Hedging Real Estate Equity Portfolios

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Commercial real estate derivatives used for hedging strategies usually have national aggregate indices as the underlying asset. In most cases, the IPD in the United Kingdom or the NPI in the United States are used as the underlying asset. This paper demonstrates that, despite the significant correlation between different property type returns or regional returns and nationwide returns, national aggregates do not constitute an appropriate underlying asset when using real estate derivates to hedge price risk in commercial real estate equity portfolios. Due to the lack of futures and options transactional data, the proof is done by estimating the basis risk of simulated derivative securities at fair market value on the NPI index, to hedge property type and regional subindexes portfolios market price processes.

Keywords: real estate derivatives; basis risk; portfolio hedging

Hedging commercial real estate equity portfolios has many complexities involved, and there is not a unique way to approach this challenge. Although investment managers have organic tools to manage their risks, such as property type or geographical diversification for systematic risk and property level financial liquidity and debt management for property-specific risk, there is still an open exposure to price risk that cannot be hedge with these tools. For instance, a core-equity hospitality private fund exposed to high room vacancy and low average daily rates due to an economic recession (as seen with the recent COVID-19 pandemic), has limited organic tools to hedge systematic risk. Diversifying with other asset classes requires vast amounts of capital (and changing the portfolio strategy), while selling the assets to repurchase them after the crisis implies a lot of transactional costs that may wipe out any profits. Therefore, the portfolio or asset manager would have to assume losses while the negative macroeconomic turmoil vanishes. This stress scenario has strong implications in compensation and shows fragility in the real estate private equity business.

A sound alternative would be to take a short position on a similar real estate portfolio, but this is not possible in private (incomplete) markets, and public markets are constrained due to liquidity problems (Bertin, Kofman, Michayluk, & Prather, 2005), i.e., short positions would affect prices rapidly making the hedge inefficient and REITs high dividend yield policy would be affected by systematic risks as well. A good substitute that academics have widely proposed - see for example (Fabozzi, Shiller, & Tunaru, A 30-Year Perspective on Property Derivatives: What Can Be Done to Tame Property Price Risk?, 2020) - is the use of real estate derivatives like futures or forwards, plain vanilla options and total return swaps, with price indices as underlying assets, to hedge the real estate portfolio. These derivatives have been used since the 1990’s and are commonly written on the IPD in the United Kingdom and on the NPI in the United States. The latter since 2006.

However, real estate derivatives lack of liquidity and volume. Even though exchange-traded residential real estate derivatives exist, like the Case-Shiller Index futures traded at CME, commercial real estate derivatives are mostly traded OTC with no exchange alternatives in the US. There are multiple plausible reasons why the latter derivatives have not enough deepness in the market. One interesting study performed by (Yoon Lim & Zhang, 2006) interviewed investment managers and other investors on their use of real estate derivatives. Although the usefulness for hedging purposes of these financial products was recognized by most of them, they were cautious about using them because of their lack of liquidity, the lack of dealers in the marketplace, and their lack of knowledge on how to price them. Even more, they recognized the importance of using regional property price indices rather than just nation-wide indices.

Therefore, real estate derivatives seem to fill an essential gap in the real estate public and private equity markets. They constitute the financial engineering approach to hedge systematic risks given that the natural way of hedging this asset class is idiosyncratic or face many constraints on the real estate equity markets. However, inefficiencies and the lack of an standardize methodology to price these derivatives limit their growth in financial markets. This paper contributes to the academic effort of creating a practical framework of structuring and pricing real estate derivatives, based on a pragmatic view of how to use them for hedging purposes. It shows that nationwide appraisal-based indices, although highly correlated with specific property type or regional indices, do not constitute a good hedge in the sense that basis risk is not properly covered.

In consequence, the following section presents a review of related literature on the different frameworks of real estate pricing, as well as their use for hedging real estate equity portfolios. Then, the methodology for measuring basis risk, how to determine whether a real estate equity portfolio is properly hedged or not by real estate derivatives, and a description of BFSP model for pricing futures and plain vanilla put options and simulate efficient market price processes for the underlying portfolios are thoroughly described. In the third section the data used for the empirical study is described, which in summary constitutes NCREIF NPI and property type and regional subindexes. The results and conclusions are presented in the two final sections.

# Literature review

The 1970’s can be seen as the birth year of financial engineering. In 1973 options moved from let alone over-the-counter negotiations to be formally traded in an exchange. The same year Black and Scholes published the first options pricing model (Fischer & Scholes, 1973), creating the no-arbitrage framework, which was extended by Robert Merton the same year (Merton, 1973). Since then, this model has been a reference for the industry, and multiple methodologies and extensions were created to price options on stocks and indices, among many other asset classes. However, within the pricing model assumptions there are two that both commercial real estate academics and practitioners have been reluctant to accept. In fact, real estate derivatives pricing models are lagged for almost two decades (or more) compared to derivatives in other asset classes.

First, real estate does not allow for short selling and trading is infrequent, so any arbitrage used for pricing a derivative would be just a theoretical idea rather than a factual reality. Moreover, assets are not homogeneous, and would not be even feasible or practical to create a derivative whose underlying asset is one property. Many authors, following (Case, Shiller, & Weiss, Index-Based Futures and Options Markets in Real Estate, 1993) have proposed to structure derivatives over indices instead of real assets to avoid such issues. This way, indices would be eligible for short selling through their futures market, and real estate price risk can be hedged with these derivatives.

Nevertheless, the construction of an appropriate index is also an open subject. Real estate price indices are categorized between transactional-based and appraisal-based (Fisher, Geltner, & Webb, 1994). The most common transactional-based methodology to track home prices, the repeated-sales index, was initially proposed by (Case & Shiller, The Efficiency of the Market for Single-Family Homes, 1989) for hedging residential equity value, with the novel idea of creating an equity insurance that homeowners could buy with their properties, to hedge their equity when homes prices were falling and avoid defaulting on their mortgages. This type of index is based on the price change of properties traded twice in time interval, so it is based on actual real estate prices. A transactional-based alternative would be the hedonic model (Adelman & Griliches, 1963) that regress property values based on multiple characteristics, such as location, size, or quality. On the other hand, appraisal-based indices require contributing participants to perform periodical valuations of their properties to calculate a periodic (usually quarterly) return based on capital growth and income yield. This type of index is based on (Giliberto, 1994) proxy of the internal rate of return to calculate a holding period return with intermediate cash flows, such NOI and capital improvements.

While transaction-based indices outstand for using actual transactional data, capturing price changes faster, and accessing a more significant sample than appraisal-based indices, they lack capturing all capital improvements and other expenses incurred by the property owner between transactions. Appraisal-based indices catch these improvements and income changes throughout time, but they are usually lagged and underestimate volatility since they depend on properties owner’s valuation process (Tunaru, 2017). The good news is that independent of the type of index used, they share some common stylized facts that are fundamental for derivatives pricing. As demonstrated by (Fabozzi, Shiller, & Tunaru, A Pricing Framework for Real Estate Derivatives, 2011) using UK IPD (transaction-based) and NCREIF NPI (appraisal-based), indices historical returns possess short-term positive serial correlation and long-term negative serial correlation, which leads to the second failure of Black and Scholes pricing framework for real estate derivatives pricing.

While in the Black and Scholes model stocks returns are assumed to be independent so they can be reproduced with a Brownian motion, serial correlation in real estate property prices cannot follow this type of stochastic process. Models for pricing derivatives with serial correlation can be traced to (Lo & Wang, 1995) and usually use an Ornstein-Uhlenbeck process instead. Moreover, the effect of interest rates in real estate prices has led many researchers to use bivariate models instead like (Bjork & Clapham, 2002) and (Patel & Pereira, 2008). Another common approach is risk-neutral valuation first proposed by (Titman & Torous, 1989). An attractive risk-neutral model was developed by (Van Bragt, Francke, Singor, & Pelsser, 2015), also known as the BFSP model. It uses a risk-neutral framework based on random departures of the observed index from an unobserved underlying index, to describe the underlying asset price process and price futures, plain vanilla options, and total return swaps. This model is of great interest because it captures all three mentioned stylized facts and recognizes significant changes in derivatives prices when the underlying is volatile.

It is worth mentioning that the models described so far are under the umbrella of the no-arbitrage principle. In fact, (Fabozzi, Shiller, & Tunaru, A Pricing Framework for Real Estate Derivatives, 2011) created a pricing framework that has been the baseline for these types of valuation models. However, there is an alternative school of thought that does not rely on arbitrage, given its alleged departure from reality and the complexity of selecting an appropriate stochastic process for the underlying asset. The equilibrium models prominent work of (Geltner & Fisher, 2007) extended by (Lizieri, Marcato, Ogden, & Baum, 2012), was written to price futures, options and total return swaps on the NPI; and concluded on similar results as standard derivatives pricing formulae, two years after the first NPI derivative was issued by Credit Suisse First Boston (Fisher J. , 2005). The principal critique to these models is that the link between futures prices and options prices was derived from the no-arbitrage principal for standard formulae and may not be supported by the equilibrium conditions, as (Fabozzi, Shiller, & Tunaru, A 30-Year Perspective on Property Derivatives: What Can Be Done to Tame Property Price Risk?, 2020) summarizes.

Finally, it has been assumed so far that real estate derivatives on price indices would provide a good hedge of private real estate equity portfolios. Although there is no strong evidence otherwise, (Ducoulombier, 2007) argues that this is a fallacy since derivatives on indices would not coincide with the actual portfolio of properties. This creates a cross-hedge basis risk exposure, a fundamental problem that was already identified by (Park & Switzer, 1996). Moreover, these derivatives would not expire at the same time the properties are disposed, leading to a time-basis risk. In other words, the incompleteness of the market would lead to a basis risk that eliminates the efficiency of the hedging strategy. Measuring the cross-hedge basis risk is the primary purpose of this article. Time-basis risk will not be discussed.

# Methodology

## Measuring Basis Risk

There are multiple reasons why hedging strategies using derivatives might not be perfect. Real estate private equity portfolios are unique, and there is no possibility of shorting a share on these assets. Therefore, investment managers using derivates to hedge their real estate position shall use a similar asset (usually a property price index) for their short position, what is known as cross-hedging strategy. Also, while the investment horizon of the properties could go as far as ten years (or more), the hedging strategy might last for at most five years, only while the economic turmoil remains. This is known as the horizon of the hedge. Following (Jorion, 2011), these imperfections on a hedging strategy are measured by the basis. The basis is the difference between the spot portfolio price and the position on the hedging derivative. The periodical change in the total value (V) of the strategy, including the hedge, is:

(1)

S stands for the spot price of the portfolio, F for an individual derivative contract price and N for the number of contracts written. The optimal number of contracts used (N\*) must be the one that minimizes the variance of the total strategy. For the scope of this study, it is important to mention that it can be derived from the slope of the equation:

(2)

Therefore, the methodology followed in this study to assess whether using nation-wide aggregates to hedge real estate equity portfolio is appropriate or not, is to evaluate the statistical significance of the slope in equation (2). If there are no statistical reasons to believe that the slope is different from 0, the hedge will be considered inappropriate since the optimal number of contracts is 0.

NCREIF property type and regional subindexes are used to calculate the market price processes for each subindex, leading 877 + XXX investment managers simulated portfolios. These portfolios are built by varying the weight of each subindex by 10% in each portfolio, such that there is always a 100% weight in real estate assets. For example, one portfolio could be 40% Apartments, 30% Office, and 30% Hospitality, assuming no short-selling and no leverage. Property type subindices lead to 877 portfolios, while regional subindices lead to XXX portfolios. Real estate index derivatives usually last from 3 to 5 years, while there are more than 30 years of history for the NPI. Thus, multiple 3- and 5-year windows will be used to calculate the slope of equation (2) and its p-value for each of the replication portfolios, using expiration dates raging 2000-Q1 to 2020-Q1 for the 3 years derivatives and from 1998-Q1 to 2020-Q1 for the 5 years derivatives. Notice that the spot price is represented by the different portfolios of property type (or regional) subindexes, while the derivative price is the one for the contract written on the NPI. The number of adequately hedged portfolios for each of the hedging strategies, relative to the total number of evaluated portfolios, for each expiration date and time to maturity, is the key metric in this study.

The only variable that has not been well defined to this point is the price of the derivative (F) that will be used on each case. That is the topic of the next subsection.

## Pricing real estate derivatives

Two strategies are used to hedge the simulated portfolios: futures and protective puts. The derivatives are priced using the BFSP methodology developed by (Van Bragt D. , Francke, Kramer, & Pelsser, 2009) and (Van Bragt D. , Francke, Singor, & Pelsser, 2015). This risk-neutral model is of particular interest for this work because it allows for high-frequency (monthly or quarterly) real estate derivatives in incomplete markets with serial correlation. The dynamics of the model rely on the index price update rule proposed by (Blundell & Ward, 1987) that weights the current time (t) real estate efficient market value and the lagged index value, which leads to an EWMA risk-neutral process for the price index given by

(3)

where represents the accrued index value at time t, while is the accrued efficient market price of the real estate assets, and is the weight parameter for the market price process . The accrued values depend on the expected rate of return, which correspond to the risk-free rate in a risk-neutral valuation model. Therefore,

(4)

(5)

Here, is a continuous-time measure, denoted differently than the discrete-time t. Although BFSP propose the one-factor no-arbitrage Hull-White (HW) model for the stochastic process for interest rate , in this work the (Vasicek, 1977) model is used for the sake of simplicity. The latter is also a no-arbitrage model, but the mean-reversion level is constant. The process is given by equation (6):

(6)

Where , and corresponds to a Weiner process on the risk-neutral probability space. The model is calibrated for three months T-Bills as the risk-free rate using least squares regression. Given this framework, the price of a forward contract that expires at time with an agreed-upon delivery price is:

(7)

is the price of a zero-coupon bond at time t that matures at time T and Since the primary use of these pricing models in this work is measuring basis risk in commercial real estate portfolio hedging strategies, Monte Carlo simulation is used for marked to market futures contracts instead, and the AR(p) model parameters are recalculated for each quarter using unconstrained OLS using R stats package (stats::ar.ols), with a maximum of 13 lags and selecting the model with the least Akaike Information Criterion. Notice that BFSP also provide a closed-form pricing formula for European put options on futures, but Monte Carlos simulation are used. The pricing formula is given by:

(8)

where is the strike price and

(9)

The option strike price for the hedging strategy can affect its performance of it and should depend on the asset manager expectations. Therefore, the simulation will be run with ATM and ITM puts shifted by one and two tenths of volatility.

Finally, basis risk is measured only for those portfolios where hedge was necessary, i.e., when the derivatives have intrinsic value at expiration. For example, a protective put in a bull market may have been a smart but unnecessary hedge, the important question is how good would it have performed in a bearish market.

# Data

Data used to determine the slope coefficients and their p-values are the NCREIF NPI and property type and regional subindexes values and quarterly returns, from 1990-Q1 to 2020-Q1. It corresponds to one nation-wide index (the NPI), five property subtype subindexes (Apartments, Industrial, Office, Retail, and Hospitality), and four regional subindexes (East, South, Midwest and West). It is important to mention that total returns were used for calibrating the model.

One key aspect of NCREIF returns is that they present a positive serial correlation, which is a basic stylized fact assumed on the pricing model used for measuring basis risks. As shown in Figure 1, the Ljung-box test shows that, as of 2020-Q1, quarterly NPI returns were significant for the first five lags, decreasing slowly to zero. Figure 1 shows that the serial correlation vanishes after five quarters. Also, NCREIF returns are left-skewed since negative returns are more significant than positive returns in absolute terms.

(Include Figure 1 here)

Another relevant aspect is the strong linear correlation between the NPI and the different property type subindexes. In fact, following the Pearson test, the linear correlation is positive and statistically significant between the NPI and each property type (see Figure 2). However, it is smaller for Hospitality which represents less than 2% of the total market value of the index and has a business model entirely different from the other types of properties. Hospitality weight on the NPI has decreased in the last three years.

(Include Figure 2 here)

One final aspect of real estate properties returns is that the total return is mostly explained by the capital component. Although there is an income component, which is positive in most of the cases. Hospitality is the only property type showing negative income returns during the period mentioned. Moreover, the volatility and the shape of the distribution is driven mainly by the capital component.

# Results

Paragraph:

There are three non-exclusive reasons why hedging strategies could have failed during the big economic crisis period: (i) the derivatives pricing and simulation model is not appropriate, (ii) measuring linear basis risk is not accurate or (iii) the hedging strategy proposed was not effective. The methodology and results provided in this study strongly suggest that the unique reason must be (iii). While is true that prices where not observed but simulated, given that there is no publicly available information in commercial real estate derivatives transactions on the NPI, the BFSP model ensures that all stylized facts for the underlying asset are correctly considered and modelled for the Monte Carlo simulation. Also, the calculation for the slope of the basis risk calculation is standard and thoroughly accepted in the industry and the academia.

Derivatives on the NPI miss important pieces in portfolio managers asset allocation that led to important basis risk exposure due to imperfect cross-hedging. Despite property types and regions are highly correlated, the weights on managers portfolios, their individual volatilities and their response to tail events is not that like those of the NPI.

# Conclusions

Paragraph:

# Appendix

The aim of this appendix is to give a general view of the code used to create the results of this work. It was written using an RStudio project (.Rproj) in R version 4.0.1 (2020-06-06) and Tidyverse 1.3.0. (Wickham, et al., 2019) for CRAN implementation, and it is stored on a private folder in GitHub. The code implementation consists of three folders. The “Source” folder contains raw data, where an Excel file provided by NCREIF stores all NPI and subindices quarterly data (for different property types and regions) from 1987 to 2020, and a .csv has all 3 Months T-Bills monthly rates provided by the Federal Reserve Economic Data (FRED) since1939 to 2020.

The “Model” folder contains all relevant scripts to create the derivatives simulated prices and the deltas for the basis risk calculations, returning the p-value of the slope in equation (2) for each case. The model\_main.R script reads the source data, creates the pay-off functions for the derivatives studied, builds the weights and returns of all asset manager simulated portfolios using the *main\_portfolios()* function, and calls *main\_pvalues()* that store the p-values in .csv files. The latter functions are stored in the model\_process.R file, along with other functions necessary for the simulation: *main\_prices()* and *main\_deltas()*. The calculation of the p-values uses the *basis\_risk()* function of the script model\_basis.R, which determines directly the needed values of the linear regression and avoid unnecessary calculation that base R functions could have done, such as *lm()*.

(Include Figure 3 here)

Notice that p-values need from both ΔS and ΔF, which are calculated by *main\_deltas()* given the spot prices (NPI) and the derivatives prices, determined by *main\_prices()*. The latter function is the main piece of the methodology of this study. It creates the prices path for each derivative until maturity. It leverages on the function *dvt\_price()* of model\_pricing.R that values derivatives based on the BFSP model. This pricing model requires of stochastic simulation for the efficient underlying asset price process, the underlying index value process and the interest rates process, which are created by functions in model\_underlying.R and model\_rates.R. The script model\_parameters.R calculates the AR(p) model parameters for the underlying asset using the stats::ar.ols() function of R, that selects the best model based on the Akaike Criterion information for up to 13 lags.

All prices and p-values are stored in a folder for each derivative in the “Output” folder. This folder also contains a script for each of the tables and figures presented along the study, except for Figure 3, which was built using Microsoft Power Point.

# Tables and Figures

A picture containing chart

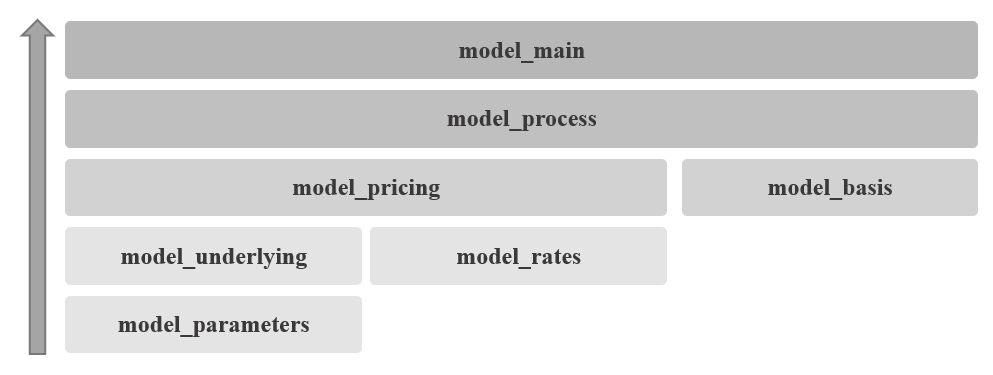
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Figure 1. NCREIF returns short-term positive serial correlation.

A picture containing chart

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Figure 2. Strong correlation between the NPI index and property type subindexes returns, with negative skewness.

 Figure 3. R Project modelling scripts architecture used for the simulation of derivatives prices and calculation of the p-values of each hedging strategy.