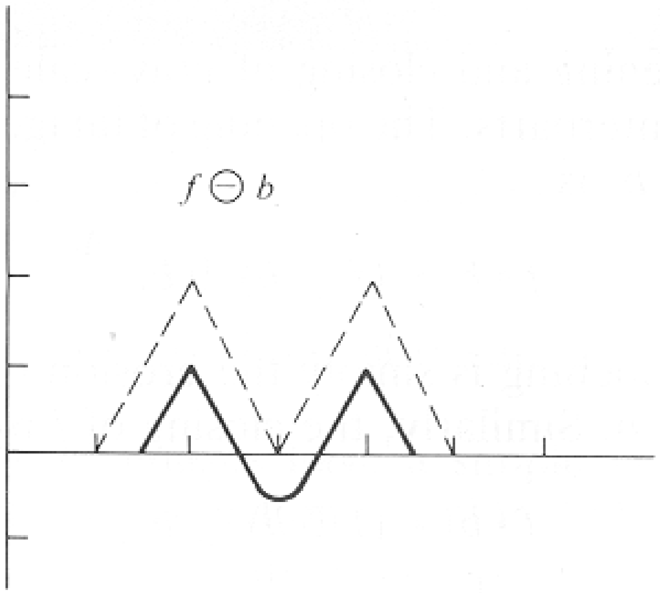
Grayscale Erosion

- Similarly, Grayscale erosion of f by b, denoted f⊖b, is defined by

$$(f \ominus b)(s,t) =$$
$$\min\{f(s-x,t-y)+b(x,y) \mid (s-x),(t-y) \in D_f;(x,y) \in D_b\}$$

- In other words, we find the minimum of the function **f+b** in a neighbourhood defined by the structuring element **b** as we slide **b** over **f**.

Erosion Example



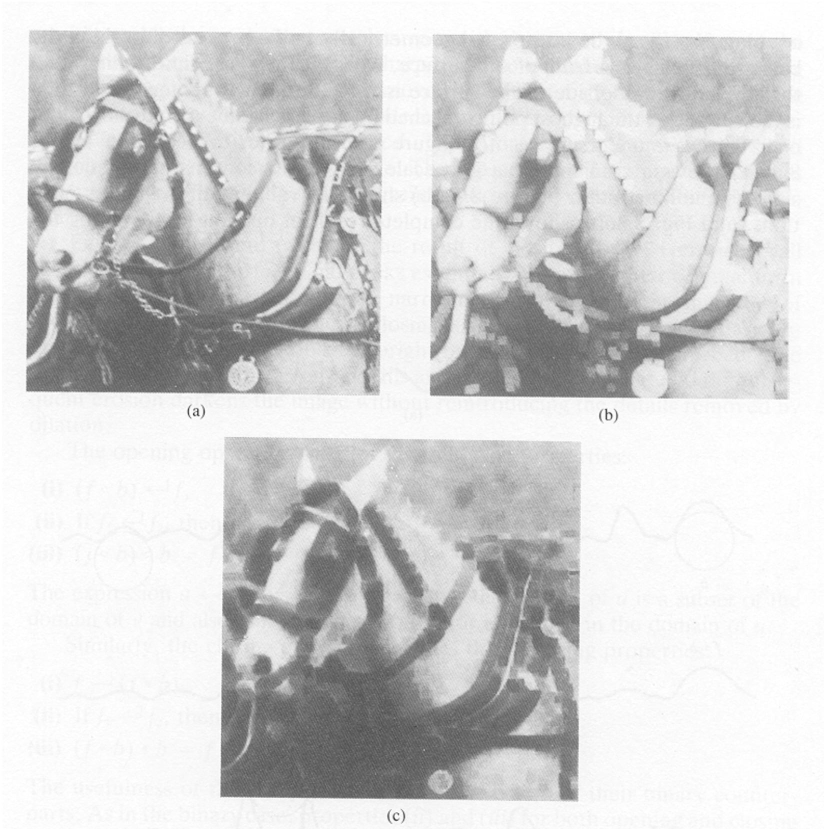
Same example as before

Image Example

Original

Dilated

Eroded



Grayscale Opening and Closing

- The expression for grayscale opening and closing is the same as for binary
- The expression for opening is

$$f \circ b = (f \ominus b) \oplus b$$

which is simply erosion followed by dilation

- The expression for closing is

$$f \bullet b = (f \oplus b) \ominus b$$

which is simply dilation followed by erosion

Geometric Interpretation

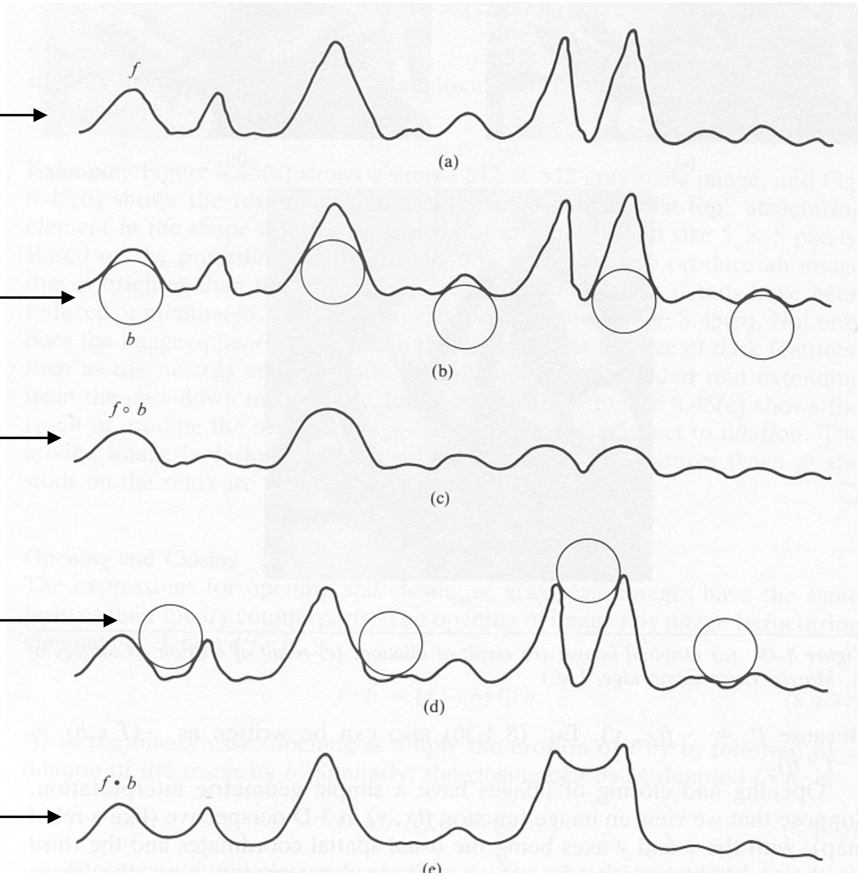
scanline

rolling ball below

opening

rolling ball above

closing



Rolling Ball Interpretation

Comments

- Openings are used to remove small light details, while leaving the overall gray levels and larger bright features relatively undisturbed
- Closing is generally used to remove dark details from an image while leaving bright features relatively undisturbed

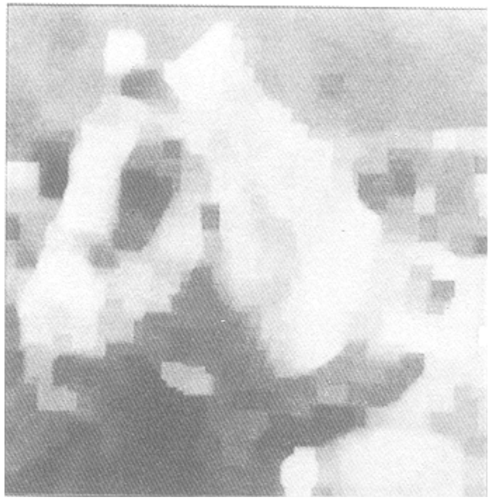
Applications

- Morphological smoothing
  - Opening followed by a closing
  - removes or attenuates both bright and dark artifacts or noise
- Morphological gradient
  - This is the difference between dilation and erosion

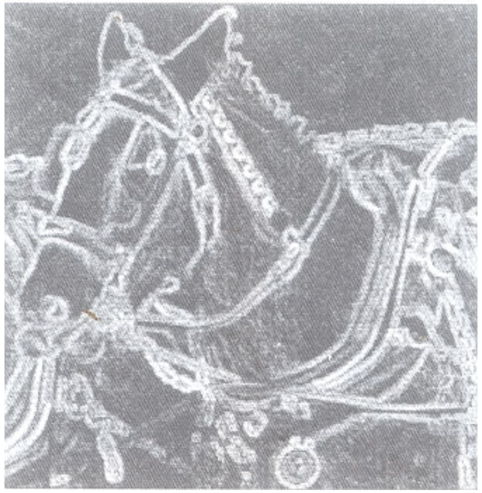
$$g = (f \oplus b) - (f \ominus b)$$

  - highlights sharp gray level transitions in the image
  - depends less on edge direction than Sobel etc

Examples

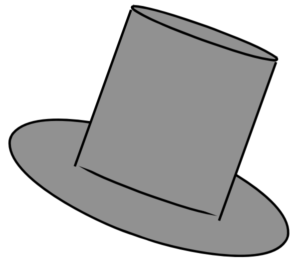


Morphological Smoothing



Morphological Gradient

Applications



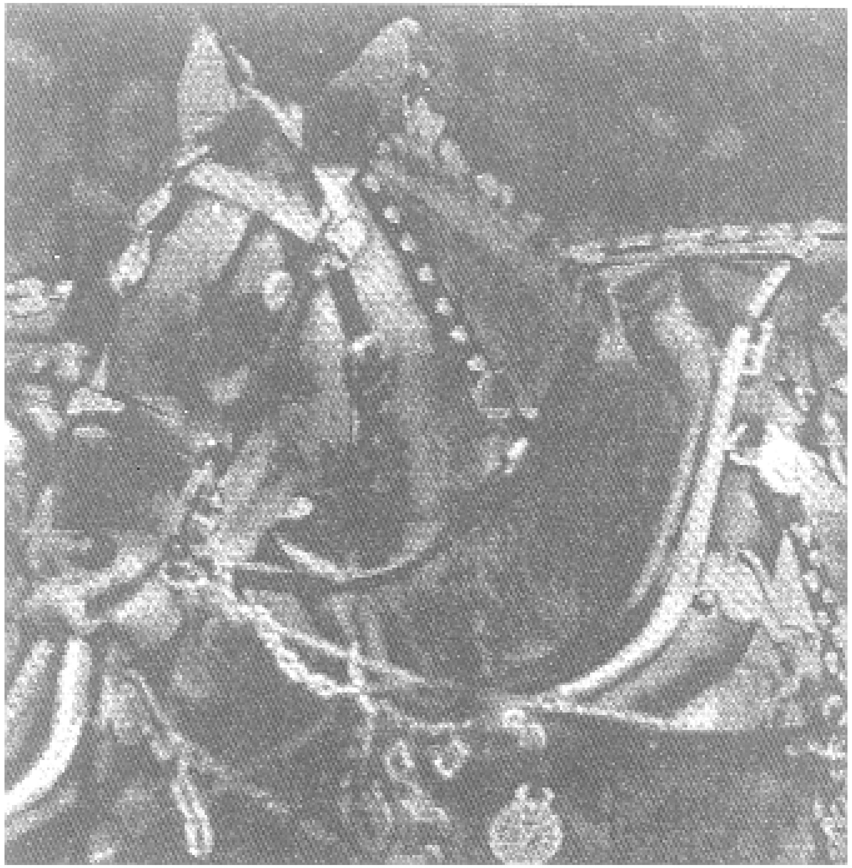
- Top Hat Transformation
  - The Morphological top-hat transformation is defined by

$$h = f - (f \circ b)$$

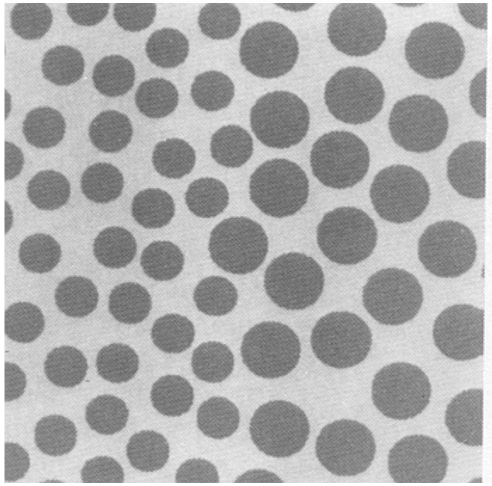
  - That is the image f minus its opening with a structuring element b, which is often of the form of a “top-hat.” That is, a cylinder attached to a disk.
  - This transformation is useful for enhancing detail in the presence of shading
  - Also good in 1D for finding peaks that are, say, greater than a certain width and more than a certain depth (significant peaks)

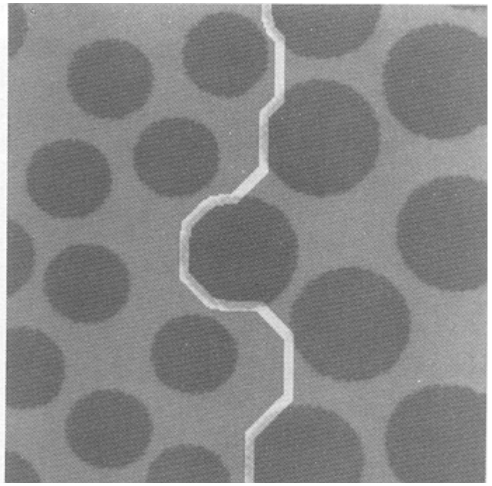
Example

Top hat transformation of image
Note the enhanced detail



Texture Segmentation





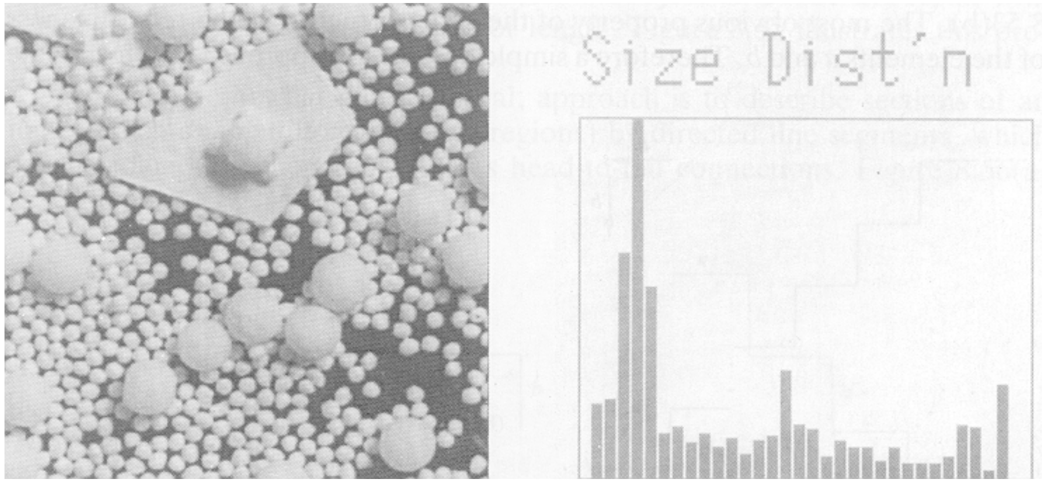
Method: Close with successively larger structuring elements until small dots disappear. Open remaining image with large structuring element and then threshold to determine textural boundary.

Consider interpretation with rolling ball – process resembles coin sorter

Granulometry

- Granulometry is a field that deals with the size distribution of particles
- In the example image (next slide), there are light objects of three different sizes
- The objects are overlapping and are too cluttered to detect individual objects

Example



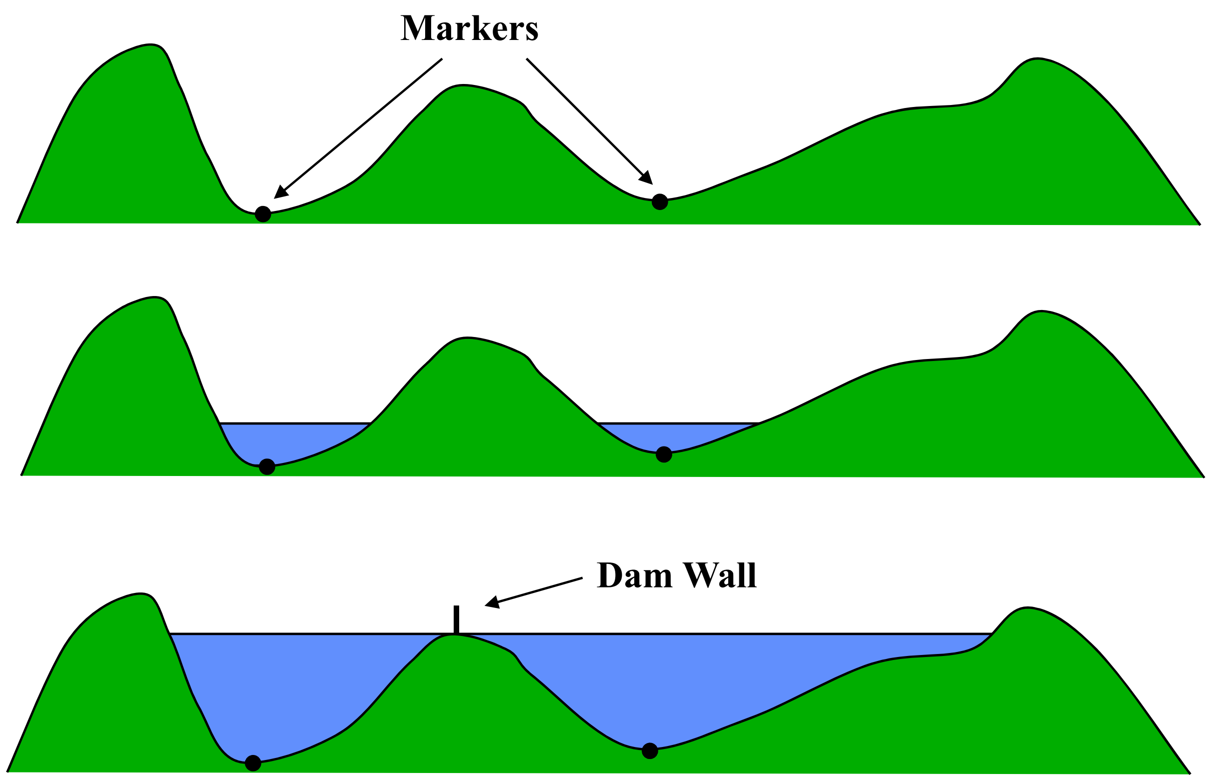
Method:

- opening operations with structuring elements of increasing size
- the difference between the original image and its opening is computed on each pass
- at the end of the process the differences are normalized and used to construct a histogram of particle size distribution

Watersheds

- Create markers on image (usually image gradient) to indicate regions of interest
- Visualize image as a topological surface (mountains and valleys)
- Flood object with a deluge of rain
- When waters from different regions meet, construct dams
- Once surface is completely flooded, the dam walls are our watershed segmentation.

Watershed Example



Problems with Watersheds

- Usually operates on the gradient image rather than the image itself
  - gradient accentuates noise
- Often difficult to determine appropriate markers in many applications
  - may lead to poor segmentation
- Found to be unsuitable for cell image segmentation application