Faraday_Amp_Moments_Pop_test

January 1, 2026

[12]:
```
import subprocess, re
import numpy as np
import matplotlib.pyplot as plt

def run(exe):
    out = subprocess.check_output([exe], text=True)
    print(out)
    return out

out_far = run("./build/tests/faraday/test-faraday")
out_amp = run("./build/tests/ampere/test-ampere")
out_mom = run("./build/tests/moments/test-moments")
```

```
Testing Faraday's law implementation in 1D…

i=1     B.y = 0.00998334        Expected: 0.00998334    Error: 1.73472e-18
        i=1     B.z = 0.000499583       Expected: 0.000499583   Error: 0
i=2     B.y = 0.00988359        Expected: 0.00988359    Error: 1.73472e-18
        i=2     B.z = 0.00149376        Expected: 0.00149376    Error: 0
i=3     B.y = 0.00968509        Expected: 0.00968509    Error: 0
        i=3     B.z = 0.00247301        Expected: 0.00247301    Error: 0
i=4     B.y = 0.00938981        Expected: 0.00938981    Error: 0
        i=4     B.z = 0.00342755        Expected: 0.00342755    Error: 0
i=5     B.y = 0.00900072        Expected: 0.00900072    Error: 1.04083e-17
        i=5     B.z = 0.00434784        Expected: 0.00434784    Error:
1.04083e-17
i=6     B.y = 0.00852169        Expected: 0.00852169    Error: 1.73472e-18
        i=6     B.z = 0.00522469        Expected: 0.00522469    Error:
8.67362e-19
i=7     B.y = 0.00795752        Expected: 0.00795752    Error: 0
        i=7     B.z = 0.00604934        Expected: 0.00604934    Error:
8.67362e-19
i=8     B.y = 0.00731384        Expected: 0.00731384    Error: 8.67362e-19
        i=8     B.z = 0.00681355        Expected: 0.00681355    Error: 0
i=9     B.y = 0.00659708        Expected: 0.00659708    Error: 0
        i=9     B.z = 0.00750967        Expected: 0.00750967    Error:
8.67362e-19
i=10    B.y = 0.00581441        Expected: 0.00581441    Error: 8.67362e-19
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        i=10    B.z = 0.00813077        Expected: 0.00813077    Error: 0
i=11    B.y = 0.00497364        Expected: 0.00497364    Error: 0
        i=11    B.z = 0.00867062        Expected: 0.00867062    Error:
1.73472e-18
i=12    B.y = 0.00408317        Expected: 0.00408317    Error: 1.12757e-17
        i=12    B.z = 0.00912384        Expected: 0.00912384    Error:
2.25514e-17
i=13    B.y = 0.00315191        Expected: 0.00315191    Error: 0
        i=13    B.z = 0.00948589        Expected: 0.00948589    Error: 0
i=14    B.y = 0.00218915        Expected: 0.00218915    Error: 1.12757e-17
        i=14    B.z = 0.00975317        Expected: 0.00975317    Error:
2.25514e-17
i=15    B.y = 0.00120453        Expected: 0.00120453    Error: 0
        i=15    B.z = 0.00992299        Expected: 0.00992299    Error: 0
i=16    B.y = 0.000207862       Expected: 0.000207862   Error: 0
        i=16    B.z = 0.00999367        Expected: 0.00999367    Error: 0
i=17    B.y = -0.000790879      Expected: -0.000790879  Error: 1.0842e-19
        i=17    B.z = 0.0099645 Expected: 0.0099645     Error: 2.08167e-17
i=18    B.y = -0.00178172       Expected: -0.00178172   Error: 2.1684e-19
        i=18    B.z = 0.00983576        Expected: 0.00983576    Error: 0
i=19    B.y = -0.00275475       Expected: -0.00275475   Error: 0
        i=19    B.z = 0.00960875        Expected: 0.00960875    Error:
2.25514e-17
i=20    B.y = -0.00370027       Expected: -0.00370027   Error: 0
        i=20    B.z = 0.00928573        Expected: 0.00928573    Error: 0
i=21    B.y = -0.00460881       Expected: -0.00460881   Error: 8.67362e-19
        i=21    B.z = 0.00886993        Expected: 0.00886993    Error: 0
i=22    B.y = -0.0054713        Expected: -0.0054713    Error: 0
        i=22    B.z = 0.0083655 Expected: 0.0083655     Error: 0
i=23    B.y = -0.00627912       Expected: -0.00627912   Error: 8.67362e-19
        i=23    B.z = 0.00777749        Expected: 0.00777749    Error: 0
i=24    B.y = -0.0070242        Expected: -0.0070242    Error: 3.38271e-17
        i=24    B.z = 0.00711177        Expected: 0.00711177    Error:
3.38271e-17
i=25    B.y = -0.0076991        Expected: -0.0076991    Error: 0
        i=25    B.z = 0.00637499        Expected: 0.00637499    Error: 0
i=26    B.y = -0.00829708       Expected: -0.00829708   Error: 0
        i=26    B.z = 0.00557451        Expected: 0.00557451    Error: 0
i=27    B.y = -0.00881215       Expected: -0.00881215   Error: 0
        i=27    B.z = 0.00471834        Expected: 0.00471834    Error:
8.67362e-19
i=28    B.y = -0.00923917       Expected: -0.00923917   Error: 0
        i=28    B.z = 0.00381502        Expected: 0.00381502    Error:
4.33681e-19
i=29    B.y = -0.00957388       Expected: -0.00957388   Error: 4.33681e-17
        i=29    B.z = 0.00287358        Expected: 0.00287358    Error:
1.12757e-17
i=30    B.y = -0.00981293       Expected: -0.00981293   Error: 1.73472e-18
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        i=30    B.z = 0.00190343        Expected: 0.00190343    Error: 0
i=31    B.y = -0.00995393       Expected: -0.00995393   Error: 0
        i=31    B.z = 0.000914265       Expected: 0.000914265   Error: 0
i=32    B.y = -0.00999548       Expected: -0.00999548   Error: 0
        i=32    B.z = -8.40374e-05      Expected: -8.40374e-05  Error: 0
i=33    B.y = -0.00993716       Expected: -0.00993716   Error: 0
        i=33    B.z = -0.0010815        Expected: -0.0010815    Error:
2.1684e-19
i=34    B.y = -0.00977954       Expected: -0.00977954   Error: 4.33681e-17
        i=34    B.z = -0.00206816       Expected: -0.00206816   Error:
2.25514e-17
i=35    B.y = -0.00952421       Expected: -0.00952421   Error: 0
        i=35    B.z = -0.00303415       Expected: -0.00303415   Error:
4.33681e-19
i=36    B.y = -0.00917372       Expected: -0.00917372   Error: 0
        i=36    B.z = -0.00396983       Expected: -0.00396983   Error: 0
i=37    B.y = -0.00873157       Expected: -0.00873157   Error: 0
        i=37    B.z = -0.00486584       Expected: -0.00486584   Error:
8.67362e-19
i=38    B.y = -0.00820218       Expected: -0.00820218   Error: 0
        i=38    B.z = -0.00571323       Expected: -0.00571323   Error: 0
i=39    B.y = -0.00759083       Expected: -0.00759083   Error: 3.29597e-17
        i=39    B.z = -0.00650354       Expected: -0.00650354   Error:
3.29597e-17
i=40    B.y = -0.00690363       Expected: -0.00690363   Error: 6.59195e-17
        i=40    B.z = -0.00722887       Expected: -0.00722887   Error:
6.67869e-17
i=41    B.y = -0.00614746       Expected: -0.00614746   Error: 0
        i=41    B.z = -0.00788197       Expected: -0.00788197   Error: 0
i=42    B.y = -0.00532987       Expected: -0.00532987   Error: 8.67362e-19
        i=42    B.z = -0.00845631       Expected: -0.00845631   Error: 0
i=43    B.y = -0.00445902       Expected: -0.00445902   Error: 3.29597e-17
        i=43    B.z = -0.00894616       Expected: -0.00894616   Error:
8.32667e-17
i=44    B.y = -0.00354361       Expected: -0.00354361   Error: 0
        i=44    B.z = -0.00934663       Expected: -0.00934663   Error: 0
i=45    B.y = -0.0025928        Expected: -0.0025928    Error: 2.1684e-17
        i=45    B.z = -0.00965371       Expected: -0.00965371   Error:
8.50015e-17
i=46    B.y = -0.00161609       Expected: -0.00161609   Error: 2.1684e-19
        i=46    B.z = -0.00986433       Expected: -0.00986433   Error: 0
i=47    B.y = -0.000623225      Expected: -0.000623225  Error: 1.09504e-17
        i=47    B.z = -0.00997639       Expected: -0.00997639   Error:
8.84709e-17
i=48    B.y = 0.000375865       Expected: 0.000375865   Error: 0
        i=48    B.z = -0.00998876       Expected: -0.00998876   Error:
1.73472e-18
i=49    B.y = 0.0013712 Expected: 0.0013712     Error: 2.1684e-19
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        i=49    B.z = -0.00990134       Expected: -0.00990134   Error: 0
i=50    B.y = 0.00235283        Expected: 0.00235283    Error: 2.21177e-17
        i=50    B.z = -0.00971498       Expected: -0.00971498   Error:
8.67362e-17
i=51    B.y = 0.00331096        Expected: 0.00331096    Error: 4.33681e-19
        i=51    B.z = -0.00943156       Expected: -0.00943156   Error: 0
i=52    B.y = 0.004236  Expected: 0.004236      Error: 3.38271e-17
        i=52    B.z = -0.00905389       Expected: -0.00905389   Error:
7.63278e-17
i=53    B.y = 0.00511872        Expected: 0.00511872    Error: 0
        i=53    B.z = -0.00858577       Expected: -0.00858577   Error: 0
i=54    B.y = 0.0059503 Expected: 0.0059503     Error: 0
        i=54    B.z = -0.00803185       Expected: -0.00803185   Error: 0
i=55    B.y = 0.00672242        Expected: 0.00672242    Error: 6.59195e-17
        i=55    B.z = -0.00739769       Expected: -0.00739769   Error:
6.59195e-17
i=56    B.y = 0.00742737        Expected: 0.00742737    Error: 0
        i=56    B.z = -0.00668961       Expected: -0.00668961   Error:
8.67362e-19
i=57    B.y = 0.00805811        Expected: 0.00805811    Error: 6.59195e-17
        i=57    B.z = -0.00591469       Expected: -0.00591469   Error:
4.42354e-17
i=58    B.y = 0.00860834        Expected: 0.00860834    Error: 0
        i=58    B.z = -0.00508067       Expected: -0.00508067   Error:
8.67362e-19
i=59    B.y = 0.00907255        Expected: 0.00907255    Error: 1.73472e-18
        i=59    B.z = -0.00419589       Expected: -0.00419589   Error: 0
i=60    B.y = 0.00944612        Expected: 0.00944612    Error: 8.1532e-17
        i=60    B.z = -0.00326919       Expected: -0.00326919   Error:
3.33934e-17
i=61    B.y = 0.0097253 Expected: 0.0097253     Error: 1.73472e-18
        i=61    B.z = -0.00230982       Expected: -0.00230982   Error:
4.33681e-19
i=62    B.y = 0.00990731        Expected: 0.00990731    Error: 8.67362e-17
        i=62    B.z = -0.00132737       Expected: -0.00132737   Error:
1.10589e-17
i=63    B.y = 0.00999033        Expected: 0.00999033    Error: 0
        i=63    B.z = -0.000331654      Expected: -0.000331654  Error: 0
i=64    B.y = 0.00997353        Expected: 0.00997353    Error: 1.73472e-18
        i=64    B.z = 0.000667372       Expected: 0.000667372   Error: 0
i=65    B.y = 0.00985708        Expected: 0.00985708    Error: 8.84709e-17
        i=65    B.z = 0.00165973        Expected: 0.00165973    Error:
2.21177e-17
i=66    B.y = 0.00964214        Expected: 0.00964214    Error: 1.73472e-18
        i=66    B.z = 0.0026355 Expected: 0.0026355     Error: 0
i=67    B.y = 0.00933086        Expected: 0.00933086    Error: 8.1532e-17
        i=67    B.z = 0.00358494        Expected: 0.00358494    Error:
4.42354e-17
```

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i=68    B.y = 0.00892634        Expected: 0.00892634    Error: 1.73472e-18
        i=68    B.z = 0.00449857        Expected: 0.00449857    Error: 0
i=69    B.y = 0.00843264        Expected: 0.00843264    Error: 1.73472e-18
        i=69    B.z = 0.00536724        Expected: 0.00536724    Error:
8.67362e-19
i=70    B.y = 0.00785468        Expected: 0.00785468    Error: 6.59195e-17
        i=70    B.z = 0.00618228        Expected: 0.00618228    Error:
5.55112e-17
i=71    B.y = 0.00719824        Expected: 0.00719824    Error: 0
        i=71    B.z = 0.00693556        Expected: 0.00693556    Error: 0
i=72    B.y = 0.00646988        Expected: 0.00646988    Error: 5.46438e-17
        i=72    B.z = 0.00761954        Expected: 0.00761954    Error:
7.71952e-17
i=73    B.y = 0.00567688        Expected: 0.00567688    Error: 0
        i=73    B.z = 0.00822738        Expected: 0.00822738    Error: 0
i=74    B.y = 0.00482715        Expected: 0.00482715    Error: 0
        i=74    B.z = 0.00875302        Expected: 0.00875302    Error:
1.73472e-18
i=75    B.y = 0.00392919        Expected: 0.00392919    Error: 2.25514e-17
        i=75    B.z = 0.0091912 Expected: 0.0091912     Error: 8.1532e-17
i=76    B.y = 0.00299197        Expected: 0.00299197    Error: 0
        i=76    B.z = 0.00953755        Expected: 0.00953755    Error: 0
i=77    B.y = 0.00202486        Expected: 0.00202486    Error: 1.12757e-17
        i=77    B.z = 0.0097886 Expected: 0.0097886     Error: 9.02056e-17
i=78    B.y = 0.00103751        Expected: 0.00103751    Error: 2.1684e-19
        i=78    B.z = 0.00994184        Expected: 0.00994184    Error: 0
i=79    B.y = 3.97996e-05       Expected: 3.97996e-05   Error: 6.77626e-21
        i=79    B.z = 0.00999575        Expected: 0.00999575    Error:
1.73472e-18
i=80    B.y = -0.00095831       Expected: -0.00095831   Error: 0
        i=80    B.z = 0.00994979        Expected: 0.00994979    Error: 0
i=81    B.y = -0.00194684       Expected: -0.00194684   Error: 4.44523e-17
        i=81    B.z = 0.00980441        Expected: 0.00980441    Error:
1.75207e-16
i=82    B.y = -0.00291593       Expected: -0.00291593   Error: 4.33681e-19
        i=82    B.z = 0.00956107        Expected: 0.00956107    Error: 0
i=83    B.y = -0.00385587       Expected: -0.00385587   Error: 0
        i=83    B.z = 0.0092222 Expected: 0.0092222     Error: 0
i=84    B.y = -0.00475729       Expected: -0.00475729   Error: 0
        i=84    B.z = 0.00879118        Expected: 0.00879118    Error:
1.73472e-18
i=85    B.y = -0.00561118       Expected: -0.00561118   Error: 8.67362e-19
        i=85    B.z = 0.00827232        Expected: 0.00827232    Error: 0
i=86    B.y = -0.006409 Expected: -0.006409     Error: 1.10155e-16
        i=86    B.z = 0.00767081        Expected: 0.00767081    Error:
1.32706e-16
i=87    B.y = -0.00714279       Expected: -0.00714279   Error: 8.67362e-19
        i=87    B.z = 0.00699266        Expected: 0.00699266    Error: 0
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i=88    B.y = -0.0078052        Expected: -0.0078052    Error: 0
        i=88    B.z = 0.00624464        Expected: 0.00624464    Error: 0
i=89    B.y = -0.00838963       Expected: -0.00838963   Error: 1.73472e-18
        i=89    B.z = 0.00543422        Expected: 0.00543422    Error:
8.67362e-19
i=90    B.y = -0.00889024       Expected: -0.00889024   Error: 0
        i=90    B.z = 0.00456951        Expected: 0.00456951    Error: 0
i=91    B.y = -0.00930201       Expected: -0.00930201   Error: 1.70003e-16
        i=91    B.z = 0.00365913        Expected: 0.00365913    Error:
5.55112e-17
i=92    B.y = -0.00962084       Expected: -0.00962084   Error: 0
        i=92    B.z = 0.0027122 Expected: 0.0027122     Error: 0
i=93    B.y = -0.00984355       Expected: -0.00984355   Error: 0
        i=93    B.z = 0.00173817        Expected: 0.00173817    Error:
2.1684e-19
i=94    B.y = -0.0099679        Expected: -0.0099679    Error: 1.73472e-18
        i=94    B.z = 0.000746772       Expected: 0.000746772   Error:
1.0842e-19
i=95    B.y = -0.00999265       Expected: -0.00999265   Error: 1.76942e-16
        i=95    B.z = -0.000252089      Expected: -0.000252089  Error:
1.11131e-17
i=96    B.y = -0.00991757       Expected: -0.00991757   Error: 0
        i=96    B.z = -0.00124843       Expected: -0.00124843   Error:
2.1684e-19
i=97    B.y = -0.00974338       Expected: -0.00974338   Error: 0
        i=97    B.z = -0.0022323        Expected: -0.0022323    Error: 0
i=98    B.y = -0.00947185       Expected: -0.00947185   Error: 0
        i=98    B.z = -0.00319386       Expected: -0.00319386   Error:
4.33681e-19
i=99    B.y = -0.00910568       Expected: -0.00910568   Error: 1.73472e-18
        i=99    B.z = -0.00412351       Expected: -0.00412351   Error: 0

Testing Ampere's law implementation in 1D…

i=1     J.y = 0.0997918 Expected: 0.0997918     Error: 0
        i=1     J.z = 0.99459   Expected: 0.99459       Error: 0
i=2     J.y = 0.198587  Expected: 0.198587      Error: 0
        i=2     J.z = 0.979658  Expected: 0.979658      Error: 5.55112e-16
i=3     J.y = 0.295397  Expected: 0.295397      Error: 0
        i=3     J.z = 0.954938  Expected: 0.954938      Error: 5.55112e-16
i=4     J.y = 0.389256  Expected: 0.389256      Error: 0
        i=4     J.z = 0.920677  Expected: 0.920677      Error: 5.55112e-16
i=5     J.y = 0.479226  Expected: 0.479226      Error: 0
        i=5     J.z = 0.877217  Expected: 0.877217      Error: 1.11022e-15
i=6     J.y = 0.564407  Expected: 0.564407      Error: 0
        i=6     J.z = 0.824992  Expected: 0.824992      Error: 0
i=7     J.y = 0.643949  Expected: 0.643949      Error: 0
        i=7     J.z = 0.764524  Expected: 0.764524      Error: 0
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i=8     J.y = 0.717057  Expected: 0.717057      Error: 0
        i=8     J.z = 0.696416  Expected: 0.696416      Error: 0
i=9     J.y = 0.783001  Expected: 0.783001      Error: 0
        i=9     J.z = 0.621351  Expected: 0.621351      Error: 0
i=10    J.y = 0.84112   Expected: 0.84112       Error: 0
        i=10    J.z = 0.540077  Expected: 0.540077      Error: 0
i=11    J.y = 0.890836  Expected: 0.890836      Error: 0
        i=11    J.z = 0.453407  Expected: 0.453407      Error: 0
i=12    J.y = 0.931651  Expected: 0.931651      Error: 2.22045e-15
        i=12    J.z = 0.362207  Expected: 0.362207      Error: 1.11022e-15
i=13    J.y = 0.963157  Expected: 0.963157      Error: 0
        i=13    J.z = 0.267387  Expected: 0.267387      Error: 0
i=14    J.y = 0.985039  Expected: 0.985039      Error: 2.22045e-15
        i=14    J.z = 0.169896  Expected: 0.169896      Error: 0
i=15    J.y = 0.997079  Expected: 0.997079      Error: 0
        i=15    J.z = 0.0707077 Expected: 0.0707077     Error: 0
i=16    J.y = 0.999157  Expected: 0.999157      Error: 0
        i=16    J.z = -0.0291874        Expected: -0.0291874    Error: 0
i=17    J.y = 0.991252  Expected: 0.991252      Error: 2.22045e-15
        i=17    J.z = -0.128791 Expected: -0.128791     Error: 1.11022e-15
i=18    J.y = 0.973442  Expected: 0.973442      Error: 0
        i=18    J.z = -0.227107 Expected: -0.227107     Error: 0
i=19    J.y = 0.945906  Expected: 0.945906      Error: 2.22045e-15
        i=19    J.z = -0.323155 Expected: -0.323155     Error: 1.11022e-15
i=20    J.y = 0.908919  Expected: 0.908919      Error: 0
        i=20    J.z = -0.415973 Expected: -0.415973     Error: 0
i=21    J.y = 0.86285   Expected: 0.86285       Error: 0
        i=21    J.z = -0.504636 Expected: -0.504636     Error: 0
i=22    J.y = 0.80816   Expected: 0.80816       Error: 0
        i=22    J.z = -0.588256 Expected: -0.588256     Error: 0
i=23    J.y = 0.745395  Expected: 0.745395      Error: 0
        i=23    J.z = -0.665998 Expected: -0.665998     Error: 0
i=24    J.y = 0.675182  Expected: 0.675182      Error: 2.22045e-15
        i=24    J.z = -0.737087 Expected: -0.737087     Error: 3.33067e-15
i=25    J.y = 0.598223  Expected: 0.598223      Error: 0
        i=25    J.z = -0.80081  Expected: -0.80081      Error: 0
i=26    J.y = 0.515287  Expected: 0.515287      Error: 0
        i=26    J.z = -0.856532 Expected: -0.856532     Error: 0
i=27    J.y = 0.427202  Expected: 0.427202      Error: 0
        i=27    J.z = -0.903695 Expected: -0.903695     Error: 0
i=28    J.y = 0.334849  Expected: 0.334849      Error: 0
        i=28    J.z = -0.94183  Expected: -0.94183      Error: 0
i=29    J.y = 0.23915   Expected: 0.23915       Error: 1.11022e-15
        i=29    J.z = -0.970554 Expected: -0.970554     Error: 4.44089e-15
i=30    J.y = 0.141061  Expected: 0.141061      Error: 0
        i=30    J.z = -0.98958  Expected: -0.98958      Error: 0
i=31    J.y = 0.0415633 Expected: 0.0415633     Error: 0
        i=31    J.z = -0.998719 Expected: -0.998719     Error: 0
```

```
i=32    J.y = -0.0583498        Expected: -0.0583498    Error: 0
        i=32    J.z = -0.997879 Expected: -0.997879     Error: 0
i=33    J.y = -0.15768  Expected: -0.15768      Error: 0
        i=33    J.z = -0.987068 Expected: -0.987068     Error: 0
i=34    J.y = -0.255435 Expected: -0.255435     Error: 1.11022e-15
        i=34    J.z = -0.966395 Expected: -0.966395     Error: 4.44089e-15
i=35    J.y = -0.350637 Expected: -0.350637     Error: 0
        i=35    J.z = -0.936067 Expected: -0.936067     Error: 0
i=36    J.y = -0.442336 Expected: -0.442336     Error: 0
        i=36    J.z = -0.896385 Expected: -0.896385     Error: 0
i=37    J.y = -0.529615 Expected: -0.529615     Error: 0
        i=37    J.z = -0.847747 Expected: -0.847747     Error: 0
i=38    J.y = -0.611603 Expected: -0.611603     Error: 0
        i=38    J.z = -0.790638 Expected: -0.790638     Error: 0
i=39    J.y = -0.68748  Expected: -0.68748      Error: 2.22045e-15
        i=39    J.z = -0.72563  Expected: -0.72563      Error: 3.33067e-15
i=40    J.y = -0.756487 Expected: -0.756487     Error: 6.66134e-15
        i=40    J.z = -0.653371 Expected: -0.653371     Error: 5.55112e-15
i=41    J.y = -0.817936 Expected: -0.817936     Error: 0
        i=41    J.z = -0.574584 Expected: -0.574584     Error: 0
i=42    J.y = -0.871213 Expected: -0.871213     Error: 0
        i=42    J.z = -0.490057 Expected: -0.490057     Error: 0
i=43    J.y = -0.915784 Expected: -0.915784     Error: 8.32667e-15
        i=43    J.z = -0.400632 Expected: -0.400632     Error: 3.33067e-15
i=44    J.y = -0.951206 Expected: -0.951206     Error: 0
        i=44    J.z = -0.307205 Expected: -0.307205     Error: 0
i=45    J.y = -0.977123 Expected: -0.977123     Error: 8.88178e-15
        i=45    J.z = -0.210708 Expected: -0.210708     Error: 1.11022e-15
i=46    J.y = -0.993277 Expected: -0.993277     Error: 0
        i=46    J.z = -0.112106 Expected: -0.112106     Error: 0
i=47    J.y = -0.999507 Expected: -0.999507     Error: 8.88178e-15
        i=47    J.z = -0.0123835        Expected: -0.0123835    Error:
1.11022e-15
i=48    J.y = -0.99575  Expected: -0.99575      Error: 0
        i=48    J.z = 0.0874625 Expected: 0.0874625     Error: 0
i=49    J.y = -0.982043 Expected: -0.982043     Error: 0
        i=49    J.z = 0.186435  Expected: 0.186435      Error: 0
i=50    J.y = -0.958525 Expected: -0.958525     Error: 8.32667e-15
        i=50    J.z = 0.283544  Expected: 0.283544      Error: 2.22045e-15
i=51    J.y = -0.925429 Expected: -0.925429     Error: 0
        i=51    J.z = 0.37782   Expected: 0.37782       Error: 0
i=52    J.y = -0.883087 Expected: -0.883087     Error: 6.66134e-15
        i=52    J.z = 0.468321  Expected: 0.468321      Error: 4.44089e-15
i=53    J.y = -0.831921 Expected: -0.831921     Error: 0
        i=53    J.z = 0.554143  Expected: 0.554143      Error: 0
i=54    J.y = -0.772443 Expected: -0.772443     Error: 0
        i=54    J.z = 0.634428  Expected: 0.634428      Error: 0
i=55    J.y = -0.705246 Expected: -0.705246     Error: 5.55112e-15
```

```
        i=55    J.z = 0.708375  Expected: 0.708375      Error: 6.66134e-15
i=56    J.y = -0.631004 Expected: -0.631004     Error: 0
        i=56    J.z = 0.775243  Expected: 0.775243      Error: 0
i=57    J.y = -0.550456 Expected: -0.550456     Error: 4.44089e-15
        i=57    J.z = 0.834365  Expected: 0.834365      Error: 7.77156e-15
i=58    J.y = -0.464409 Expected: -0.464409     Error: 0
        i=58    J.z = 0.885151  Expected: 0.885151      Error: 0
i=59    J.y = -0.373721 Expected: -0.373721     Error: 0
        i=59    J.z = 0.927092  Expected: 0.927092      Error: 0
i=60    J.y = -0.279299 Expected: -0.279299     Error: 2.22045e-15
        i=60    J.z = 0.95977   Expected: 0.95977       Error: 8.54872e-15
i=61    J.y = -0.182087 Expected: -0.182087     Error: 0
        i=61    J.z = 0.982859  Expected: 0.982859      Error: 0
i=62    J.y = -0.0830548        Expected: -0.0830548    Error: 0
        i=62    J.z = 0.996127  Expected: 0.996127      Error: 8.88178e-15
i=63    J.y = 0.0168069 Expected: 0.0168069     Error: 0
        i=63    J.z = 0.999442  Expected: 0.999442      Error: 0
i=64    J.y = 0.116501  Expected: 0.116501      Error: 0
        i=64    J.z = 0.992771  Expected: 0.992771      Error: 0
i=65    J.y = 0.21503   Expected: 0.21503       Error: 2.22045e-15
        i=65    J.z = 0.976181  Expected: 0.976181      Error: 8.32667e-15
i=66    J.y = 0.311412  Expected: 0.311412      Error: 0
        i=66    J.z = 0.949837  Expected: 0.949837      Error: 0
i=67    J.y = 0.404681  Expected: 0.404681      Error: 4.44089e-15
        i=67    J.z = 0.914002  Expected: 0.914002      Error: 7.77156e-15
i=68    J.y = 0.493907  Expected: 0.493907      Error: 0
        i=68    J.z = 0.869035  Expected: 0.869035      Error: 0
i=69    J.y = 0.578199  Expected: 0.578199      Error: 0
        i=69    J.z = 0.815385  Expected: 0.815385      Error: 0
i=70    J.y = 0.656713  Expected: 0.656713      Error: 6.66134e-15
        i=70    J.z = 0.753588  Expected: 0.753588      Error: 6.66134e-15
i=71    J.y = 0.728665  Expected: 0.728665      Error: 0
        i=71    J.z = 0.684261  Expected: 0.684261      Error: 0
i=72    J.y = 0.793337  Expected: 0.793337      Error: 6.66134e-15
        i=72    J.z = 0.608098  Expected: 0.608098      Error: 4.44089e-15
i=73    J.y = 0.850082  Expected: 0.850082      Error: 0
        i=73    J.z = 0.525858  Expected: 0.525858      Error: 0
i=74    J.y = 0.898334  Expected: 0.898334      Error: 0
        i=74    J.z = 0.438365  Expected: 0.438365      Error: 0
i=75    J.y = 0.937609  Expected: 0.937609      Error: 8.88178e-15
        i=75    J.z = 0.346491  Expected: 0.346491      Error: 3.33067e-15
i=76    J.y = 0.967516  Expected: 0.967516      Error: 0
        i=76    J.z = 0.251155  Expected: 0.251155      Error: 0
i=77    J.y = 0.987757  Expected: 0.987757      Error: 8.77076e-15
        i=77    J.z = 0.15331   Expected: 0.15331       Error: 1.11022e-15
i=78    J.y = 0.998127  Expected: 0.998127      Error: 0
        i=78    J.z = 0.0539329 Expected: 0.0539329     Error: 0
i=79    J.y = 0.998525  Expected: 0.998525      Error: 0
```

```
        i=79    J.z = -0.045983 Expected: -0.045983     Error: 0
i=80    J.y = 0.988946  Expected: 0.988946      Error: 0
        i=80    J.z = -0.145439 Expected: -0.145439     Error: 0
i=81    J.y = 0.969486  Expected: 0.969486      Error: 1.66533e-14
        i=81    J.z = -0.243443 Expected: -0.243443     Error: 5.55112e-15
i=82    J.y = 0.940339  Expected: 0.940339      Error: 0
        i=82    J.z = -0.339014 Expected: -0.339014     Error: 0
i=83    J.y = 0.901796  Expected: 0.901796      Error: 0
        i=83    J.z = -0.431197 Expected: -0.431197     Error: 0
i=84    J.y = 0.854243  Expected: 0.854243      Error: 0
        i=84    J.z = -0.519072 Expected: -0.519072     Error: 0
i=85    J.y = 0.798154  Expected: 0.798154      Error: 0
        i=85    J.z = -0.601761 Expected: -0.601761     Error: 0
i=86    J.y = 0.734091  Expected: 0.734091      Error: 1.33227e-14
        i=86    J.z = -0.678437 Expected: -0.678437     Error: 1.22125e-14
i=87    J.y = 0.662693  Expected: 0.662693      Error: 0
        i=87    J.z = -0.748335 Expected: -0.748335     Error: 0
i=88    J.y = 0.584674  Expected: 0.584674      Error: 0
        i=88    J.z = -0.810755 Expected: -0.810755     Error: 0
i=89    J.y = 0.500812  Expected: 0.500812      Error: 0
        i=89    J.z = -0.865075 Expected: -0.865075     Error: 0
i=90    J.y = 0.411947  Expected: 0.411947      Error: 0
        i=90    J.z = -0.910751 Expected: -0.910751     Error: 0
i=91    J.y = 0.318965  Expected: 0.318965      Error: 4.44089e-15
        i=91    J.z = -0.947327 Expected: -0.947327     Error: 1.72085e-14
i=92    J.y = 0.222797  Expected: 0.222797      Error: 0
        i=92    J.z = -0.974437 Expected: -0.974437     Error: 0
i=93    J.y = 0.124403  Expected: 0.124403      Error: 0
        i=93    J.z = -0.991812 Expected: -0.991812     Error: 0
i=94    J.y = 0.0247651 Expected: 0.0247651     Error: 0
        i=94    J.z = -0.999277 Expected: -0.999277     Error: 0
i=95    J.y = -0.0751198        Expected: -0.0751198    Error: 2.22045e-15
        i=95    J.z = -0.996757 Expected: -0.996757     Error: 1.76525e-14
i=96    J.y = -0.174254 Expected: -0.174254     Error: 0
        i=96    J.z = -0.984278 Expected: -0.984278     Error: 0
i=97    J.y = -0.271647 Expected: -0.271647     Error: 0
        i=97    J.z = -0.961964 Expected: -0.961964     Error: 0
i=98    J.y = -0.366326 Expected: -0.366326     Error: 0
        i=98    J.z = -0.930039 Expected: -0.930039     Error: 0
i=99    J.y = -0.457345 Expected: -0.457345     Error: 0
        i=99    J.z = -0.888821 Expected: -0.888821     Error: 0

Density:
n[0] = 0
n[1] = 0
n[2] = 0
n[3] = 0
n[4] = 0
```

```
n[5] = 1
n[6] = 1
n[7] = 0
n[8] = 0
n[9] = 0
n[10] = 0
n[11] = 0
n[12] = 0

Flux (Vx):
Vx[0] = 0
Vx[1] = 0
Vx[2] = 0
Vx[3] = 0
Vx[4] = 0
Vx[5] = 1
Vx[6] = 1
Vx[7] = 0
Vx[8] = 0
Vx[9] = 0
Vx[10] = 0
Vx[11] = 0
Vx[12] = 0

Total Density N:
N[0] = 0
N[1] = 0
N[2] = 0
N[3] = 0
N[4] = 0
N[5] = 1
N[6] = 1
N[7] = 0
N[8] = 0
N[9] = 0
N[10] = 0
N[11] = 0
N[12] = 0

Bulk Velocity Vx:
Vx[0] = 0
Vx[1] = 0
Vx[2] = 0
Vx[3] = 0
Vx[4] = 0
Vx[5] = 1
Vx[6] = 1
Vx[7] = 0
```

```
Vx[8] = 0
Vx[9] = 0
Vx[10] = 0
Vx[11] = 0
Vx[12] = 0
```

[13]:
```
import re
import numpy as np
import matplotlib.pyplot as plt

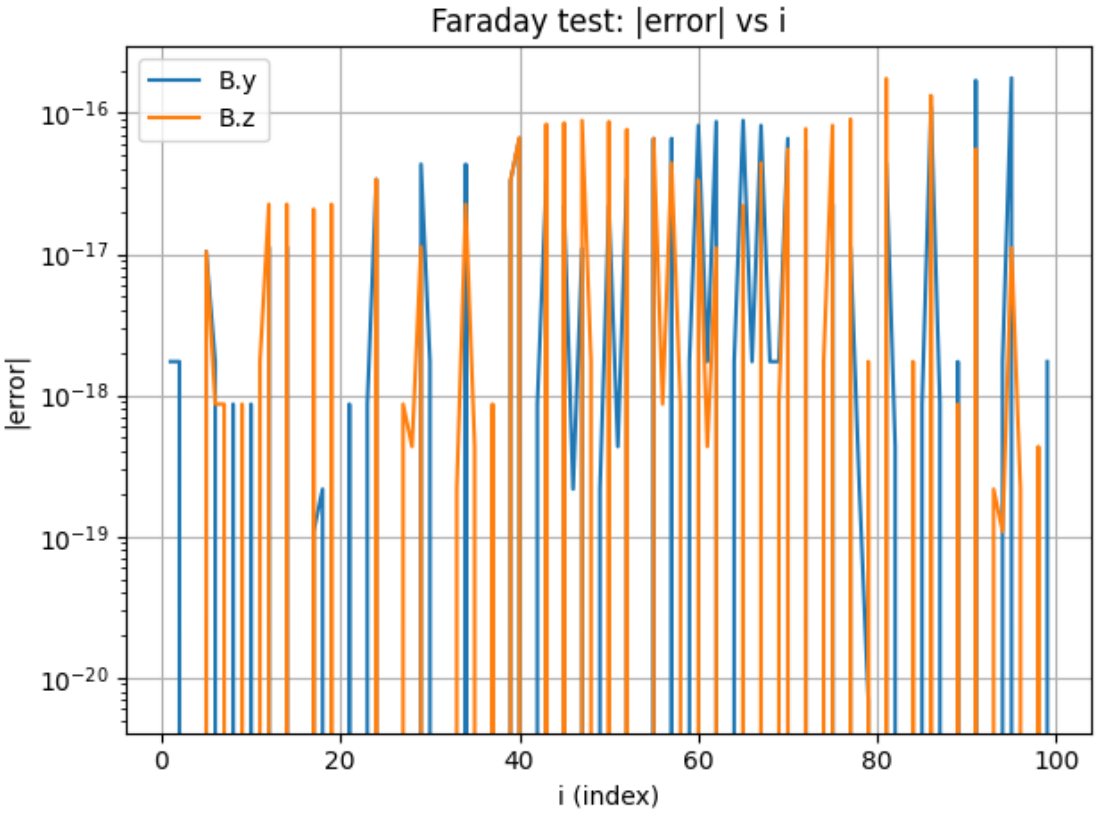
pat = re.compile(
    r"i\s*=\s*(\d+)\s+([A-Za-z0-9\.\_\-]+)\s*=\s*([+\-]?\d+(?:\.\d+)?(?:
 ↪[eE][+\-]?\d+)?)"
    r"\s+Expected:\s*([+\-]?\d+(?:\.\d+)?(?:[eE][+\-]?\d+)?)"
    r"\s+Error:\s*([+\-]?\d+(?:\.\d+)?(?:[eE][+\-]?\d+)?)"
)

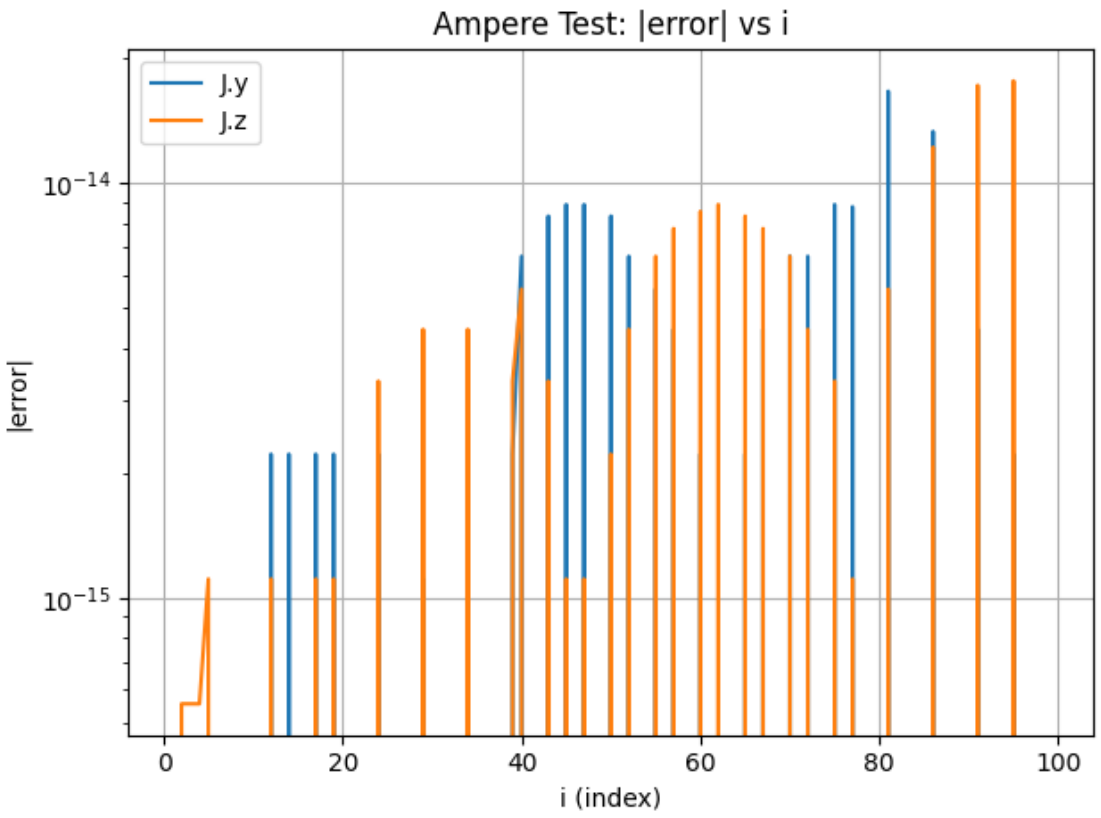
def parse_error_lines(out):
    rows = []
    for m in pat.finditer(out):
        i   = int(m.group(1))
        nm  = m.group(2)
        err = float(m.group(5))
        rows.append((i, nm, err))
    return rows

def plot_errors(rows, title):
    names = sorted(set(nm for _, nm, _ in rows))
    plt.figure()
    for nm in names:
        rr = sorted([(i, abs(e)) for i, n, e in rows if n == nm], key=lambda t:␣
 ↪t[0])
        ii = np.array([t[0] for t in rr])
        ee = np.array([t[1] for t in rr])
        plt.semilogy(ii, ee, label=nm)
    plt.xlabel("i (index)")
    plt.ylabel("|error|")
    plt.title(title)
    plt.grid(True)
    plt.legend()
    plt.tight_layout()
    plt.show()

# usage:
rows = parse_error_lines(out_far)
plot_errors(rows, "Faraday test: |error| vs i")
```

```
rows1= parse_error_lines(out_amp)
plot_errors(rows1, "Ampere Test: |error| vs i")
```





[14]:
```
def parse_array_block(out, name):
    # matches lines like: n[6] = 1
    p = re.compile(rf"^{re.escape(name)}\[(\d+)\]\s*=\s*([+\-]?\d+(?:\.\d+)?(?:
 ↪[eE][+\-]?\d+)?)\s*$",
                   re.MULTILINE)
    items = [(int(i), float(v)) for i, v in p.findall(out)]
    if not items:
        return None
    items.sort()
    idx = np.array([i for i,_ in items], dtype=int)
    val = np.array([v for _,v in items], dtype=float)
    return idx, val

out_mom = run("./build/tests/moments/test-moments")

idx_n, n  = parse_array_block(out_mom, "n")
idx_vx, vx = parse_array_block(out_mom, "Vx")

plt.figure()
plt.bar(idx_n, n)
```

```
plt.xlabel("i")
plt.ylabel("n[i]")
plt.title("Moments test: deposited density")
plt.show()
```

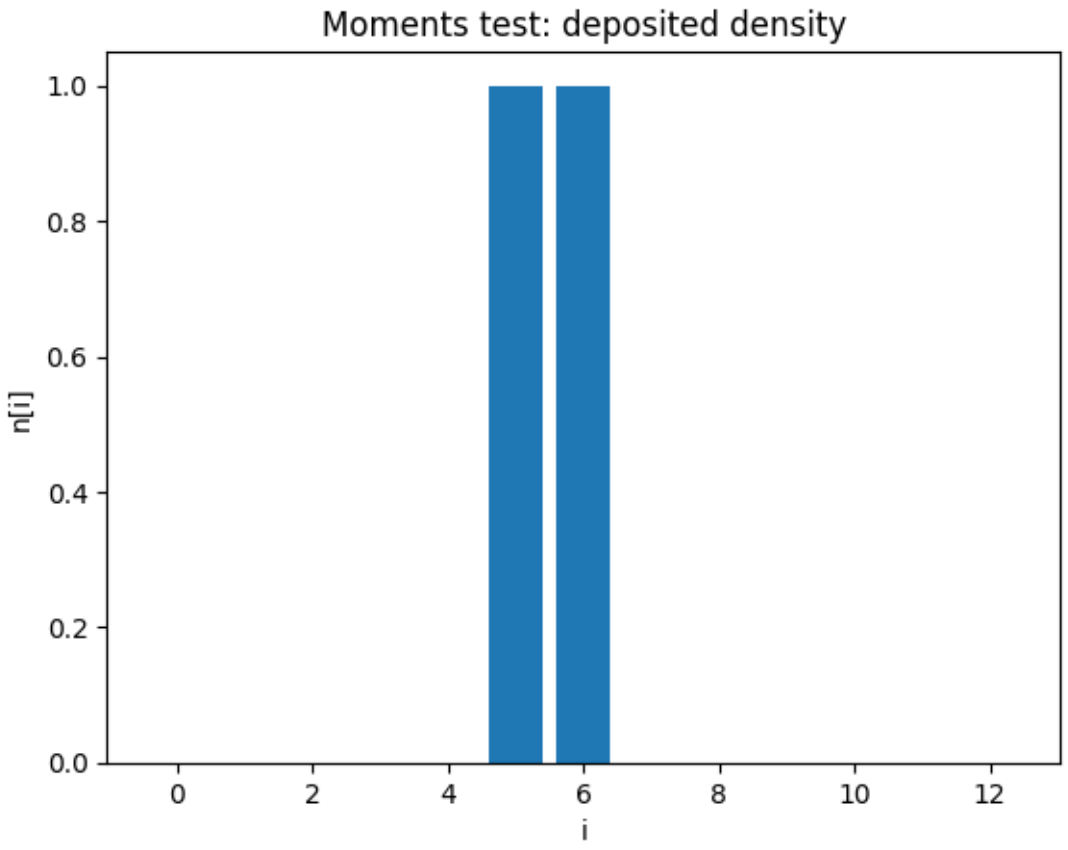
```
Density:
n[0] = 0
n[1] = 0
n[2] = 0
n[3] = 0
n[4] = 0
n[5] = 1
n[6] = 1
n[7] = 0
n[8] = 0
n[9] = 0
n[10] = 0
n[11] = 0
n[12] = 0

Flux (Vx):
Vx[0] = 0
Vx[1] = 0
Vx[2] = 0
Vx[3] = 0
Vx[4] = 0
Vx[5] = 1
Vx[6] = 1
Vx[7] = 0
Vx[8] = 0
Vx[9] = 0
Vx[10] = 0
Vx[11] = 0
Vx[12] = 0

Total Density N:
N[0] = 0
N[1] = 0
N[2] = 0
N[3] = 0
N[4] = 0
N[5] = 1
N[6] = 1
N[7] = 0
N[8] = 0
N[9] = 0
N[10] = 0
N[11] = 0
```

```
N[12] = 0

Bulk Velocity Vx:
Vx[0] = 0
Vx[1] = 0
Vx[2] = 0
Vx[3] = 0
Vx[4] = 0
Vx[5] = 1
Vx[6] = 1
Vx[7] = 0
Vx[8] = 0
Vx[9] = 0
Vx[10] = 0
Vx[11] = 0
Vx[12] = 0
```



[16]:
```
out_pop = run("./build/tests/population/test_population")
```

```
Testing Population::deposit() with a single deterministic particle…
```

```
PASS.
  particle x=1.06 (frac=0.3), W=2, vx=0.5
  density nodes: i=6 -> 1.4 , i=7 -> 0.6
  flux-x nodes:  i=6 -> 0.7 , i=7 -> 0.3
```

0.1 Numerical Validation Summary

0.1.1 Faraday Operator Test

The Faraday test verifies the discrete curl operator used to advance the magnetic field. The numerical update is compared against the analytic expectation, and the pointwise error (| B|) is plotted as a function of grid index.

The observed errors are of order ($10^{\{-18\}-10}${-16}), consistent with floating-point round-off. No systematic bias or spatial structure is visible, confirming that the Faraday implementation is correct.

0.1.2 Ampere Operator Test

The Ampere test checks the computation of the current-induced magnetic field response. The numerical current components ((J_y, J_z)) are compared to their analytical values.

The error remains at machine precision ((10^{-15})) across the domain, indicating that the discrete Ampère operator and staggering are implemented correctly.

0.1.3 Moments (Density and Flux) Test

This test validates the particle-to-grid moment deposition. A deterministic particle configuration is used so that the expected deposited density and flux can be computed analytically.

The resulting density (n(i)), flux (V_x(i)), total density (N(i)), and bulk velocity (V_x(i)) match the expected values exactly at the affected grid points, confirming correct weighting and conservation properties.

0.1.4 Population::deposit() Test

The population test validates the linear (CIC) deposition scheme implemented in `Population::deposit()`.

A single particle with known position, weight, and velocity is deposited onto the grid. The deposited density and flux are split between neighboring nodes according to the analytic linear weights. The numerical output matches the analytic expectation exactly, confirming correctness of the deposit implementation.

0.1.5 Conclusion

All core numerical operators (Faraday, Ampère, moments, and particle deposition) exhibit errors consistent with machine precision or exact agreement with analytic results. This confirms the correctness of the discretization and implementation used in the hybrid solver.

[ ]: