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Frank J. Fabozzi, Robert J. Shiller, and Radu S. Tunaru

Frank J. Fabozzi
is a professor of finance at EDHEC Business School in Nice, France.
frank.fabozzi@edhec-risk.edu

Robert J. Shiller
is Sterling Professor of Economics at Yale University in New Haven, CT.
robert.shiller@yale.edu

Radu S. Tunaru
is a professor of finance at Kent Business School, University of Kent in Canterbury, UK.
r.tunaru@kent.ac.uk

Financial derivatives play a critical economic role in financial markets. Within different sectors of the financial market, there is an ongoing debate, however, as to the necessity of certain types of derivatives. In the real estate sector, for example, there is the question of the need for real estate or property derivatives. Part of the answer is that the spot real estate market represents approximately half of the wealth in developed economies. In the United States, for example, $21.6 trillion was invested in residential property as of April 2006 (Labuszewski 2006). Furthermore, the global financial crisis of 2007–2009 and a few small financial crises related to real estate misvaluations strongly suggest that real estate risk is embedded in new financial products that are traded in nonproperty investment markets.

Approximately a third of the total global commercial real estate properties can be found in the United States, representing more than $10 trillion right after the global financial crisis. Investigating the optimal composition of household investment portfolios comprising common stocks, stocks in real estate investment trusts (REITs), housing, bonds and Treasury bills, Englund et al. (2002) found that, for short-term investment horizons, housing is not part of the optimal portfolio. For longer investment horizons, however, low-risk portfolios had between 15% to 50% allocated to housing. Hence, they demonstrate that being able to hedge the exposure to real estate risk would be highly beneficial to society. Similar ideas were put forward by Iacoviello and Ortalo-Magné (2003), who argued that by hedging with housing price derivatives, households would improve their portfolio wealth value that may deteriorate because of exposure to housing. Their empirical evidence was focused mainly on London housing prices.

For the United States, Bertus et al. (2008) provided evidence that households could benefit from a functional property derivatives market. For three decades, Karl Case and Robert Shiller have argued for the introduction of property derivatives for hedging against housing price risk. According to the magazine *The Economist* in 2006, it is very likely that the impact of the subprime mortgage crisis could have been softer if market participants considered more seriously the signal that house prices might burst.

In this article, we review the latest advances in real estate derivatives modeling, emphasizing the importance of research in this area and discussing various advantages and disadvantages of various methods. Our survey covers property derivatives where the underlying is residential and commercial property. In addition, we highlight a discrete-time model from the ARMA-EGARCH family that can be employed to price options on a real estate price index considered frequently by banks and insurers in modeling housing price risk.

WHY DO WE NEED REAL ESTATE DERIVATIVES?

Homeowners bear the risk of a house price market drop. The vast majority of people in developed economies invest the largest portion of their wealth in housing. One early solution to this housing price risk hedging problems was offered by Case et al. (1993), who proposed using futures contracts linked to regional house price indexes. There is usually a high degree of heterogeneity in householders present in an economy with respect to their mortgage financing decisions and risk-preferences, which are contingent on their net wealth as well as on their income supply and borrowing (Campbell 2006). Hence, the mortgage design will interact greatly with funding constraints and the solvency risk of individual homeowners. Consequently, homeowners are naturally looking to connect their mortgage to a house price index. House price index derivatives were viewed as a viable solution to allow householders to disentangle pure consumption decisions from housing investment decisions.[1]

It is not only homeowners that are primarily interested in real estate derivatives. To have a liquid real estate derivative contract, the market must have investors on both sides of the real estate risk. Investment banks and mutual funds should be interested in property derivatives to allow them to obtain positive exposure to real estate prices and ultimately reach economically substantial and significant diversification.

Although Voicu and Seiler (2013) and De Jong et al. (2008) analyzed from a theoretical perspective the role of housing futures in hedging house price risk, they did not consider the impact of adding a functional property derivatives market on actual house prices or whether the introduction of a property derivative financial instrument would solve the standard problems encountered in the housing market, such as lack of granularity of trading, impossibility of short selling, and high transaction costs.

Focusing only on the forward housing price contract, Fan et al. (2012) developed a utility indifference model to investigate the role of forward house sales in hedging against house price risk in the spot market, searching for the optimal investment portfolio mix comprised of the house as an asset, risk-free bond, and stock. Uluc (2018) analyzed the impact of residential property futures on housing demand, house price volatility, and housing price bubbles, including a utility consumption characteristic for the risky asset and taking into account the well-known feature that short-selling is not feasible for houses. Noise traders will introduce uncertainty in house prices, potentially leading to temporary deviations of house prices from their fundamental values. In the presence of housing futures contracts, house prices are stabilized through three channels by 1) allowing investors in residential properties to separate their housing consumption decisions from investment decisions; 2) introducing short-selling; and 3) opening up trading in the risky asset to pure speculators who will invest in houses in order to diversify their investment portfolio, hence being short housing price risk. Moreover, Uluc (2018) demonstrated that, for a large set of admissible parameter values, housing futures trading will reduce the volatility of house prices and boost the welfare of investors in the housing market. The stabilizing role played by real estate futures on property markets was also supported empirically in other studies such as Lee et al. (2014) and Wong et al. (2006).

EVOLUTION OF REAL ESTATE DERIVATIVES AS FINANCIAL INSTRUMENTS

The First Developments

According to Shiller (2003), the first house price securities were stocks and bonds linked to real estate on the New York Real Estate Securities Exchange (NYRESE) in 1929. This contract collapsed in 1941. In modern times, the first public development of a real estate derivative contract traded on an exchange was related to commercial property, and it was introduced in May 1991 on the London Futures and Options Exchange (FOX). This derivative was a commercial property futures contract linked to the capital component of the Investment Property Databank (IPD) index and the IPD commercial rent index. The lack of trading volume induced insider dealings and artificial trading to create the illusion of a liquid market. The scandalous demise of the first real estate derivative in the United Kingdom resulted in a negative view by investors regarding property derivatives. Even more importantly, UK regulators barred

[1] See Chapter 6 in Englund (2010).

institutional investors from using this new market and applied an unfavorable tax treatment to this instrument.

Three new types of derivatives contracts marketed by Barclays Capital followed in the 1990s: a property index certificate, a property index note, and a property index forward. The property index certificate, which commenced trading in 1994, was a term note linked to the total yield of the IPD index and structured as a Eurobond that sold at par and replicated IPD returns if held to maturity (Lizieri et al. 2012). The property index note, introduced in 1999, paid the current yield on the IPD index and promised a redemption value determined by the capital component of the IPD index. Introduced in 1996, the underlying for the standard property index forward was the capital component of the IPD index.

Shiller (2008a) reviewed earlier efforts to launch property derivatives style contracts which we briefly describe here. In the 1990s, despite several attempts, no financial instrument established itself as a tool for hedging property risk. Because of the need for such instruments, new contract designs appeared at the beginning of the 21st century in both the United States and the United Kingdom. Contracts were started for single-family homes in the United Kingdom by two providers of financial spread betting products:[2] City Index in 2001 and the IG Index in 2002. For several reasons, the two markets were closed in 2004. Another attempt at spread betting on UK house prices was the Cantor Index formulated by Cantor Fitzgerald Europe. This contract was terminated at the end of December 2008 as a result of the global financial crisis. Goldman Sachs introduced a market for covered warrants on UK home price indexes that was supported on the London Stock Exchange in 2003 based on the Halifax home price indexes. Hedgestreet.com introduced a market for betting on the direction of home prices with the expectation that the contract would attract home-owners seeking to hedge against a property price collapse. None of these attempts succeeded, however, suggesting that it is difficult to develop hedging markets started for real estate (Shiller 2008b, p. 7). Using an analogy with the classical discounted value stock equity models, Patel and Sing (2000) conceptualized a property's market price as a contingent claim on the property's perpetual stream of rental income. This model was aimed at commercial properties where rental cash flows are intrinsic to the commercial property market. One downside of the model is that it was based on the assumption that rental amounts follow a geometric Brownian motion.

In the United Kingdom, the Financial Services Authority (FSA) in 2002 encouraged the use of property derivatives by allowing life insurance companies to use real-estate swaps and real-estate forwards as assets for solvency ratio calculations. Moreover, since 2004, the UK's Inland Revenue harmonized the taxation on property derivatives for all participants in financial markets, permitting the offsetting of the losses realized on derivatives contracts against capital gains. This resulted in more than 20 investment banks obtaining licenses to use the IPD family of indexes to market property derivatives in the United Kingdom. Most regulatory and fiscal barriers on trading property derivatives were removed by 2005 in the United Kingdom for the majority of investors.

Property derivatives trades based on IPD indexes were made in countries like France in 2006 and Australia, Germany, Japan, and Switzerland in 2007. The IPD total return swaps market expanded rapidly after 2004, reaching GBP 3.5 billion of notional value traded in the first quarter of 2008. In the United Kingdom, the property derivatives market almost doubled in the space of one year, from GBP 3.9 billion in 2006 to GBP 7.2 billion in 2007. At the end of 2009, there were over 2,100 trades in total return swaps on the IPD index with a notional value reaching USD 35 billion from inception in 2004 (Lizieri et al. 2012).

The introduction of commercial property derivatives in the United States was slower. Tullet Prebon tried first to trade derivatives on the National Council of Real Estate Investment Fiduciaries NCREIF Property Index (NPI) in 2003 but without much success. The first property derivative in the United States was probably traded by Credit Suisse on the NPI in 2005 and in 2006 (see Torous 2017). Because this was viewed as a line of business with significant potential, in October 2006, six other large investment banks (Bank of America, Deutsche Bank, Goldman Sachs, Lehman Brothers, Morgan Stanley, and Merrill Lynch) bought licenses for trading derivatives on the NPI. By the end of 2007, there were trades on NPI with a total notional of only USD500 million, suggesting that this market was very much in its nascency. Unfortunately, the emergence of commercial property derivatives on exchanges coincided with the eruption

[2] Spread betting is a leveraged trading activity that is based on making a bet on the price movement of a security. There is a market maker quoting a bid and an ask defining the spread, and investors bet whether the price of the underlying security/index will be lower than the bid or higher than the offer. Neither the market maker nor the investor owns the underlying security in spread betting.

of the subprime mortgage crisis. In October 2007, the Chicago Mercantile Exchange (CME) started trading futures and options on the S&P/Global Real Analytics commercial real estate indexes (SPCREX). The CME launched the year before, on May 22, 2006, futures and options on the S&P/Case-Shiller Home Price 10 city real-estate indexes and a national composite index (Deng and Quigley 2008). Futures on the Case-Shiller index were traded on the CME electronic market. Futures options on the Case-Shiller index were traded on the Goldman Sachs facilities.

Real Capital Analytics and MIT developed in 2006 a set of real estate indexes tracking US commercial prices with the stated intent to trade derivatives on these indexes. A milestone in the development of property derivatives markets was the launch on February 9, 2009, of the EUREX of futures on the IPD UK Annual All property index for each of the next five years December maturities.[3] There is scattered evidence of housing derivatives in Switzerland and Sweden and the derivatives on the IPD family of indexes expanding to Australia, Canada, France, Germany, and Japan, but with mixed success. The most successful real estate derivatives contract is the IPD UK futures contract traded on the EUREX exchange in London, the only European exchange to list property index futures. These financial instruments are annual contracts based on the total returns of MSCI-IPD UK Quarterly Indices for individual calendar years. Investors can trade futures on the following MSCI-IPD UK Total Return Indexes: All Property, three sectors (All Retail, All Office, All Industrial), and five subsectors (Shopping Centre, Retail Warehouse, City Office, West End Office, South East Industrial). Before MSCI acquired the IPD database generating the various commercial property indexes, the futures were contingent on an IPD index that was published annually. There were five yearly maturities with a December market calendar roll. The new MSCI index is reset quarterly in an effort to reflect changes in the market on a more frequent basis and ultimately increase liquidity in the near future.

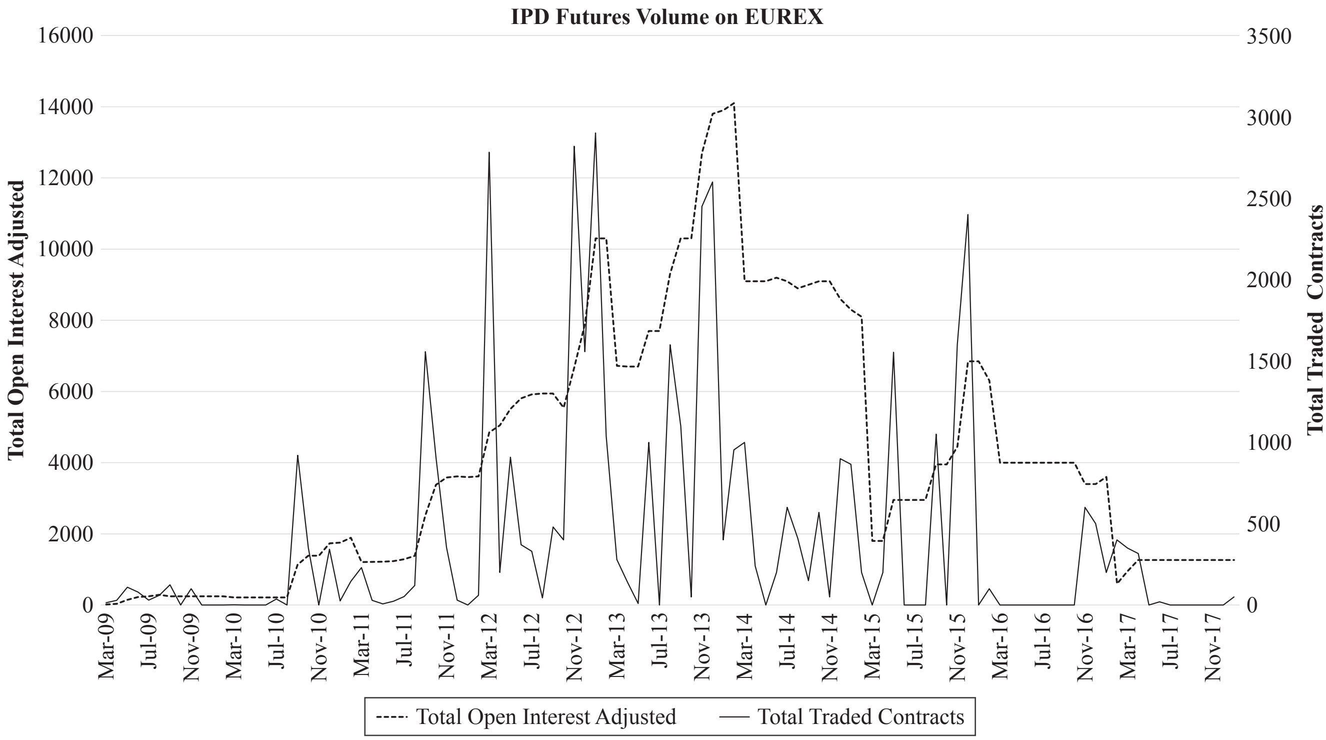
In Exhibit 1, the total volume of IPD Total Return All Property UK futures contracts is illustrated. There was a clear increase in demand in the aftermath of the subprime crisis that peaked toward the end of 2013 and then a downward trading trend to the end of 2017. The sudden drop in the volume of futures can be attributed to the Brexit vote of June 23, 2016. In Exhibit 2, we illustrate the evolution of the IPD UK All Property futures prices, with five annual December maturities between March 9, 2015, and November 20, 2018. There is a clear structural break caused by the Brexit vote that led to a change of futures level from 140 to 120, implying less confidence in the commercial real estate sector in the United Kingdom over the next five-year period.

Real-Estate Derivatives Embedded in Other Financial Contracts

Real-estate derivatives may be embedded within other important financial contracts. They are embedded within the covenants of those contracts. For example, lease and rental agreements carry options attached or embedded in them that depend on the consumer price index, a real estate index, and/or the value of the real estate property itself. Buetow and Albert (1998) looked at the value of an embedded option to renew a five-year lease on class A office space in each of the 25 markets for which the National Real Estate Index estimates quarterly rental data. The pricing is done under the basic assumption that both real estate prices and rents have a lognormal distribution at maturity. However, quite innovatively, they compared the standard geometric Brownian motion model with a mean-reverting model with stochastic volatility. They priced the five-year renewal option for 26 locations in the United States and found evidence that rent volatility is more significant in some market areas than in others. In the United States, the landlords and tenants of most shopping centers have options to cancel the lease and tenants have, in addition, options to expand, contract, and sublet. The value of these options may lead to an adjustment of rent. Hendershot and Ward (2003) analyzed the overage rent lease and renewal options. The former is a string of landlord options that entitles a landlord to higher annual rents when tenant sales exceed a specified benchmark. The renewal options allow the tenants to renew their lease at the original terms, compounded at the current inflation rate or by cumulated inflation since the lease was issued, or at the market. Hendershot and Ward (2003) showed that the call options that landlords have on overages and the renewal put options that the tenants

[3] A description of the usefulness of this contract, together with a pricing framework and other applications, can be found in Fabozzi, Shiller, and Tunaru (2009, 2010, 2012), Fabozzi, Stanescu, and Tunaru (2013), Fabozzi and Tunaru (2017), and Tunaru (2017).

Exhibit 1
Volume of Trade for IPD UK Total Return All Property Futures Traded on EUREX



Source: Created by the authors from data obtained from EUREX London.

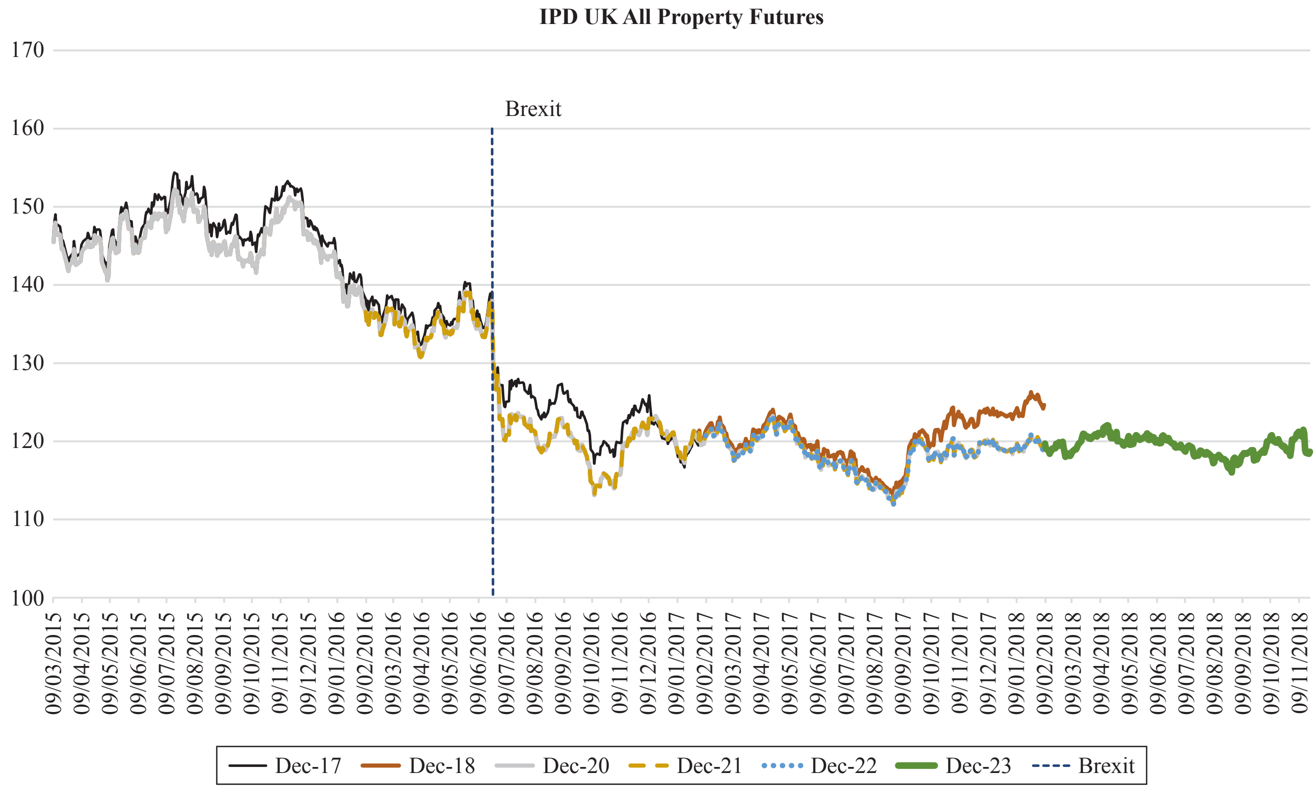
have can be structured such that the two options offset each other in a given state of the economy.

An interesting class of commercial real estate derivatives emerged a few years before the global financial crisis that started in 2007: CMBX contracts. These property derivatives are credit default swaps on a set portfolio of 25 reference commercial mortgage-backed securities (CMBS). A CMBX contract is a structured (synthetic) tradable basket that references 25 equally weighted CMBS tranches that pays a fixed rate over an approximately six-month time period. It has a "pay as you go" settlement process based on three possible events related to the reference securities: principal writedowns, principal shortfalls due to failures to pay on an underlying mortgage, and interest shortfalls arising when current cash flows pay less than the CMBX coupon. The tranches associated with a particular CMBX series reference a static portfolio of 25 CMBS deals that were most important at the time when the CMBX series was issued.

Driessen and Hemert (2012) constructed a Merton-style structural pricing model that they calibrated on stock and option prices for the S&P 500 Index and several REITs, so not directly on CMBX market prices. This choice was motivated by the fact that predictability of CMBX prices from REIT prices is statistically and economically significant. At the time of the eruption of the subprime crisis, Syz (2008) introduced a new financial product, a housing price index linked mortgage. This is a loan with regular mortgage payments determined directly by the housing market performance. This instrument is supposed to immunize the housing markets from default by borrowers. In February 2006, Syz et al. (2008) launched a transaction-based, hedonic index covering the Zurich area, with property derivatives linked to this index.

A separate class of options related to property is real options related to property investment given by the type and number of expansion, contraction, and suspension choices. These choices may refer to remaining idle, building and operating properties, expanding, contracting, suspending, reverting to normal service or reduced service capacity, or abandoning. Paxson (2005)

EXHIBIT 2
IPD UK Futures Prices, March 9, 2015–November 20, 2018



Source: Created by the authors from data obtained from Bloomberg.

developed a valuation model for valuing up to eight different options, each with a distinct trigger.

Non-Negative Equity Guarantee in Equity Release Mortgages

Equity release mortgages or reverse mortgages have been traded worldwide for many years. The reverse mortgage is a financial instrument that can be tracked back to the 1960s in the United States, with more activity reintroduced during the early 1980s,[4] before spreading to the United Kingdom where it has been referred to as an equity release mortgage (ERM) since the 1980s. It has been revamped in the aftermath of the subprime crisis in the United States, in the United Kingdom, and in some European countries (France and Italy), and it is very popular in Far East countries (Japan, Korea, Hong Kong, Singapore, and Australia).

Reverse mortgages have been endorsed by Robert Merton as a viable source of funding for the elderly. Moreover, Merton and Lai (2016) discussed a structural design of reverse mortgages that is meant to improve the risk sharing between the borrower and the lender while also examining the role of the regulator in the reverse mortgage process. Cocco and Lopes (2014) discussed improvements to reverse mortgage design.

In the United Kingdom, there is often a guarantee embedded in the ERM contract stipulating that any excess of the accrued loan amount above the sale value of the property after the exit event will be written off by the lender, subject to certain conditions. This no-negative-equity-guaranteed (NNEG) condition is the primary concern with ERMs. In the United Kingdom, a reverse mortgage must incorporate an NNEG in order to meet the Product Standards within the

[4] The Federal Home Loan Bank Board approved reverse mortgages in 1979.

Statement of Principles of the Equity Release Council.[5] There are many variants of the main equity release mortgage product that is based on lending initially a lump sum,[6] all of them being exposed to house price risk.

The NNEG can be viewed in essence as a put option that the lender is passing on to the loan borrower. This is not a standard put option for several reasons. First, the exercise time is a stochastic variable that is spanned by the first arrival of three stochastic processes: mortality, long-term care movement, and prepayment. Second, the exercise price accrues over time at a fixed or adjustable rate. Finally, the underlying asset is the borrower's house that is collateral for the loan.

The main driver of an NNEG is the house price risk. The technology for pricing this guarantee is similar to real estate derivatives valuation and the same technical issues are encountered in both areas.

REAL-ESTATE DERIVATIVES MODELING

Main Features and Applications

House price volatility can vary considerably between a 12% per annum decrease in Houston in the mid-1980s and 28% per annum increase in Seattle in the 1990s as reported in Case et al. (1993). Hinkelman and Swidler (2008) analyzed whether CME futures on the S&P/Case-Shiller house price index can be used for hedging housing price risk. The results had various degrees of success geographically, suggesting that idiosyncratic risk is relevant in this context.

When employing appraisal-based indexes for designing property derivatives, one should be aware of two main problems. The first problem is noise, defined as the deviation between the index true value and the actual market price. The second problem is related to the temporal lag that causes index inertia and predictability, leading the expected future returns of the index to differ from the equilibrium property market return expectations (Firstenberg et al. 1988). Another important characteristic is the nontradability of the real estate index, implying that it cannot be perfectly replicated with standard assets in the economy. Consequently, the classic arbitrage mechanisms cannot be imposed.

The forecasts or expectations of housing prices play a fundamental role in real estate research in general and in mortgage research in particular (Leventis 2008). The studies in the literature have typically used backward-looking proxies constructed from historical time series. An improved approach was advanced by Zhu et al. (2014), who proposed using the transaction prices of Case-Shiller housing futures as a forward-looking proxy for future house prices. When compared with other available proxies in the literature, they found that the Case-Shiller futures-based proxy had better accuracy and the futures contracts contained additional information that was not available in the backward-looking proxies.

One important potential application of property derivatives is as an early warning signal for possible downfalls in property markets. Regulators and central bankers could take rapid measures to attenuate the impact of a sudden decrease in property prices. The information recovered from total-return swaps on real estate indexes may be affected by liquidity issues, which may even lead to arbitrage vis-a-vis futures markets based on the same real estate index as described in Stanescu et al. (2014) for the IPD index. Bond and Mitchell (2011) argued that with the introduction of IPD futures in the UK on the EUREX exchange, futures prices could be used as a predictor of future returns that would be superior to a consensus forecast of industry experts.

A Review of Modeling Approaches

Early literature on real estate derivatives pricing focused on the risk-neutral approach as developed in Titman and Torous (1989), Buttimer et al. (1997), Bjork and Clapham (2002), and Clapham et al. (2006). Quite early on, Park and Switzer (1996) identified pricing problems caused by the basis risk introduced when an asset's return is not perfectly correlated with the market index return. To deal with basis risk, Otaka and Kawaguchi (2002) proposed a three-factor model to address issues posed for commercial real estate total return swap pricing caused by the incomplete market character when the underlying asset is not fungible. The total return swaps on commercial real estate indexes are traded usually against LIBOR plus or minus a spread. The spreads effectively are the prices of these total return swaps and, in contrast to the equity index swaps, are very large in real estate markets, in the order of magnitude of a few percentage points. Drouhin et al. (2016) analyzed the IPD All Property Total Return Swaps using a four-year UK database. After stripping

[5] See www.equityreleasecouncil.com.

[6] See Hosty et al. (2008) for an inventory of these products.

the swaps into the equivalent portfolios of forward contracts, they used a panel model with clustering to gauge the risk factors driving those derivatives prices. They confirm that the standard spot-forward relationship does not hold for these property derivatives and that the information contained in equity REITs is significant for the extracted forwards.

Arguably the simplest model for the valuation of forward contracts on real estate prices has been suggested by Syz (2008). If H_t is a property price index, then the forward at time t for maturity T is

$$F_t(T) = H_t exp((r + \rho)(T - t)) \tag{1}$$

where r is a constant risk-free rate covering the period $[t, T]$ and ρ is property spread similar to a convenience yield in commodity markets. This property spread absorbs possible property specific transaction costs,[7] limitations on capital loss deductions, and other possible tax-related constraints (Cornell 1985).

Furthermore, Syz (2008) prices options on the real-estate index using a Black-type formula. The formula for a European call contingent on the index H_t with maturity T and exercise price K is

$$C(F_t, t) = exp(-r(T-t))[F_t\Phi(d_1) - K\Phi(d_2)]$$

$$d_1 = \frac{\ln(F_t/K) + 0.5\sigma^2(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t}$$

This approach ignores the empirical features of the property index, autocorrelation in particular.

Geltner and Fisher (2007) offered a critique of pricing considerations for total return swaps for commercial real estate indexes that were constructed with the appraisal method. They pointed out that in these markets, investors cannot hold the underlying portfolio and the method of index construction itself may introduce noise and lagging effects. Recognizing that the market index value is different from the fundamental index value due to market sentiment, they proposed a standard appraisal adjustment model that simultaneously smoothes market volatility, producing momentum effects. An important feature of real estate derivatives markets is that the underlying index cannot be traded. Hence, Geltner (2007) showed how to identify a trading range for the underlying index by considering simultaneously a long investor and a short investor. The long investor may expect higher growth in the index than would be necessary to deliver the equilibrium return, and hence the long investor will be prepared to pay a spread (positive or negative) on top of LIBOR; the short investor may hold a portfolio hoping for positive alpha returns greater than the index returns. The Geltner–Fisher model focuses on pricing a real estate forward contract and this price is calculated as the minimum between the minimum and maximum prices in the trading range after transaction costs.

The Geltner–Fisher approach has been extended by Lizieri et al. (2012) by imposing four initial assumptions. First, the equilibrium spread is not explained by differences in expected returns. Then, the second assumption is that arbitrage and covered trade spreads must be equal to the difference between LIBOR and the borrowing costs of the representative investor. Consequently, if the funding rate is LIBOR, this spread is naturally zero, as also determined by Bjork and Clapham (2002). Third, total return swap contracts are intermediated by a swap dealer that is assumed to remove counterparty risk. Finally, any nonzero spreads are attributed to real estate asset market distortions and inefficiencies. Based on these four assumptions, Bjork and Clapham focused on whether it is possible to construct a self-financing arbitrage portfolio. Their approach employs a risk-adjusted cash flow model based on the certainty equivalence approach.

An interesting method also based on unsmoothing the observed market price of a real estate index into an efficient underlying market and additional market sentiment has been proposed by van Bragt et al. (2015). They demonstrate how to derive a closed formula for pricing forwards, swaps, and options taking into account that the value of the real estate index at maturity is a weighted sum of lognormal variables; consequently, from a derivatives pricing perspective, the techniques known for pricing Asian basket options are applicable.

A different modeling approach is based on equilibrium models. Motivated by the idea that the lack of liquidity in real estate derivatives markets is caused by the absence of sufficient pricing models, Cao and Wei (2010) proposed a flexible equilibrium model that leads to closed-form solutions.

It has been found that the risk-neutral valuation models applied to real estate derivatives, such as total return swaps, produce significantly lower model

[7] We thank the editor Joe Pimbley for pointing out this interpretation to us.

prices than market prices. To solve this problem, Pu et al. (2014) suggested a utility indifference-based model that can be employed to calculate the reservation spreads of swap receivers and payers using the principle of expected wealth utility equivalence. This model can be adapted to take into account the incomplete character of the real estate asset class and the heterogeneity of the representative agents who can have different beliefs about the expected future property prices.

Ciurlia and Gheno (2009) introduced a two-factor model where the real estate asset value and the spot rate dynamics are jointly modeled in order to price both European and American options using a bidimensional binomial lattice framework. The model proposed allows calibration to the interest rate and volatility term structures. Another two-factor model in this area was proposed by Baran et al. (2008), who calibrated the Schwartz–Smith two-factor commodity model to Case–Shiller futures prices and rental rates.

Expanding the valuation framework by Fabozzi et al. (2012) to the case when interest rates are stochastic, Zou and Gong (2017) proposed a general binomial lattice model that is computationally fast and numerically accurate for pricing real estate derivatives. Gong et al. (2018) considered further numerical improvements in this context, applying the mesh-free radial basis point interpolation for pricing real estate derivatives contingent on a real estate index. An interesting approach for the valuation of property derivatives has been advanced by Syz and Vanini (2011). Recognizing the importance of market frictions (i.e., transaction costs, transaction time, and short sale constraints) that effectively deter the application of a standard no-arbitrage approach, they design a framework that allows the calculation of arbitrage-free price bounds for property derivatives.

ARMA-EGARCH MODEL

One feasible solution to issues such as serial correlations and volatility clustering is to consider the ARMA-EGARCH model family. Denoting by $Y_t = \ln\left(\frac{H_t}{H_{t-1}}\right)$ the log-return of the house price index at time t, an ARMA(m, M) is specified first for the log-returns

$$Y_t = c + \sum_{i=1}^{m} \phi_i Y_{t-i} + \sum_{j=1}^{M} \theta_j \epsilon_{t-j} + \epsilon_t \tag{2}$$

where $\epsilon_t \sim N(0, h_t)$. This is followed by a second step for the conditional variance h_t, where an EGARCH (P, Q) model is specified:

$$\ln(h_t) = k + \sum_{i=1}^{P} \alpha_i \ln(h_{t-i}) + \sum_{j=1}^{Q} \beta_j [\,|\widetilde{\epsilon}_{t-j}| - E\,|\widetilde{\epsilon}_{t-j}|\,] + \sum_{j=1}^{Q} \gamma_j \widetilde{\epsilon}_{t-j} \tag{3}$$

with $\widetilde{\epsilon}_t = \frac{\epsilon_t}{h_t}$ as the standardized innovation at time t. It then follows that under the real-world measure $\mathbb{P}_t$ $Y_t | \mathcal{F}_{t-1} \sim N(\mu_t, h_t)$ where

$$\mu_t = c + \sum_{i=1}^{m} \phi_i Y_{t-i} + \sum_{j=1}^{M} \theta_j \epsilon_{t-j} + \epsilon_t \tag{4}$$

For pricing real-estate derivatives contingent on the index $\{H_t\}_{t\geq 0}$ with a given maturity T, we need to identify a risk-neutral measure. If $\mathbb{P}$ is the probability measure associated with the information set $\mathcal{F}_T$ described by an economy, we construct $\mathbb{P}_t$ as the projected measure $\mathbb{P}$ on the smaller information set $\mathcal{F}_t$. Following Buhlman et al. (1996) and Siu et al. (2004), for any given sequence of constants $\lambda_1, \lambda_2, \ldots, \lambda_t, \ldots$ the conditional Esscher distribution $\tilde{\mathbb{P}}_t$ is defined computationally through

$$F_{\tilde{\mathbb{P}}_t}(y; \lambda_t | \mathcal{F}_t) = \frac{\int_{-\infty}^{y} e^{\lambda_t x} dF_{\mathbb{P}_t}(x | \mathcal{F}_t)}{E_{\mathbb{P}_t}(e^{\lambda_t Y_t} | \mathcal{F}_t)} \tag{5}$$

The key to the risk neutralization under the conditional Esscher measure is to observe that the moment-generating function of Y_t given $\mathcal{F}_{t-1}$ under $\tilde{\mathbb{P}}_t$ is calculated from

$$E_{\tilde{\mathbb{P}}_t}(e^{zY_t}; \lambda_t | \mathcal{F}_{t-1}) = \frac{E_{\mathbb{P}_t}(e^{(z+\lambda_t)Y_t} | \mathcal{F}_t)}{E_{\mathbb{P}_t}(e^{\lambda_t Y_t} | \mathcal{F}_t)} \tag{6}$$

Since $Y_t | \mathcal{F}_{t-1} \sim N(\mu_t, h_t)$ it follows that

$$E_{\tilde{\mathbb{P}}_t}(e^{zY_t}; \lambda_t | \mathcal{F}_{t-1}) = e^{(\mu_t + h_t \lambda_t) z + \frac{1}{2} h_t z^2} \tag{7}$$

The risk-neutral measure would be identifiable if a real estate market forward curve $\{F_{t-1}^{mkt}(t)\}_{t\geq 0}$ is available.

In the absence of forward curves,[8] we work with a proxy for the forward curve such that the risk-neutral measure is identified by finding those λ_t^q that solve the martingale equation

$$E_{\tilde{\mathbb{P}}_t}(e^{Y_t};\lambda_t^q|\mathcal{F}_{t-1}) = e^{r-g} \tag{8}$$

with r being the risk-free rate and g the rental yield. The risk-neutralizing constants can be then computed analytically as follows:

$$\lambda_t^q = \frac{r-g-\mu_t-\frac{1}{2}h_t}{h_t} \tag{9}$$

In this way, we obtain a sequence of risk-neutral measures $\mathbb{Q}_t$, such that

$$E_{\tilde{\mathbb{Q}}_t}(e^{zY_t};\lambda_t^q|\mathcal{F}_{t-1}) = e^{(r-g-\frac{1}{2}h_t)z+\frac{1}{2}h_t z^2} \tag{10}$$

which shows that the risk-neutralization effect is to keep the same type of normal distribution but change by translating the parameters. Hence, under the risk-neutral measure $\mathbb{Q}_t$,

$$Y_t|\mathcal{F}_{t-1} \sim N\left(r-g-\frac{1}{2}h_t, h_t\right) \tag{11}$$

Because the monthly log-returns produced by the ARMA-EGARCH model are not independent, in order to price real estate derivatives at some given maturity T, we need to take a Monte Carlo pathway simulation approach. The paths are easy to simulate using Equation (11) and then calculating $H_T = H_0 exp(\Sigma_{i=1}^{i=n} Y_{t_i})$, where $t_n = T$. The advantage of constructing the risk-neutral measure blockwise is that we can price immediately European options and also path-dependent options. In Exhibit 3, we illustrate the evolution of a monthly series for the Nationwide house price index monthly between January 1991 and September 2018.

Consider here that we would like to price European calls and puts with monthly maturities ahead one month to six months, contingent on a house with the current price 500,000 in the United Kingdom. Under this scenario we take the risk-free rate as $r = 1.5\%$, $g = 2\%$, and the *unobservable* returns on the house are described by the observable returns on the Nationwide house price index. Using Occam's razor for model selection guidance in a forward model selection search, we identify the ARMA(4,3)-EGARCH(1,1) as a suitable model for the Nationwide return series. Estimates of the parameters of the model are given in Exhibit 4.

The European call and put options prices for at-the-money, in-the-money, and out-of-the-money strike prices, for near-term maturities up to six months, are reported in Exhibit 5. Because the European put at-the-money options are more expensive than the European call at-the-money options, one can infer that, under this market scenario, it is more likely to see a house price decline than a house price increase. However, this is driven by the forward/futures prices that are determining the risk-neutral pricing measure. When the difference $r - g$ is negative, the forward prices project a downward movement.

Exhibit 6 describes the same real estate derivatives calculations for the scenario when $r - g$ is positive. Here, we take $r = 3.5\%$ and we keep $g = 2\%$ as before. In this scenario, the call options prices at the money are larger than the put options prices at the money, reflecting the upward trend imposed by the forward prices. Revisiting formula (11) we can see that it is possible to have larger put prices under this model even when $r - g > 0$. This is likely to occur when $r - g - \frac{1}{2}h_t < 0$ (i.e., when the volatility of house prices outweighs the difference between the risk-free rate and the rental yield).

CONCLUSIONS

The modeling research on real estate derivatives has expanded in the last decade in the aftermath of the subprime crisis. However, financial markets are still in a nascent stage, with the notable exceptions being the CME Case-Shiller futures and the IPD (MSCI) UK futures.

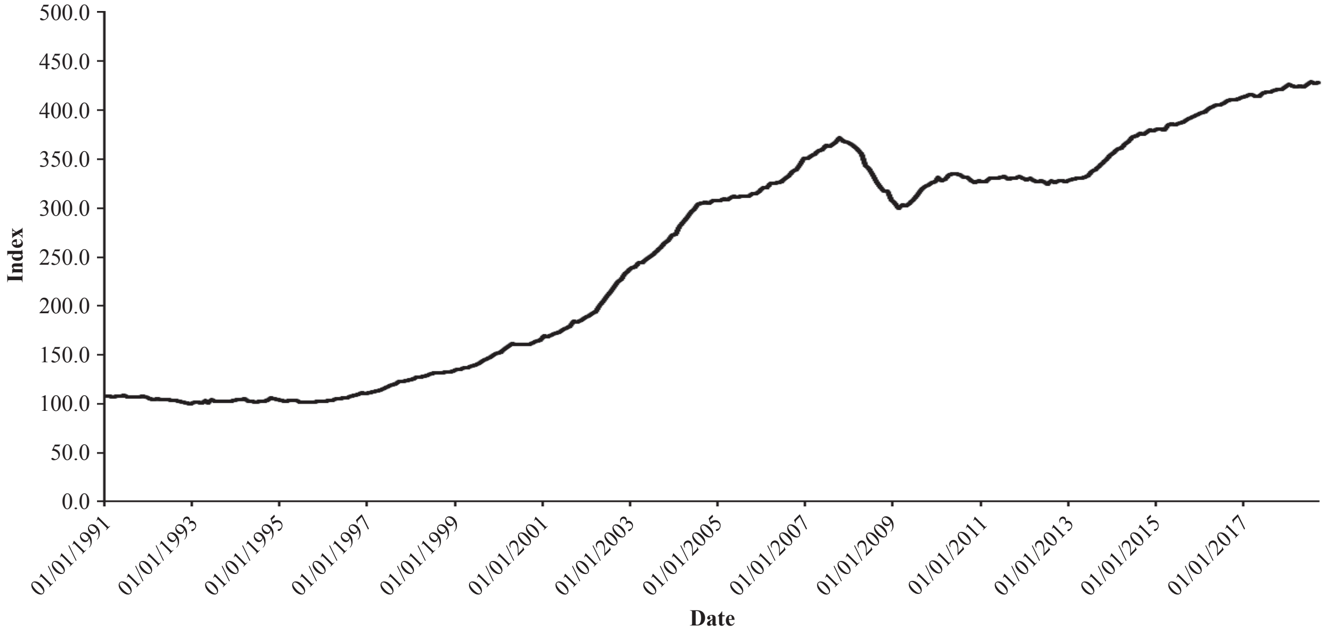
There is an evident need for real estate derivatives that would help hedge the property risk in many other asset classes (insurance products, mortgage products, inflation products) and that would contribute to the stability of financial markets. Although the real estate cash market represents arguably the largest asset class by notional value, the development of real estate derivatives

[8] One may determine forward prices using other methods, such as the equilibrium models described in Cao and Wei (2010) and Lizieri et al. (2012).

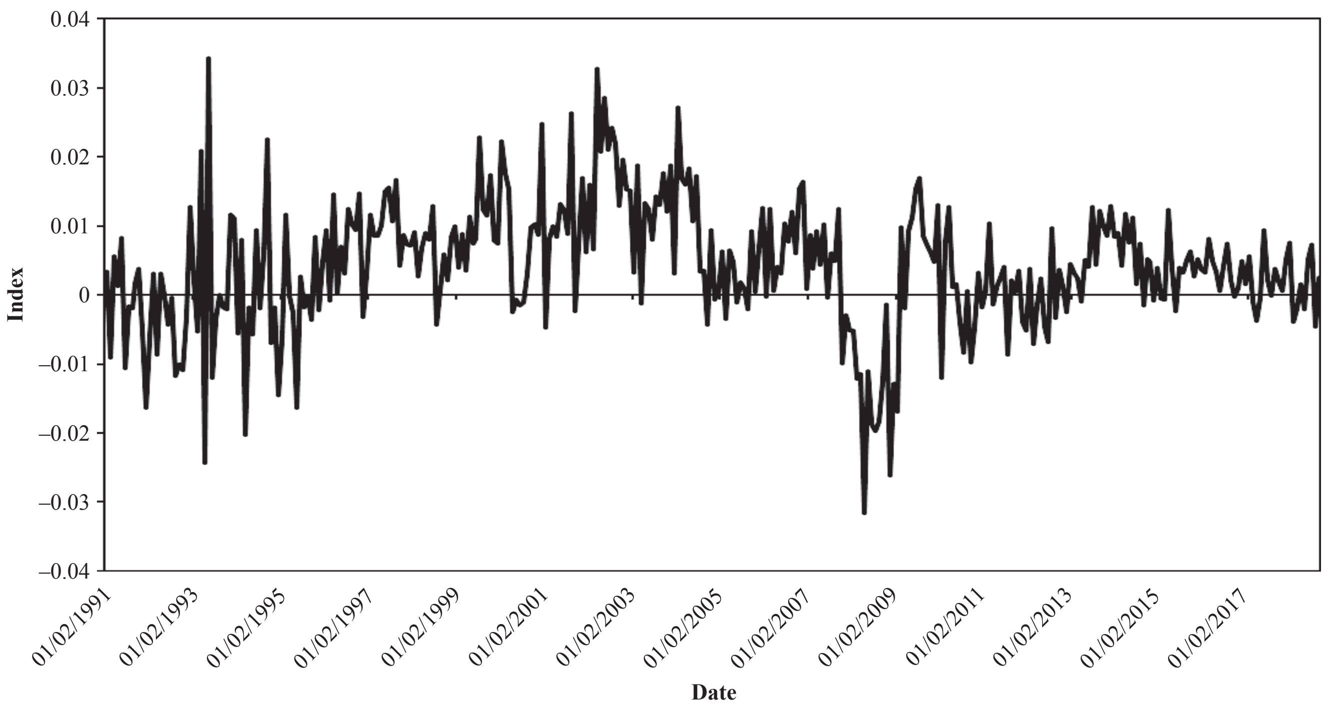
EXHIBIT 3

Nationwide Series and Log-Returns Monthly, January 1991–September 2018

Panel A: Seasonally Adjusted Nationwide HPI—Jan 1991–Sept 2018



Panel B: Seasonally Adjusted Nationwide HPI Returns—Jan 1991–Sept 2018



Source: Created by the authors from data obtained from Nationwide.

EXHIBIT 4

Parameter Estimates of ARMA(4,3)-EGARCH(1,1) Model for the Nationwide Monthly Price Series, January 1991–September 2018

Parameter	Estimate	Std. Error	*t*-Value
c	0.0196	0.0025	7.8174
ϕ_1	0.3543	0.0277	12.7967
ϕ_2	–0.1517	0.0182	–8.3276
ϕ_3	0.0802	0.0074	10.8261
ϕ_4	0.3274	0.0280	11.7062
θ_1	0.0453	0.0059	7.6161
θ_2	0.1838	0.0195	9.4265
θ_3	0.3059	0.0374	8.1843
k	–0.4270	0.0930	–4.5935
α_1	0.2617	0.0453	5.7719
β_1	0.9497	0.0098	97.2610
γ_1	0.2177	0.0461	4.7217

EXHIBIT 5

European Call and Put Options Prices on a House Worth 500,000 and $r - g$ Is Negative, September 2018

	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
K = 500,000						
Call	730.15	986.08	1,277.70	1,329.56	1,461.43	1,495.54
Put	1,064.17	1,625.07	2,148.47	2,417.96	2,791.59	3,094.13
K = 520,000						
Call	4.21	3.43	3.01	3.46	6.71	9.79
Put	20,111.23	20,224.08	20,499.73	20,711.75	20,891.93	21,100.12
K = 480,000						
Call	19,717.10	19,524.49	19,149.24	18,857.93	18,705.50	18,503.12
Put	0.00	0.00	6.63	7.76	8.23	3.35

Note: The risk-free rate and the rental yield are r = 1.5% and g = 2%, respectively.

EXHIBIT 6

European Call and Put Options Prices on a House Worth 500,000 and $r - g$ Is Positive, September 2018

	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
K = 500,000						
Call	1,243.07	2,016.25	2,794.48	3,441.50	4,137.96	4,595.86
Put	574.43	713.42	864.36	874.52	945.78	1,013.78
K = 520,000						
Call	0.00	0.00	13.23	11.11	17.64	21.90
Put	19,199.99	18,432.99	17,725.40	17,015.44	16,245.88	15,550.06
K = 480,000						
Call	20,572.94	21,210.06	21,790.14	22,306.79	22,818.42	23,360.17
Put	0.00	0.00	0.00	0.00	1.23	1.05

Note: The risk-free rate and the rental yield are r = 3.5% and g = 2%, respectively.

in financial markets is hampered by the lack of flexible and yet rigorous models that can be applied for valuation and risk management.

There are now sufficient models that can offer a sensible solution to these problems. We included a new one here aimed at housing derivatives. A crucial role is played by the forward/futures contract that will help identify a suitable risk-neutral pricing measure. Thus, the most important hurdle to eliminate seems to be the establishing of a liquid futures real estate contract.

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