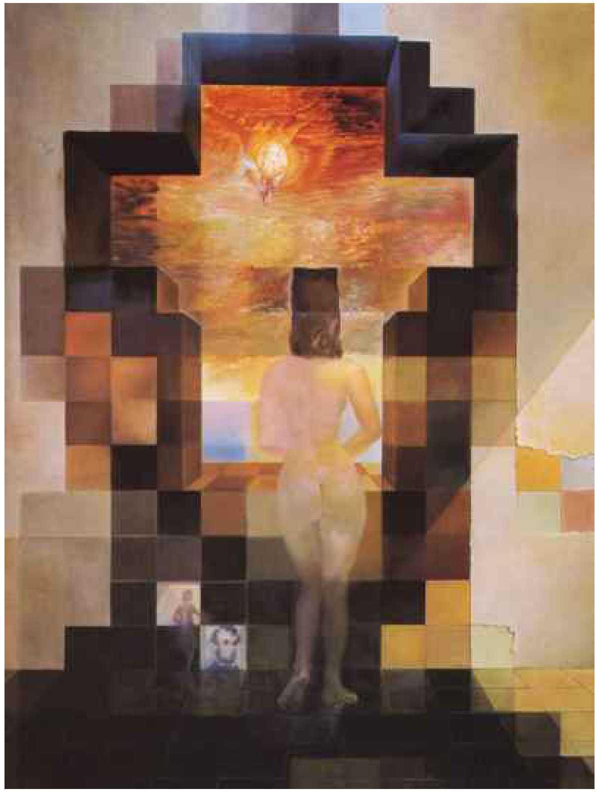
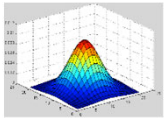
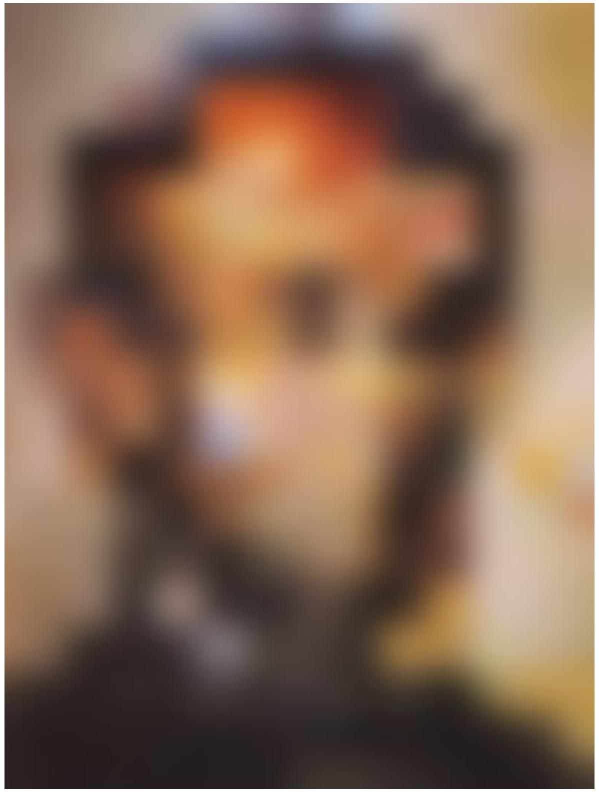
Global to Local Analysis



Dali





Linear image transformations

- In analyzing images, it's often useful to make a change of basis.

transformed image

$$\vec{F} = U\vec{f}$$

Vectorized image

Fourier transform, or
Wavelet transform, or
Steerable pyramid transform

Self-inverting transforms

Same basis functions are used for the inverse transform

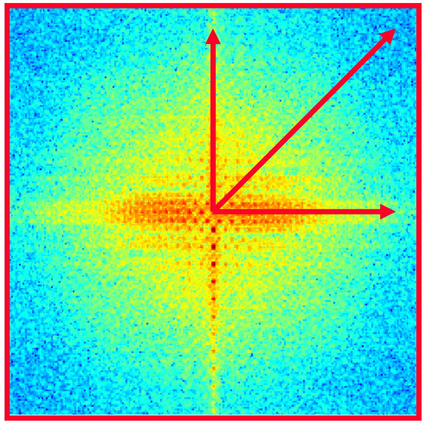
$$\vec{f} = U^{-1}\vec{F}$$

$$= U^{+}\vec{F}$$

U transpose and complex conjugate

How to interpret a Fourier Spectrum

Vertical orientation



45 deg.

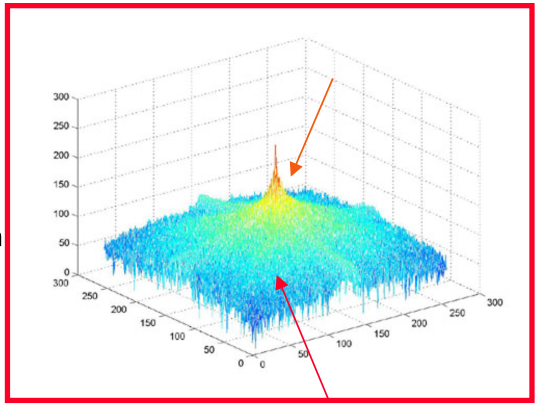
0

Horizontal orientation

0 f_{max}

fx in cycles/image

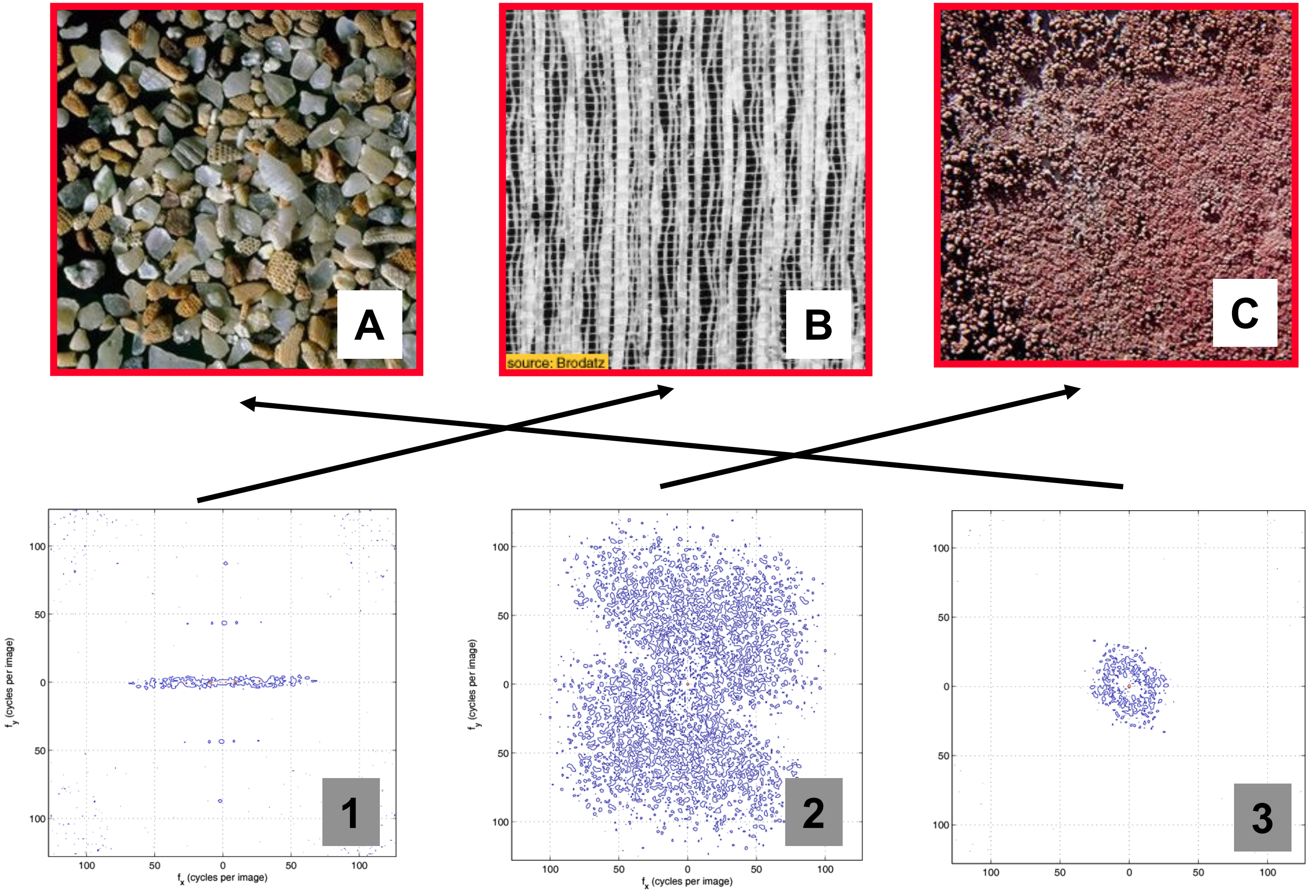
Low spatial frequencies



High spatial frequencies

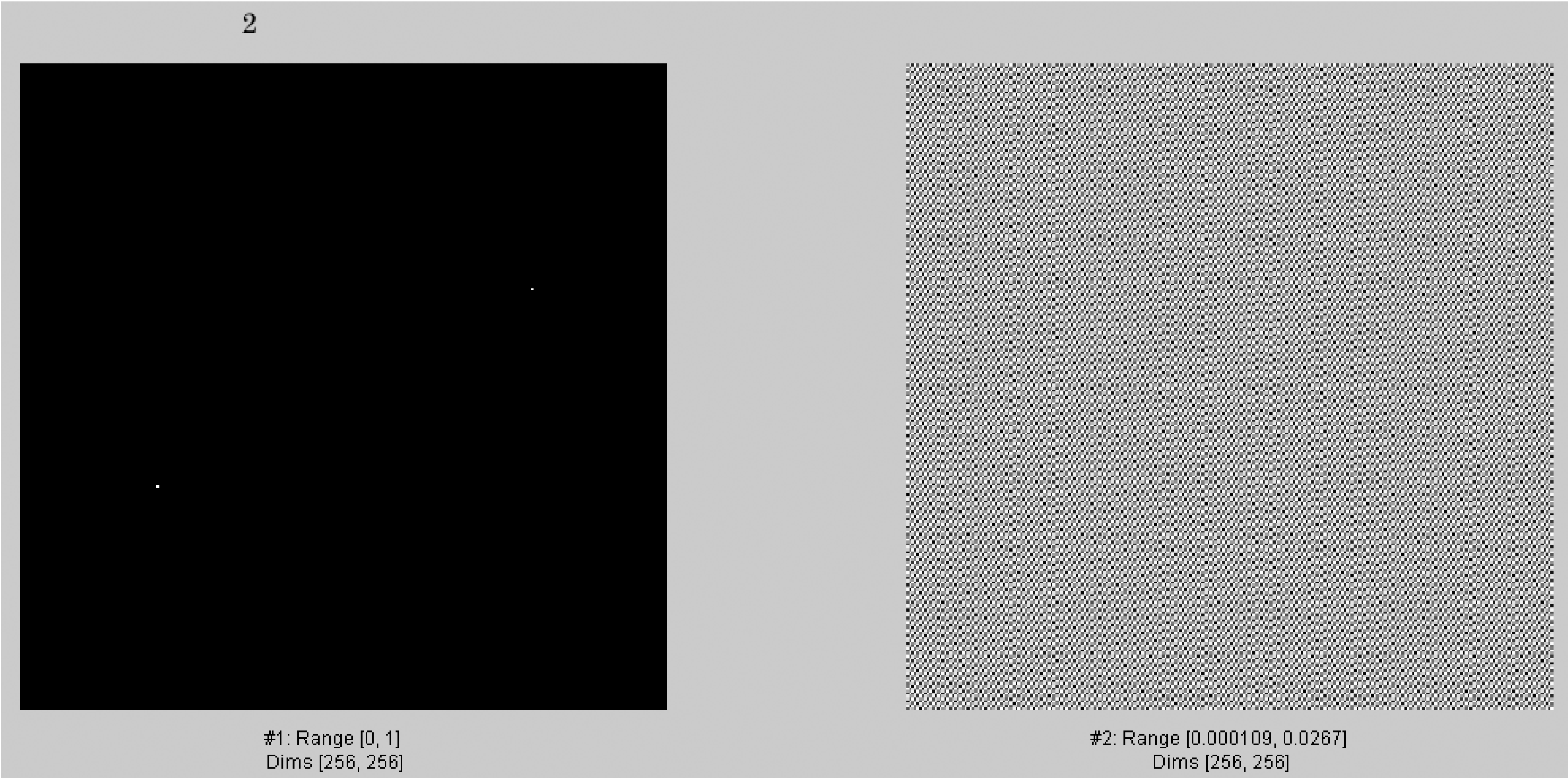
Log power spectrum

Fourier Amplitude Spectrum

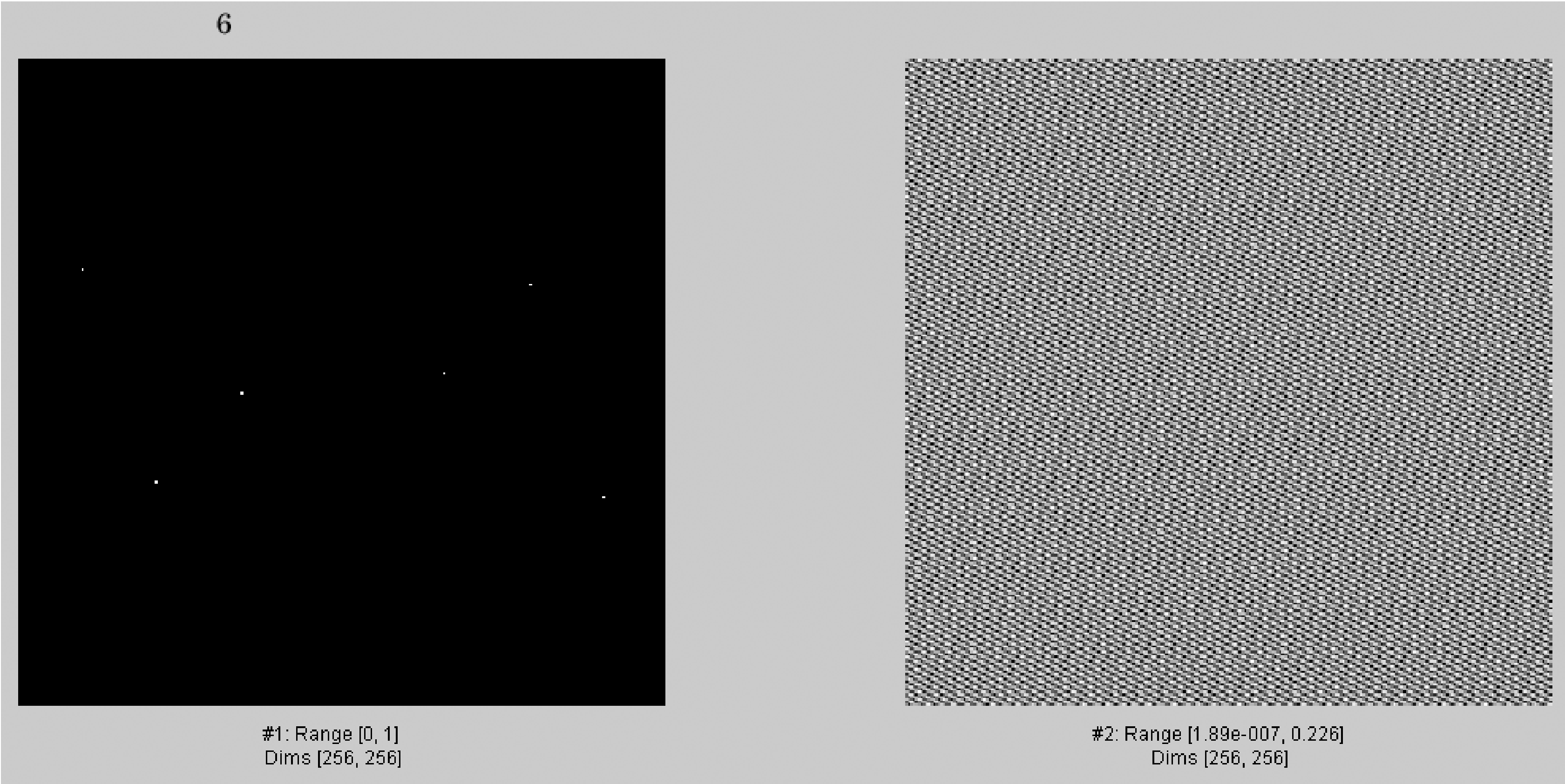


fx(cycles/image pixel size) fx(cycles/image pixel size) fx(cycles/image pixel size)

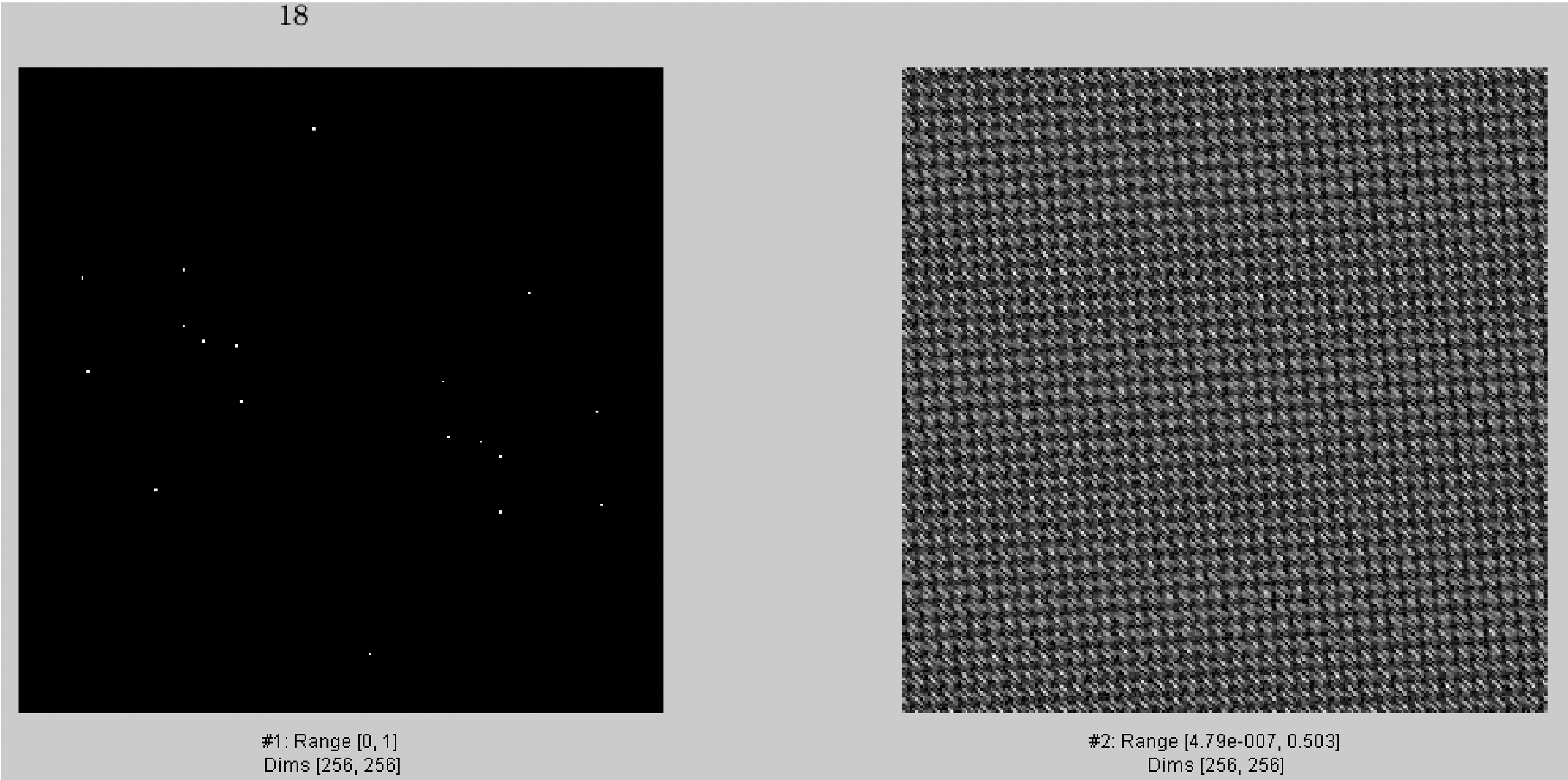
2



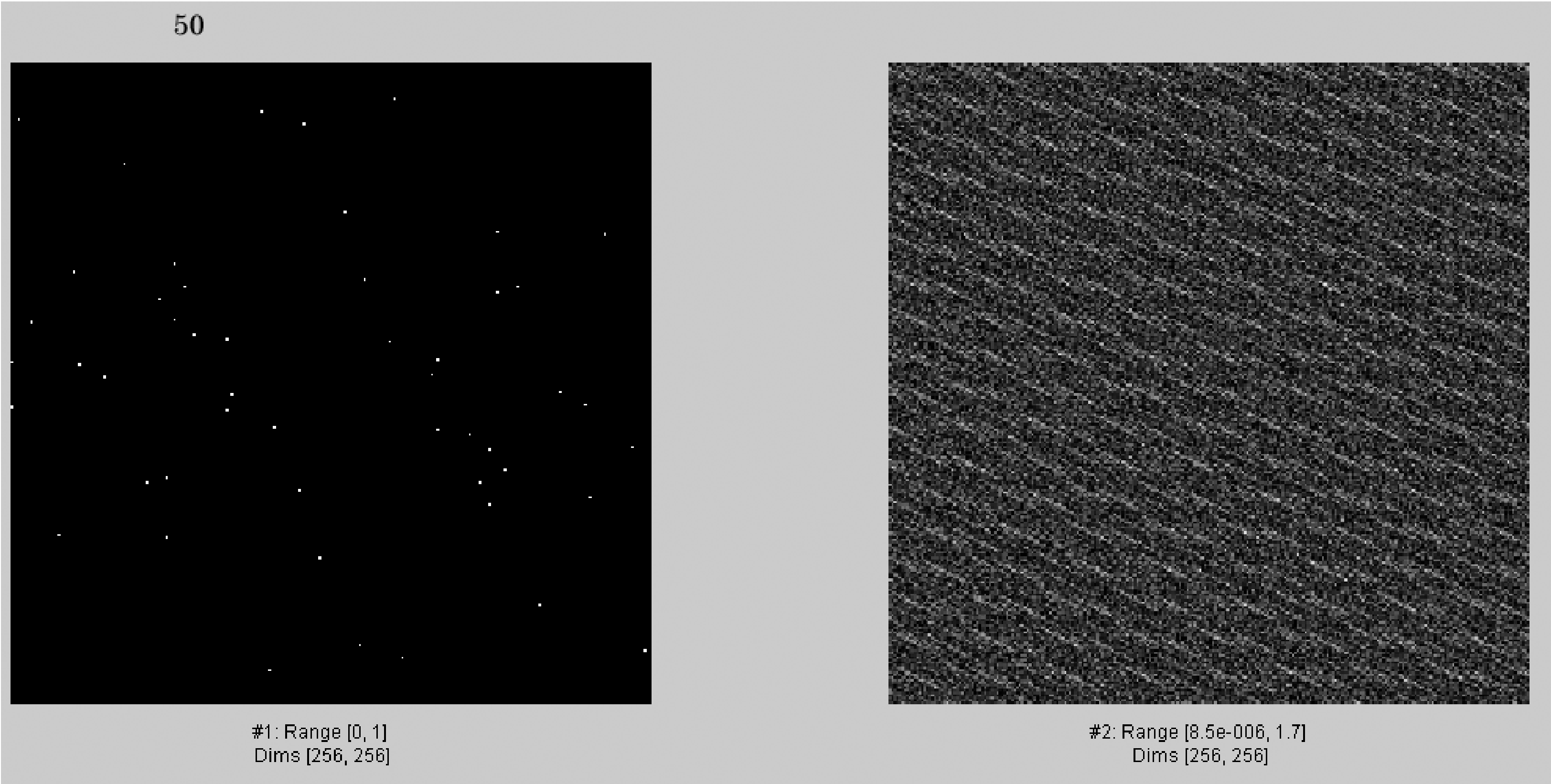
6



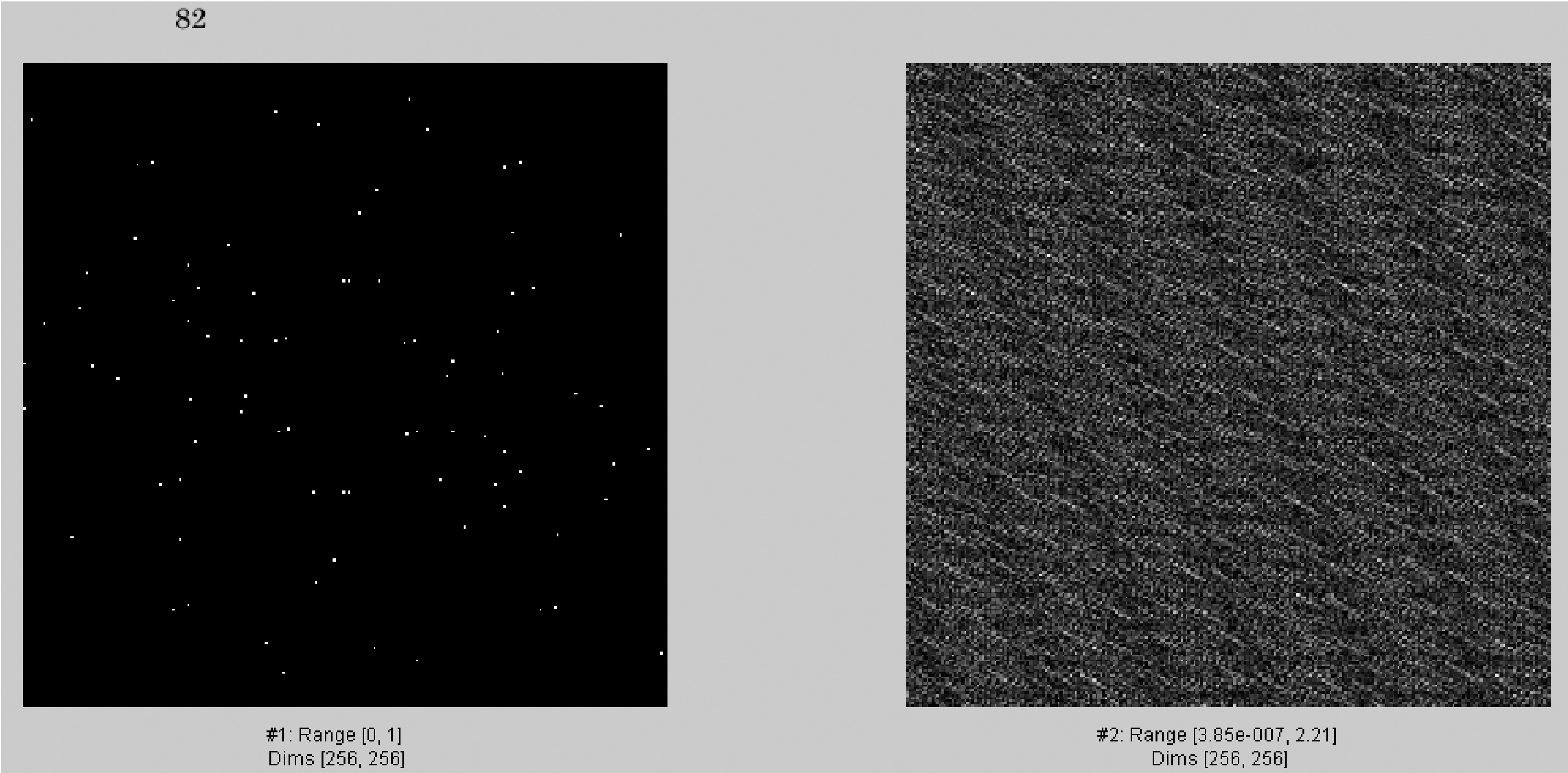
18



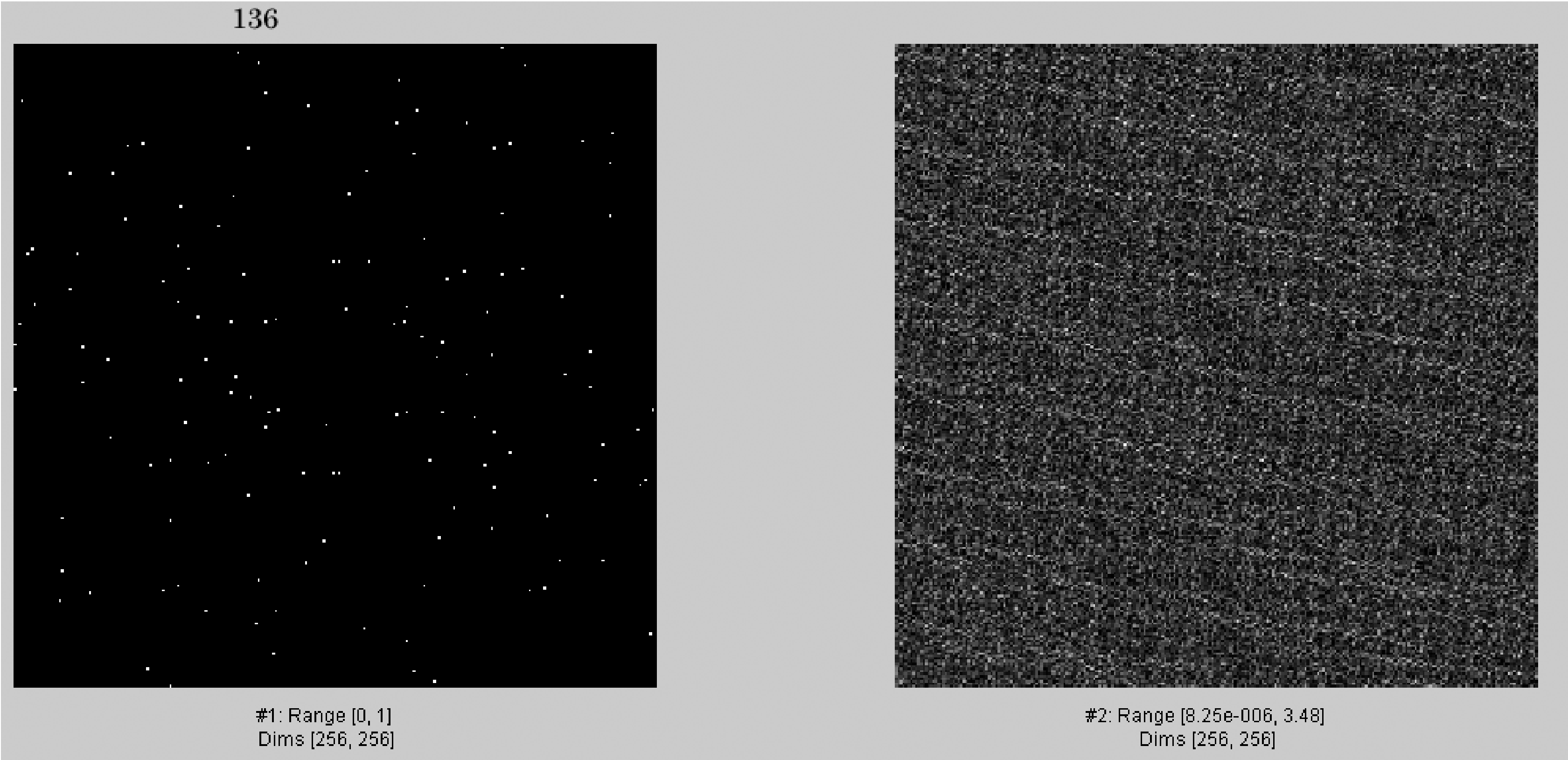
50



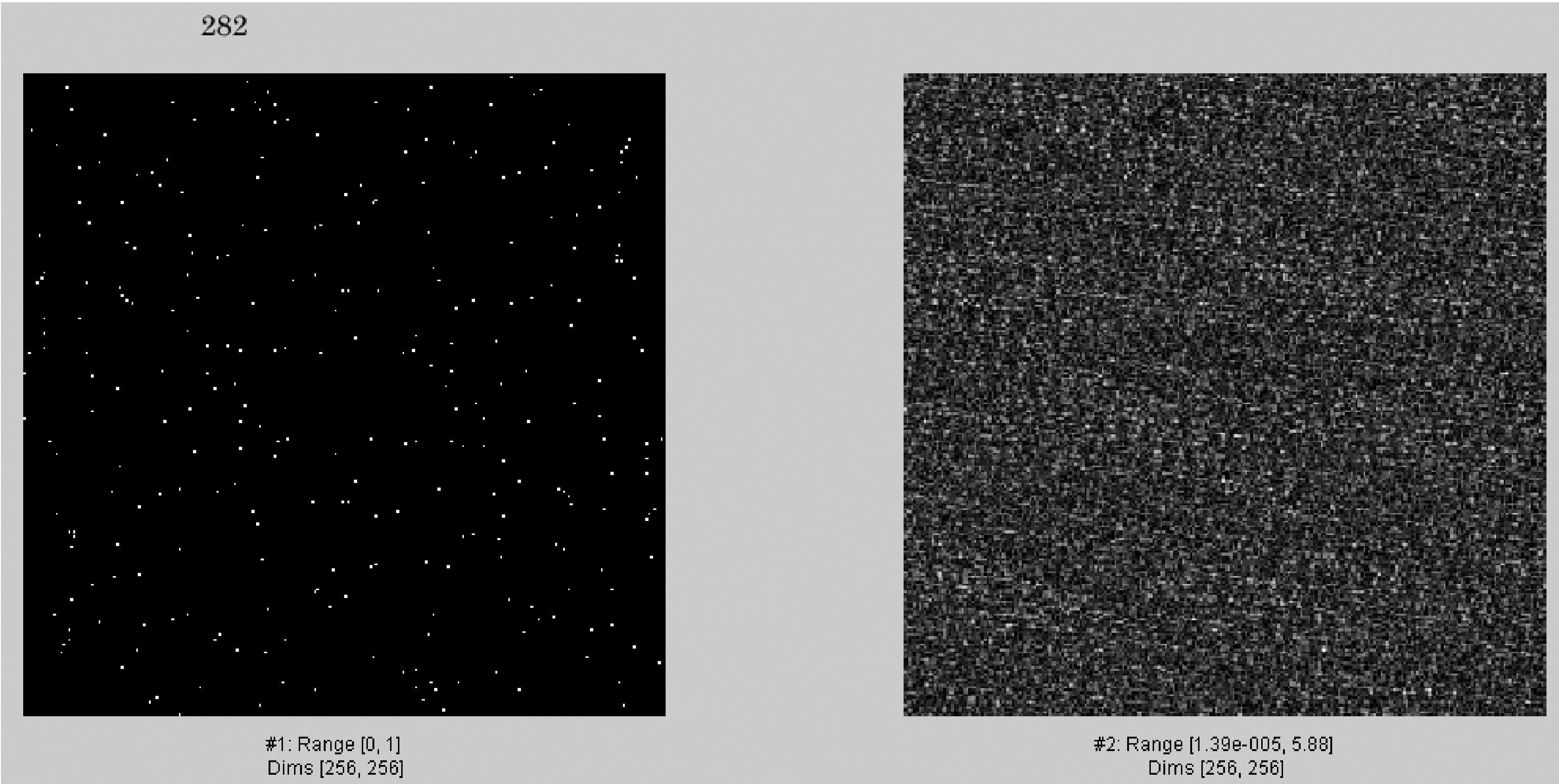
82



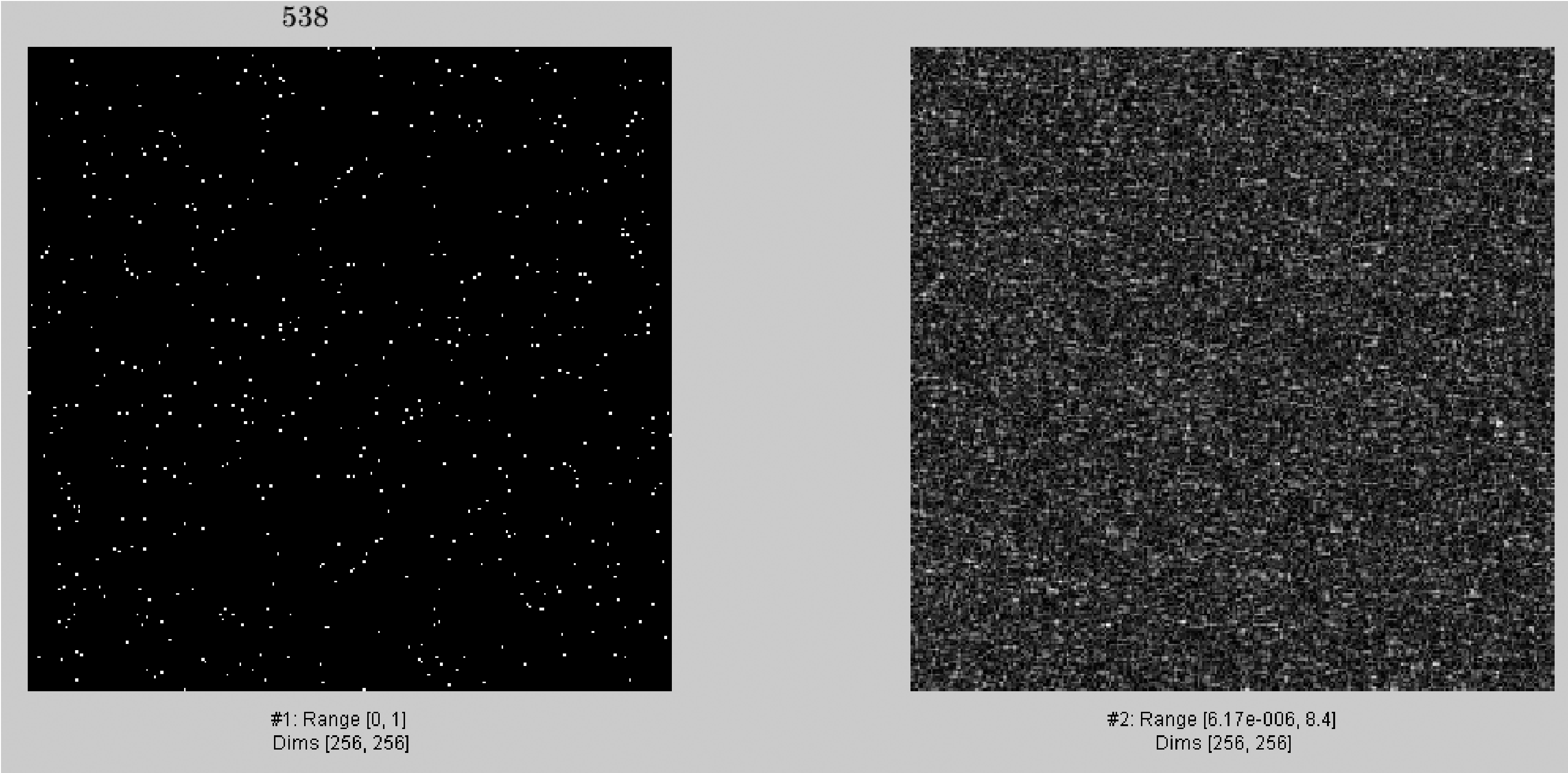
136



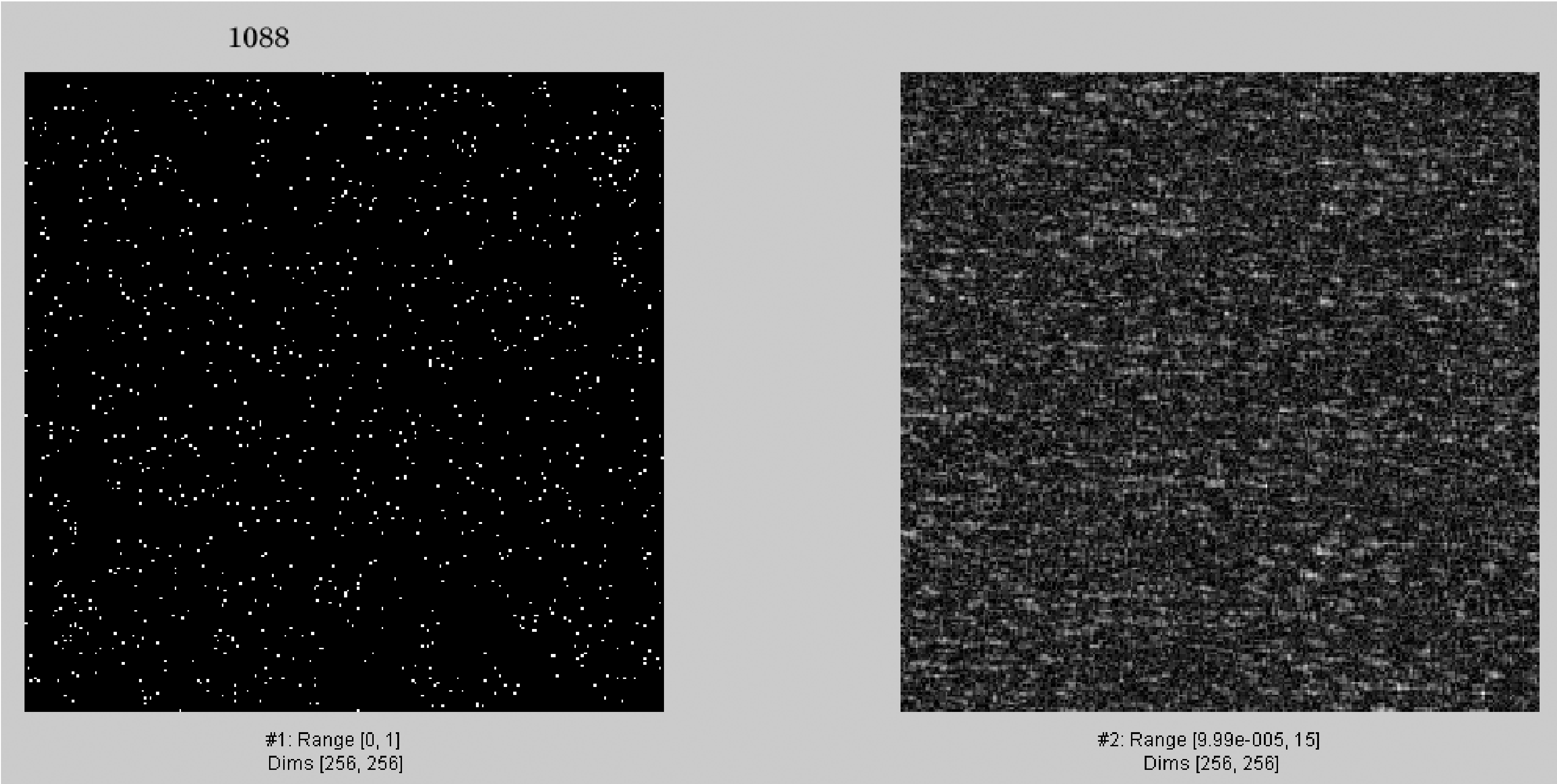
282



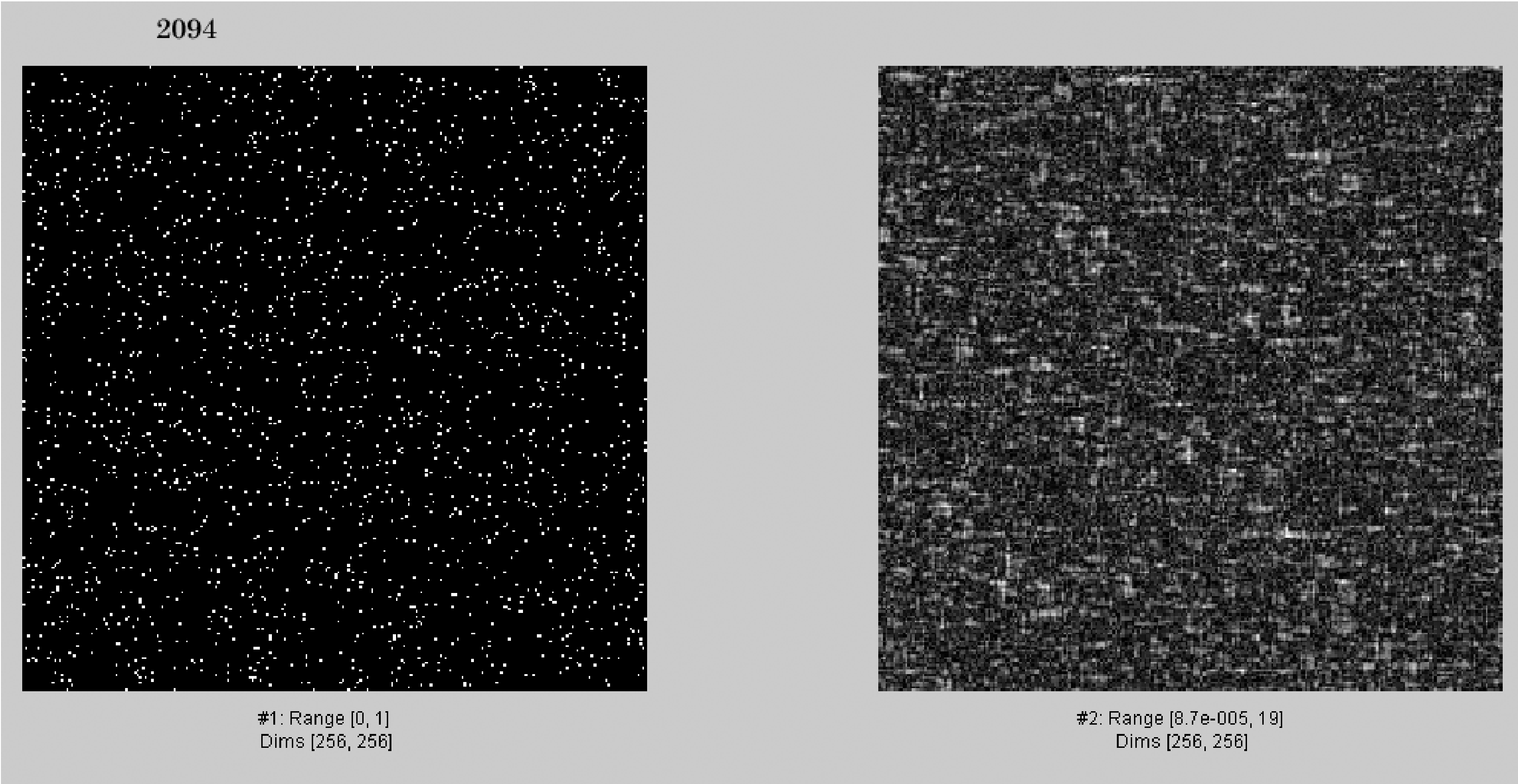
538



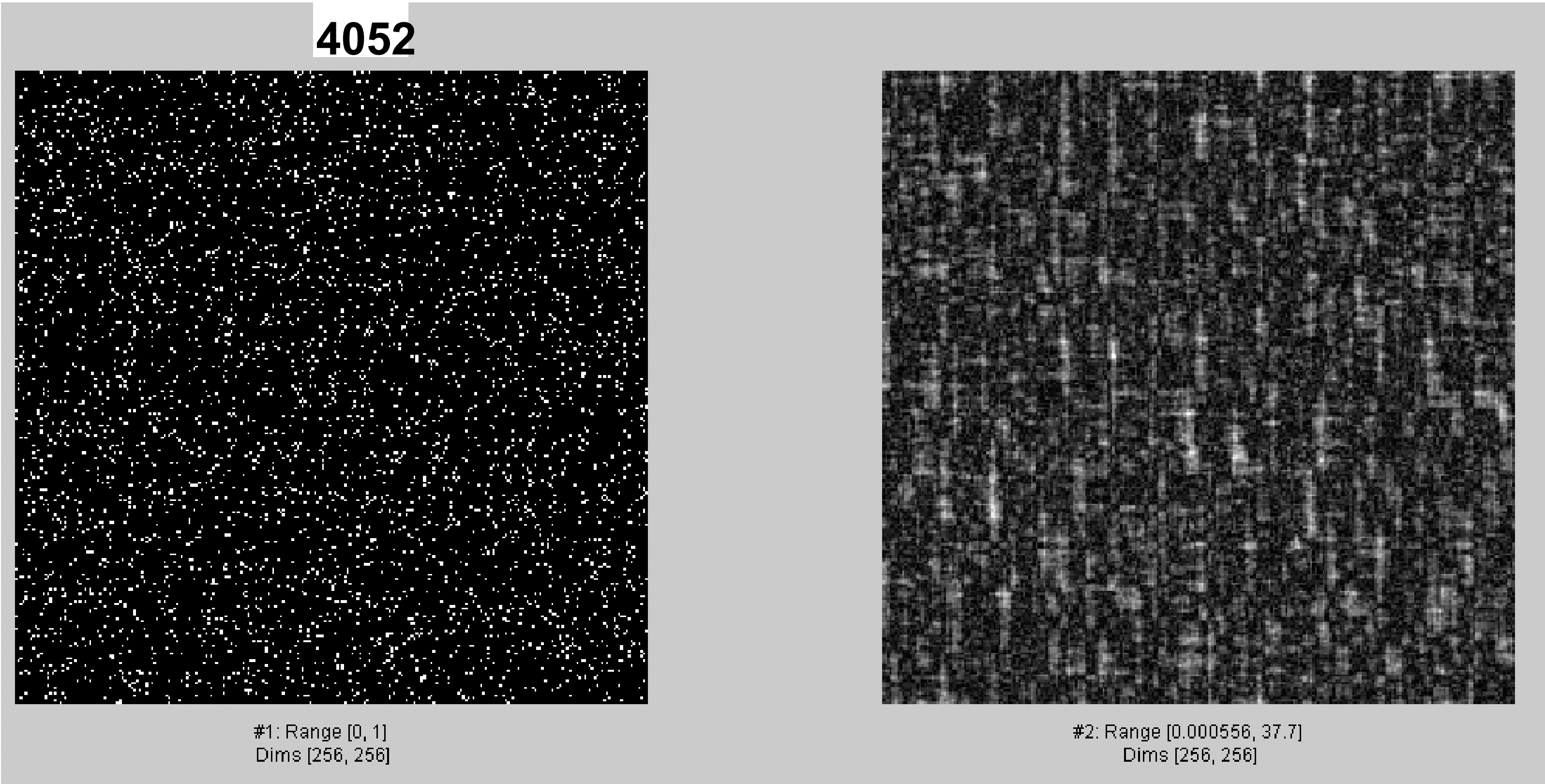
1088



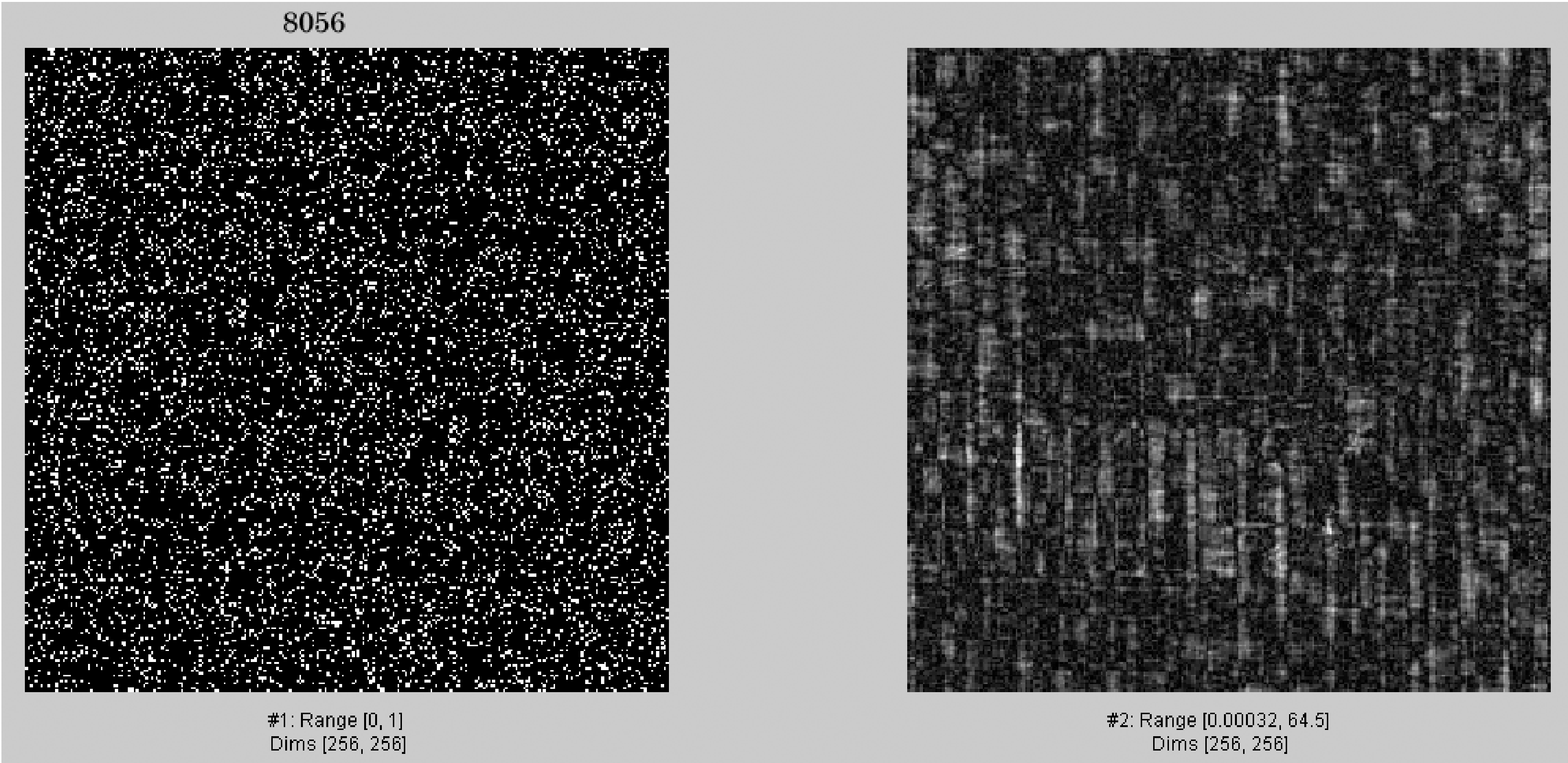
2094



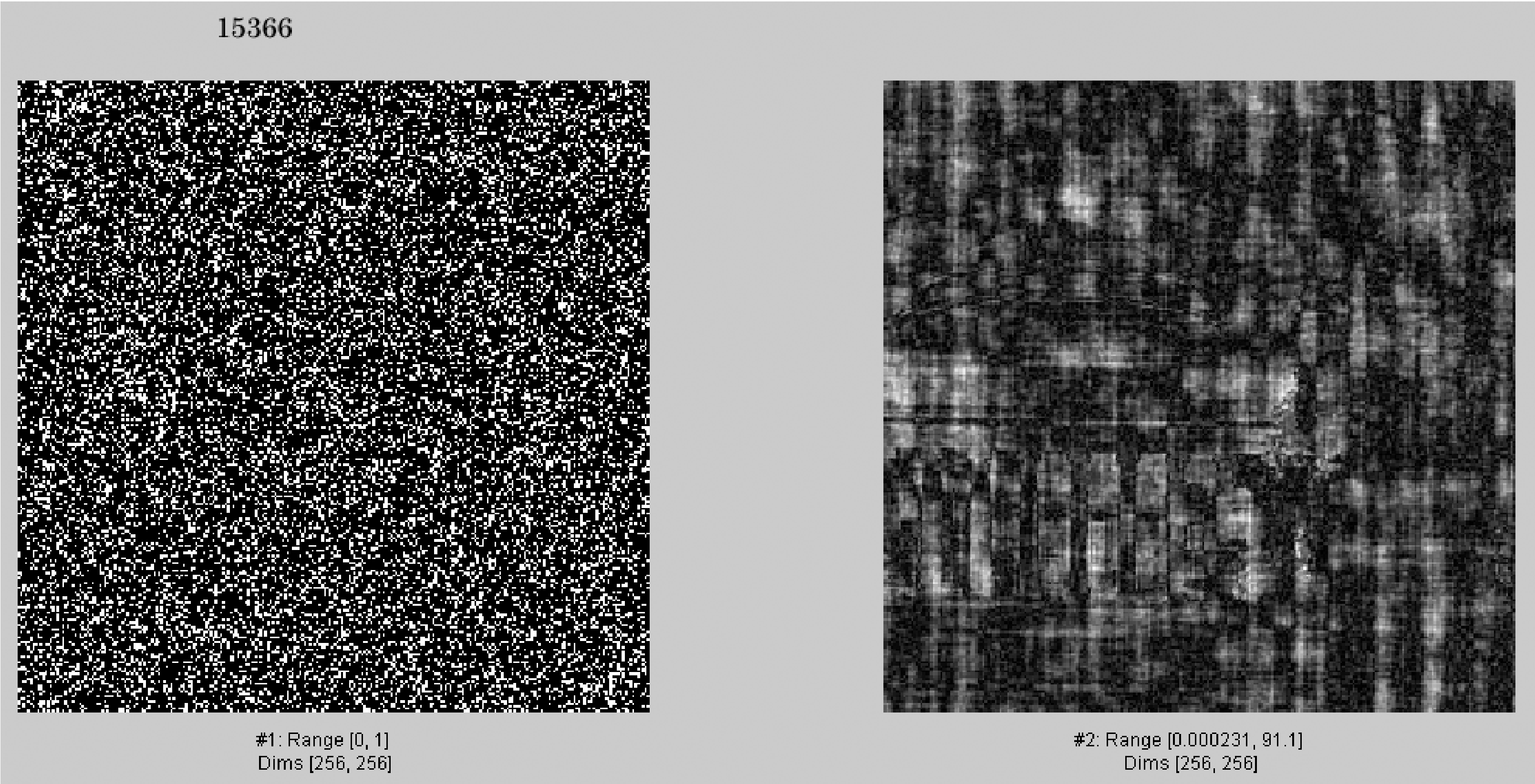
4052.



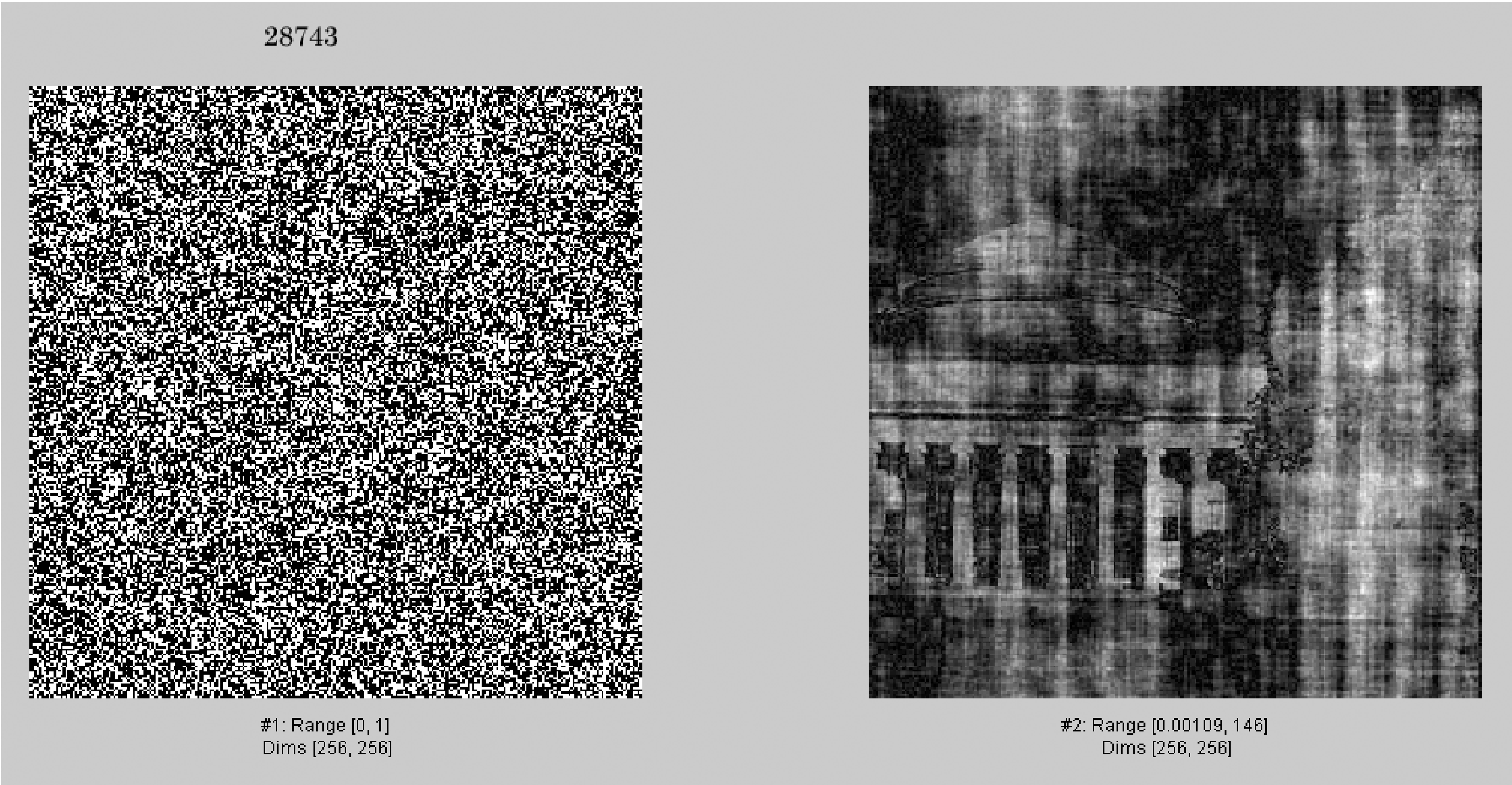
8056.



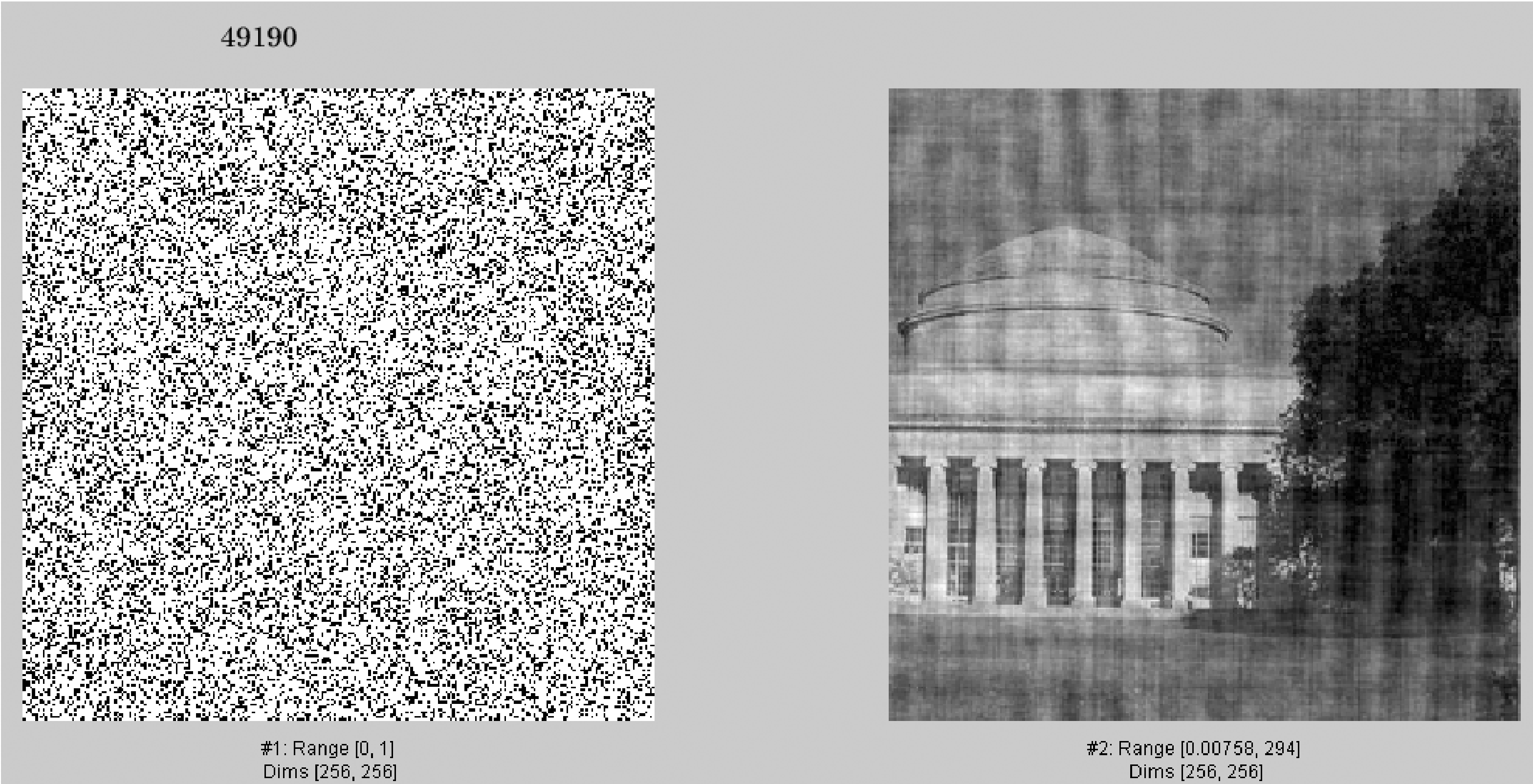
15366



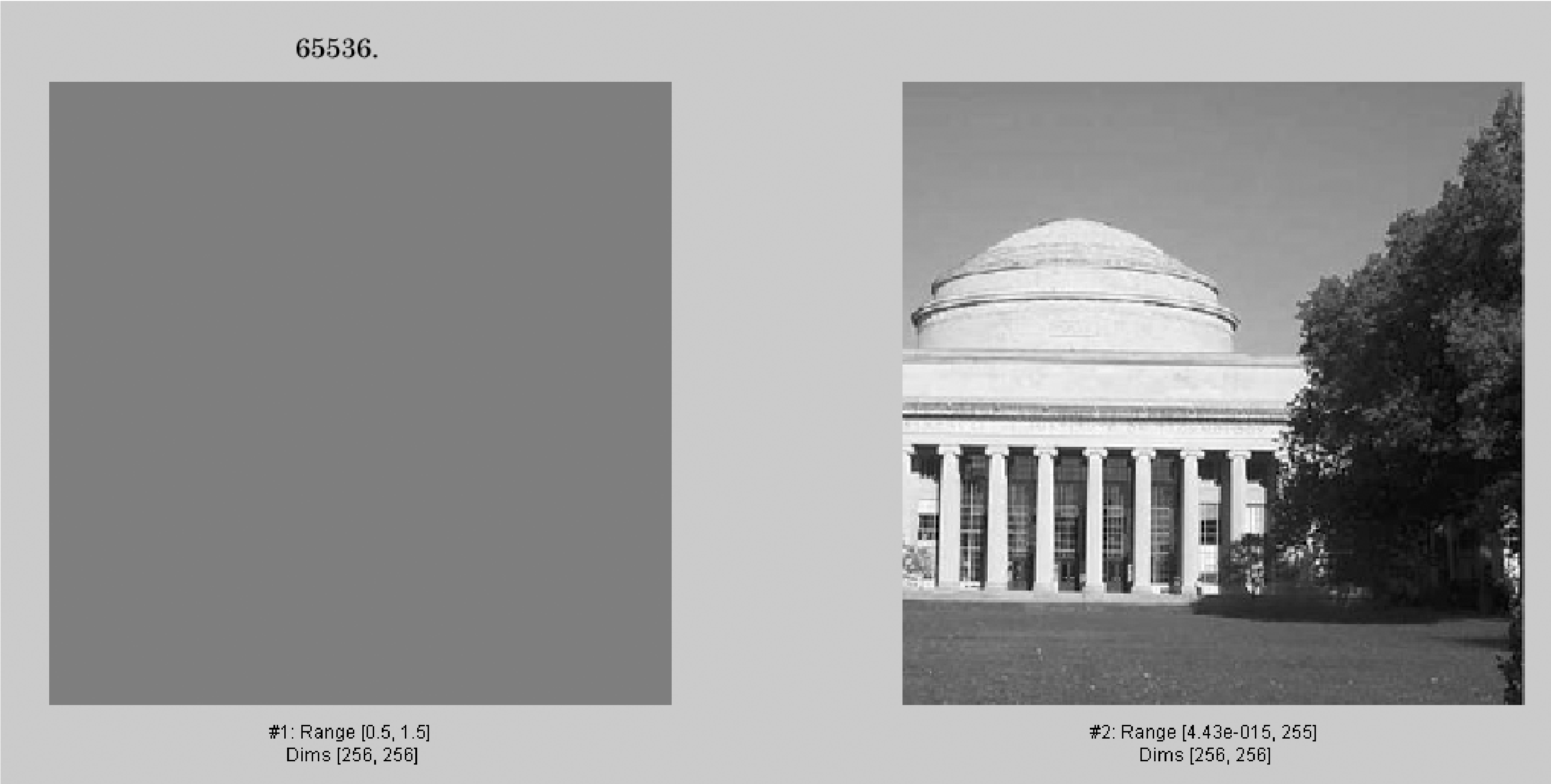
28743



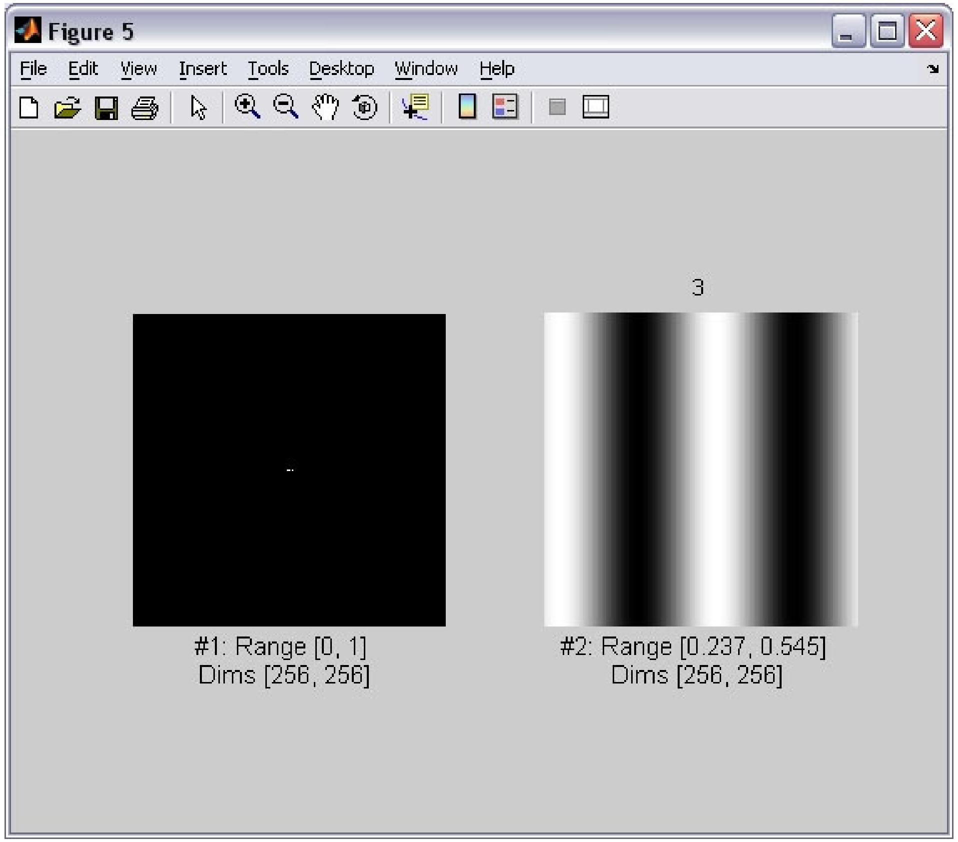
49190.



65536.


65536.
#1: Range [0.5, 1.5]
Dims [256, 256]
#2: Range [4.43e-015, 255]
Dims [256, 256]

3



Now, a similar sequence of images, but selecting Fourier components in descending order of magnitude.

5

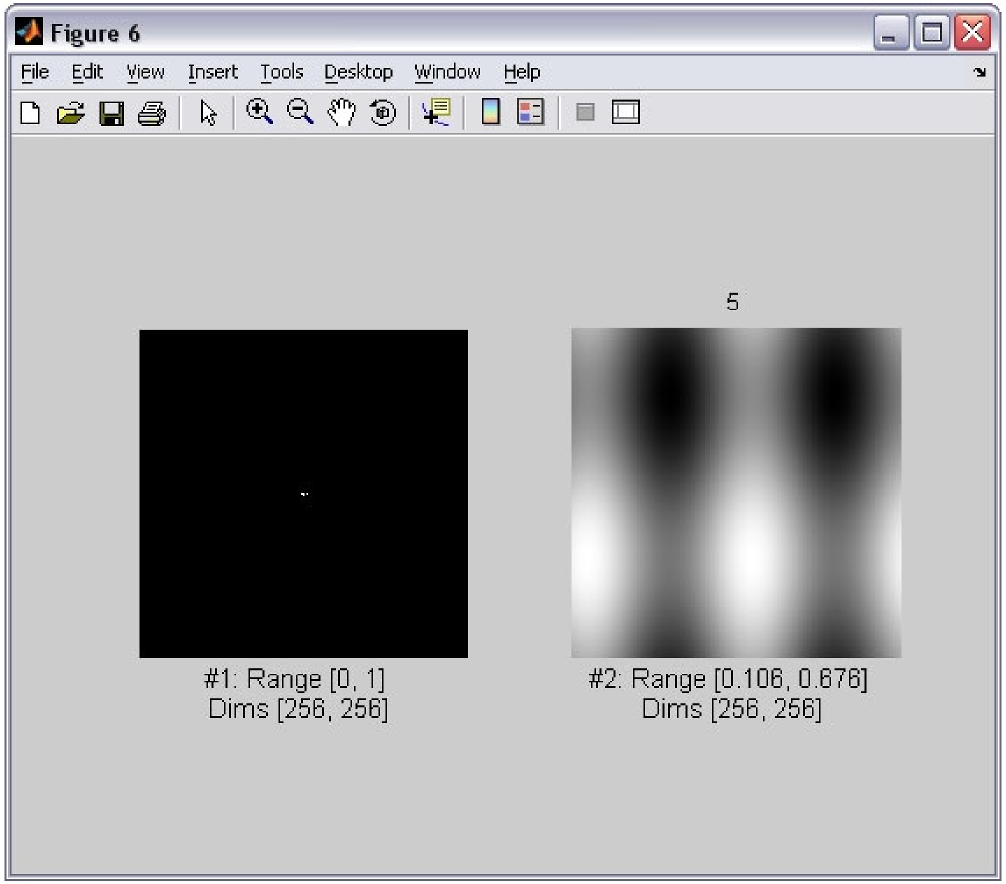


Figure 6
File Edit View Insert Tools Desktop Window Help
5
#1: Range [0, 1]
Dims [256, 256]
#2: Range [0.106, 0.676]
Dims [256, 256]

9

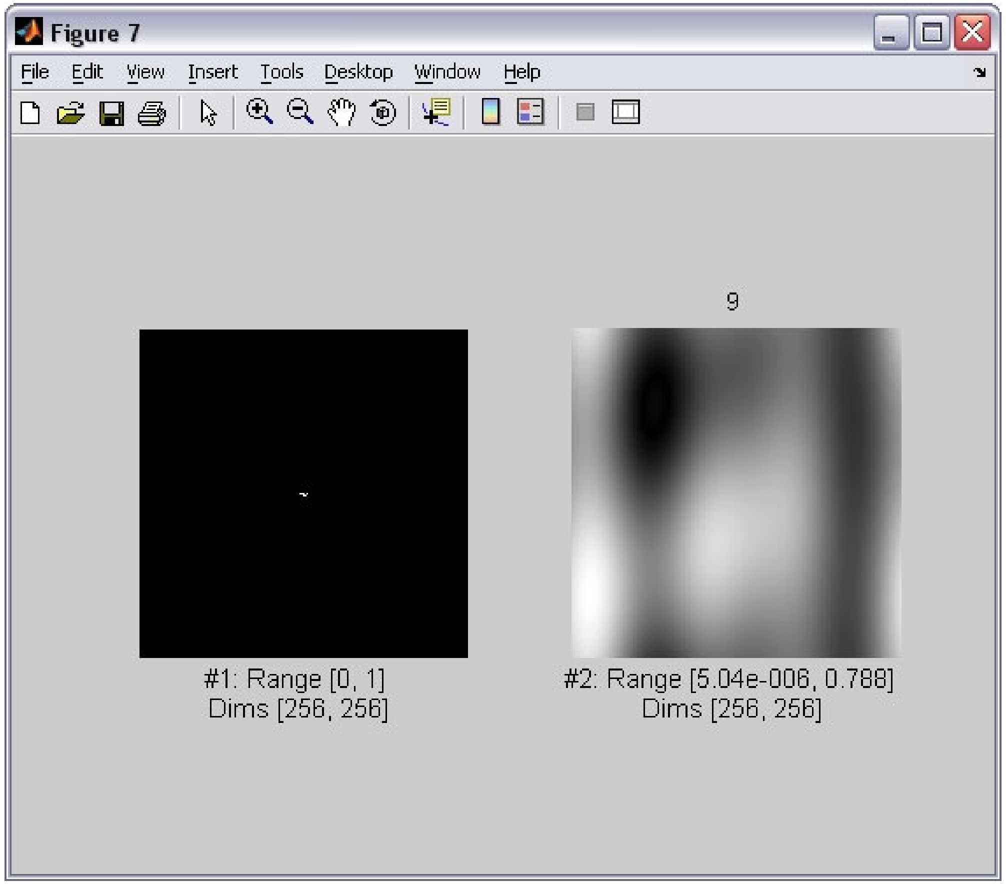


Figure 7
File Edit View Insert Tools Desktop Window Help
9
#1: Range [0, 1]
Dims [256, 256]
#2: Range [5.04e-006, 0.788]
Dims [256, 256]

17

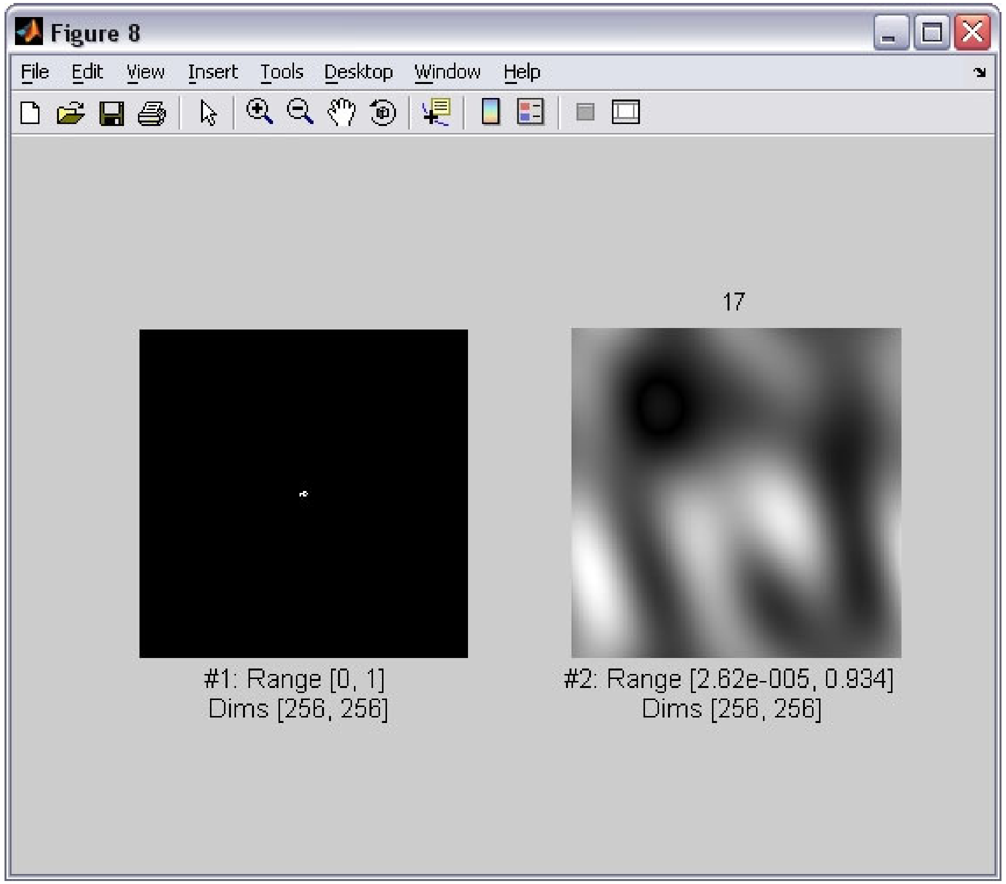


Figure 8
File Edit View Insert Tools Desktop Window Help
17
#1: Range [0, 1]
Dims [256, 256]
#2: Range [2.62e-005, 0.934]
Dims [256, 256]

33

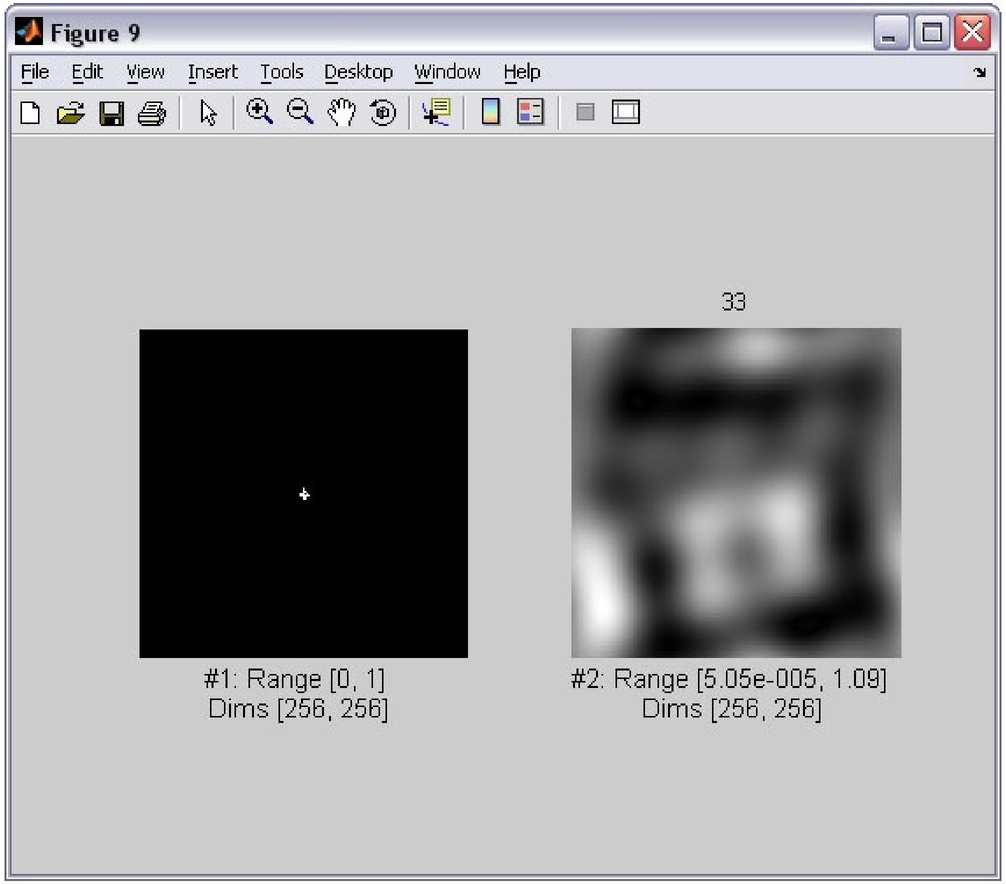


Figure 9
File Edit View Insert Tools Desktop Window Help
33
#1: Range [0, 1]
Dims [256, 256]
#2: Range [5.05e-005, 1.09]
Dims [256, 256]

65

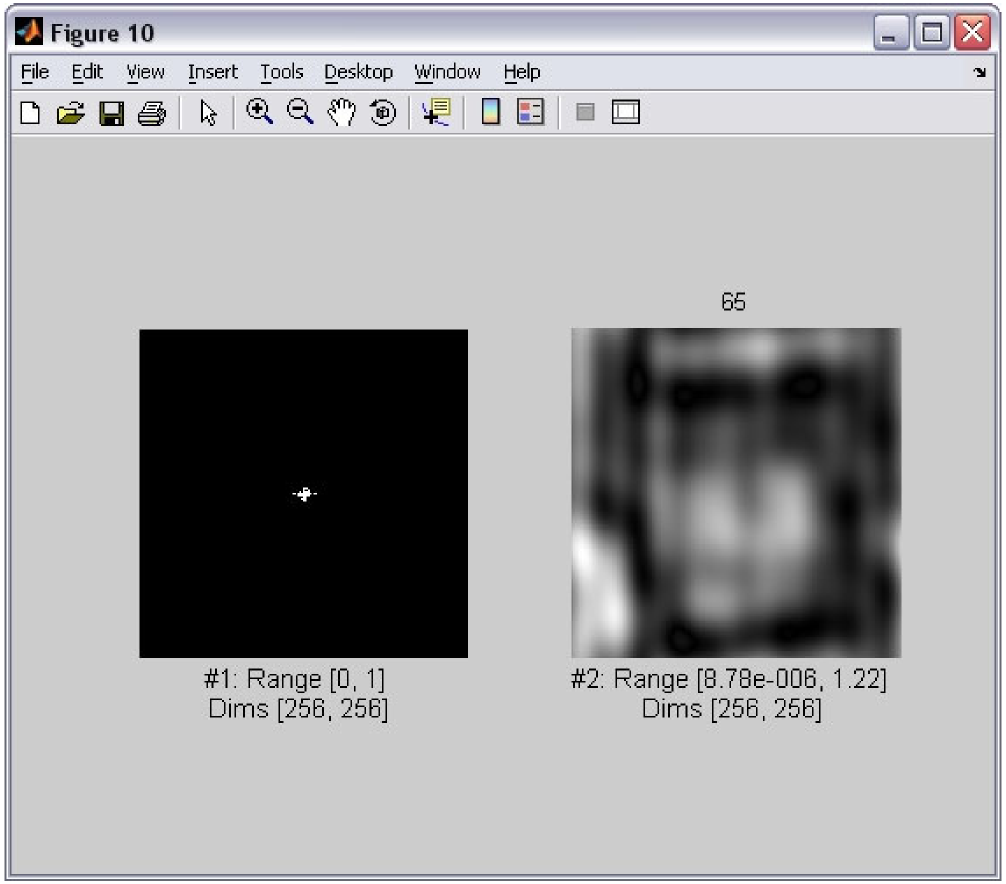


Figure 10
File Edit View Insert Tools Desktop Window Help
65
#1: Range [0, 1]
Dims [256, 256]
#2: Range [8.78e-006, 1.22]
Dims [256, 256]

129

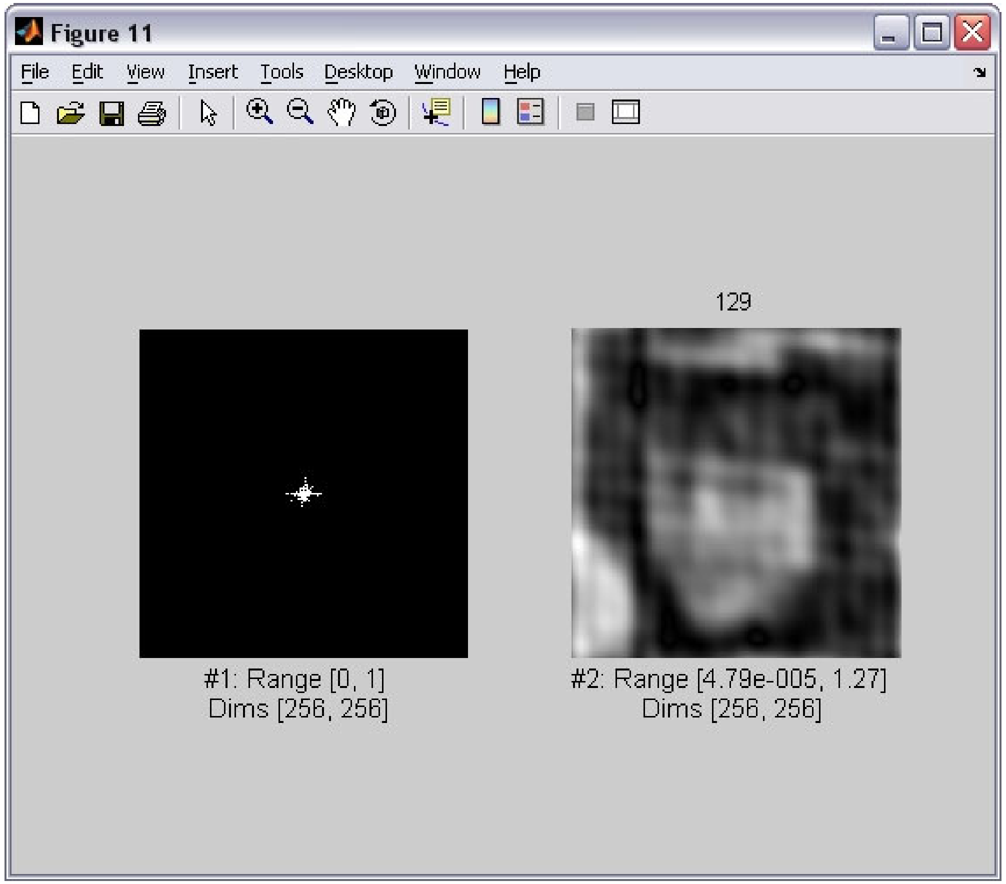


Figure 11
File Edit View Insert Tools Desktop Window Help
129
#1: Range [0, 1]
Dims [256, 256]
#2: Range [4.79e-005, 1.27]
Dims [256, 256]

257

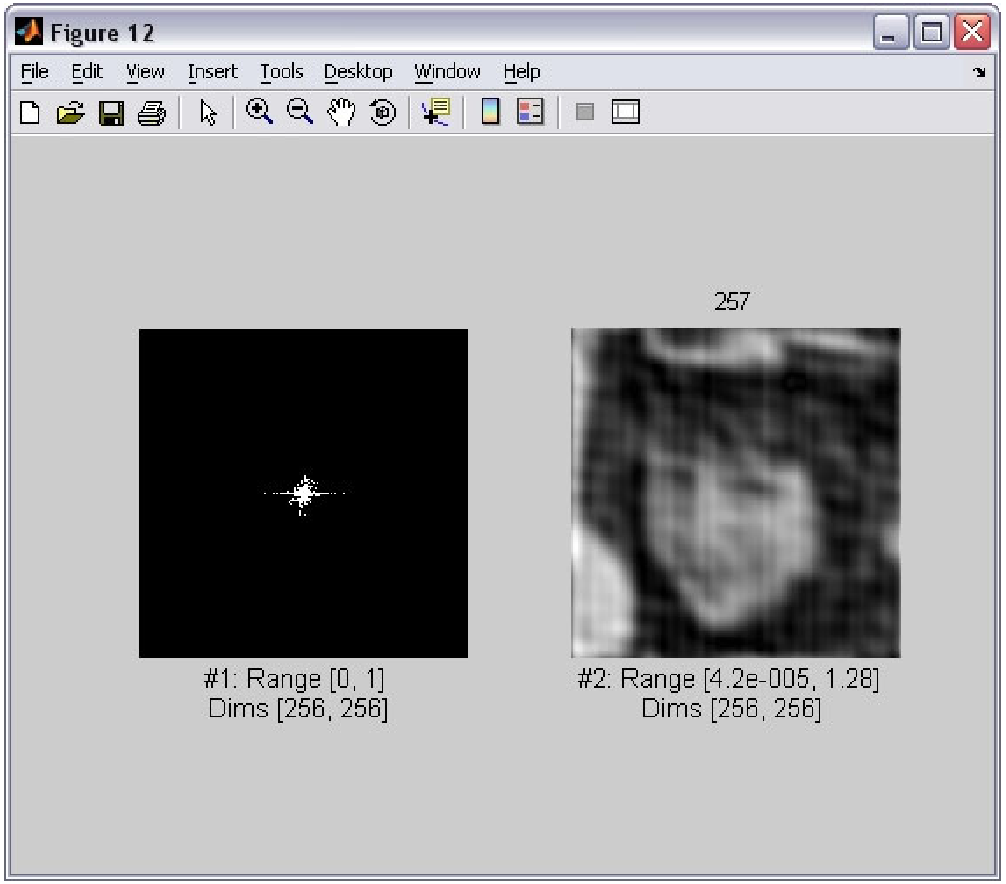


Figure 12
File Edit View Insert Tools Desktop Window Help
257
#1: Range [0, 1]
Dims [256, 256]
#2: Range [4.2e-005, 1.28]
Dims [256, 256]

513

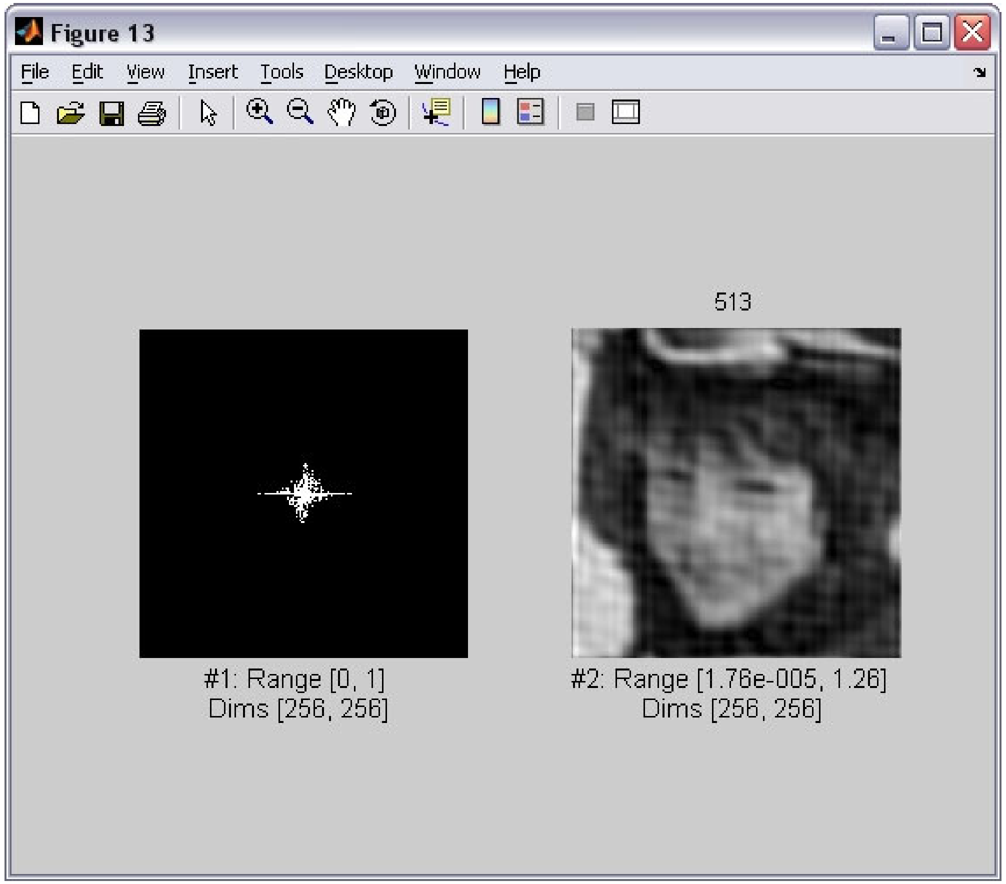


Figure 13
File Edit View Insert Tools Desktop Window Help
513
#1: Range [0, 1]
Dims [256, 256]
#2: Range [1.76e-005, 1.26]
Dims [256, 256]

1025

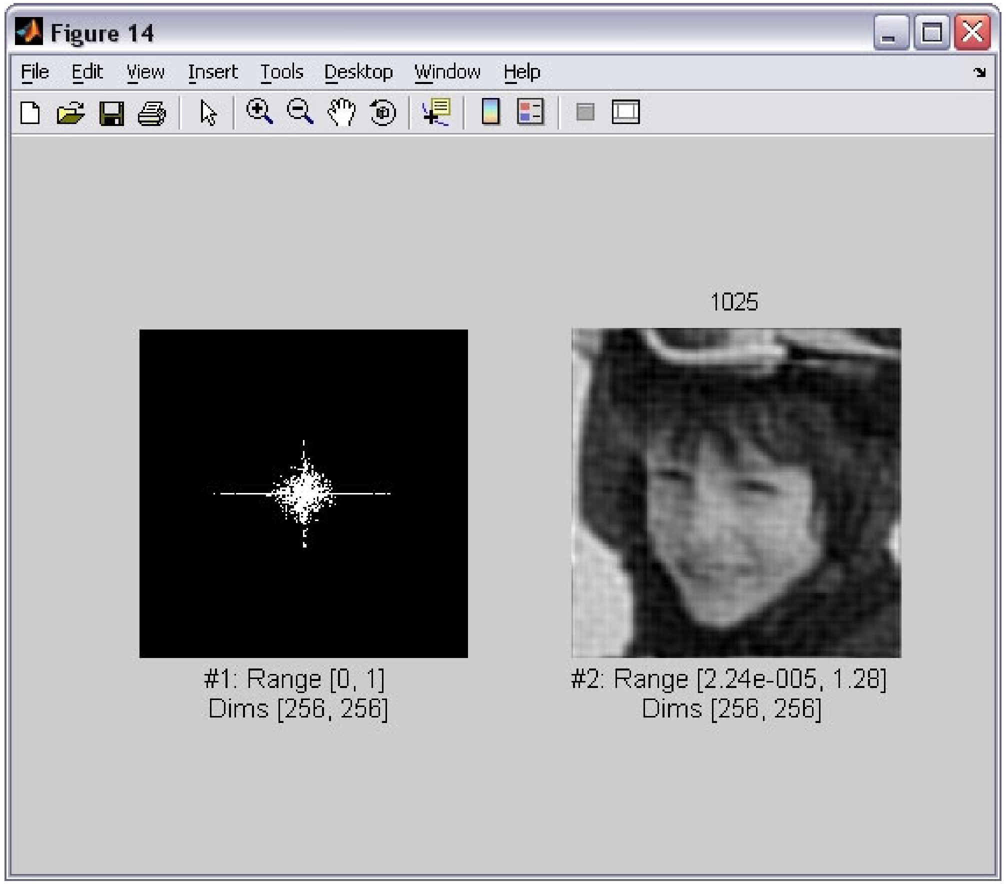


Figure 14
File Edit View Insert Tools Desktop Window Help
1025
#1: Range [0, 1]
Dims [256, 256]
#2: Range [2.24e-005, 1.28]
Dims [256, 256]

2049

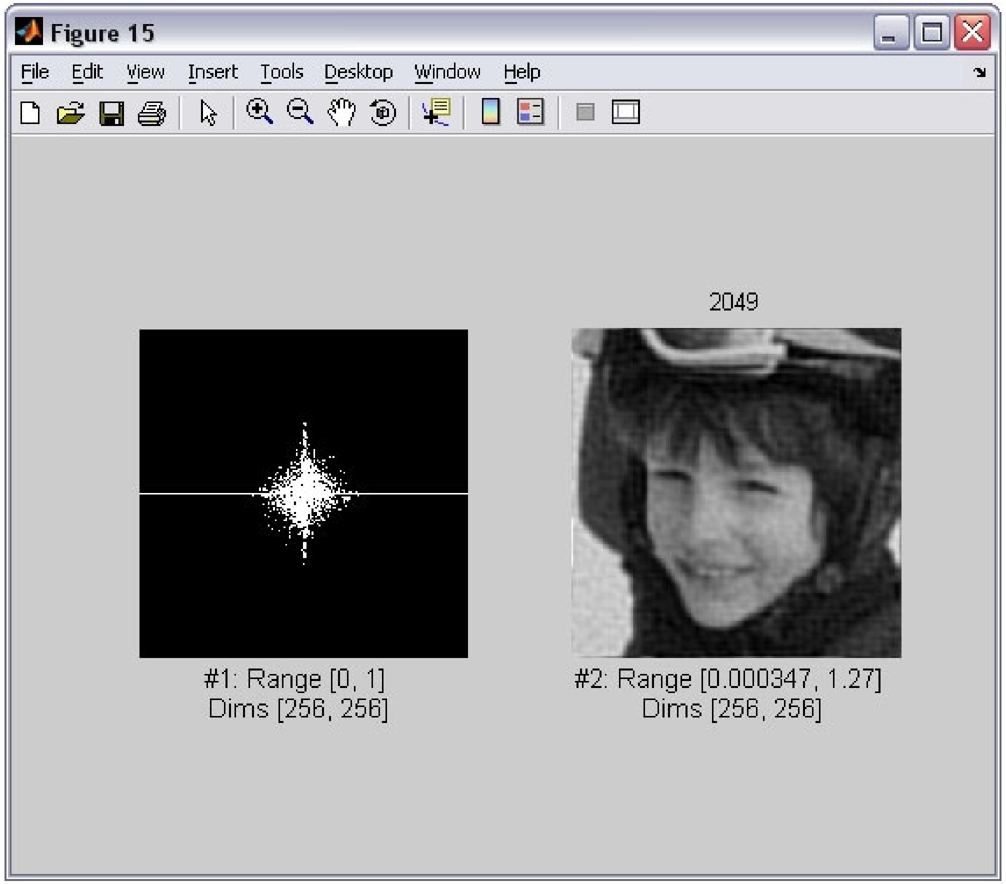


Figure 15
File Edit View Insert Tools Desktop Window Help
2049
#1: Range [0, 1]
Dims [256, 256]
#2: Range [0.000347, 1.27]
Dims [256, 256]

4097

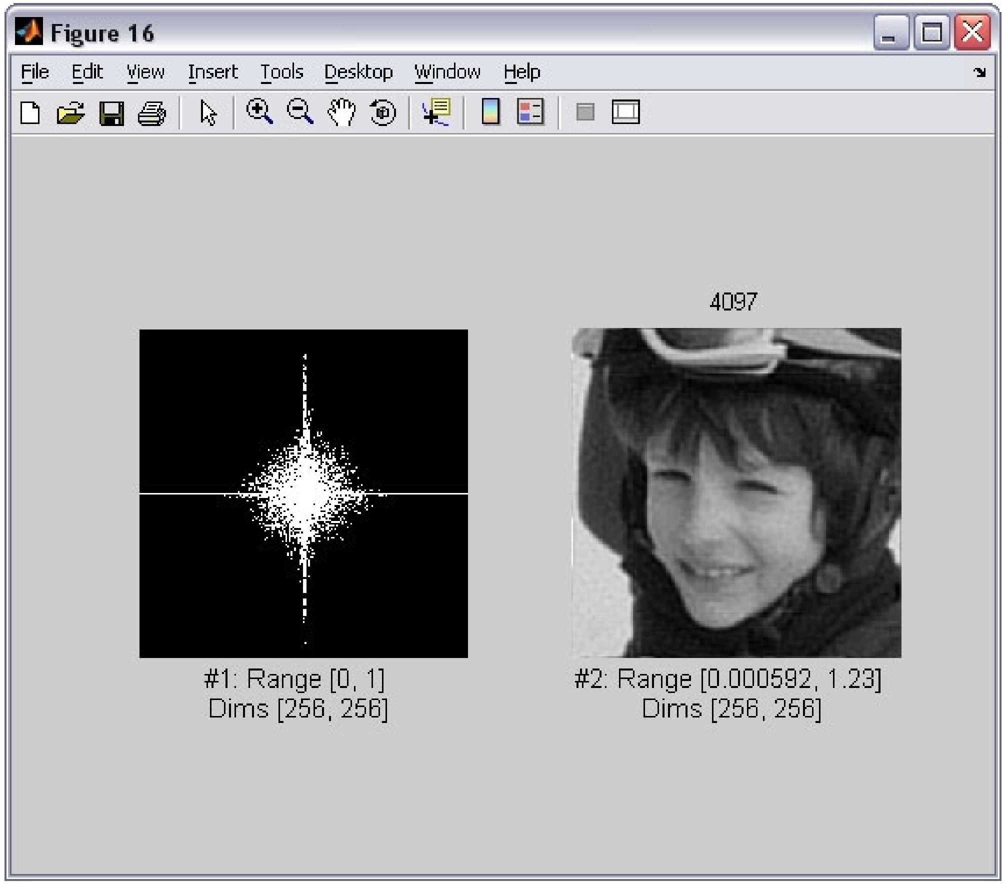


Figure 16
File Edit View Insert Tools Desktop Window Help
4097
#1: Range [0, 1]
Dims [256, 256]
#2: Range [0.000592, 1.23]
Dims [256, 256]

8193

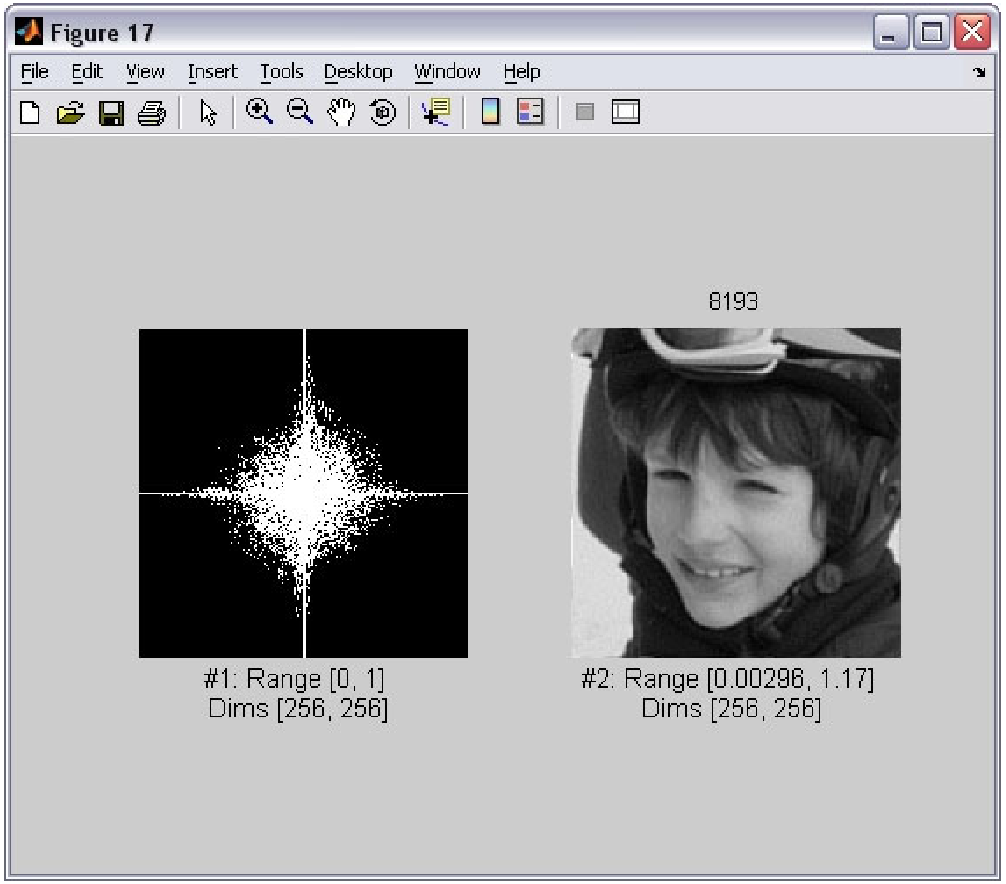


Figure 17
File Edit View Insert Tools Desktop Window Help
8193
#1: Range [0, 1]
Dims [256, 256]
#2: Range [0.00296, 1.17]
Dims [256, 256]

16385

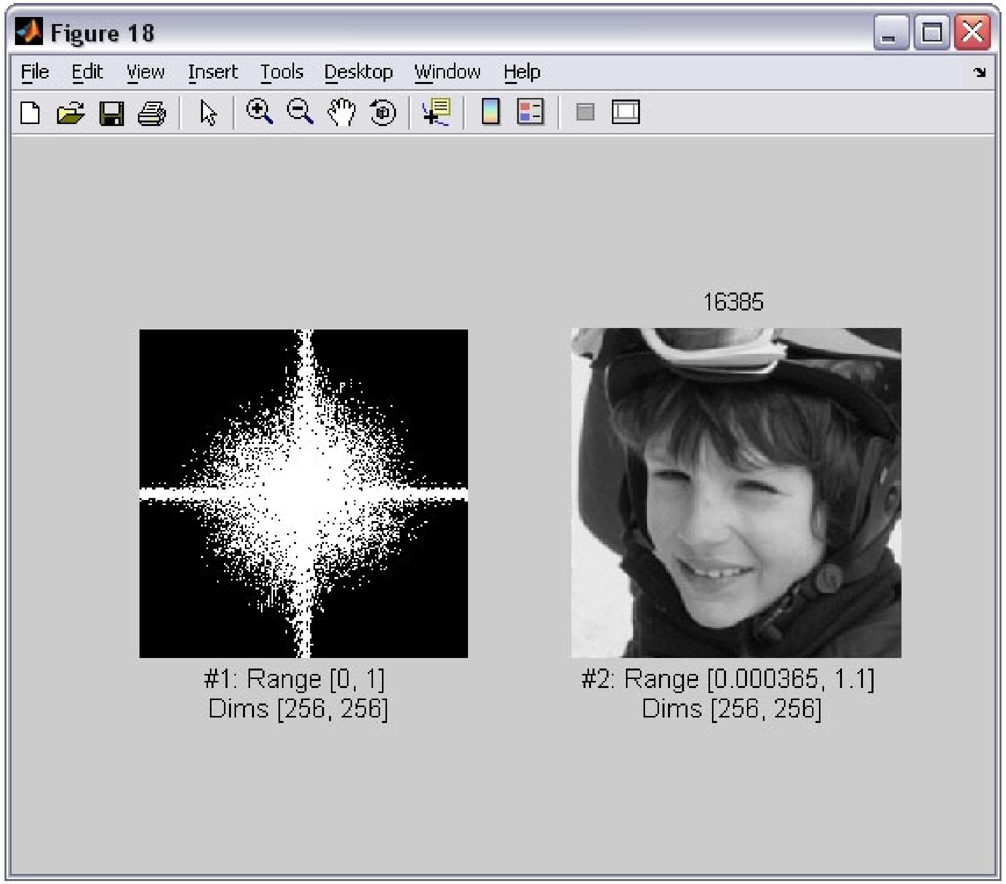


Figure 18
File Edit View Insert Tools Desktop Window Help
16385
#1: Range [0, 1]
Dims [256, 256]
#2: Range [0.000365, 1.1]
Dims [256, 256]

32769

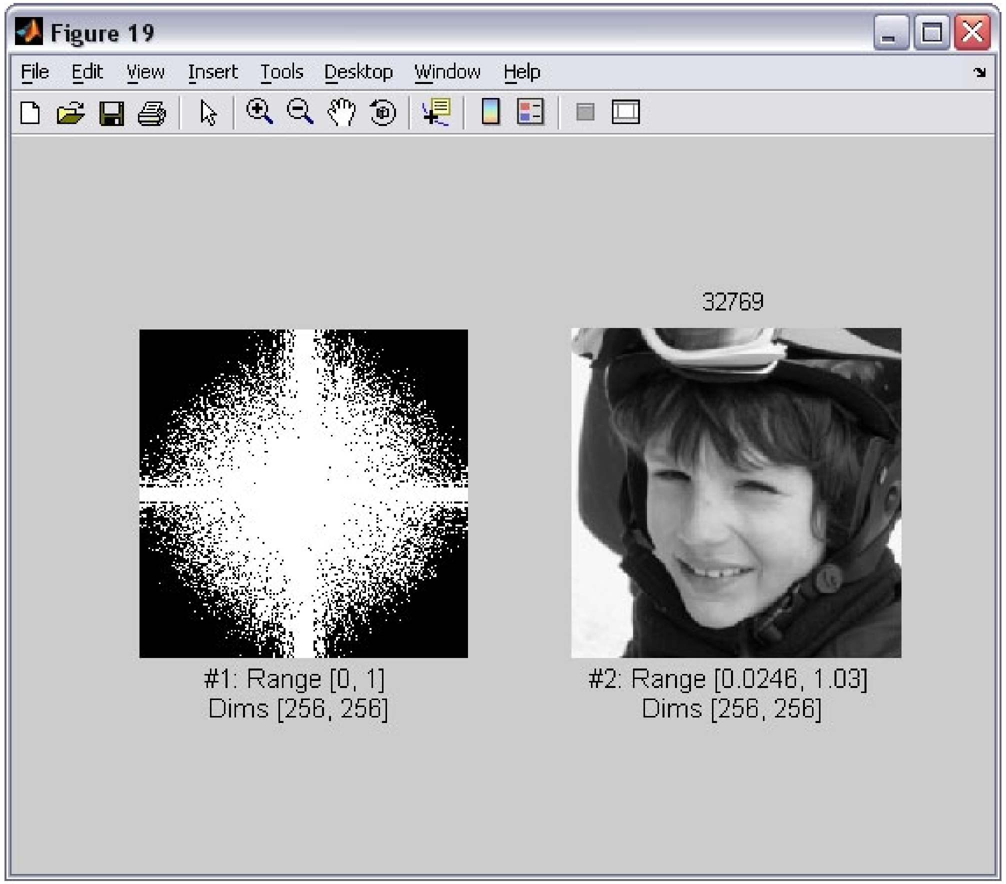


Figure 19
File Edit View Insert Tools Desktop Window Help
32769
#1: Range [0, 1]
Dims [256, 256]
#2: Range [0.0246, 1.03]
Dims [256, 256]

65536

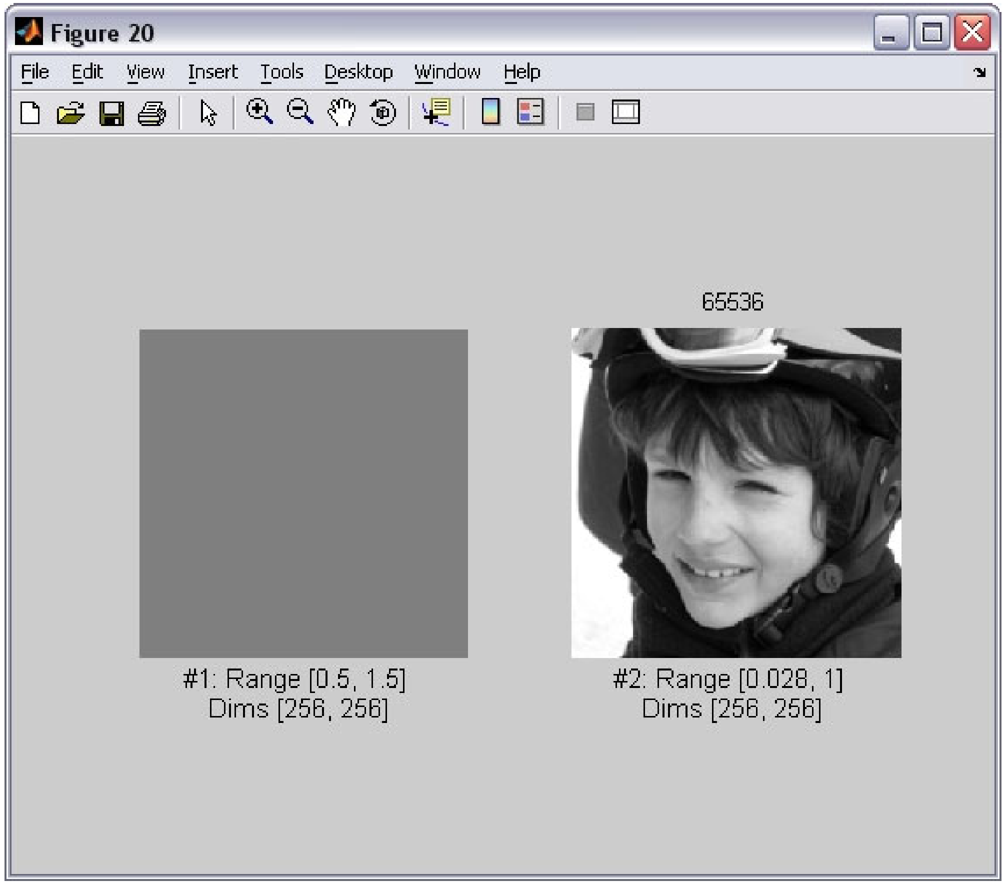
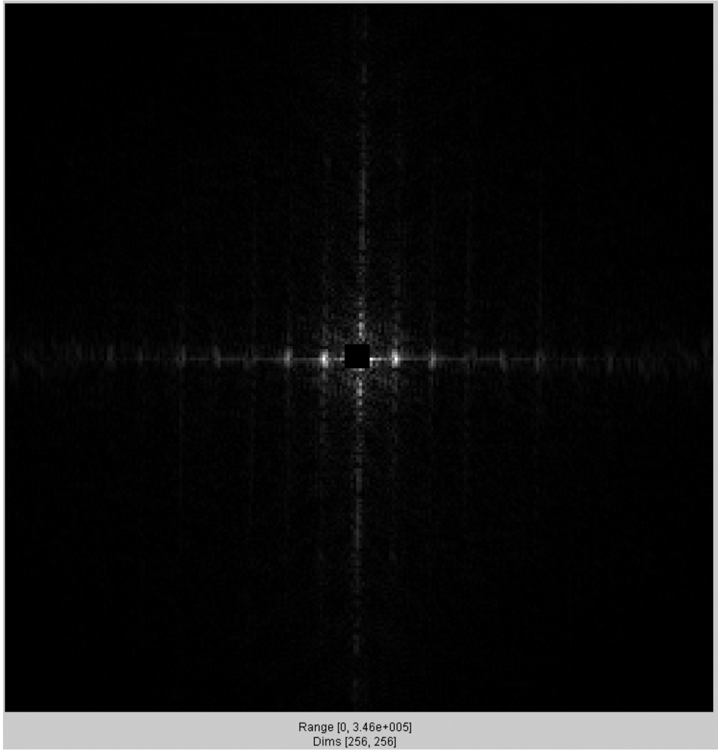
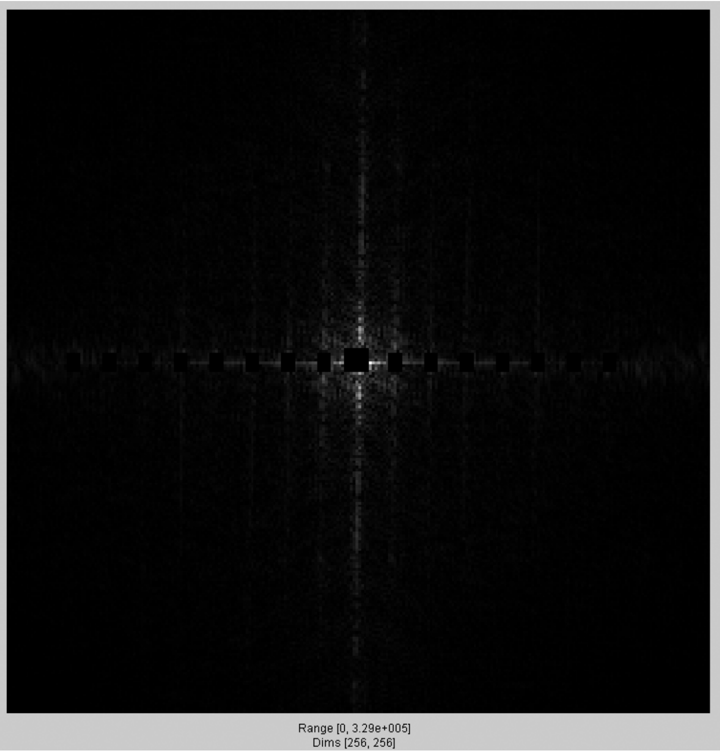


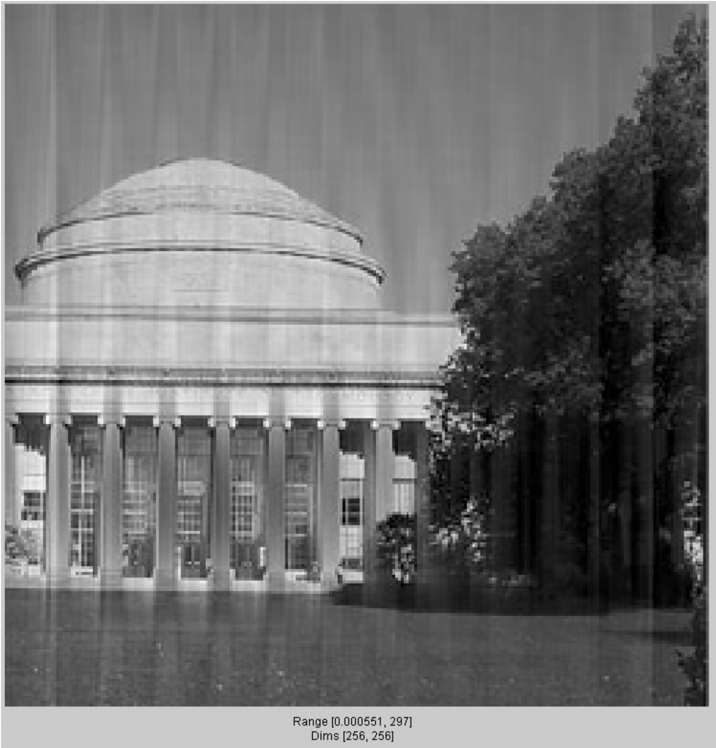
Figure 20
File Edit View Insert Tools Desktop Window Help
65536
#1: Range [0.5, 1.5]
Dims [256, 256]
#2: Range [0.028, 1]
Dims [256, 256]

Fourier transform magnitude



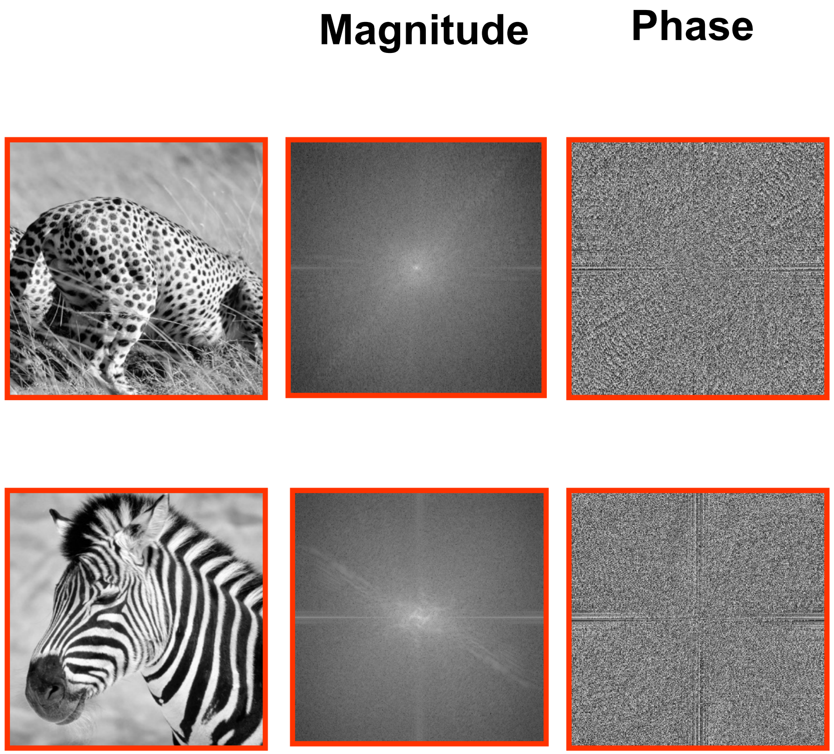
Masking out the fundamental and harmonics from periodic pillars





Fourier Transform

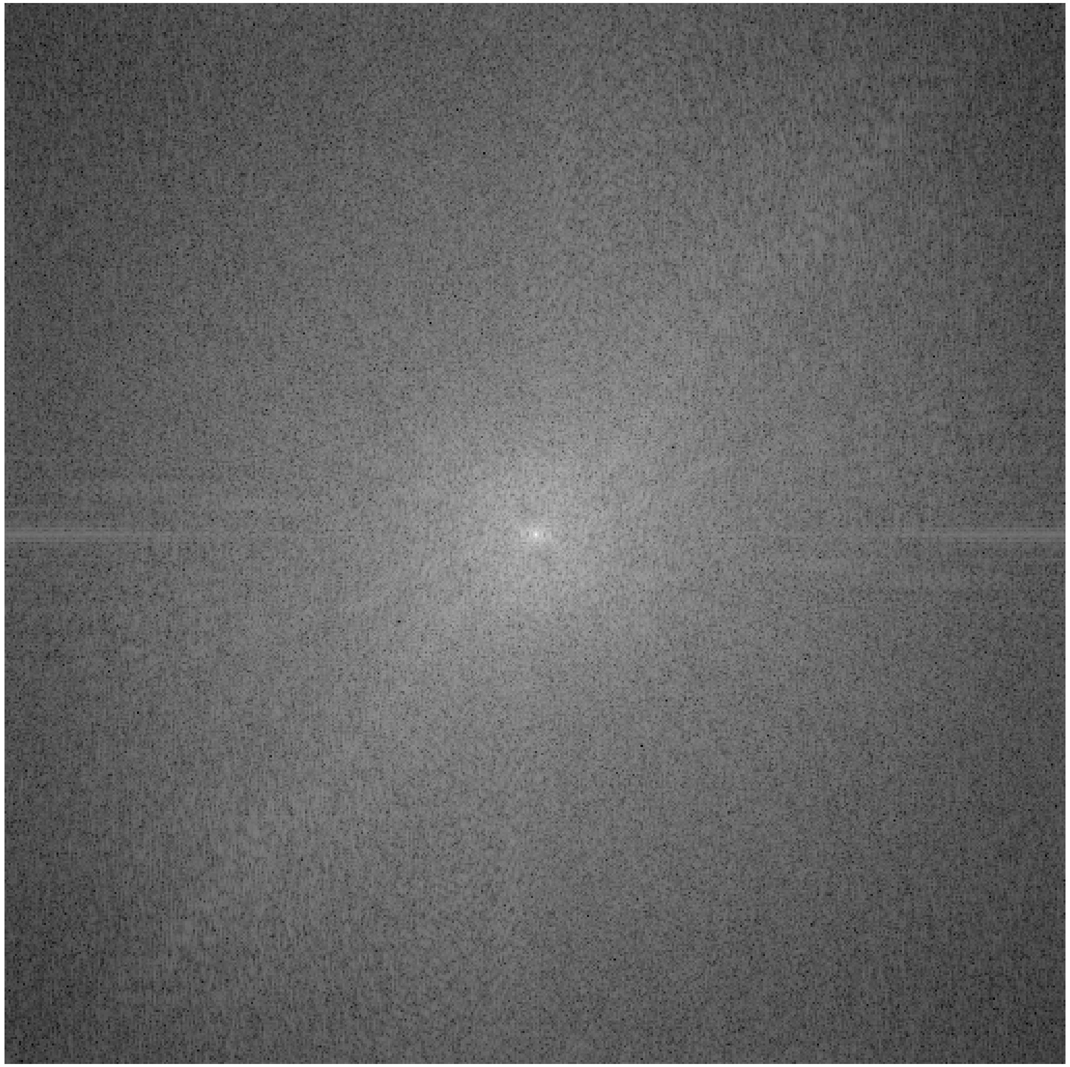
- Fourier transform of a real function is complex
  - difficult to plot, visualize
  - instead, we can think of the phase and magnitude of the transform
- Phase is the phase of the complex transform
- Magnitude is the magnitude of the complex transform



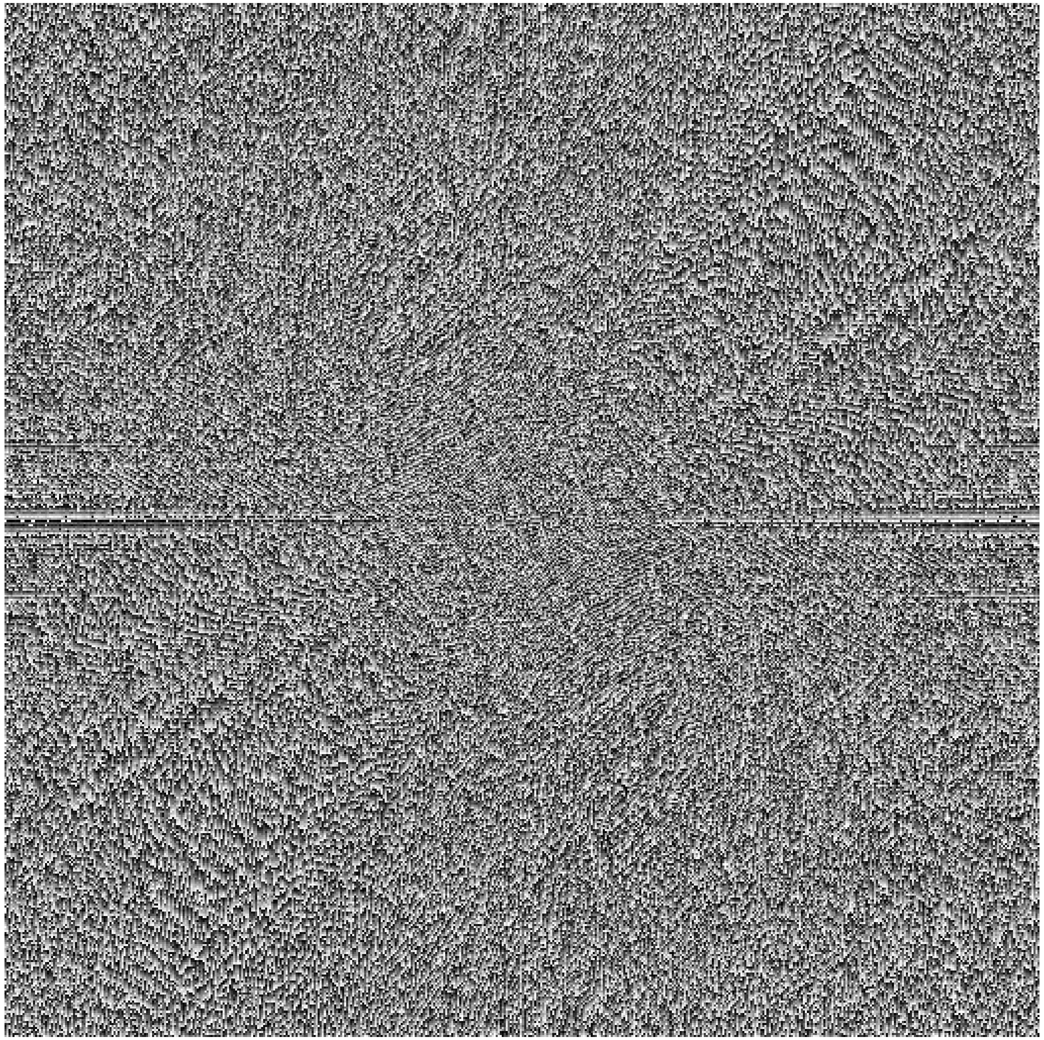
Phase and Magnitude

- Curious fact
  - all natural images have about the same magnitude transform
  - hence, phase seems to matter, but magnitude largely doesn’t
- Demonstration
  - Take two pictures, swap the phase transforms, compute the inverse - what does the result look like?

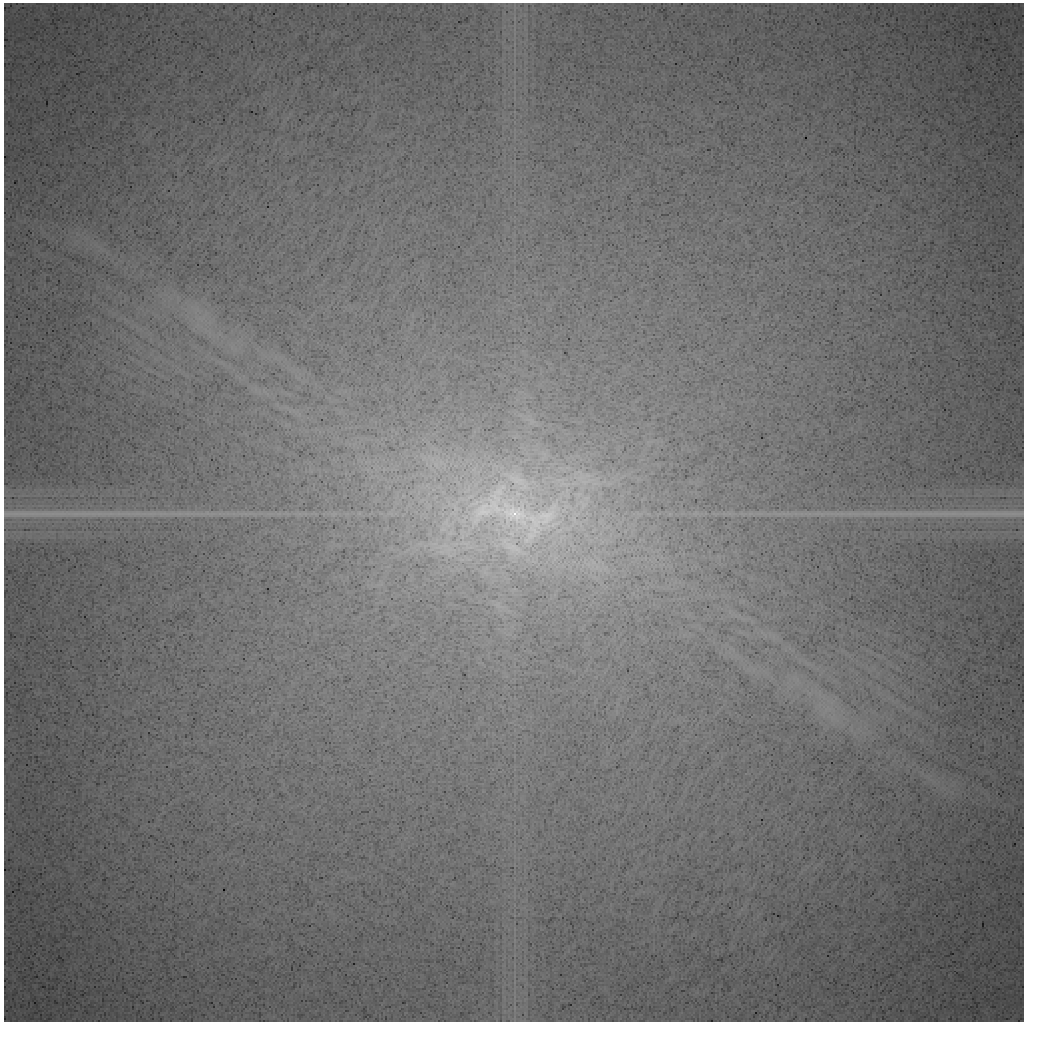
This is the magnitude transform of the cheetah pic



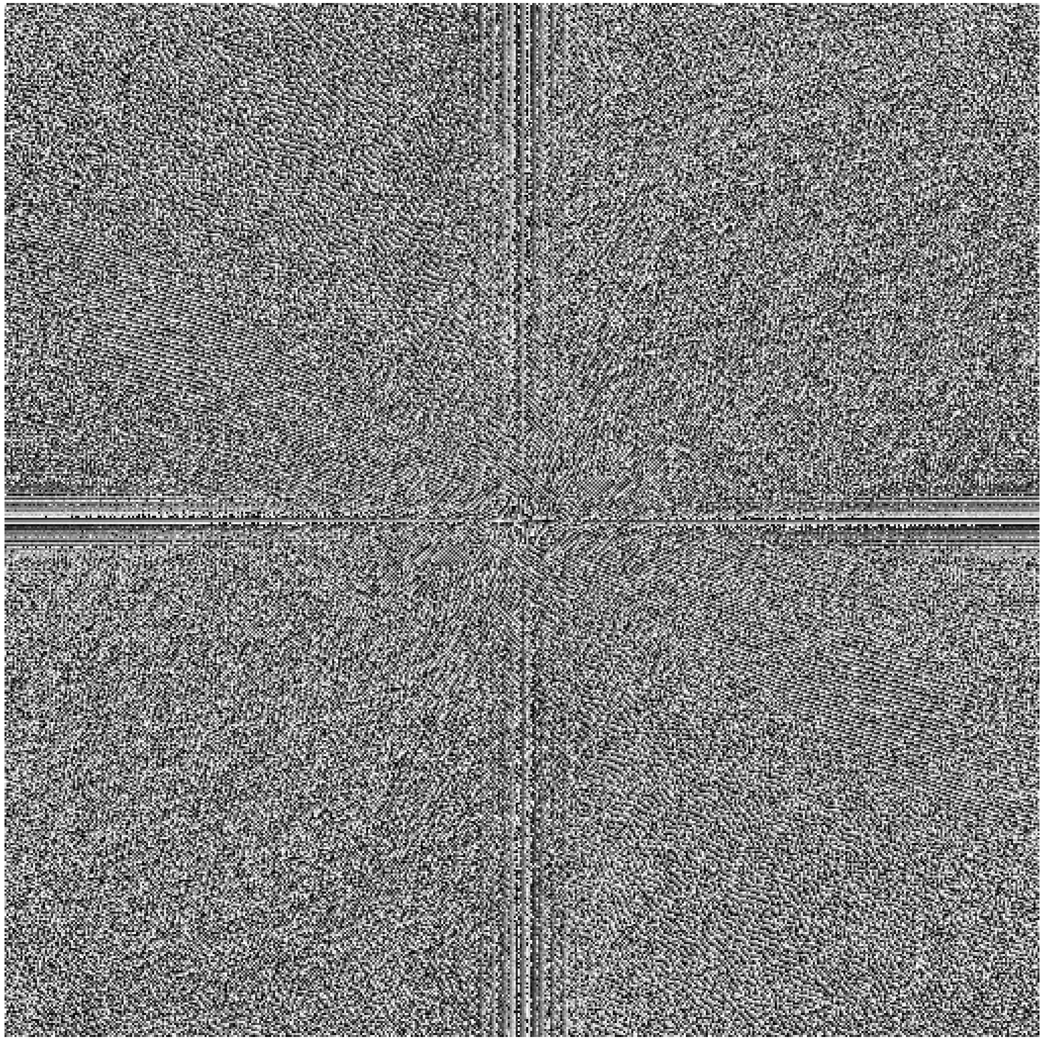
This is the phase transform of the cheetah pic



This is the magnitude transform of the zebra pic

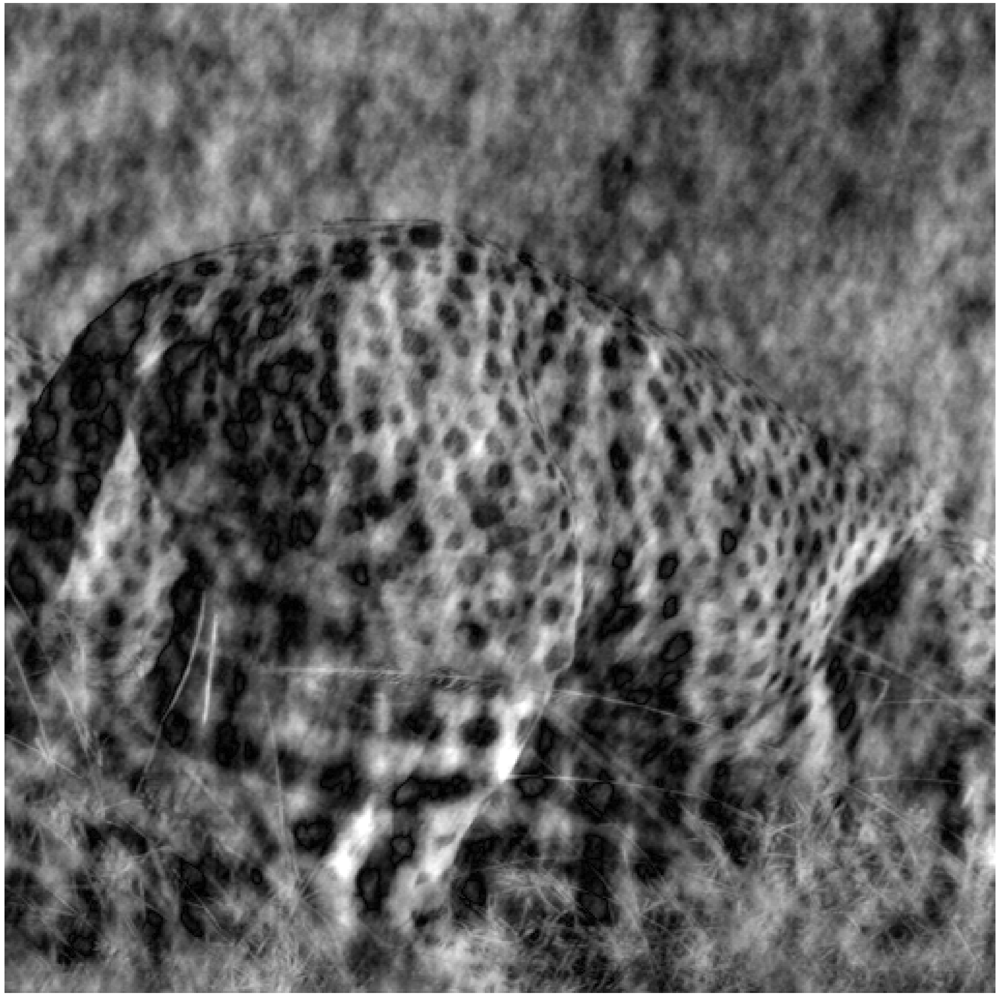


This is the phase transform of the zebra pic



Reconstruction with zebra phase, cheetah magnitude

Reconstruction with cheetah phase, zebra magnitude



Phase and Magnitude

Image with cheetah phase (and zebra magnitude)

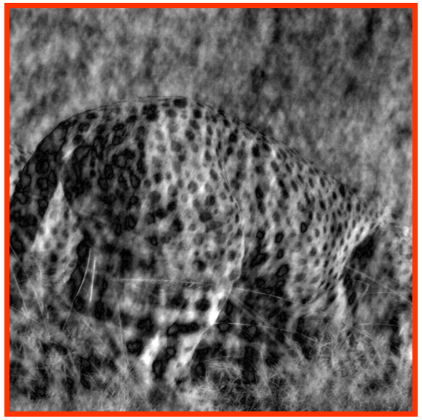
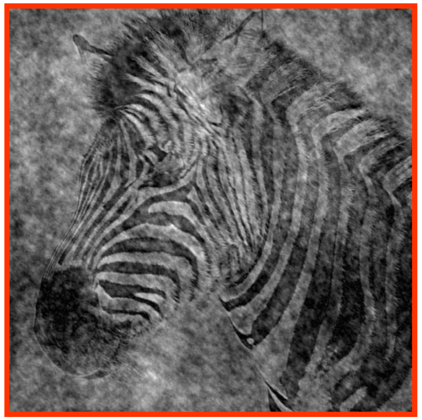
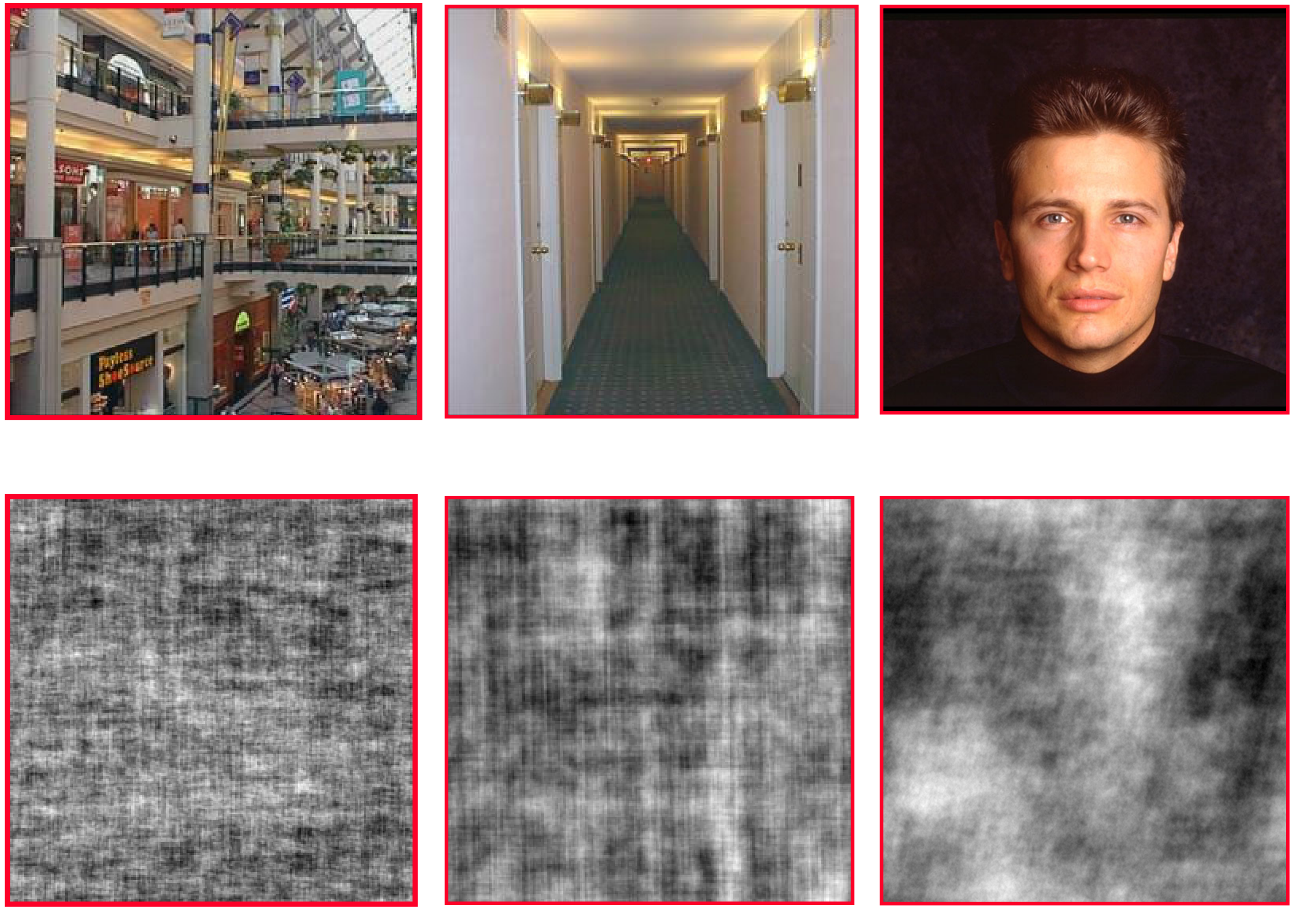


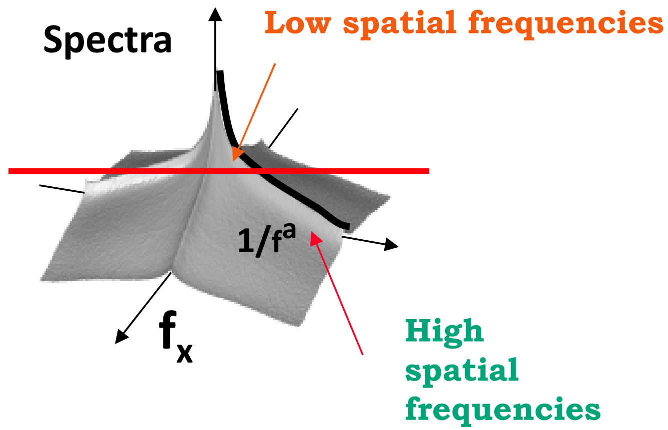
Image with zebra phase (and cheetah magnitude)

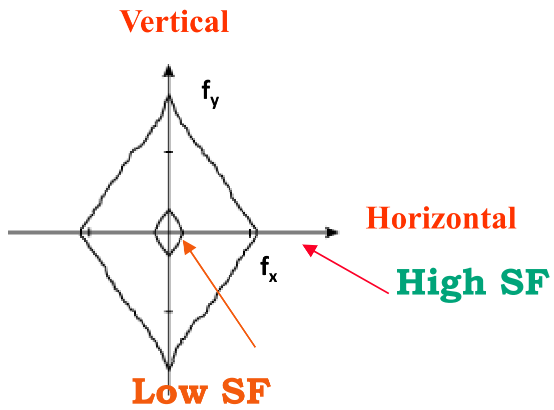


Randomizing the phase



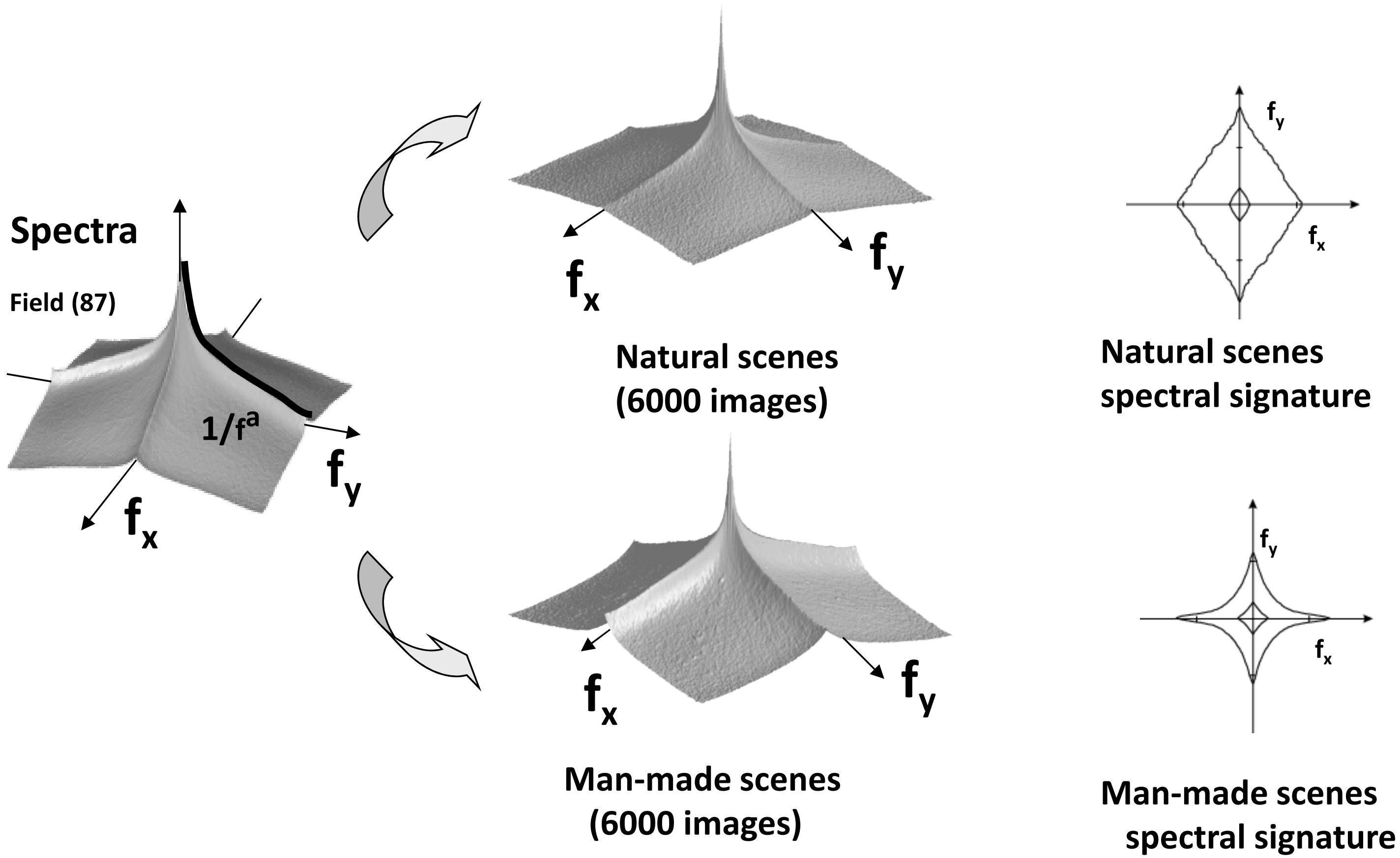
Fourier Characteristics of Natural Images





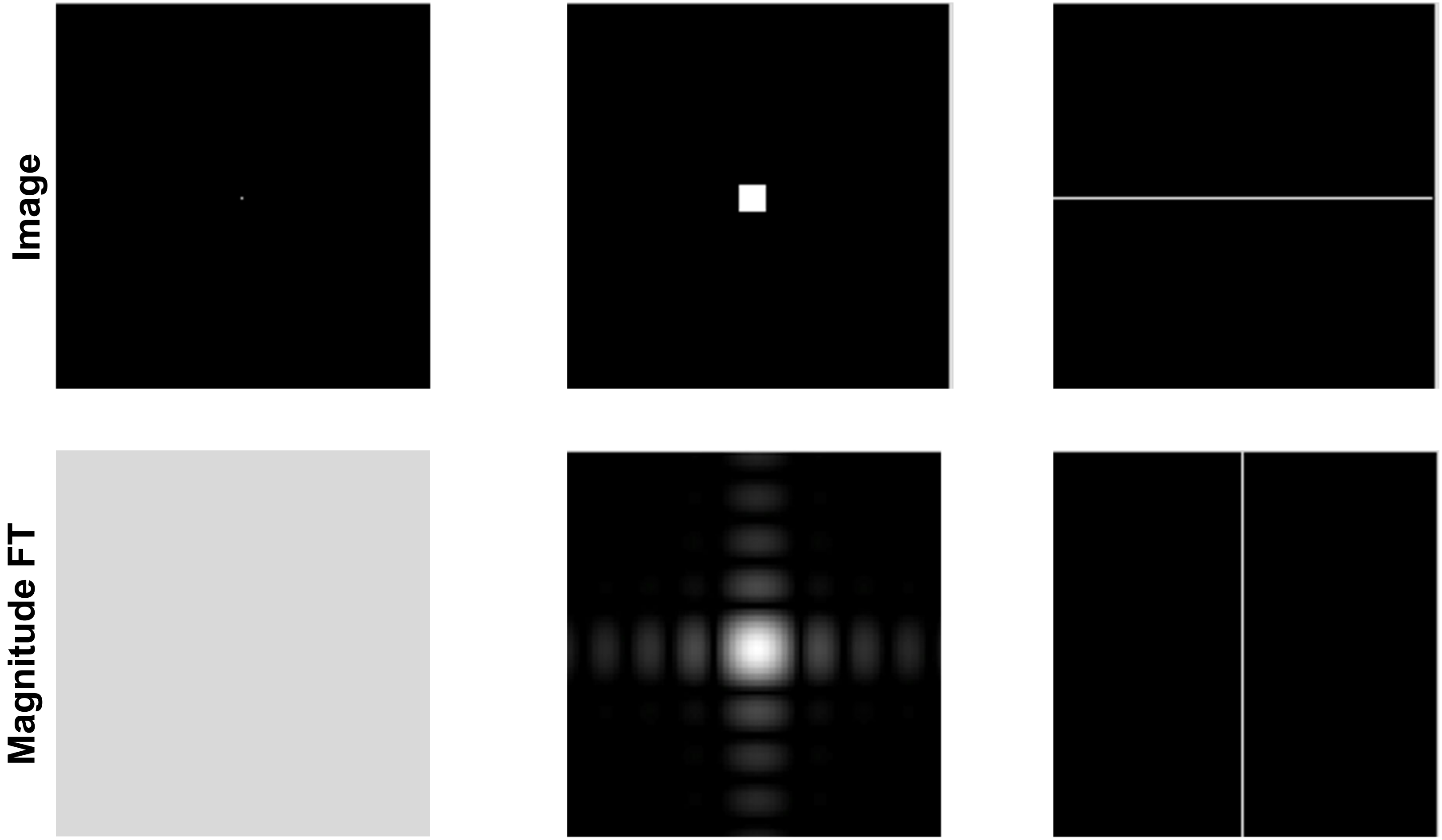
Torralba and Oliva, *Statistics of Natural Image Categories*. Network: Computation in Neural Systems 14 (2003) 391-412.

Power Spectrum of Images

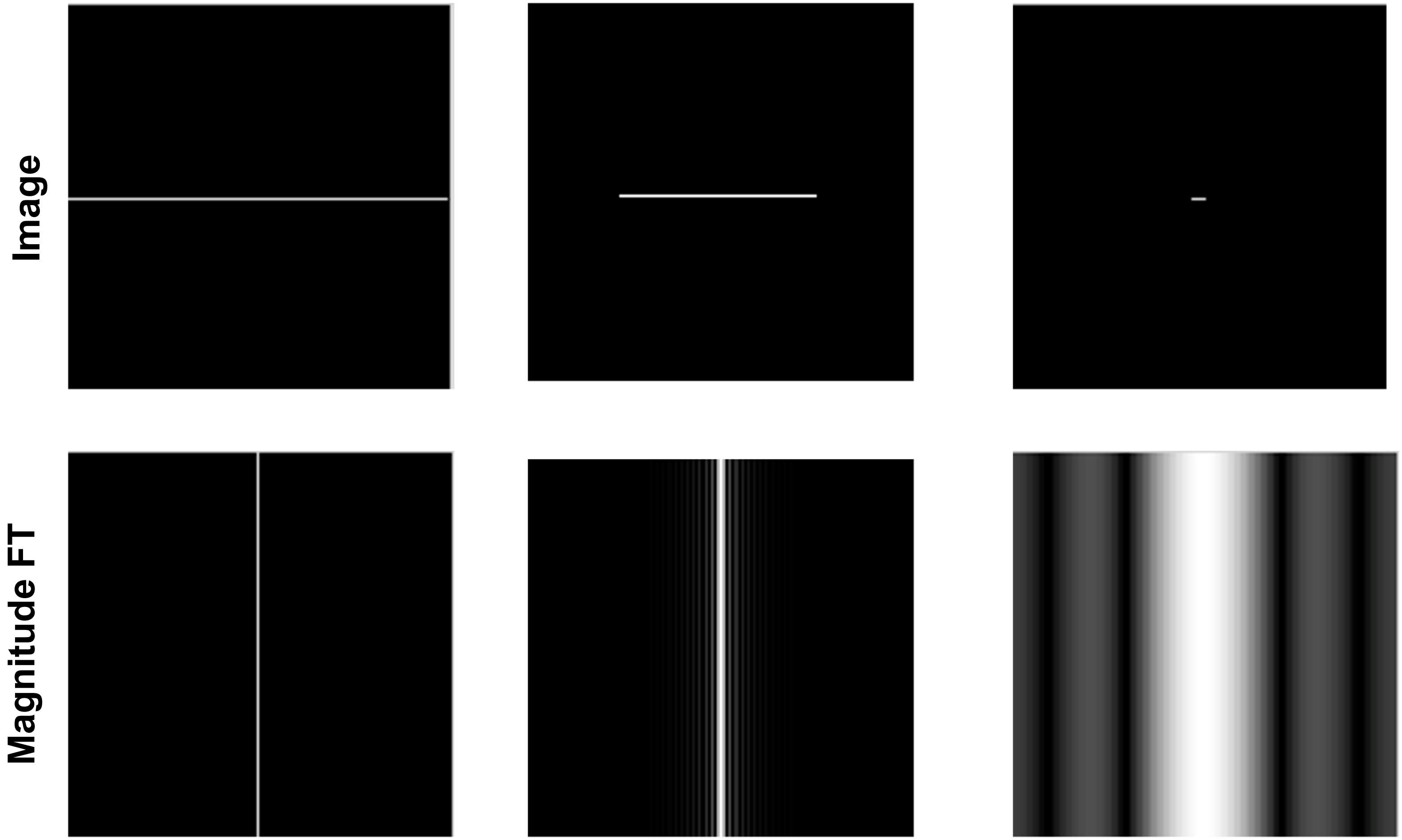


Torralba and Oliva, *Statistics of Natural Image Categories*. Network: Computation in Neural Systems 14 (2003) 391-412.

Some important Fourier Transforms

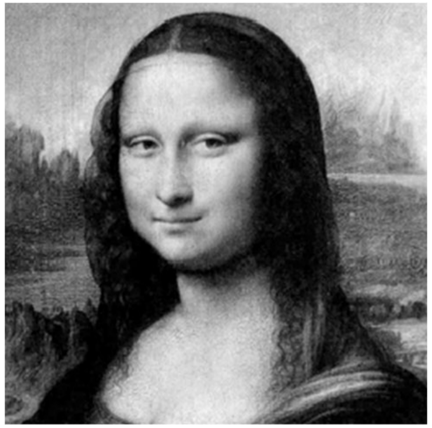


Some important Fourier Transforms



The Fourier Transform of some important images

Image



Log(1+Magnitude FT)