Futures Pair Trading Research Summary

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Executive Summary

The spread, $s = NQc1 - h \times ESc1$, was found to exhibit mean reversion properties. h, which is called the hedge ratio, was calculated using OLS on a rolling window, ensuring the absence of lookahead bias. Then, an exponential moving window z-score was applied to the spread calculated from the fitted hedge ratio, to generate buy, sell, and exit signals based on predefined z-score thresholds. The scale of mean reversion is relatively small: backtesting results without considering liquidity constraints and transaction costs indicated a high Sharpe ratio of 6.3. However, the profitability of the strategy was significantly undermined when liquidity and bid-ask spreads were factored in, leading to negative returns.

1. Exploratory analysis

Mid prices

As demonstrated in Figure 1, the price time series reveal a high correlation between them, with returns showing a correlation coefficient of 0.88. The stability of this correlation across various frequencies was then examined. Figure 2 illustrates that, generally, the correlation remains strong, even at the 15-minute interval.

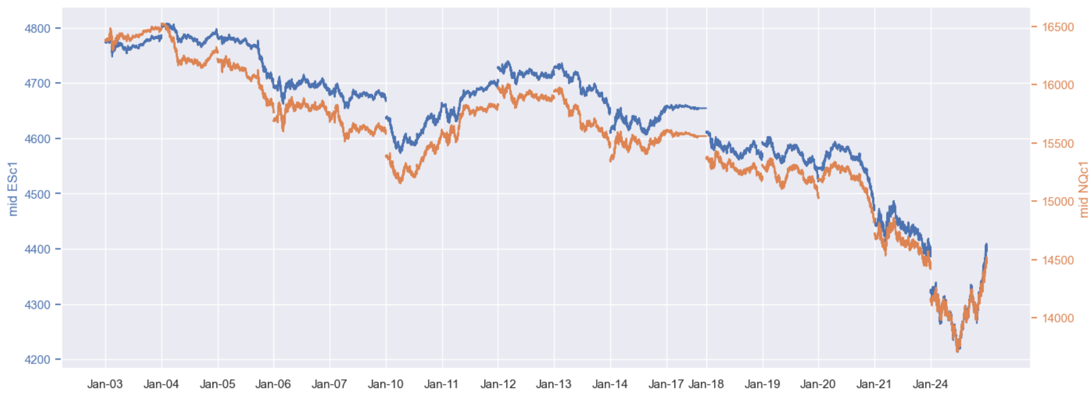

Figure 1- Mid prices (during trading hours)

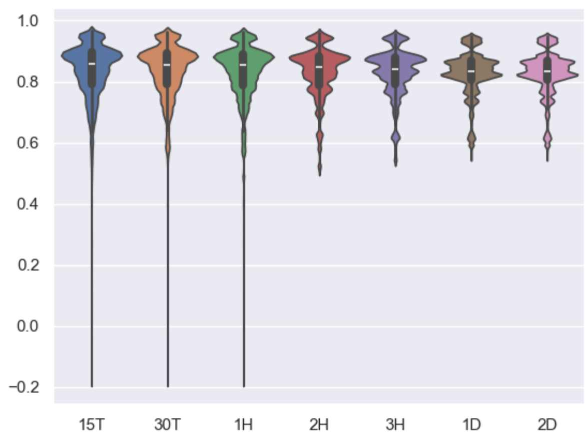

Figure 2- Distribution of rolling correlation for multiple scales

Figure 3 reveals noticeable large fluctuations occurring between close and open of next trading day, suggesting it may be better to avoid holding positions at the day's end.

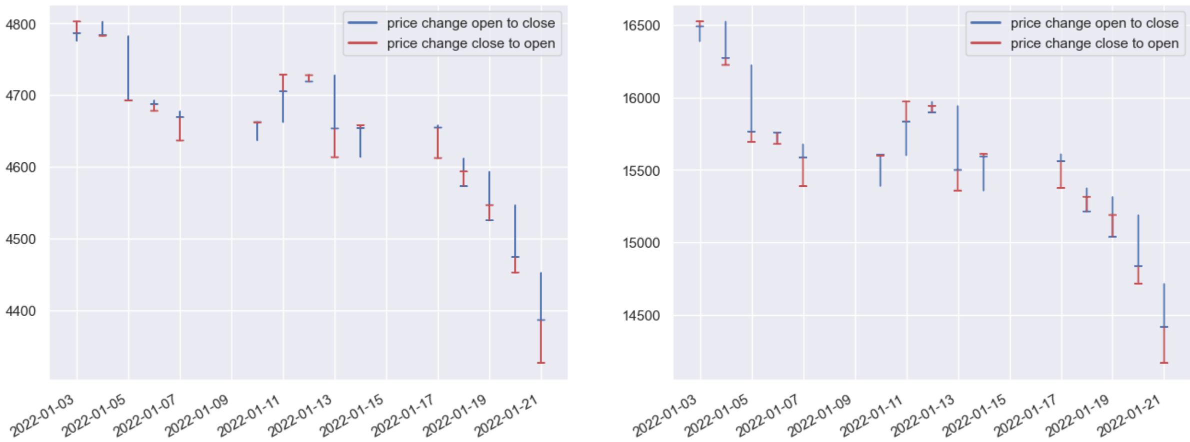

Figure 3 - ESc1 (left) and NQc1 (right) price movement inside and outside trading hours

Liquidity

Figure 4Figure 5 indicate relatively high liquidity on both the bid and ask sides (at one minute aggregations), although it is observed that liquidity for NQc1 is somewhat lower, attributed to its higher price.

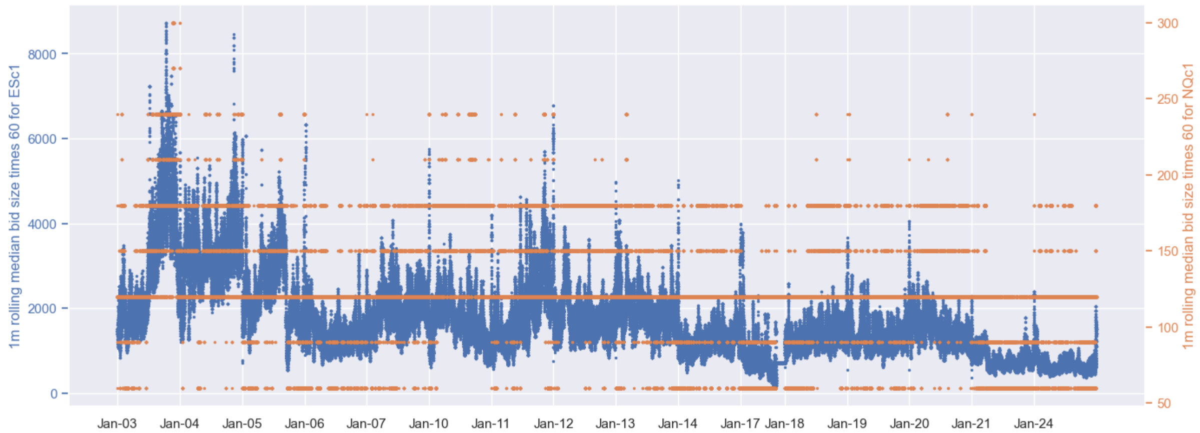

Figure 4 - 1 minute bid liquidity for both futures

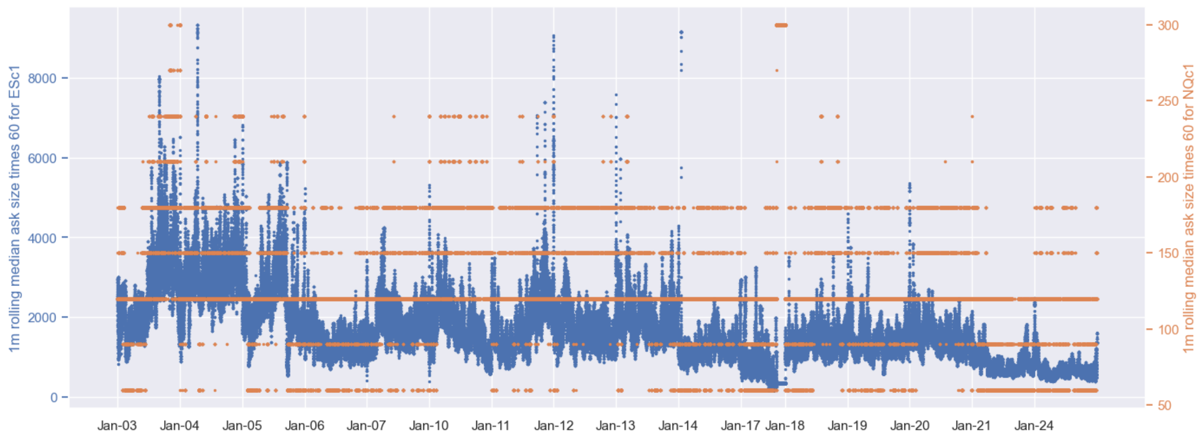

Figure 5 - 1 minute ask liquidity for both futures

Bid Ask Spread

Figure 6 illustrates that ESc1 is a large tick with a bid ask spread rarely exceeding 50 cents, in contrast to NQc1, which is a small tick, with large fluctuating bid ask spread, as depicted in Figure 7Figure 7.

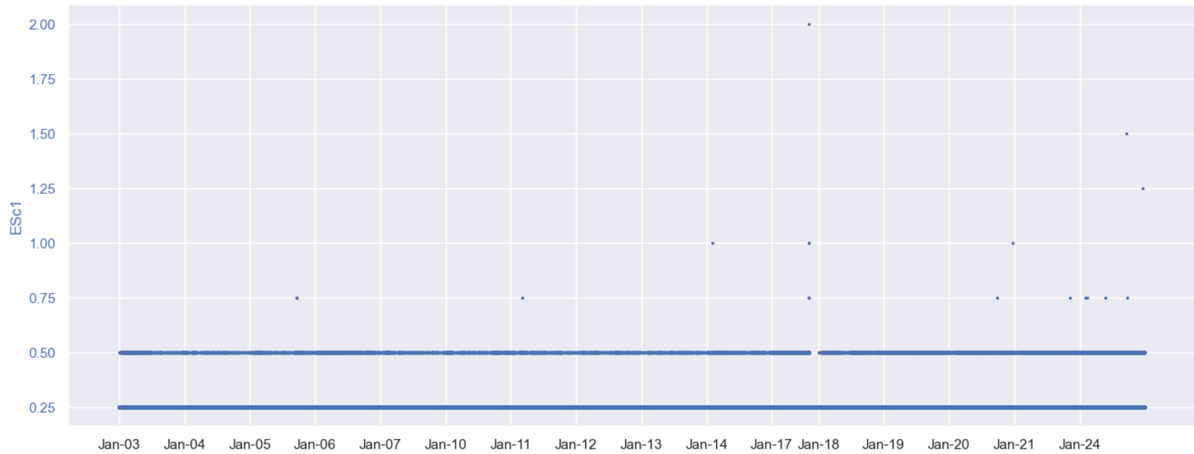

Figure 6 - Bid Ask Spread of ESc1

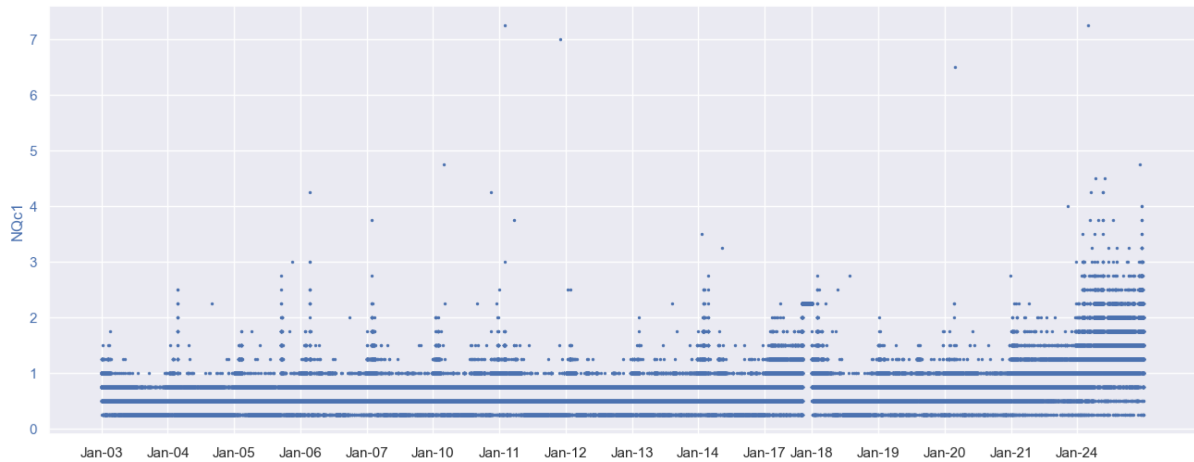

Figure 7 - Bid Ask Spread of NQc1

2. Spread

The objective is to assess whether the spread $s = NQc1 - h \times ESc1$ exhibits mean-reverting behavior. Identifying the optimal hedge ratio h is crucial, followed by monitoring the temporal progression of s. Figure 8 demonstrates that when h is fitted daily through ordinary least squares (OLS) and the residuals (which is s) are examined, and mean reversion is observed. This outcome is anticipated since the fitting occurs over the same period as the residual analysis, leading to an expected dispersion of residuals because of the OLS fitting.

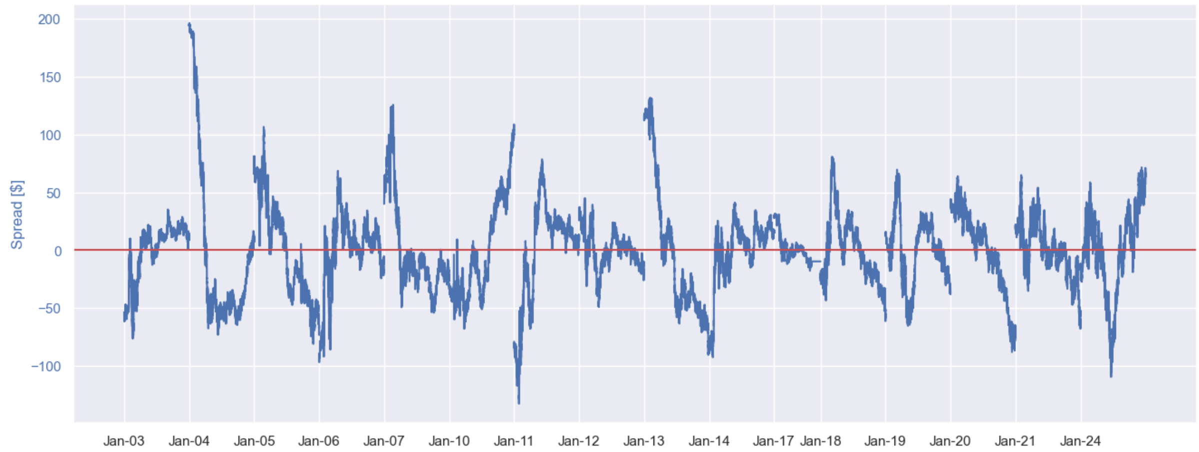

Figure 8 - Spread with daily constant hedge ratio

We eliminate leakage or lookahead bias by fitting h with OLS based on data from a preceding period before applying it to a subsequent period to derive the spread. Figure 9 shows that, employing a 2-day lookback window for hourly fits, results in clear mean reversion, though the speed of the mean reversion seems long. Consequently, exploring different frequencies for this methodology is advisable.

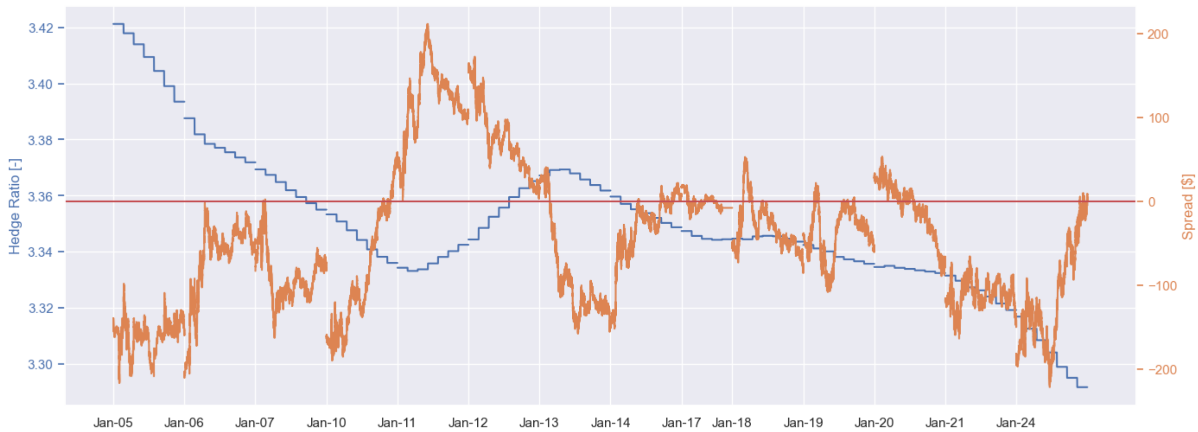

Figure 9 - Fitted Hedge Ratio and corresponding spread each 1 hour with max lookback window of 2 days

As shown in Figure 10, employing a 3 hour lookback window for 15 minute fits yields a better and faster mean reversion. A detailed examination of sub-periods shows that it seems possible to trade fast mean reversion signals even at sub-hour frequency (refer to Figure 11Figure 11).

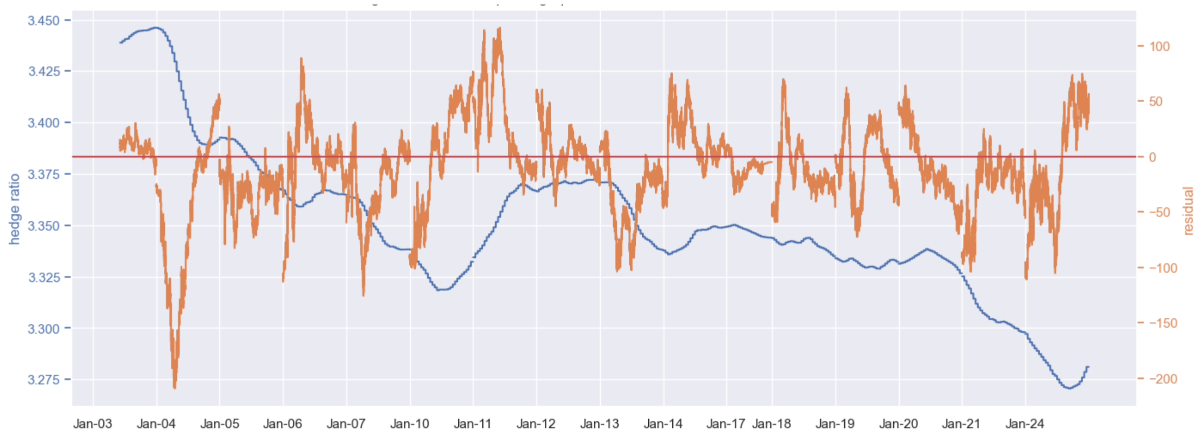

Figure 10 - Fitted hedge ration and corresponding spread each 15min with max lookback window of 3 hours

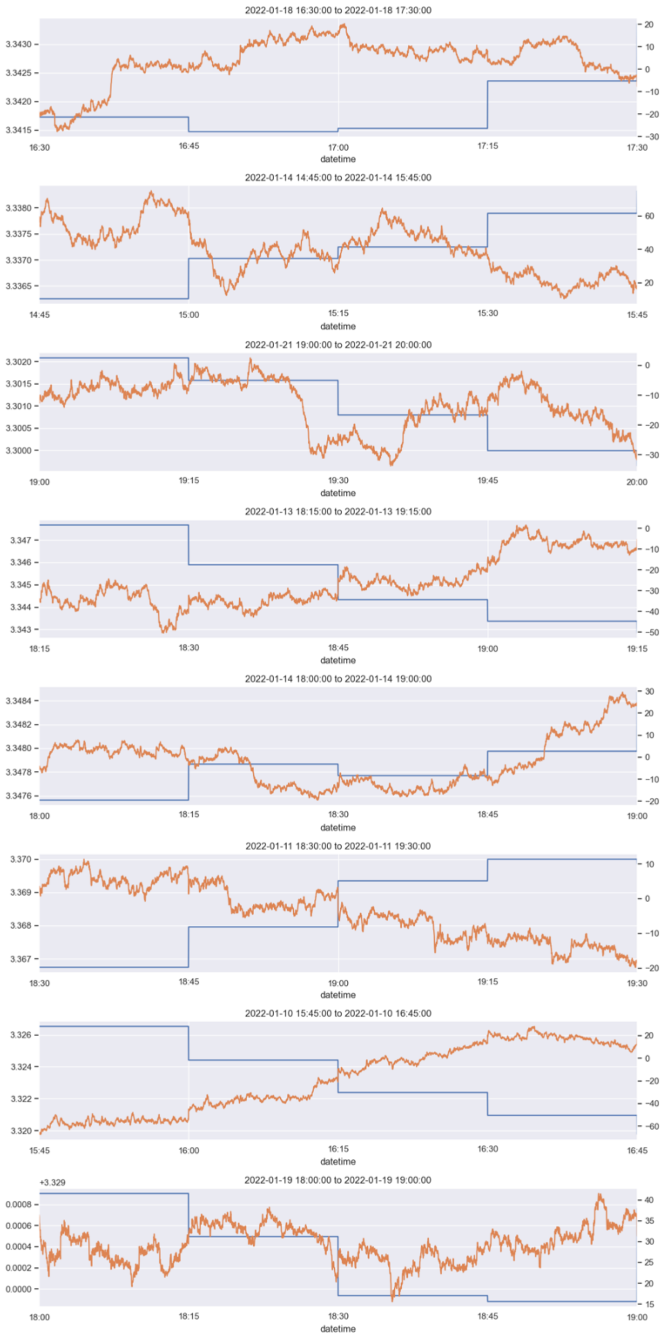

Figure 11 - XX

In each subperiod, the feasibility of executing mean-reverting strategies is evident.

Given additional time, a grid search could have been conducted to fine-tune the step size and lookback window, optimizing the trade-off between the mean reversion's magnitude/velocity and the stability of the hedge ratio.

To construct a signal, the exponential moving average *z-score* of the spread (calculated as $s = NQc1 - h \times ESc1$, with h fitted based on the last period) will be utilized.

- A high *z-score* indicates an overly high spread, signaling a potential mean reversion to the downside, prompting a sell.

- A low *z-score* suggests an excessively low spread, indicating a mean reversion to the upside, triggering a buy.

Positions are exited when the *z-score's* absolute value is small. For instance, the entry thresholds could be set at 2 and -2, with exit thresholds at 1 and -1 (refer to Figure 12Figure 12)

Figure 12 – XX

Selection of five parameters is required:

- *ewm_halflife*: the half-life for the exponential weighted moving (EWM) *z-score*.
- *z_enter_sell*: the *z-score* threshold to enter a sell order.
- *z_exit_sell*: the *z-score* threshold to exit a sell order.
- *z_enter_buy*: the *z-score* threshold to enter a buy order.
- *z_exit_buy*: the *z-score* threshold to exit a buy order.

Again, ideally a grid search would be conducted for optimization, but through trial and error, the following parameters were identified: 720, 1.7, 0.6, -1.7, -0.6. Figure 13 illustrates the distribution of the difference in spread between entry and exit points: a positive *z-score* correlates with negative spread change (indicating mean reversion downwards), with the inverse occurring for negative *z-scores*.

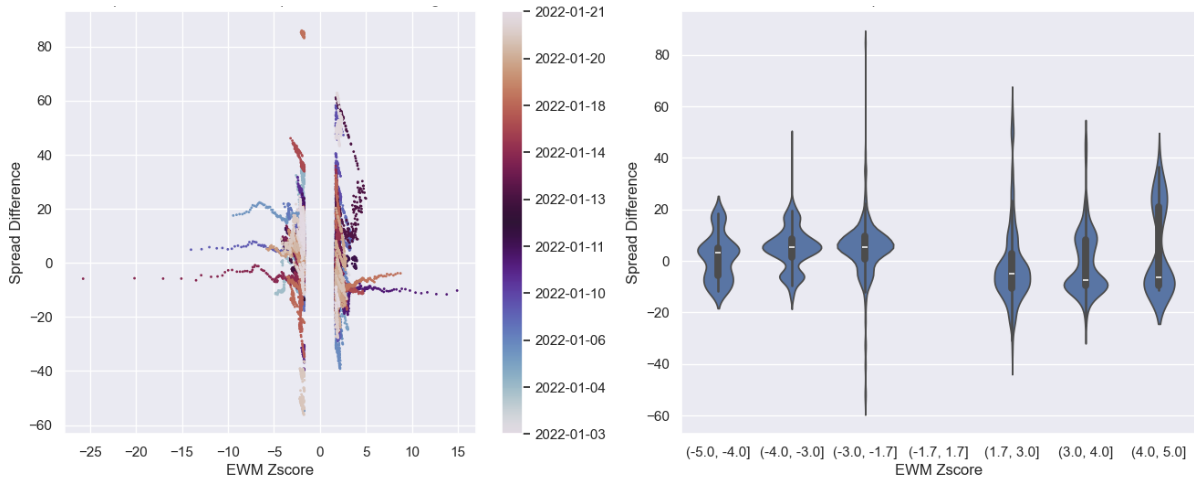

Figure 13 - Relationship between z-score and spread difference throughout time (left); Distribution of spread difference for sized of z-score (right)

3. Backtest

The outlined strategy operates as follows: trades are executed based on the *ewm z-score* of the spread. When the *z-score* exceeds a certain threshold, the size of trades in dollars is determined by linear interpolation between a predefined minimum and maximum size of trades:

$$trade\_size\ [\$] = \frac{(\text{max\_sizing} - \text{min\_sizing})}{(z_{max} - z_{enter})} \times (z - z_{enter}) + \text{min\_sizing}$$

The quantities in to trade in the futures Q_{ESc1} and $Q_{NQc1}$are calculated as follows are calculated as follows:

$$Q_{NQc1} > 0, Q_{ESc1} < 0\ if\ z < z_{enter_{buy}}$$
$$Q_{NQc1} < 0, Q_{ESc1} > 0\ if\ z > z_{enter_{sell}}$$
$$\frac{Q_{ESc1}}{Q_{NQc1}} = -h$$
$$|Q_{NQc1}| \times P_{NQc1} + |Q_{ESc1}| \times P_{ESc1} = trade\_size$$

Two backtests were conducted:

1) Under unrealistic trading conditions: unlimited liquidity and no bid ask.
2) Under realistic conditions: liquidity is the one at bid/ask, trades are executed at bid/ask prices and constant transaction cost in dollar.

Backtest 1

As demonstrated in Figure 14 in unrealistic trading conditions, the outcomes are favorable, with a yearly return of 7.9% and Sharpe ratio of 6.3. However, the challenge arises from the necessity to invest approximately 1.5M $ per trade, which on average generates a return of only $300, indicating substantial positions are required for minimal pnl movements.

Figure 14 - PNL with market-to-market positions

Backtest 2

Under realistic trading conditions as shown in Figure 15, the previously observed alpha vanishes, resulting in negative returns. This outcome is attributed to the fact that slippage per trade under such conditions is on par with the returns generated in the first backtest, rendering the alpha untradeable.

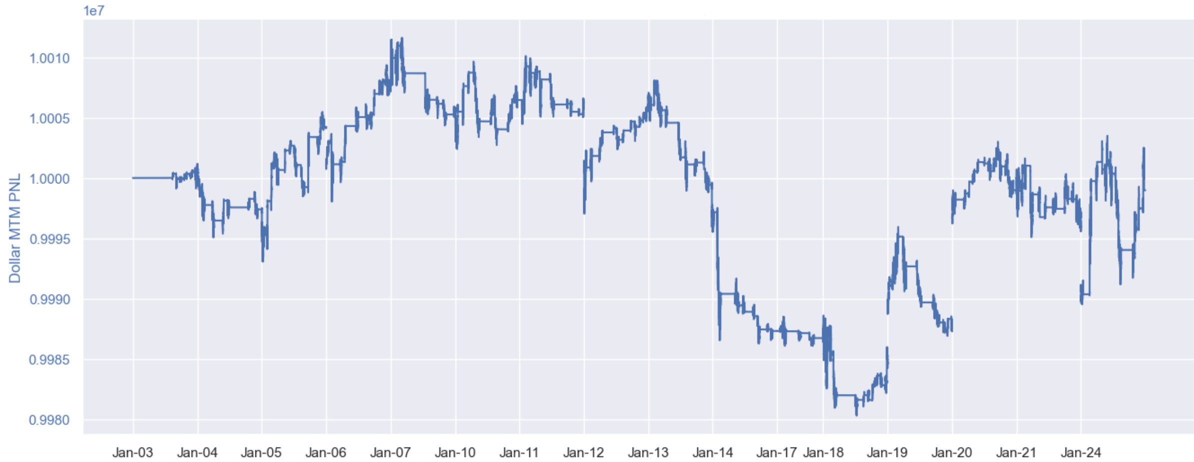

Figure 15 - PNL with market-to-market positions