Coffee Model April 2019

How long will it take for my ideal coffee to cool down. Use law of cooling T' = k(T(t)-Tinf) ; T Temperature and Tinf ambient temperature

```
clear;clc; syms t k T(t) Tinf b T0 Tsol(t);

% The Basic System
diff(T)+k*T == k*Tinf
```

```
ans(t) =
```

$$\frac{\partial}{\partial t} T(t) + k\,T(t) = \mathrm{Tinf}\,k$$

```
dsolve( diff(T)+k*T == k*Tinf, T(0)==T0)
```

```
ans = Tinf + e^(-k*t)*(T0 - Tinf)
```

$$\mathrm{Tinf} + \mathrm{e}^{-k\,t}\,(T_0 - \mathrm{Tinf})$$

```
% The coffee's initially 95 degrees and the ambient temperature's 25 degrees
T0 = 90; Tinf = 30;
Tsol(t) = dsolve( diff(T)+k*T == k*Tinf, T(0)==T0)
```

```
Tsol(t) = 60*exp(-k*t) + 30
```

$$60\,\mathrm{e}^{-k\,t} + 30$$

```
% Solve for k with another datapoint T(1/6)=48
Tsol(t) = dsolve( diff(T)+k*T == k*Tinf, T(0)==T0)
```

```
Tsol(t) = 60*exp(-k*t) + 30
```

$$60\,\mathrm{e}^{-k\,t} + 30$$

```
% If the ambient temperature oscillates it gets crazy
T0 = 95; Tinf = 7*sin(pi*t/12)+25;
Tsol(t) = dsolve( diff(T)+k*T == k*Tinf, T(0)==T0)
```

```
Tsol(t) =
```

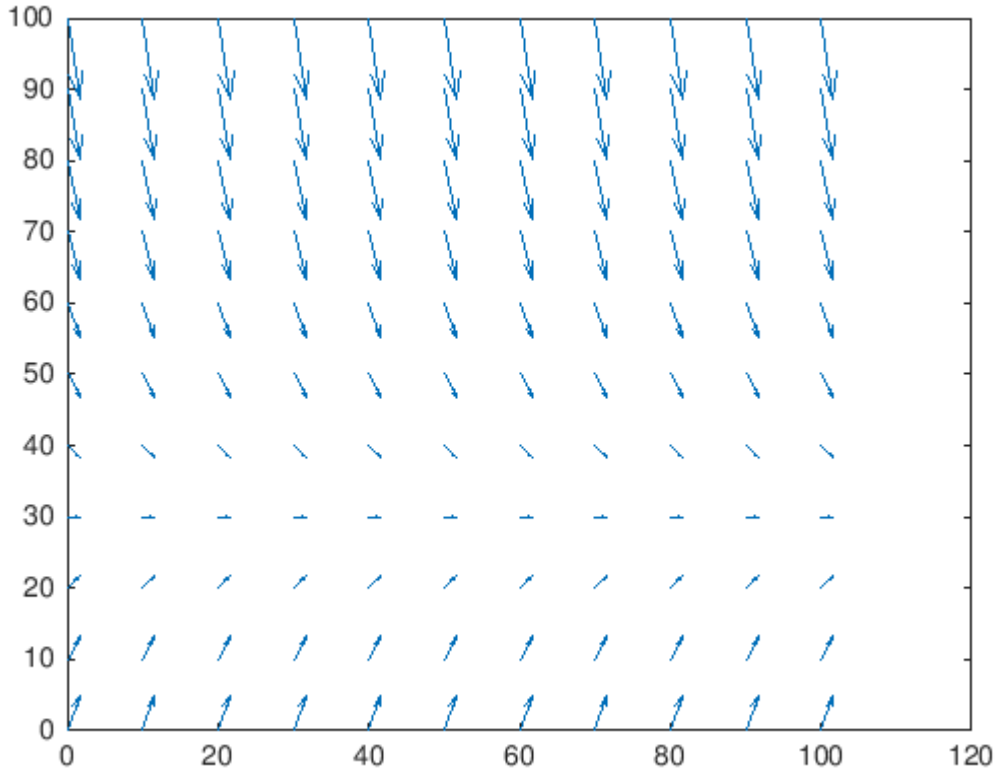
$$\frac{25\,\pi^2 + 1008\,k^2 \sin\left(\frac{\pi\,t}{12}\right) + 3600\,k^2 - 84\,\pi\,k\cos\left(\frac{\pi\,t}{12}\right)}{\sigma_1} + \frac{\mathrm{e}^{-k\,t}\,(10080\,k^2 + 84\,\pi\,k + 70\,\pi^2)}{\sigma_1}$$

where

$$\sigma_1 = 144\,k^2 + \pi^2$$

```
% If you're too lazy to find a solution, or
% I don't want one position I want all positions

[t, T] = meshgrid(linspace(0,100,11),linspace(0,100,11));
k = 0.1; %-6*log(3/10); % modified so it looks nicer
T0 = 90; Tinf = 30;
figure();
quiver(t, T, ones(length(t)), -k*(T-Tinf))
```



Plot Solution with forcing function

```
clear;clc; syms t k T0 T(t) Tinf(t) ;
digits(4)

% k has been changed for a nicer plot
T0 = 90; Tinf(t) = 7*sin(pi*t/12)+25; k = 0.1; % should be vpa(-6*log(3/10));
Tsol(t) = dsolve( diff(T)+k*T == k*Tinf, T(0)==T0)
```

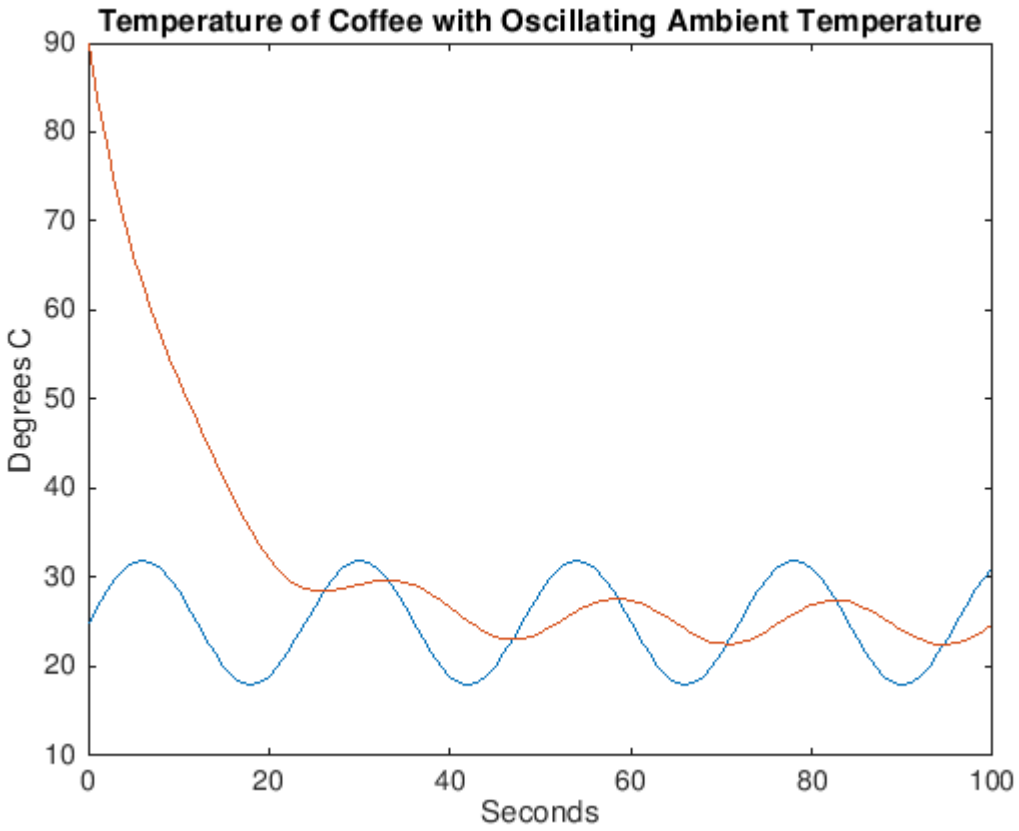
```
Tsol(t) =
```

$$e^{-\frac{t}{10}}\left(\frac{210\,\pi}{25\,\pi^2+36}+65\right)+e^{-\frac{t}{10}}\left(25\,e^{t/10}+\frac{42\,e^{t/10}\left(6\sin\left(\frac{\pi\,t}{12}\right)-5\,\pi\cos\left(\frac{\pi\,t}{12}\right)\right)}{25\,\pi^2+36}\right)$$

```
T = linspace(0,100,101); Temps = []; Ambs = [];
for t = T
    Ambs = [Ambs; Tinf(t)];
    Temps = [Temps; Tsol(t)];

end
plot(T, Ambs, T, Temps);
```

```
title("Temperature of Coffee with Oscillating Ambient Temperature");
xlabel("Seconds"); ylabel("Degrees C");
```



References

```
% MATLAB Online R2019a
% https://matlab.mathworks.com/
%
% (18) Symbolic Mathematics in Matlab - YouTube
% https://www.youtube.com/watch?v=Anrm7B6yD18
%
% notes - Online LaTeX Editor Overleaf
% https://www.overleaf.com/project/5cb3f98ee51a152e6d1dd2bb
%
% How do I get pretty symbolic expressions in MATLAB using fancy? - MATLAB Answers - MA
% https://au.mathworks.com/matlabcentral/answers/650-how-do-i-get-pretty-symbolic-expre
%
% How to display answers in original form, rather than automatically in pretty() form ?
% https://au.mathworks.com/matlabcentral/answers/262113-how-to-display-answers-in-origi
%
% Pretty Equation Viewer - File Exchange - MATLAB Central
% https://au.mathworks.com/matlabcentral/fileexchange/55477-pretty-equation-viewer
```

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%
% Polynomial Regression filter implementation - MATLAB Answers - MATLAB Central
% https://au.mathworks.com/matlabcentral/answers/285570-polynomial-regression-filter-im
%
% 8 Handy MATLAB Shortcuts That Will Save You a Ton of Time
% https://interestingengineering.com/8-handy-matlab-shortcuts-that-will-save-you-a-ton-
```