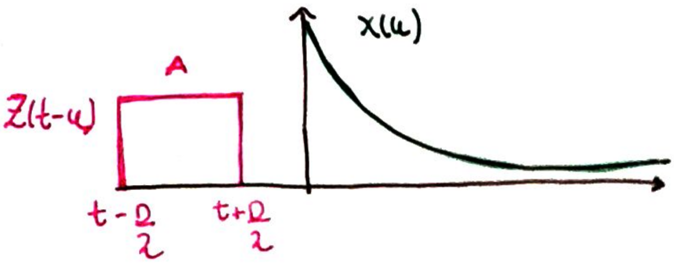
Ejercicios Convolución:

1. $x(t) = e^{-t}\,u(t)$, $z(t) = A\,\mathrm{rect}\left(\frac{t}{D}\right)$

$y(t) = x(t) * z(t)$

$$y(t) = \int_{-\infty}^{\infty} x(u)\,z(t-u)\,du$$

Caso I.



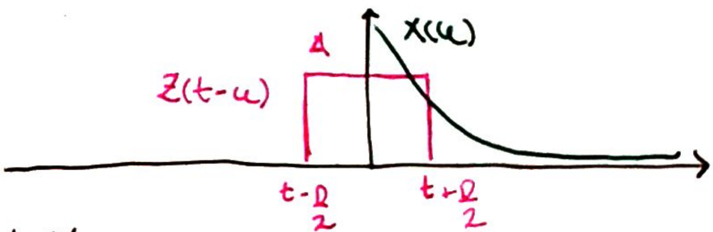
$t + \frac{D}{2} < 0$

$t < -\frac{D}{2}$

$(-\infty, -D/2)$

$y(t) = 0.$

Caso II.



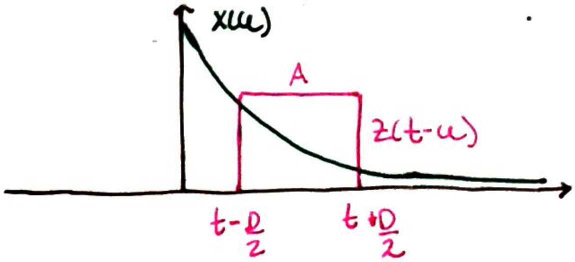
$t+\frac{D}{2} \geq 0 \quad \wedge \quad t - \frac{D}{2} < 0$

$t \geq -\frac{D}{2} \quad \wedge \quad t < \frac{D}{2}$

$[-D/2, D/2)$

$$y(t) = \int_{0}^{t+D/2} A \cdot e^{-u}\,du = -A \cdot e^{-D/2} \cdot e^{-t} + A$$

Caso III



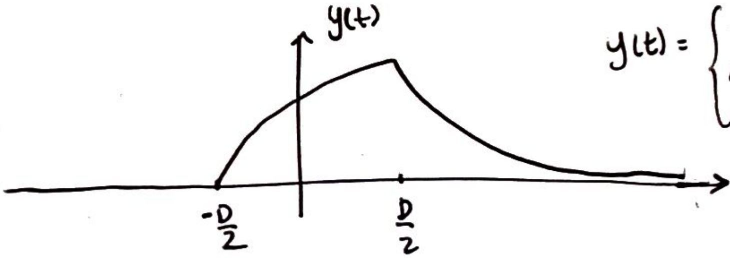
$t - \frac{D}{2} \geq 0$

$t \geq \frac{D}{2}$

$[D/2, \infty)$

$$y(t) = \int_{t-D/2}^{t+D/2} A e^{-u}\,du = -A\left[e^{-t-D/2} - e^{-t+D/2}\right]$$

$$= A e^{-t}\left(e^{D/2} - e^{-D/2}\right) = A\left(e^{D/2} - e^{-D/2}\right)e^{-t}$$

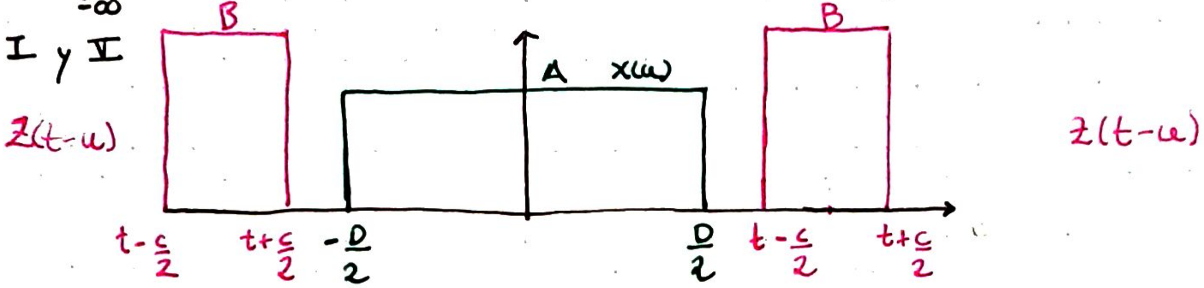


$$y(t) = \begin{cases} A(1-e^{-(t-D/2)})\,; & -D/2 \leq t < D/2 \\ A(e^{D/2} - e^{-D/2})e^{-t}\,; & D/2 \leq t \\ 0\,; & t > -D/2 \end{cases}$$

2. $x(t) = A\,\text{rect}\left(\frac{t}{D}\right) \qquad z(t) = B\,\text{rect}\left(\frac{t}{C}\right) \quad ; \quad D > C$

$$y(t) = \int_{-\infty}^{\infty} x(u)\,z(t-u)\,du$$

Caso I y II



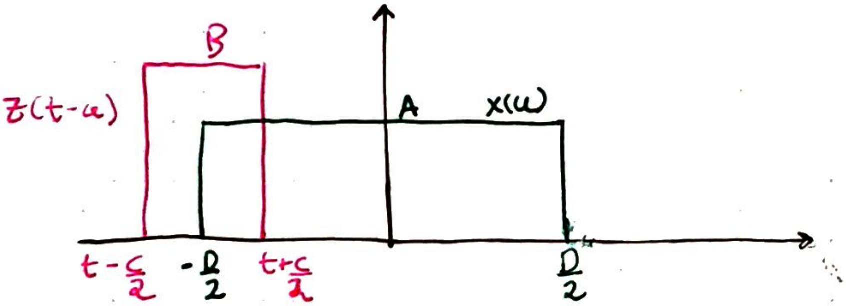
$t + \frac{c}{2} < -\frac{D}{2}$ $\qquad\qquad$ $t - \frac{c}{2} > \frac{D}{2}$

$t < -\left(\frac{D}{2} + \frac{c}{2}\right)$ $\qquad\qquad$ $t > \frac{D}{2} + \frac{c}{2}$

$\left(-\infty, -\frac{1}{2}(D+c)\right)$ $\qquad\qquad$ $\left(\frac{1}{2}(D+c), \infty\right)$

$\underbrace{y(t) = 0.}$

Caso II

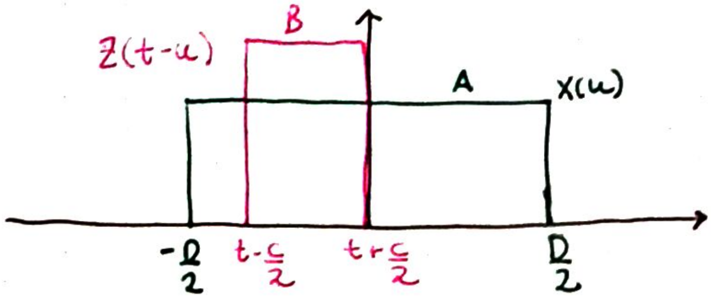


$t + \frac{c}{2} \geq -\frac{D}{2} \;\wedge\; t - \frac{c}{2} < -\frac{D}{2} \rightarrow t \geq -\frac{1}{2}(D+c) \;\wedge\; t < -\frac{1}{2}(D-c)$

$\left[-\frac{1}{2}(D+c), -\frac{1}{2}(D-c)\right)$

$$y(t) = \int_{-D/2}^{t+c/2} A \cdot B\,du = \underbrace{\frac{AB}{2}(C+D) + ABt}$$

Caso III

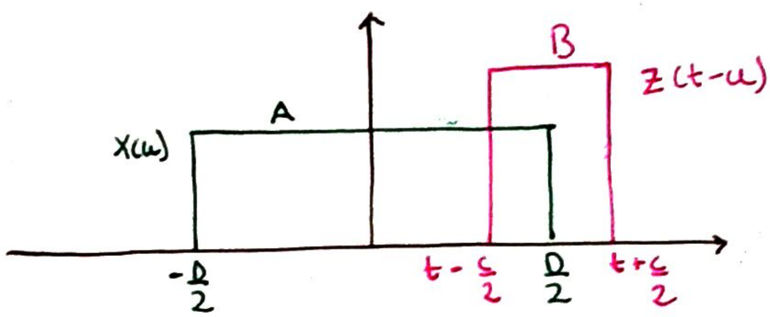


$$t-\frac{c}{2} \geq -\frac{D}{2} \quad \wedge \quad t+\frac{c}{2} \leq \frac{D}{2} \rightarrow t \geqslant -\frac{1}{2}(D-c) \quad \wedge \quad t \leq \frac{1}{2}(D-c)$$

$$\left[-\frac{1}{2}(D-c),\ \frac{1}{2}(D-c)\right]$$

$$y(t)=\int_{t-\frac{c}{2}}^{t+\frac{c}{2}} A \cdot B \cdot du = A \cdot B\left(t+\frac{c}{2}-t+\frac{c}{2}\right) = \underline{ABC}$$

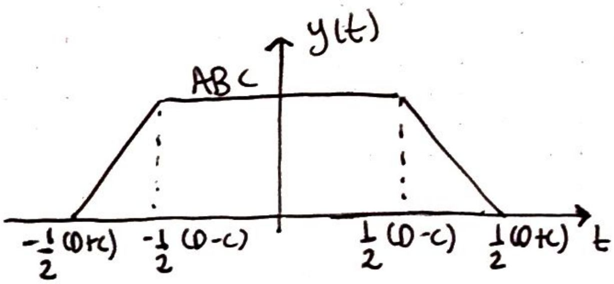
Caso IV



$$t-\frac{c}{2} \leq \frac{D}{2} \quad \wedge \quad t+\frac{c}{2} > \frac{D}{2} \rightarrow t \leq \frac{1}{2}(D+C) \quad \wedge \quad t > \frac{1}{2}(D-C)$$

$$\left(\frac{1}{2}(D-c),\ \frac{1}{2}(D+c)\right]$$

$$y(t)=\int_{t-\frac{c}{2}}^{D/2} A \cdot B \cdot du = AB\left(\frac{D}{2}-t+\frac{c}{2}\right) = \frac{AB}{2}(D+C) - ABt$$



$$y(t)=\begin{cases} \frac{AB}{2}(C+D)+ABt; & -\frac{1}{2}(D+C) \leq t < -\frac{1}{2}(D-C) \\ ABC; & -\frac{1}{2}(D-C) \leq t \leq \frac{1}{2}(D-C) \\ \frac{AB}{2}(C+D)-ABt; & \frac{1}{2}(D-C) < t \leq \frac{1}{2}(D+C) \\ 0; & \text{p.d.v.} \end{cases}$$