



$$T_0 = 5s \quad f_0 = \frac{1}{5} \text{ Hz}$$

$$x(t) = \begin{cases} 0 & -3/2 \leq t \leq -1/2 \\ \frac{1}{4}t + \frac{1}{8} & -1/2 \leq t \leq 3/2 \end{cases}$$

$$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos(2\pi n f_0 t) dt$$

$$a_n = \frac{2}{5} \int_{-3/2}^{3/2} x(t) \cos(2\pi n f_0 t) dt$$

$$a_n = \frac{2}{5} \int_{-3/2}^{1/2} 0 \cos(2\pi n f_0 t) dt + \frac{2}{5} \int_{-1/2}^{7/2} \left(\frac{1}{4}t + \frac{1}{8}\right) \cos(2\pi n f_0 t) dt$$

$$a_n = \frac{2}{5} (\cancel{0}) + \frac{2}{5} \int_{-1/2}^{7/2} \frac{1}{4} t \cos(2\pi n f_0 t) dt + \frac{2}{5} \int_{-1/2}^{7/2} \frac{1}{8} \cos(2\pi n f_0 t) dt$$

$$a_n = \frac{1}{10} \int_{-1/2}^{7/2} t \cos(2\pi n f_0 t) dt + \frac{1}{20} \int_{-1/2}^{7/2} \cos(2\pi n f_0 t) dt$$

$$\hookrightarrow \int_{-1/2}^{7/2} t \cos(2\pi n f_0 t) dt \quad u = t \quad dv = \cos(2\pi n f_0 t) dt$$

$$du = dt \quad v = \frac{\sin(2\pi n f_0 t)}{2\pi n f_0}$$

$$= \left. \frac{t \sin(2\pi n f_0 t)}{2\pi n f_0} \right|_{-1/2}^{7/2} - \int_{-1/2}^{7/2} \frac{\sin(2\pi n f_0 t)}{2\pi n f_0} dt$$

$$= \frac{7}{4\pi n \frac{1}{5}} \sin\left(\frac{7\pi n}{5}\right) - \frac{1}{4\pi n \frac{1}{5}} \sin\left(\frac{\pi n}{5}\right) - \frac{1}{2\pi n f_0} \left(-\frac{\cos(2\pi n f_0 t)}{2\pi n f_0} \right) \bigg|_{-1/2}^{7/2}$$

$$= \frac{35}{4\pi n} \sin\left(\frac{7\pi n}{5}\right) - \frac{5}{4\pi n} \sin\left(\frac{\pi n}{5}\right) + \frac{25}{4\pi^2 n^2} \cos\left(\frac{7\pi n}{5}\right)$$

$$- \frac{25}{4\pi^2 n^2} \cos\left(\frac{\pi n}{5}\right)$$

$$L \int_{-1/2}^{1/2} \cos(2\pi n f_0 t) dt = \frac{\sin(2\pi n f_0 t)}{2\pi n f_0} \Big|_{-1/2}^{1/2}$$

$$= \left[\frac{5}{2\pi n} \sin\left(\frac{7\pi n}{5}\right) + \frac{5}{2\pi n} \sin\left(\frac{\pi n}{5}\right) \right]$$

$$a_n = \frac{1}{10} \left[\frac{35}{4\pi n} \sin\left(\frac{7\pi n}{5}\right) - \frac{5}{4\pi n} \sin\left(\frac{\pi n}{5}\right) + \frac{25}{4\pi^2 n^2} \cos\left(\frac{7\pi n}{5}\right) \right.$$

$$\left. - \frac{25}{4\pi^2 n^2} \cos\left(\frac{\pi n}{5}\right) \right] + \frac{1}{20} \left[\frac{5}{2\pi n} \sin\left(\frac{7\pi n}{5}\right) + \frac{5}{2\pi n} \sin\left(\frac{\pi n}{5}\right) \right]$$

$$a_n = \frac{7}{8\pi n} \sin\left(\frac{7\pi n}{5}\right) - \frac{7}{8\pi n} \sin\left(\frac{\pi n}{5}\right) + \frac{5}{8\pi^2 n^2} \cos\left(\frac{7\pi n}{5}\right)$$

$$- \frac{5}{8\pi^2 n^2} \cos\left(\frac{\pi n}{5}\right) + \frac{1}{8\pi n} \sin\left(\frac{7\pi n}{5}\right) + \frac{1}{8\pi n} \sin\left(\frac{\pi n}{5}\right)$$

$$a_n = \frac{1}{\pi n} \sin\left(\frac{7\pi n}{5}\right) - \frac{5}{8\pi^2 n^2} \cos\left(\frac{\pi n}{5}\right) + \frac{5}{8\pi^2 n^2} \cos\left(\frac{7\pi n}{5}\right)$$

$$a_n = \frac{5}{8\pi^2 n^2} \left[\frac{8\pi n}{5} \sin\left(\frac{7\pi n}{5}\right) - 5\pi n \cos\left(\frac{\pi n}{5}\right) + 5\pi n \cos\left(\frac{7\pi n}{5}\right) \right]$$

$$b_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} x(t) \sin(2\pi n f_0 t) dt$$

$$b_n = \frac{2}{5} \int_{-3/2}^{7/2} x(t) \sin(2\pi n f_0 t) dt$$

$$b_n = \frac{2}{5} \int_{-3/2}^{-1/2} 0 \sin(2\pi n f_0 t) dt + \frac{2}{5} \int_{-1/2}^{7/2} \left(\frac{1}{4}t + \frac{1}{8}\right) \sin(2\pi n f_0 t) dt$$

$$b_n = \frac{2}{5} (0) + \frac{2}{5} \int_{-1/2}^{7/2} \frac{1}{4}t \sin(2\pi n f_0 t) dt + \frac{2}{5} \int_{-1/2}^{7/2} \frac{1}{8} \sin(2\pi n f_0 t) dt$$

$$b_n = \frac{1}{10} \int_{-1/2}^{7/2} t \sin(2\pi n f_0 t) dt + \frac{1}{20} \int_{-1/2}^{7/2} \sin(2\pi n f_0 t) dt$$

$$\hookrightarrow \int_{-1/2}^{7/2} t \sin(2\pi n f_0 t) dt$$

$$u = t \quad du = \sin(2\pi n f_0 t) dt$$

$$du = dt \quad v = -\frac{\cos(2\pi n f_0 t)}{2\pi n f_0}$$

$$= -\frac{t \cos(2\pi n f_0 t)}{2\pi n f_0} \Big|_{-1/2}^{7/2} - \int_{-1/2}^{7/2} -\frac{\cos(2\pi n f_0 t)}{2\pi n f_0} dt$$

$$= -\frac{1}{2\pi n f_0} \left[\frac{7}{2} \cos(7\pi n f_0) + \frac{1}{2} \cos(\pi n f_0) \right] + \frac{1}{4\pi^2 n^2 f_0^2} \sin(2\pi n f_0 t) \Big|_{-1/2}^{7/2}$$

$$= -\frac{7}{4\pi n \frac{1}{5}} \cos\left(\frac{7\pi n}{5}\right) - \frac{1}{4\pi n \frac{1}{5}} \cos\left(\frac{\pi n}{5}\right) + \frac{25}{4\pi^2 n^2} \sin\left(\frac{7\pi n}{5}\right)$$

$$+ \frac{25}{4\pi^2 n^2} \sin\left(\frac{\pi n}{5}\right)$$

$$\hookrightarrow \int_{-1/2}^{1/2} \sin(2\pi n f_0 t) dt = \left. \frac{-\cos(2\pi n f_0 t)}{2\pi n f_0} \right|_{-1/2}^{1/2}$$

$$= -\frac{\cos(7\pi n f_0)}{2\pi n f_0} + \frac{\cos(\pi n f_0)}{2\pi n f_0}$$

$$= -\frac{5}{2\pi n} \cos\left(\frac{7\pi n}{5}\right) + \frac{5}{2\pi n} \cos\left(\frac{\pi n}{5}\right)$$

$$b_n = \frac{1}{10} \left[-\frac{35}{4\pi n} \cos\left(\frac{7\pi n}{5}\right) - \frac{5}{4\pi n} \cos\left(\frac{\pi n}{5}\right) + \frac{25}{4\pi^2 n^2} \sin\left(\frac{7\pi n}{5}\right) \right.$$

$$\left. + \frac{25}{4\pi^2 n^2} \sin\left(\frac{\pi n}{5}\right) \right] + \frac{1}{20} \left[-\frac{5}{2\pi n} \cos\left(\frac{7\pi n}{5}\right) + \frac{5}{2\pi n} \cos\left(\frac{\pi n}{5}\right) \right]$$

$$= -\frac{7}{8\pi n} \cos\left(\frac{7\pi n}{5}\right) - \frac{1}{8\pi n} \cos\left(\frac{\pi n}{5}\right) + \frac{5}{8\pi^2 n^2} \sin\left(\frac{7\pi n}{5}\right)$$

$$+ \frac{5}{8\pi^2 n^2} \sin\left(\frac{\pi n}{5}\right) + \frac{1}{8\pi n} \cos\left(\frac{7\pi n}{5}\right) + \frac{1}{8\pi n} \cos\left(\frac{\pi n}{5}\right)$$

$$b_n = -\frac{1}{8\pi n} \cos\left(\frac{7\pi n}{5}\right) + \frac{5}{8\pi^2 n^2} \sin\left(\frac{7\pi n}{5}\right) + \frac{5}{8\pi^2 n^2} \sin\left(\frac{\pi n}{5}\right)$$

$$a_0 = \frac{1}{T_0} \int_{T_0} x(t) dt = \frac{1}{5} \int_{-3/2}^{7/2} x(t) dt$$

$$= \frac{1}{5} \int_{-3/2}^{-1/2} 0 dt + \frac{1}{5} \int_{-1/2}^{7/2} \left(\frac{1}{4}t + \frac{1}{8} \right) dt$$

$$= \frac{1}{5} (0) + \frac{1}{5} \int_{-1/2}^{7/2} \frac{1}{4}t dt + \frac{1}{5} \int_{-1/2}^{7/2} \frac{1}{8} dt$$

$$= \frac{1}{20} \left. \frac{t^2}{2} \right|_{-1/2}^{7/2} + \frac{1}{40} \left. t \right|_{-1/2}^{7/2} = \frac{1}{40} \frac{49}{4} - \frac{1}{40} \frac{1}{4} + \frac{7}{80} + \frac{1}{80}$$

$$= \frac{49}{160} - \frac{1}{160} + \frac{7}{80} - \frac{1}{80} = \frac{48}{160} + \frac{6}{80} = \frac{3}{10} + \frac{3}{40} = \frac{3}{8}$$

$$b_n = \frac{5}{8\pi^2 n^2} \left[-\frac{8\pi n}{5} \cos\left(\frac{7\pi n}{5}\right) + 5 \sin\left(\frac{7\pi n}{5}\right) + 5 \sin\left(\frac{\pi n}{5}\right) \right]$$