Online Appendix to: New fertility patterns: The role of human versus physical capital

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1 Empirical Analysis

To run the panel regressions we first built a data set containing the ASFRs (four age intervals, $i = 20-24, 25-29, 30-34, 35-39$), the GDP per capita (measured in PPP, 2015 US \$), and the average years of schooling for women for 32 European countries indexed by c and listed in footnote 1[1] over 14 periods of 5-year interval indexed by t, covering the whole range of 1950-2015 years. Indeed, from the data set on educational attainment for women provided by Barro and Lee (2013), we have available the average years of schooling for women aged of 20-24, 25-29, 30-34 and 35-39 years old, every five years from 1950. To build consistent observations for the dependent variable, we comput a 5-year rolling mean for both ASFRs and GDP variables, each corresponding to the considered year in the Barro and Lee (2013) data set so

[1]Our main source is the United Nations 2024 World Population Prospect database, which covers the period 1950-2023 and provides age-specific fertility rates for European countries among others. ASFRs per 1000 women are measured as the number of births to women in a particular single age, divided by the number of women in that age, multiplied by 1000. https://population.un.org/wpp/Download/Standard/Fertility/. For the sake of clarity, we choose to report data for four cohorts for four wide European areas, namely Northern (including Denmark, Estonia, Finland, Iceland, Ireland, Latvia, Lithuania, Norway, Sweden, United Kingdom), Western (including Austria, Belgium, France, Germany, Luxembourg, Netherlands, Switzerland), Southern (including Albania, Croatia, Greece, Italy, Portugal, Serbia, Slovenia, Spain) and Eastern Europe (including Bulgaria, Czech Republic, Hungary, Poland, Romania, Slovakia, Ukraine).

that, for each regression we have run, we have 249 observations. We perform three regression models, for which results are reported in Tables 1 and 2 below:

	20-24 (1)	**20-24 (2)**	**20-24 (3)**	**25-29 (1)**	**25-29 (2)**	**25-29 (3)**
Dep. Var.	ASFR	ASFR	ASFR	ASFR	ASFR	ASFR
Estimator	RandomEff.	PanelOLS	PanelOLS	RandomEff.	PanelOLS	PanelOLS
Obs.	249	249	249	249	249	249
Cov. Est.	Unadjusted	Unadjusted	Unadjusted	Unadjusted	Unadjusted	Unadjusted
R-squared	0.6655	0.7244	0.0700	0.5440	0.5910	0.1098
R-Squared (Within)	0.7138	0.7244	-0.2494	0.5869	0.5910	0.3513
R-Squared (Between)	-0.2659	-0.8910	-0.5123	0.1049	-0.0586	0.2185
R-Squared (Overall)	0.4810	0.3639	-0.3129	0.3272	0.2774	0.2707
F-statistic	162.48	187.53	5.1146	97.422	103.09	8.3843
P-value (F-stat)	0.0000	0.0000	0.0020	0.0000	0.0000	0.0000
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const	215.73	228.20	56.412	254.86	261.68	198.30
	(9.9396)	(10.904)	(2.9124)	(16.715)	(17.615)	(8.4566)
yr_sch	-5.4540	-7.1542	4.5259	-18.734	-19.702	-13.922
	(-1.1453)	(-1.5364)	(1.3106)	(-5.6913)	(-5.9841)	(-3.5600)
yr_sch_sq	-0.3821	-0.2281	-0.2696	0.5055	0.5989	0.5022
	(-1.4824)	(-0.8847)	(-1.4891)	(2.8136)	(3.2695)	(2.4889)
GDP_cap	-0.0010	-0.0015	0.0006	-8.474e-05	-0.0003	0.0002
	(-8.4997)	(-9.0336)	(3.5723)	(-0.6874)	(-1.9749)	(1.1612)
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Effects		Entity	Entity Time		Entity	Entity Time

T-stats reported in parentheses

Table 1: Model Comparison

	30-34 (1)	30-34 (2)	30-34 (3)	35-39 (1)	35-39 (2)	35-39 (3)
Dep. Var.	ASFR	ASFR	ASFR	ASFR	ASFR	ASFR
Estimator	RandomEff.	PanelOLS	PanelOLS	RandomEff.	PanelOLS	PanelOLS
Obs.	249	249	249	249	249	249
Cov. Est.	Unadjusted	Unadjusted	Unadjusted	Unadjusted	Unadjusted	Unadjusted
R-squared	0.3323	0.3614	0.2201	0.3946	0.4362	0.3008
R-Squared (Within)	0.3597	0.3614	0.2135	0.4321	0.4362	0.3528
R-Squared (Between)	0.2890	0.2342	0.3067	0.1572	0.0116	0.2283
R-Squared (Overall)	0.2661	0.2643	0.2149	0.2633	0.2389	0.2420
F-statistic	40.648	40.362	19.191	53.240	55.197	29.256
P-value (F-stat)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
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const	188.64	192.66	170.75	118.42	121.55	83.201
	(14.376)	(14.934)	(8.8581)	(15.130)	(15.977)	(7.7878)
yr_sch	-27.400	-28.085	-18.310	-21.420	-22.036	-12.322
	(-9.5074)	(-9.6032)	(-5.8235)	(-12.063)	(-12.290)	(-6.9262)
yr_sch_sq	1.4122	1.4808	0.8073	1.1543	1.2244	0.6318
	(8.8436)	(8.9144)	(4.9385)	(11.401)	(11.623)	(6.5210)
GDP_cap	0.0007	0.0006	0.0004	0.0003	0.0002	0.0004
	(5.8235)	(3.6962)	(2.4382)	(4.4220)	(2.1638)	(3.6135)
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Effects		Entity	Entity Time		Entity	Entity Time

T-stats reported in parentheses

Table 2: Model Comparison

2 Additional Details about the Quantitative Analysis

From equations (33)-(34) and (37)-(38), we (implicitly) define the perfect-foresight intertemporal equilibrium:

$$\begin{array}{ll} g^L(z_{t+1}, z_t, z_{t-1}, n_{t-1}, n_{t-2}, A^j) = 0 & \text{if } w_t < \tilde{w} \\ g^H(w_t, w_{t-1}, w_{t-2}, n_{t-1}, n_{t-2}, A^j) = 0 & \text{else} \end{array} \tag{1}$$

In order to obtain a one-period lag system, we add $\hat{w}_t = w_{t-1}$ and $\hat{z}_t = z_{t-1}$ to consider $\hat{g}^j(x_t^j, x_{t-1}^j, A^j) = 0$ for $j = L, H$, where $x_t^H = [n_{t-1}, w_t, \hat{w}_t]$ and $x_t^L = [n_{t-1}, z_t, \hat{z}_t]$.

Using our calibration in Section 7, we compute the steady state by solving $\hat{g}^j(x^j, x^j, A^j) = 0$ with x^j the steady state solutions.

To study the local stability in the neighborhood of the steady state in both regime, the vectorized linear model writes:

$$C^j dx_t^j = B^j dx_{t-1}^j \tag{2}$$

with $C \equiv \frac{\partial \hat{g}^j(x^j, x^j, A^j)}{\partial x_t}$ and $B \equiv -\frac{\partial \hat{g}^j(x^j, x^j, A^j)}{\partial x_{t-1}}$.

From (2), we derive the eigenvalues. Based on our calibration in Section 7, we obtain Figures **??**. As stated after Propositions 2 and 3, the steady state is locally a saddle in the low-income regime and it is a sink in the high-income regime steady state.

We can perform a comparative statics exercise by varying A^j (e.g. a 10 per cent increase). Figure 7 replicates the results stated in Corollaries 1 and 2.

To solve for the transition dynamics between the low-income and the high-income regime, we use the extended-path method.[2] In system (1), we allow A to be a time-dependent exogenous process $A = A_t$, $t = 0, 1, .., T$. We also substitute $z_t = \frac{w_t^{\frac{1}{\alpha}}}{w_{t-1}}$ in the low-income regime. Taking these changes into account, an equilibrium path satisfies:

$$\begin{array}{ll} g^L(w_{t+1}, w_t, w_{t-1}, w_{t-2}, n_{t-1}, n_{t-2}, A_t, A_{t-1}) = 0 & \text{if } w_t < \tilde{w} \\ g^H(w_t, w_{t-1}, w_{t-2}, n_{t-1}, n_{t-2}, A_t) = 0 & \text{else} \end{array} \tag{3}$$

[2]See Judd (1998) for further details on the extended-path method.

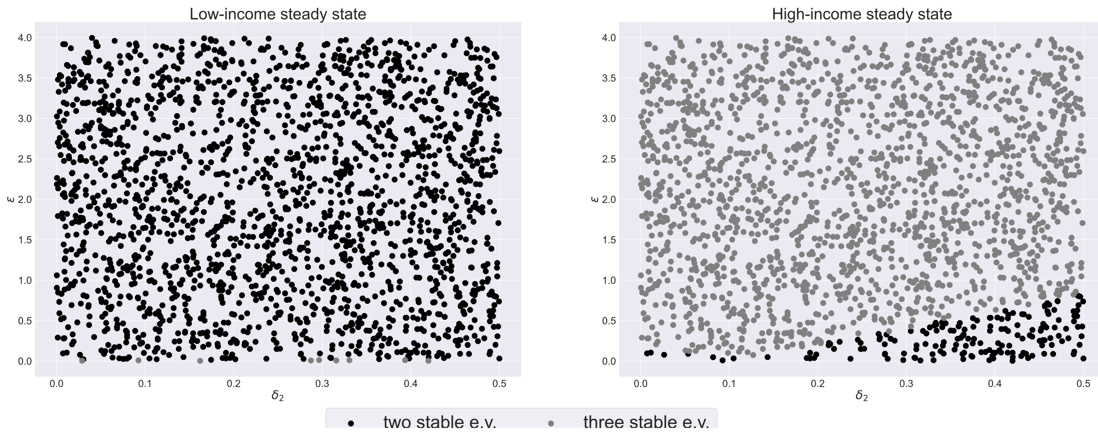

Figure 1: The effects of a productivity shock on stationary wages and population growth factors.

for $t = 0, 1, .., T$. The extended-path method defines this problem as a $2 \times (T + 1)$ system of non-linear equations with given initial and terminal conditions n_{-1} and w_T, and exogenous path of A_t solved by vectors $\mathbf{w} = (w_0, w_1, ..., w_{T-1})$ and $\mathbf{n} = (n_0, n_1, ..., n_{T-1})$ satisfying (3).

References

Barro, R. J. and Lee, J. W. (2013). A new data set of educational attainment in the world, 1950–2010. *Journal of Development Economics*, 104:184–198.

Judd, K. L. (1998). *Numerical Methods in Economics*, volume 1 of *MIT Press Books*. The MIT Press.