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Property portfolio allocation: a multi-factor model

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Summary

The objective of the paper is to describe a method of portfolio allocation currently being developed in the property sector. The model applies a multi-index approach to portfolio decision making and enables investors to explore the effects of changing their allocational decisions. In estimating the parameters required for the model, we explain the method which can be used to adjust for smoothing effects on returns caused by the valuation process. Although the model is still at the development stage, the approach is general and can be applied both to allocating funds within a mixed-asset portfolio or between different types of property within a wholly property-orientated portfolio.

Keywords: Portfolio, multi-factor, asset allocation risk, property.

1. Introduction

In a recent review of US pension sponsors by Arnott (1985), the view was expressed that 'strategic' decisions were of prime importance in affecting investment performance. Strategic decisions concern the allocation of assets between markets. In the case of property, strategic decisions would determine (a) the proportion of the total portfolio devoted to property and (b) the allocation of funds to different types of property (e.g. shops, offices or industrial properties).

Arnott's findings also reflect the strategic decisions of the larger UK property investors. Where formulated, these typically focus on the spread between types and property and, sometimes, between locations. Generally the policy aims to avoid 'too many eggs in too few baskets', or to 'buy the index'. This, it will be recognized, can be a less than satisfactory solution to the asset allocation problem.

In this paper, we discuss one method which institutional investors could use in analysing the asset allocation problem. In the first part of the paper we briefly describe the characteristics of the capital asset pricing model. This model has been advocated as a practical method of analysing diversification issues, but there are serious problems that arise in applying it. There follows a discussion of the type of model that can be used for normative portfolio analysis when the assets are traded in markets which are subject to diffuse influences. We discuss, in the following section, the way in which returns can be transformed so that different assets may be consistently compared and finally we describe the way in which the model operates and future improvements which are being planned.

2. Part 1: Portfolio decision-making and the capital asset pricing

2.1. *Model*

It must be admitted that the traditional expertise of Chartered Surveyors has not usually been concerned with generating analytical recommendations for asset allocation. Indeed it is only in the last decade that the larger surveying firms have attempted to bring analysis to bear on the problem. But times change, and the competition implied by the 'Big Bang' has increased the need to justify the advice on how much to buy (or indeed sell) within institutions' portfolios. Almost inevitably the basis of this justification has been eclectic and in particular has been drawn from the finance/investment literature.

Despite much development, the essentials of portfolio analysis remain unchanged and stem from the seminal works of Markowitz (1952) and Sharpe (1964). The bases of the model are the assumptions that (a) investors are risk averse, i.e. they will require a higher return to compensate them for bearing additional risk in their portfolio; (b) that risk can be measured by analysing the likely divergence between the returns from a portfolio and its expected return; (c) that investors, when considering whether or not to buy an asset, will be rationally concerned with the net effect on their portfolio and will not consider the risk of the asset in isolation from other assets held.

Based on these restrictive assumptions Markowitz developed a model which investors could use to construct efficient portfolios. Although the model was designed to be used for ordinary shares it was, and is, a general model Provided the inputs are correctly estimated, it can be used to construct a portfolio containing many kinds of assets.

However Sharpe (1964) and other writers quickly saw that if investors used the Markowitz model, the markets in which shares were traded would begin to reflect portfolio considerations. In particular, Sharpe (1964) asked the question, 'What would be the result if all investors used Markowitz-type reasoning?' The answer became widely known as the capital asset pricing model (CAPM) and is the basis of many hundreds of articles in the finance research-oriented journals in the last twenty years. Importantly it became used to evaluate the performance of investment managers because it was argued that in the stock market, the optimal decision in an efficient CAPM-like world would be to keep all one's assets in a portfolio which was spread around the market in an entirely neutral way, by maintaining allocations based on current share valuations (e.g. if BP plc accounts for 5% of the market, one should invest 5% of one's own portfolio in BP plc). Thus the task of portfolio managers was to manage their portfolio in such a way as to achieve a return at least as great as that earned on the theoretically optimal portfolio (suitably adjusted for risk).

The adjustment for risk is an important contribution of the CAPM to modern investment writing: the model asserts that whilst total risk of an efficient portfolio can be measured by the variability of returns, risk of a single asset *within the portfolio* should be measured by a constant, referred to as 'beta'.

The beta of an asset represents the linkage between the returns from the asset and the returns from the market portfolio referred to above. If returns from an asset magnify and tend to keep in step with returns from the market portfolio, the beta of the asset will be greater than one. If on the other hand, the returns from the asset are not related at all, period by period with the returns from the market portfolio, the beta will be zero. Once we know the beta of an asset, we can, following the CAPM, calculate the returns that we should expect to earn from the asset (if the CAPM holds). We can then compare this expected return with the figure we derive from analysing our forecasted cash flows and own valuation of the asset. If our own forecasted return is higher than that 'expected' by the CAPM we should then buy the asset because it will add value to our portfolio.

2.2. *Problem with betas*

The problem with this approach is that its use involves us in taking on board all the assumptions required for CAPM. The list for assumptions is formidable and includes the ability of all investors to borrow or lend money in unlimited quantities at a single rate of interest, a single period decision in which the rate of interest is known and at the end of which, an entirely new set of decisions may be taken. The plain fact is that many investors, when they are faced with the assumptions, reject the model as being hopelessly unrealistic. To make matters worse, the record of CAPM, even judged by academic researchers, is not too good. Most of the tests of its validity have investigated shares traded on the New York Stock Exchange and curious anomalies have wreaked havoc in the necessary theorem that the market portfolio is optimal.

The first anomaly is that small firms have *consistently* earned more than the return 'required' by the CAPM. *Prima facie*, it invalidates the CAPM, at least as far as the New York Stock Exchange is concerned.

The other anomaly is even stranger. A recent study by Tinic and West (1984) revealed that although high beta shares do indeed earn more than low beta shares (as required by the CAPM), the difference is very sensitive to the time of the year. In particular, if returns over the period covering the last few days of December and the first few days of January are excluded, the tests invalidate the CAPM.

In applying CAPM to the UK scene, a further problem applies since there have been no tests published which lend support to the model. It is used by academics in examining the behaviour of share prices but, at the time of writing, the authors know only of three (unpublished) studies of the London Stock Exchange, none of which support the CAPM. Given the anomalies reported in the US and the lack of empirical support in the UK, it seems simply an act of faith to assume that the CAPM holds in the UK capital market. To extend the assumption into markets, such as the property investment market, in which the pre-conditions for market efficiency appear to be lacking, seems to be foolhardy[1].

3. Part 2: Alternative approaches – the asset allocation model

But if CAPM is inappropriate, what should be used in its place? The principles of diversification are sound, so too is Markowitz's model of portfolio analysis. Where the CAPM fails is in ignoring more than one source of systematic influence. In the case of the UK Stock Exchange, for example, most investors would recognize that there are many companies which depend largely on the state of markets outside the UK. These companies will therefore tend to be systematically affected by world economic factors to a much greater extent than a company trading entirely within Britain. Similarly a large number of companies may be strongly affected by changes in the rate of interest. Others may be unaffected. We emphasize that CAPM takes these separate influences into account *only* to the extent that the market portfolio reflects them. Thus if the separate factors are not each strongly evident in the period in which the betas are *calculated*, subsequent large changes in one factor will cause large differences between the returns expected by the CAPM and all those firms directly affected by the relevant factor.

3.1. *Proposed method*

One technique by which some of the shortcomings of CAPM can be overcome is to identify the separate factors which influence the shares or assets traded in the market. This approach which is largely identified with Ross (1976) leads to a model which shares some common ground with the CAPM but avoids some of the difficulties established in the empirical tests. The factors can be identified either by statistical analysis of the historical returns from each asset or can be represented by various economic indicators. Thus industrial properties might reasonably be expected to be influenced by the index of industrial production, shops might be more sensitive to changes in personal disposable income or consumer expenditure. We have identified plausible economic indicators and have had some limited success in estimating the relationship between property and the statistically related factors.

3.2. *Data problem*

The difficulty of carrying out this last step in the property context arises because of the lack of historical data in returns from different types of property. However with the development of performance analysis in the property market this situation has changed. The Jones Lang Wootton Property Performance Analysis System which started in 1980 was one of the first in the field and has developed a quarterly data base back to 1970, a uniquely long time period. Other data banks have been established by Richard Ellis–Wood Mackenzie (quarterly from 1978) and IPD (annual from 1980). Although these data are not all in the public domain, research using the data has become possible and will influence the future direction of property research.

3.3. *Efficient market problem*

If one is to discover the effect of the economic factors on the returns from different properties, it is essential that one establishes first that the returns are behaving reasonably consistently with those expected in an 'efficient' market. If this was not the case, valuations (returns) would reflect the subjective biases of a small group of investors and it would be correspondingly more difficult to isolate the effects caused by changes in the external economic factors. This argument underpins the need to establish that the market should be operating efficiently in the economic sense.

While more data has become more available in the last few years, tests reported by Brown (1985) on the returns from one relatively large group of properties revealed some remaining problems. In particular Brown reports that the returns from the property market do not form a random walk but do comply with a 'fair game'. A 'fair game' results if the actual returns randomly differ from the expected returns (if expectations are derived from past observations). Thus if share prices have increased by an average of 0.5% each month over the last 12 months the market would be behaving like a 'fair game' if the prices next month were, with equal probability, either 0.75% *or* 0.25% higher than this month. But the fair game definition does not have anything to say about the way in which the expectations are derived from past data, nor how large fluctuations have to be around the expected return.

In practice there would be considerable uncertainty about whether a fair game is a strong enough criterion to use in judging if a market is efficient. Much depends on the capacity of investors to trade out the foreseeable patterns in returns. Judgements about the market efficiency would also depend on the returns which investors could earn from alternative assets. It is possible, but unlikely, that all assets might fluctuate in value from one month to another. The fact that the fluctuations were foreseeable might not enable investors to profit from the regularities. Thus the interpretation of any pattern is historical returns must be made in the context of the trading possibilities. A 'fair game' is not therefore a *sufficient* condition for market efficiency.

A stronger and sufficient condition of market efficiency is if returns from investment behave like a random walk. In this case, no pattern from one period to another can be observed and returns over time fluctuate around an average level (which may even be zero).

This model is usually chosen when researchers test the efficiency of share trading – those for which the returns behave like a random walk are assumed to be efficiently traded. In an ideal world, one would like to observe returns estimated over small intervals of time stretching over a long period. In the US stock market studies, daily returns are available for hundreds of companies for more than fifty years. in the UK property market, the situation is not so satisfactory! Brown (1985) was fortunate in being able to use a sample of properties, from which he could calculate the hypothetical returns on the basis of monthly valuations over a four-year period. In a recent study, we used quarterly returns over a fifteen-year period (1970 to 1984). The data represented properties valued in aggregate over £2 billion and consisted of valuation-derived returns on portfolios comprising distinct types of property (shops, offices and industrials etc.)

On analysing the returns from our data, we rejected, like Brown (1985) the hypothesis that the returns behaved like a random walk. However following the Box-Jenkins (1976) approach, we diagnosed that the returns on our portfolios were strongly auto-correlated and exhibited the behaviour expected if they were generated by a 'first-order auto-regressive process'. This is sometimes to be expected when returns from portfolios are estimated by averaging individual asset returns collected at different times during the period. But the maximum effect that this averaging process can have, is to increase the first order serial-correlation to 0.25 [see Brown (1985)]. In the case of the quarterly-based returns, we initially found an average value significantly higher than 0.25. Furthermore on checking with those responsible for obtaining the valuations, it was established that the valuations had been carried out within a few days of the end of each quarter. Clearly some smoothing process was either implicitly or explicitly reflected in the valuations. It does therefore suggest that in order to make use of returns from these valuations, some analysis and transformation is required.

The transformation was achieved by applying the statistical methodology referred to under the general heading of the Box-Jenkins approach, and is briefly described in Appendix A. Suffice it to say that our initial analysis proceeded in empirical terms with transformed returns that appeared to behave consistently with the requirements of a random walk. Since the time of our initial analysis, we have explored other samples of returns and found similar problems to greater or lesser extents. The JLW index for example seems to suffer from a degree of smoothing which could be transformed by the method advocated.

3.4 *Factor selection*

In selecting the generating factors that might be expected to influence returns there were three criteria which dominated the process:

1. the link between the factors and the returns from property portfolios should be economically plausible,
2. they should be relatively easy to predict and
3. they should, as far as possible, be orthogonal.

The justification for the first criterion is that it is possible to identify, from any given data set containing many time series, a number of variables that over a specific period of time appeared to be related to the returns from one particular portfolio. However unless there was a logical association between the variables and property returns, it would be unlikely that the association observed in one period would persist into another period. An economically-rationalized link would provide some justification for the belief that the association was more than temporary or fortuitous.

The predictability feature is a useful but not strictly necessary criterion. But since the object is to employ the model for forecasting portfolio returns as well as to modify the composition, it is advantageous that the generating factors are easier to predict than the 'raw' returns of the properties. Similarly orthogonality is not a necessary criterion; it is easy to create a set of orthogonal indices from the same number of non-orthogonal indices[2].

Initial work on the selection of plausible generating factors stems from the work of Roll and Ross (1984) who identified four factors which jointly defined the generating process of stock market returns. The factors identified by Roll and Ross were (a) inflation, (b) industrial production, (c) risk premium (the difference between the yields of high and low 'quality' bonds) and (d) changes in the shape of the term structure of interest rates. The application of this research to either mixed asset portfolios or to different types of properties suggests that additional factors might include (a) the returns on a stock market index and (b) parameters relating to forecasts of the property market. Accordingly the models we have so far explored have incorporated the FT-All share index and the expected rental growth on 'prime' property. The technical and mathematical background of the economic indicator approach is presented in Appendix B.

3.5. *Using the model*

In developing the model, the general decision was taken to ensure that a portfolio manager would find it intuitively appealing as well as relatively simple to apply. Accordingly the model, which was set up on an IBM-PC, invites the user to override the forecasts of the economic indicators, thus facilitating a sensitivity analysis to the manager's own forecasts. This leads to the analysis of portfolio efficiency: given the current asset-weighting of the portfolio, could performance be improved by changing the components? Conventionally an improvement is said to be achieved if either the same expected return can be forecasted with a higher degree of accuracy (i.e. less risk) or a higher return can be achieved with the same level of risk. This analysis requires estimates of the expected return and standard deviation of returns from an infinitely large set of portfolios. Given the portfolio approach, the benefits of diversification can be demonstrated by (a) the calculation of the standard deviation of the portfolio returns and (b) the direct calculation of the effect of forecasted changes in the individual assets. (The derivation of the standard deviation of the portfolio return is given in Appendix B.)

The model will accept constraints of the amount of movement into and out of each type of asset. Thus if a portfolio consists of 60% equities, 30% property and 10% cash (on deposit), a portfolio manager may be unwilling or unable to consider selling or buying more than 10% of the property assets yet be able to liquidate his entire equity stock if conditions justify such a decision.

From some initial experiments with the model we have found that the forecasting aspects tend to be less interesting than the portfolio management aspects. Thus fund managers are interested in the effect of exploring the allocation of assets rather than revising the expected returns of each indicator. We have also found that the relationship between the property returns and some economic indicators is not robust. This of course might be expected if the property returns were 'noisy' since systematic relationships would only be established over periods longer than our data availability allowed (1974 to 1984). In the short term therefore we are developing the model by using factor analysis to identify common generating factors. In the simplest case, a one-factor model will correspond to a market model as developed by Sharpe (1964). We are also investigating the multiple-factor case which can then be related to economic indicators.

3.6. *Future development*

In the UK we have found the pattern of time weighted returns by property type to be fairly synchronous[3]. This limits the potential to lower portfolio return standard deviations by diversification strategies based on property type classifications. It is likely therefore that future developments of the model will incorporate analyses of other forms of property classification from the diversification viewpoint. These forms may include size of investment, age of asset, financing arrangements and location.

We are now engaged in preliminary investigations on the last of these, location. There are a number of conceptual problems, not least of which are the most appropriate level of spatial disaggregation and the availability of data. Nevertheless Fig. 1 gives an indication of the potential for simultaneously analysing portfolio spread by type and location. The standard deviation of the averaged portfolios (the bold line) is significantly less than either of its

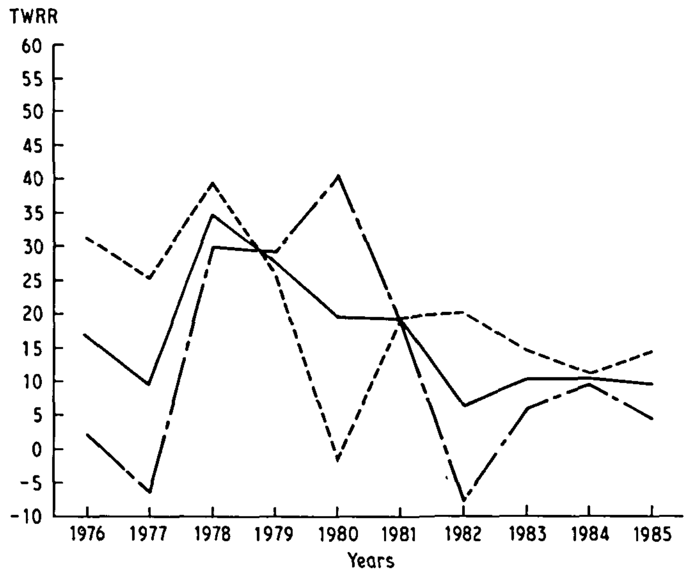

Fig. 1. Diversification analysis: type and location. Dashed line: shops (Hertfordshire) return = 20.0 ± 10.8. Broken line: offices (EC 4) return = 12.6 ± 15.5. Solid line: average, return = 16.3 ± 8.6. (Source-JLW Research, 1986.)

component parts[4]. This of course simply reflects the low correlation between the returns from the components. More importantly, preliminary research suggests that the correlations between, say, location-based components is both lower *and more stable* than correlations between asset-type components. This distinction may, if confirmed by more extensive research, prove to be of practical importance for portfolio analysis and management of property portfolios.

There are many ways in which portfolio managers are increasingly experiencing pressure from trustees to achieve high performance. In the US, portfolio managers are evaluated on their performance, not only on the basis of their achieved returns, but also on whether or not their actions have been consistent with prudent fund management. One response has been to develop an increasingly sophisticated approach to portfolio investment by which portfolios can be 'insured' against future changes in the component markets. The common and necessary requirement is that these changes have the ability to explain the performance of existing assets and it is in this area that much work remains to be done in analysing property investment. There is a lamentable amount of information on the factors which determine the performance of property. There is also a continuing lack of published research on the actual returns earned by property when traded. One of the most important developments which will radically affect this situation is the establishment of a market in which units of property may by publicly traded. The publication of prices will profoundly affect the conventional views held by the property investors on the characteristics of property and it is in this context that portfolio management techniques such as those discussed above will be especially valuable.

Notes

1. The 'efficiency', required by the partial equilibrium of the capital asset pricing model, assumes that all assets are perfectly divisible and marketable, no transaction costs, no taxes, single-period decisions taken by market participants with homogeneous expectations. Violation of any of these assumptions can be taken into account and a model derived. The result is always a model which introduces either additional (and usually unobservable) variables or parameters which are investor-specific.

2. An example in a two-index model would be

$$R_i = a + bI_1 + cI_2 + e$$

where R = return; I_i = return of index i; e = residual error and a, b, c are constants. To orthogonalize I_2 and I_1, we first regress I_2 against I_1;

$$I_2 = w + vI_1 + u$$

and form the orthogonal index from u. Thus by substitution for I_2 we find

$$R_i = (a + cw) + (b + cv)I_1 + cu + e$$

where I_1 and u are now orthogonal indices. The same technique can be extended to three and more indices.

3. See Fig. 8 of JLW Occasional Paper 'Returns to Property' Winter 1986 which graphically illustrates the similarity in return trends to type through time.

4. Returns were averaged rather than recalculated for the joint portfolios so as to avoid reductions in standard deviation due to the portfolio effect.

Appendix A: The *transformation* from valuations to market price proxies

It was found that the returns based on raw valuations tended to be too 'smooth', to a greater extent than could be attributable to the averaging process used in compiling the indices. It can however be hypothesized that valuers through this training, caution and the lack of access to market transaction data, might themselves cause some of the observed returns behaviour. An example might usefully clarify this view. One report of a sale 10% below expected value would not normally cause valuers to reduce the valuations on comparable properties by 10%. Further evidence would be required and this demand for corroborating evidence would tend to cause some delay if sudden changes occurred in the market which was relatively thin. The delay and subsequent smoothing would depend on the degree of caution exercised by the valuer.

Presumably a very cautious valuer would only minimally refer to observed prices in updating his valuation. A more sensitive valuation would reflect all the relevant changes in the market and the resultant value would thus reflect the underlying market value of the asset and be independent of the provious valuation.

We therefore suggest that a simple model of valuation would be given by

$$V_t = (1 - A)P_t + AV_{t-1} \tag{A1}$$

where V_t is the valuation at time t, P_t the 'true' market valuation at t, A is a constant, $0 \leqslant A \leqslant 1$ and V, P are measured, for mathematical simplicity in logarithms.

In this equation, the valuation is a weighted average of (a) the 'true' market information and (b) last period's valuation. The higher the value of A, the more cautious the valuer.
At time $t-1$.

$$V_{t-1}=(1-A)P_{t-1}+AV_{t-2} \tag{A2}$$

and subtracting (A2) from (A1) the following results

$$V_t-V_{t-1}=(1-A)(P_t-P_{t-1})+A(V_{t-1}-V_{t-2}) \tag{A3}$$

Since V and P are both expressed in logarithms, the difference between V_{t-1} and V_t can be taken as the returns (continuously compounded) in the interval $t-1$ to t. Thus we have

$$R_t=(1-A)RM_t+AR_{t-1} \tag{A4}$$

where R_t is the return derived from valuation and RM_t the return which would be observed if the market prices were correctly captured by valuations.

In order to proceed some assumptions about the stochastic behaviour of RM must be made. The evidence from studies of other investment markets suggests that in an efficient (weak form) market, random walk behaviour would be expected and we therefore assume the same underlying process for RM.

$$RM_t=B+e_t \tag{A5}$$

where B is a constant and e_t is a normally distributed variable with $E(e_t)=0$, $E(e_t, e_{t-k})=0$ for $K=1 \ldots \infty$ and $E(e_t^2)=\sigma_e^2$ for all t.

$$\text{Thus } E(RM)=B \tag{A6}$$

$$\text{Var}(RM)=\sigma_e^2 \tag{A7}$$

Substituting for RM_t in (A4) we have

$$R_t=(1-A)(B+e_t)+AR_{t-1} \tag{A8}$$

$$R_t=B(1-A)+AR_{t-1}+(1-A)e_t \tag{A9}$$

In analysing the behaviour of R_t we find

$$E(R_t)=B(1-A)+AE(R_{t-1})+(1-A)E(e_t)$$

which, if the series R_{t-1}, $R_t \ldots$, is stationary yields

$$E(R)(1-A)=B(1-A)$$

$$E(R)=B \tag{A10}$$

Therefore we conclude that the expected return of our observed series R will be equal to the expected value of the underlying market series RM (cf Equations A6 and A10).

Similarly

$$\text{Var}(R_t)=A^2\,\text{Var}(R_{t-1})+(1-A)^2\text{Var}(e_t)$$

$$\text{Var}(R)=\frac{(1-A)^2\text{Var}(e)}{1-A^2} \tag{A11}$$

or by Equation A7

$$\mathrm{Var}(R)=\frac{(1-A)^2\mathrm{Var}(RM)}{1-A^2}$$

For values of A between 0 and 1 the variability of the returns derived from valuations will be less than the variability of the underlying market returns. Furthermore it would appear from Equation (A9) that by analysis of the stochastic behaviour of R_t, estimates of A and B can be obtained.

Finally it will be seen that the returns series of interest is RM_t which from Equation (A4) can be estimated by

$$\widehat{RM}_t=\frac{R_t}{1-A}-\frac{A}{1-A}R_{t-1} \tag{A12}$$

which will have the same mean but a larger variance than R_t.

Appendix B: Multi-economic indicator model of portfolio construction

We assume that for a property type i,

$$R_{it}=a_i+b_{i1}I_{1t}+b_{i2}\mathrm{I}_{2t}+\cdots+e_{it} \tag{B1}$$

where a_i, b_i, b_{i2} are constants, I_1, I_2, I_3 are economic indicators and e_{it} is the component of R_it not influenced/captured by the economic indicators.
We need to derive expressions for

(a) the mean of R_i
(b) the variance of R_i
(c) the covariance of R_i and R_j (where j = another type of property)

The indicators are usually chosen to reflect *changes* in the economy or market.

(a) From Equation B1

$$E(R_i)=a_i+B_{i1}E(I_1)+b_{i2}E(I_2)+\cdots+E(e_i)$$

$$E(R_i)=a_i+b_{i1}E(I_1)+b_{i2}E(I_2)+\cdots+E(I_n) \tag{B2}$$

Equation B2 follows from the assumption that if $E(e_i)\neq 0$, it would be subsumed in the constant a_i. Thus knowing the historical relationship between R_i and the economic indicators allows us to use externally published forecasts of these indicators to derive an independent forecast of $E(R_i)$. However it will be seen that a_i is a constant that can be manipulated quite independently of the externally driven expectation. Thus users of the model may modify the input by manipulating a_i in accordance with their own expectations.

(b) From Equation B1

$$\mathrm{Var}(R_i)=E[b_{i1}I_1+b_{i2}I_2+\cdots+e_i-b_{i1}E(I_1)-b_{12}E(I_2)\ldots]^2 \tag{B3}$$

$$=b_{i1}^2\ \mathrm{Var}(I_1)+b_{i2}^2\ \mathrm{Var}(I_2)+\cdots\ \mathrm{Var}(e_i)+2b_{i1}b_{i2}\mathrm{Cov}(I_1I_2)+2b_{i1}b_{i3}\mathrm{Cov}(I_1I_3)+\cdots$$

Thus

$$\mathrm{Var}(R_i) = \sum_{k=1}^{n} \sum_{k=1}^{n} b_{ik} b_{i1} \mathrm{Cov}(I_k I_l) + \mathrm{Var}(e_i) \tag{B4}$$

(Recognizing that $\mathrm{Cov}(I_k I_k) = \mathrm{Var}(I_k)$).

In practice, economic indicators may be chosen which are orthogonal. In which case the covariance terms disappear if $k \neq l$.

(c) From Equation B1

$$\mathrm{Cov}(R_i R_j) = E[b_{i1} I_1 + \cdots - b_{i1} E(I_1) - \cdots] \, [b_{j1} I_1 + \cdots - b_{j1} E(I_1) \ldots] \tag{B5}$$

$$+ E[b_{i1} I_1 b_{ji} I_1 + b_{i1} I_1 b_{j2} I_2 + \cdots - b_{i1} E(I_1) b_{j1} I_1 - b_{i1} E(I_1) b_{j2} I_2 \ldots$$
$$+ b_{i1} E(I_1) b_{j1} E(I_1) \cdots + b_{i1} E(I_1) b_{j2} E(I_2) \ldots] \tag{B6}$$

By simplification it can be seen that

$$\mathrm{Cov}(R_i R_j) = \sum_{k=1}^{n} \sum_{l=1}^{n} b_{ik} b_{jl} \, \mathrm{Cov}(I_k I_l) \tag{B7}$$

In the case where the economic indicators are orthogonal,

$$\mathrm{Cov}(R_i R_j) = \sum_{k=1}^{n} b_{ik} b_{jk} \, \sigma^2(I_k) \tag{B8}$$

Having established expressions for the mean, variance and covariance of each property type, we can refer to Markowitz's portfolio approach to generate the most efficient combinations in mean/variance terms. For reference purposes only we remind readers that the objective is to minimize the variance of the portfolio subject to the constraint that the mean return of the portfolio should equal $E(R_p)$ and that the sum of the weights of each asset should be equal to one.

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