$$x(t) = \frac{1}{4}\cancel{t} + \frac{1}{8}$$

$$C_n = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-j2\pi n f_0 t}\, dt$$

$$C_n = \frac{1}{4}\int_{-1/2}^{7/2}\left(\frac{1}{4}t + \frac{1}{8}\right) e^{-j2\pi n f_0 t}\, dt$$

$$C_n = \frac{1}{4}\int_{-1/2}^{7/2} \frac{1}{4}\cancel{t}\, e^{-j2\pi n f_0 t}\, dt + \frac{1}{4}\int_{-1/2}^{7/2} \frac{1}{8} e^{-j2\pi n f_0 t}\, dt$$

$$C_n = \frac{1}{16}\int_{-1/2}^{7/2} t\, e^{-j2\pi n f_0 t}\, dt + \frac{1}{32}\int_{-1/2}^{7/2} e^{-j2\pi n f_0 t}\, dt$$

$$\hookrightarrow \int_{-1/2}^{7/2} t\, e^{-j2\pi n f_0 t}\, dt$$

$$u = t \qquad v' = e^{-j2\pi n f_0 t}\, dt$$

$$u' = dt \qquad v = -\frac{1}{j2\pi n f_0} e^{-j2\pi n f_0 t}$$

$$= \frac{1}{16}\left(-\frac{t}{j2\pi n f_0} e^{-j2\pi n f_0 t}\Big|_{-1/2}^{7/2} - \int_{-1/2}^{7/2} -\frac{1}{j2\pi n f_0} e^{-j2\pi n f_0 t}\, dt\right)$$

$$= \frac{1}{16}\left(-\frac{7}{j4\pi n f_0} e^{-j7\pi n f_0} - \frac{1}{j4\pi n f_0} e^{+j\pi n f_0} \right.$$

$$\left. + \frac{1}{j2\pi n f_0} \int_{-1/2}^{7/2} e^{-j2\pi n f_0 t} \, dt \right)$$

$$= \frac{1}{16}\left(-\frac{7}{j4\pi n f_0} e^{-j7\pi n f_0} - \frac{1}{j4\pi n f_0} e^{j\pi n f_0} + \frac{1}{j2\pi n f_0} \right.$$

$$\left. \left(-\frac{1}{j2\pi n f_0} e^{-j2\pi n f_0 t} \Big|_{-1/2}^{7/2} \right) \right)$$

$$= \frac{1}{16}\left(-\frac{7}{j4\pi n f_0} e^{-j7\pi n f_0} - \frac{1}{j4\pi n f_0} e^{j\pi n f_0} - \left(\frac{1}{j2\pi n f_0}\right)^2 \right.$$

$$\left. \left(e^{-j7\pi n f_0} - e^{j\pi n f_0} \right) \right)$$

$$= \frac{1}{16}\left(+ \frac{j7}{\pi n} e^{-j\frac{7\pi n}{4}} + \frac{j1}{\pi n} e^{j\frac{\pi n}{4}} + \frac{4}{\pi^2 n^2} e^{-j\frac{7\pi n}{4}} \right.$$

$$\left. - \frac{4}{\pi^2 n^2} e^{j\pi n/4} \right)$$

$$= \frac{1}{16}\left(\frac{j7}{\pi n} e^{-j\frac{7\pi n}{4}} + \frac{j1}{\pi n} e^{j\frac{\pi n}{4}} + \frac{4}{\pi^2 n^2}\left(e^{-j\frac{7\pi n}{4}} - e^{j\frac{\pi n}{4}} \right) \right)$$

$$= \frac{1}{16}\left(\frac{j7}{\pi n}\left(\cos\left(\frac{7\pi n}{4}\right) - j\sin\left(\frac{7\pi n}{4}\right)\right) + \frac{j1}{\pi n}\left(\cos\left(\frac{\pi n}{4}\right) + j\sin\left(\frac{\pi n}{4}\right)\right)\right.$$

$$\left. + \frac{4}{\pi^2 n^2}\left(\cos\left(\frac{7\pi n}{4}\right) - j\sin\left(\frac{7\pi n}{4}\right) - \cos\left(\frac{\pi n}{4}\right) - j\sin\left(\frac{\pi n}{4}\right)\right)\right)$$

$$= \frac{1}{4\pi^2 n^2}\left(j\frac{7\pi n}{4}\cos\left(\frac{7\pi n}{4}\right) + \frac{7\pi n}{4}\sin\left(\frac{7\pi n}{4}\right) + j\frac{\pi n}{4}\cos\left(\frac{\pi n}{4}\right) + \dots\right.$$

$$\underbrace{\left. - \frac{\pi n}{4}\sin\left(\frac{\pi n}{4}\right) + \cos\left(\frac{7\pi n}{4}\right) - j\sin\left(\frac{7\pi n}{4}\right) - \cos\left(\frac{\pi n}{4}\right) - j\sin\left(\frac{\pi n}{4}\right)\right)}_{\frac{1}{16}\int_{-1/2}^{7/2} t e^{-2\pi n f_0 t}\, dt}$$

$$\hookrightarrow \frac{1}{32}\int_{-1/2}^{7/2} e^{-j2\pi n f_0 t}\, dt = \frac{1}{32}\left[-\frac{1}{j2\pi n f_0} e^{-j2\pi n f_0 t}\right]_{-1/2}^{7/2}$$

$$= \frac{j}{64\pi n f_0}\left(e^{-j7\pi n f_0} - e^{j\pi n f_0}\right) = \frac{j}{16\pi n} e^{-j\frac{7\pi n}{4}} - \frac{j}{16\pi n} e^{j\frac{\pi n}{4}}$$

$$= \frac{j}{16\pi n}\cos\left(\frac{7\pi n}{4}\right) + \frac{1}{16\pi n}\sin\left(\frac{7\pi n}{4}\right) - \frac{j}{16\pi n}\cos\left(\frac{\pi n}{4}\right) + \frac{1}{16\pi n}\sin\left(\frac{\pi n}{4}\right)$$

$$= \cancel{j\frac{7}{16\pi n}\cos\left(\frac{7\pi n}{4}\right)} + \cancel{\frac{7}{16\pi n}\sin\left(\frac{7\pi n}{4}\right)} + \cancel{j\frac{1}{16\pi n}\cos\left(\frac{\pi n}{4}\right)} \dots$$

$$\cancel{j\frac{1}{16\pi n}\sin\left(\frac{\pi n}{4}\right)} + \frac{1}{4\pi^2 n^2}\cos\left(\frac{7\pi n}{4}\right) - j\frac{1}{4\pi^2 n^2}\sin\left(\frac{7\pi n}{4}\right) \dots$$

$$\frac{1}{4\pi^2 n^2}\cos\left(\frac{\pi n}{4}\right) - j\frac{1}{4\pi^2 n^2}\sin\left(\frac{\pi n}{4}\right) + j\frac{1}{16\pi n}\cos\left(\frac{7\pi n}{4}\right)$$

$$+ \cancel{\frac{1}{16\pi n}\sin\left(\frac{7\pi n}{4}\right)} - \cancel{j\frac{1}{16\pi n}\cos\left(\frac{\pi n}{4}\right)} + \cancel{\frac{1}{16\pi n}\sin\left(\frac{\pi n}{4}\right)}$$

$$= j\frac{1}{2\pi n}\cos\left(\frac{7\pi n}{4}\right) + \frac{1}{2\pi n}\sin\left(\frac{7\pi n}{4}\right) + \frac{1}{4\pi^2 n^2}\cos\left(\frac{7\pi n}{4}\right)$$

$$- j\frac{1}{4\pi^2 n^2}\sin\left(\frac{7\pi n}{4}\right) - \frac{1}{4\pi^2 n^2}\cos\left(\frac{\pi n}{4}\right) - j\frac{1}{4\pi^2 n^2}\sin\left(\frac{\pi n}{4}\right)$$

$$= \frac{1}{4\pi^2 n^2}\Big[j\,2\pi n\cos\left(\frac{7\pi n}{4}\right) + 2\pi n\sin\left(\frac{7\pi n}{4}\right) + \cos\left(\frac{7\pi n}{4}\right)$$

$$- j\sin\left(\frac{7\pi n}{4}\right) - \cos\left(\frac{\pi n}{4}\right) - j\sin\left(\frac{\pi n}{4}\right)\Big]$$

$$a_n = 2\,\mathrm{Re}\{c_n\} \rightarrow a_n = 2\left(\frac{1}{4\pi^2 n^2}\left(2\pi n\sin\left(\frac{7\pi n}{4}\right) + \cos\left(\frac{7\pi n}{4}\right)\right.\right.$$

$$\left.\left. - \cos\left(\frac{\pi n}{4}\right)\right)\right)$$

$$a_n = \frac{1}{2\pi^2 n^2}\left(2\pi n\sin\left(\frac{7\pi n}{4}\right) + \cos\left(\frac{7\pi n}{4}\right) - \cos\left(\frac{\pi n}{4}\right)\right)$$

$$b_n = -2\,\mathrm{Im}\{C_n\} \to b_n = -2\left(\frac{1}{4\pi^2 n^2}\left(2\pi n \cos\left(\frac{7\pi n}{4}\right)\right.\right.$$

$$\left.\left.-\sin\left(\frac{7\pi n}{4}\right) - \sin\left(\frac{\pi n}{4}\right)\right)\right)$$

$$b_n = \frac{1}{2\pi^2 n^2}\left(\sin\left(\frac{7\pi n}{4}\right) + \sin\left(\frac{\pi n}{4}\right) - 2\pi n \cos\left(\frac{7\pi n}{4}\right)\right)$$

$$a_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt \to a_0 = \frac{1}{4}\int_{-1/2}^{7/2} \frac{1}{4}t + \frac{1}{8}\,dt$$

$$= \frac{1}{16}\left.\frac{t^2}{2}\right|_{-1/2}^{7/2} + \frac{1}{32}\left.t\right|_{-1/2}^{7/2} = \frac{1}{16}\cdot\frac{49}{8} - \frac{1}{16}\cdot\frac{1}{8} + \frac{1}{32}\cdot\frac{7}{2} + \frac{1}{32}\cdot\frac{1}{2}$$

$$= \frac{49}{128} - \frac{1}{128} + \frac{7}{64} + \frac{1}{64} = \frac{48}{128} + \frac{1}{8} = \frac{1}{2}$$

$$a_0 = \frac{1}{2}$$