$$x(t) = \left(\frac{1}{4}t + \frac{1}{8}\right) \text{rect}\left(\frac{t}{4} + \frac{3}{8}\right)$$

$D = 3/2$

$B = 2$

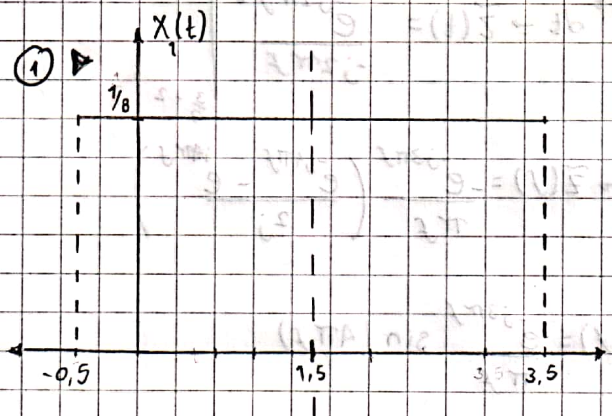
$\hookrightarrow \text{rect}\left(\frac{t+D}{2B}\right) \cdot A$

$$= \underbrace{\frac{1}{4} t \,\text{rect}\left(\frac{t}{4} + \frac{3}{8}\right)}_{2} + \underbrace{\frac{1}{8} \text{rect}\left(\frac{t}{4} + \frac{3}{8}\right)}_{1}$$

(2) Multiplicación por t

(1) Translación en el tiempo

(1) ▸



$$\tilde{X}_1(f) = \int_{-\infty}^{\infty} x_1(t) e^{-j2\pi f t} dt$$

$$\tilde{X}_1(f) = \int_{3/2-2}^{3/2+2} \frac{1}{8} e^{-j2\pi f t} dt$$

$$\tilde{X}_1(f) = \frac{1}{8} \frac{e^{-j2\pi f t}}{-j2\pi f} \Bigg|_{\frac{3}{2}-2}^{\frac{3}{2}+2} \rightarrow \tilde{X}_1(f) = \frac{j}{16\pi f}\left(e^{-j2\pi f(\frac{3}{2}+2)} - e^{-j2\pi f(\frac{3}{2}-2)}\right)$$

$$\tilde{X}_1(f) = \frac{j\, e^{-j3\pi f}}{16\pi f}\left(e^{-j4\pi f} - e^{j4\pi f}\right) \rightarrow \tilde{X}_1(f) = -\frac{e^{-j3\pi f}}{j8\pi f}\left(\frac{e^{j4\pi f} - e^{-j4\pi f}}{2}\right)$$

$$\tilde{X}(f) = \frac{e^{-j3\pi f}}{8\pi f}\left(\frac{e^{j4\pi f} - e^{-j4\pi f}}{2j}\right)$$

$\cdot \cos(a) = \frac{e^{aj} + e^{-aj}}{2}$

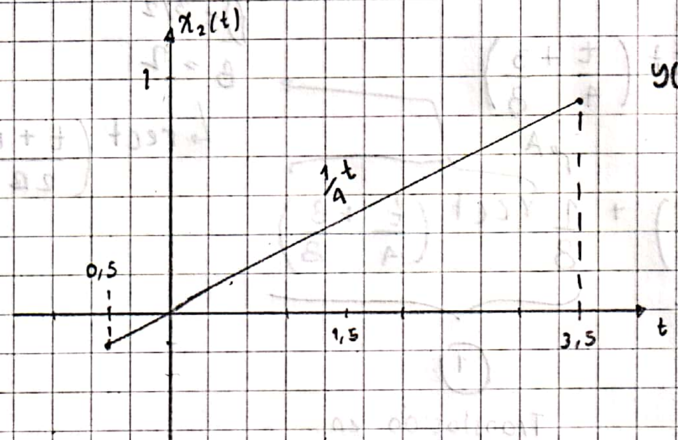
$$\tilde{X}(f) = \frac{e^{-j3\pi f}}{8\pi f} \sin(4\pi f) \rightarrow \tilde{X}(f) = \underbrace{\frac{e^{-j3\pi f}}{2(4\pi f)} \sin(4\pi f)}$$

$\cdot \sin(a) = \frac{e^{aj} - e^{-aj}}{2j}$

$\cdot \text{sinc}(B) = \frac{\sin(\pi B)}{\pi B}$

$$\tilde{X}_1(f) = \frac{e^{-j3\pi f}}{2} \text{sinc}(f)$$

② ▶



$$y(t) = \frac{1}{4} t \, rec\left(\frac{t}{4} + \frac{3}{8}\right)$$

$$\left. y(t) = \frac{1}{4} t \, z(t) \right\} t^n x(t) \xrightarrow{\mathcal{F}}$$

$$z(t) = rec\left(\frac{t}{4} + \frac{3}{8}\right)$$

$$\tilde{Z}(f) = \int_{-\infty}^{\infty} z(t) e^{-j2\pi f t} dt \rightarrow \tilde{Z}(f) = \int_{3/2-2}^{3/2+2} e^{-j2\pi f t} dt \rightarrow \tilde{Z}(t) = \left. \frac{e^{-j2\pi f t}}{-j2\pi f} \right|_{\frac{3}{2}-2}^{3/2+2}$$

$$\tilde{Z}(f) = \frac{e^{-j2\pi f(\frac{3}{2}+2)} - e^{-j2\pi f(\frac{3}{2}-2)}}{-j2\pi f} \rightarrow \tilde{Z}(f) = \frac{-e^{-j3\pi f}}{\pi f}\left(\frac{e^{-j4\pi f} - e^{j4\pi f}}{2j}\right)$$

$$\tilde{Z}(f) = \frac{e^{-j3\pi f}}{\pi f}\left(\frac{e^{j4\pi f} - e^{-j4\pi f}}{2j}\right) \rightarrow \tilde{Z}(f) = \frac{e^{-j3\pi f}}{\pi f} \sin(4\pi f)$$

$$\underbrace{\tilde{Z}(f) = 4 e^{-j3\pi f} \operatorname{sinc}(f)} \longrightarrow \tilde{Z}(f) = \frac{e^{-j3\pi f}}{\pi f} \sin(4\pi f)$$

$$\xrightarrow{\mathcal{F}} \left(\frac{j}{2\pi}\right) \frac{d}{df} \tilde{Z}(f) = \frac{j}{2\pi} \frac{d}{df}\left(\frac{e^{-j3\pi f} \sin(4\pi f)}{\pi f}\right)$$

$$= \frac{j}{2\pi}\left[\frac{\pi f\left(e^{-j3\pi f}\cos(4\pi f)(4\pi) + (-j3\pi) e^{-j3\pi f} \sin(4\pi f)\right) - \pi e^{-j3\pi f} \sin(4\pi f)}{\pi^2 f^2}\right]$$

$$= \frac{j}{2\pi}\left[\frac{4}{f}\cos(4\pi f) e^{-j3\pi f} - \frac{j3}{f} \sin(4\pi f) e^{-j3\pi f} - \frac{1}{\pi f^2} \sin(4\pi f) e^{-j3\pi f}\right]$$

$$= \frac{j}{2\pi}\left[\frac{4}{f} \cos(4\pi f) e^{-j3\pi f} - \frac{j12\pi}{4\pi f} \sin(4\pi f) e^{-j3\pi f} - \frac{4}{4\pi f^2} \sin(4\pi f) e^{-j3\pi f}\right]$$

$$= \frac{j}{2\pi}\left[\frac{4}{f} \cos(4\pi f) e^{-j3\pi f} - \frac{1}{\pi f^2}\left[\sin(4\pi f) e^{-j3\pi f}\left(\frac{j3\pi f}{f} + 1\right)\right.\right.$$

$$= \frac{j2}{\pi f} \cos(4\pi f) e^{-j3\pi f} - \frac{j}{2\pi f}\left[\operatorname{sinc}(f) e^{-j3\pi f}(j12\pi f + 4)\right]$$

$$= \frac{j2}{\pi f}\cos(4\pi f)e^{-j3\pi f} - \frac{j}{\pi f}\left[\operatorname{sinc}(f)e^{-j3\pi f}(j6\pi f+2)\right]$$

$$\tilde{x}(f) = \frac{e^{-j3\pi f}}{2}\operatorname{sinc}(f) + \frac{j2}{\pi f}\cos(4\pi f)e^{-j3\pi f} - \frac{j}{\pi f}\left[\operatorname{sinc}(f)e^{-j3\pi f}(j6\pi f+2)\right]$$

$$\tilde{x}(f) = \frac{e^{-j3\pi f}}{2}\operatorname{sinc}(f) + \frac{j2}{\pi f}\cos(4\pi f)e^{-j3\pi f} - \frac{j}{\pi f}\operatorname{sinc}(f)e^{-j3\pi f}(j6\pi f+2)$$

$$\tilde{x}(f) = \frac{\left[\cos(3\pi f) - j\,\mathrm{sen}(3\pi f)\right]}{2}\operatorname{sinc}(f) + \frac{j2}{\pi f}\cos(4\pi f)\left(\cos(3\pi f) - j\,\mathrm{sen}(3\pi f)\right)$$

$$+\, 6\operatorname{sinc}(f)\left(\cos(3\pi f) - j\,\mathrm{sen}(3\pi f)\right) - j\frac{2}{\pi f}\operatorname{sinc}(f)\left(\cos(3\pi f) - j\,\mathrm{sen}(3\pi f)\right)$$

$$\tilde{x}(f) = \frac{\operatorname{sinc}(f)\cos(3\pi f)}{2} - j\frac{\operatorname{sinc}(f)}{2}\sin(3\pi f) + \frac{j2}{\pi f}\cos(4\pi f)\cos(3\pi f)$$

$$+\, \frac{2}{\pi f}\cos(4\pi f)\,\mathrm{sen}(3\pi f) + 6\operatorname{sinc}(f)\cos(3\pi f) - j\,6\operatorname{sinc}(f)\,\mathrm{sen}(3\pi f)$$

$$-\, j\frac{2}{\pi f}\operatorname{sinc}(f)\cos(3\pi f) - \frac{2}{\pi f}\operatorname{sinc}(f)\,\mathrm{sen}(3\pi f)$$

$$\tilde{x}(f) = \frac{\operatorname{sinc}(f)\cos(3\pi f)}{2} - j\frac{3\pi f}{2}\operatorname{sinc}^2(f) + \frac{j2}{\pi f}\cos(4\pi f)\cos(3\pi f)$$

$$+\, 6\operatorname{sinc}(f)\cos(4\pi f) + 6\operatorname{sinc}(f)\cos(3\pi f) - j\,18\pi f\operatorname{sinc}^2(f)$$

$$-\, j\frac{2}{\pi f}\operatorname{sinc}(f)\cos(3\pi f) - 6\operatorname{sinc}^2(f)$$

$$\tilde{x}(f) = \cos(3\pi f)\left[\operatorname{sinc}(f)\left(\frac{13}{2} - j\frac{2}{\pi f}\right) + \frac{j2}{\pi f}\cos(4\pi f)\right] - \operatorname{sinc}^2(f)\left(6 + j\frac{39}{2}\pi f\right)$$

$$+\, 6\operatorname{sinc}(f)\cos(4\pi f)$$

$$x(t) = \begin{cases} \frac{1}{4}t + \frac{1}{8} ; \frac{3}{2} - 2 \leq t \leq \frac{3}{2} + 2 \\ \emptyset ; \text{ o.p.} \end{cases}$$

$$\tilde{X}(t) = \int_{\frac{3}{2}-2}^{\frac{3}{2}+2} \left(\frac{1}{4}t + \frac{1}{8}\right) e^{-j2\pi f t}\, dt = \frac{1}{4}\int_{\frac{3}{2}-2}^{\frac{3}{2}+2} t e^{-j2\pi f t}\, dt + \frac{1}{8}\left[\frac{e^{-j2\pi f t}}{-j2\pi f}\right|_{3/2-2}^{3/2+2}$$

$$= \frac{jt}{8\pi f} e^{-j2\pi f t} + \frac{1}{16\pi^2 f^2} e^{-j2\pi f t} + \frac{j}{16\pi f} e^{-j2\pi f t} \Bigg|_{3/2-2}^{3/2+2}$$

$$= \frac{j\left(\frac{3}{2}+2\right)e^{-j2\pi f\left(\frac{3}{2}+2\right)}}{8\pi f} + \frac{1}{16\pi^2 f^2} e^{-j2\pi f\left(\frac{3}{2}+2\right)} + \frac{j}{16\pi f} e^{-j2\pi f\left(\frac{3}{2}+2\right)} - \frac{j\left(\frac{3}{2}-2\right)e^{-j2\pi f\left(\frac{3}{2}-2\right)}}{8\pi f}$$

$$- \frac{1}{16\pi^2 f^2} e^{-j2\pi f\left(\frac{3}{2}-2\right)} + \frac{j}{16\pi f} e^{-j2\pi f\left(\frac{3}{2}-2\right)}$$

$$= \left(\frac{j3}{16\pi f} + \frac{j}{4\pi f} + \frac{1}{16\pi^2 f^2} + \frac{j}{16\pi f}\right) e^{-j3\pi f} e^{-j4\pi f} - e^{-j3\pi f} e^{j4\pi f}\left(\cancel{\frac{j3}{16\pi f}} - \cancel{\frac{j}{4\pi f}} + \frac{1}{16\pi^2 f^2} \ldots\right.$$

$$\left. + \cancel{\frac{j}{16\pi f}}\right)$$

$$= \left[\left(\frac{1}{(4\pi f)^2} + \frac{j}{2\pi f}\right) e^{-j4\pi f} - e^{j4\pi f}\left(\frac{1}{(4\pi f)^2} + \emptyset\right)\right] e^{-j3\pi f}$$

$$= -e^{-j3\pi f}\left[-\frac{e^{-j4\pi f}}{(4\pi f)^2} - \frac{j}{2\pi f} e^{-j4\pi f} + \frac{e^{j4\pi f}}{(4\pi f)^2}\right] = -e^{-j3\pi f}\left[\frac{1}{8\pi^2 f^2}\left(\frac{e^{j4\pi f} - e^{-j4\pi f}}{2}\right) \ldots\right.$$

$$\left. - \frac{j}{2\pi f} e^{-j4\pi f}\right] = -e^{-j3\pi f}\left[\frac{1}{8\pi^2 f^2}\cos(4\pi f) - \frac{j}{2\pi f}\left(\cos(4\pi f) - j\operatorname{sen}(4\pi f)\right)\right]$$

$$= -\frac{e^{-j3\pi f}}{8\pi^2 f^2}\cos(4\pi f) + j\frac{e^{-j3\pi f}}{2\pi f}\cos(4\pi f) + \frac{e^{-j3\pi f}}{2\pi f}\operatorname{sen}(4\pi f)$$

$$= -\frac{(\cos(3\pi f) - j\,sen(3\pi f)\cos(4\pi f)}{8\pi^2 f^2} + j\frac{(\cos(3\pi f) - j\,sen(3\pi f))\cos(4\pi f)}{2\pi f}$$

$$+\frac{(\cos(3\pi f) - j\,sen(3\pi f))\sin(4\pi f)}{2\pi f}$$

$$= -\frac{\cos(3\pi f)\cos(4\pi f)}{8\pi^2 f^2} + j\frac{3}{8\pi f}\,\text{sinc}(f)\cos(4\pi f) + \frac{j\cos(3\pi f)\cos(4\pi f)}{2\pi f}$$

$$+\frac{3}{2}\,\text{sinc}(f)\cos(4\pi f) + 2\,\text{sinc}(f)\cos(3\pi f) - j\,6\pi f\,\text{sinc}^2(f)$$

$$\tilde{X}(f) = \left(-\frac{1}{8\pi^2 f^2} + \frac{j}{2\pi f}\right)\cos(3\pi f)\cos(4\pi f) + \text{sinc}(f)\left[\left(\frac{3}{2} + \frac{j3}{8\pi f}\right)\cos(4\pi f) + \ldots\right.$$

$$\left. 2\cos(3\pi f)\right] - j\,6\pi f\,\text{sinc}^2(f)$$