International Trade - Lecture 2

The Melitz Model

August 2025

Essential Reading

- Melitz, M. J. (2003) "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71, 1695-1725.

What Does This Paper Do?

- *Dynamic* industry model with *heterogeneous firms* where *opening to trade leads to reallocations of resources within an industry*
- Opening to trade leads to
  - Reallocation of resources across firms
  - Low productivity firms exit
  - High productivity firms expand
    - Change in industry composition
  - High productivity firms enter export markets
  - Improvement in aggregate industry productivity
  - No change in firm productivity
- Broadly consistent with empirical evidence from trade liberalizations.

Theory and Evidence

- The theoretical model is consistent with a variety of other stylized facts about industries
  - Heterogeneous firm productivity
  - Ongoing entry and exit
    - Co-movement in (gross) entry and exit
    - Exiting firms are low productivity (selection effect)
  - Explains why some firms export within industries and others do not
    - Contrast with traditional theories of comparative advantage
    - Exporting firms are high productivity (selection effect)
    - *No feedback* from exporting to productivity

Where Does the Paper Fit in the Literature?

- Theoretical
  - Dynamic industry models of heterogeneous firms under perfect competition and closed economy
    - Jovanovic (1982) and Hopenhayn (1992)
  - Models of trade under imperfect competition
    - Krugman (1980)
  - First framework for modeling firm heterogeneity in international trade
- Empirical
  - Empirical literature on heterogeneous productivity, entry and exit
    - Davis and Haltiwanger (1991)
    - Dunne, Roberts and Samuelson (1989)
  - Empirical literature on exports and productivity
    - Bernard and Jensen (1995, 1999)
    - Roberts and Tybout (1996, 1997)
    - Clerides et al. (1998)
  - Empirical literature on trade liberalization
    - Levinsohn (1999)
    - Pavcnik (2002)

Road Map

- Overview of Model Structure
- Equilibrium in a Closed Economy
- Equilibrium in an Open Economy
- The impact of the opening of trade
- What to learn?

Overview of Model Structure

- Single factor: labor (numeraire, $w = 1$)
- Firms enter market by paying sunk entry cost (f_e)
- Firms observe their productivity (φ) from distribution $g(\varphi)$
  - Productivity is fixed thereafter
- Once productivity is observed, firms decide whether to produce or exit
- Firms produce horizontally-differentiated varieties, with a fixed production cost (f_d) and a constant variable cost that depends on their productivity
- Firms face an exogenous probability of death (δ) due to *force majeure* events

Closed Economy – Demand

- CES "love of variety" preferences:

$$U = \left[\int_{\omega\in\Omega} q(\omega)^{\rho} d\omega\right]^{\frac{1}{\rho}}, \qquad 0 < \rho < 1$$

- Price index for $Q \equiv U$:

$$P = \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}, \qquad \sigma = \frac{1}{1-\rho} > 1$$

- From consumer maximization (after manipulation): [SHOW]

$$q(\omega) = Q\left(\frac{p(\omega)}{P}\right)^{-\sigma}, \qquad r(\omega) = R\left(\frac{p(\omega)}{P}\right)^{1-\sigma}, \qquad R = PQ$$

Production

- Continuum of firms, each choosing a different variety
- Firms of all productivities behave symmetrically and therefore we can index firms by productivity alone
- Production technology (labor is only factor of production):

$$l = f + \frac{q}{\varphi}$$

- Profit maximization problem:

$$\max_{p(\varphi)} \left\{ r(\varphi) - w\left(f + \frac{q(\varphi)}{\varphi}\right)\right\}$$

- The first-order condition yields the equilibrium pricing rule:

$$p(\varphi) = \left(\frac{\sigma}{\sigma - 1}\right)\frac{w}{\varphi} = \frac{1}{\rho\varphi},$$

where we choose the wage for the numeraire, $w = 1$

Firm Revenue

- Substituting the pricing rule into equilibrium revenue:

$$r(\varphi) = (\rho\varphi)^{\sigma-1} RP^{\sigma-1}, \qquad \pi(\varphi) = \frac{r(\varphi)}{\sigma} - f.$$

- Therefore, the relative revenue of any two firms within the same market depends solely on their relative productivities:

$$r(\varphi'') = \left(\frac{\varphi''}{\varphi'}\right)^{\sigma-1} r(\varphi') \tag{1}$$

- The presence of a fixed production cost implies a zero-profit cutoff productivity, below which firms exit:

$$\pi(\varphi^*) = 0, \qquad \Leftrightarrow \qquad r(\varphi^*) = \sigma f \tag{2}$$

- The revenue of any firm can therefore be written as:

$$r(\varphi) = \left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} \sigma f$$

Firm Entry and Exit

- The *ex post* productivity distribution conditional on successful firm entry is therefore:

$$\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi^*)} & \text{for } \varphi \geq \varphi^* \\ 0 & \text{otherwise} \end{cases}$$

- The present value of a firm with productivity φ is:

$$v(\varphi) = \max\left\{0, \frac{\pi(\varphi)}{\delta}\right\}$$

- In equilibrium, the free entry condition requires the expected value of entry to equal the sunk entry cost

$$v_e = \frac{1 - G(\varphi^*)}{\delta}\bar{\pi} = f_e, \tag{3}$$

  - where $[1 - G(\varphi^*)]$ is the probability of successful entry
  - where $\bar{\pi}$ is expected profits conditional on successful entry

Free Entry

- Expected profits conditional on successful entry are:

$$\bar{\pi} = \int_{\varphi^*}^{\infty} \pi(\varphi) \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi,$$

which using the relationship between variety revenues and the zero-profit cutoff condition (2) can be written as:

$$\bar{\pi} = f \int_{\varphi^*}^{\infty} \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma - 1} - 1 \right] \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi$$

- Therefore the free entry condition becomes:

$$v_e = \frac{f}{\delta} \int_{\varphi^*}^{\infty} \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma - 1} - 1 \right] g(\varphi) d\varphi = f_e, \qquad (4)$$

where the l.h.s. is monotonically decreasing in φ^*

- Therefore the model has a recursive structure where φ^* can be determined from the free entry condition alone

Aggregate Variables

- Define a weighted average of firm productivity, a function of φ^*:

$$\tilde{\varphi}^{\sigma-1} \equiv \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi. \tag{5}$$

- Aggregate variables, such as the price index P, can be written as functions of the mass of (surviving) firms M and weighted average productivity:

$$P = \left[\int_{\varphi^*}^{\infty} p(\varphi)^{1-\sigma} M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi \right]^{\frac{1}{1-\sigma}}$$

$$P = \left[\int_{\varphi^*}^{\infty} (\rho\varphi)^{\sigma-1} M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi \right]^{\frac{1}{1-\sigma}}$$

$$P = M^{\frac{1}{1-\sigma}} \frac{1}{\rho} \left[\int_{\varphi^*}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi \right]^{\frac{1}{1-\sigma}}$$

$$P = M^{\frac{1}{1-\sigma}} \frac{1}{\rho\tilde{\varphi}} = M^{\frac{1}{1-\sigma}} p(\tilde{\varphi}). \tag{6}$$

Closed Economy General Equilibrium

- The closed economy general equilibrium is referenced by the triple $\{\varphi^*, P, R\}$
- All other endogenous variables can be written in terms of this triple
- The steady-state equilibrium is characterized by a constant mass of firms entering each period, M_e, a constant mass of firms producing, M, and a stationary *ex post* distribution of firm productivity, $g(\varphi)/\left[1 - G(\varphi^*)\right]$
- To determine general equilibrium, we use the recursive structure of the model
- Equilibrium φ^* follows from the free entry condition (4) alone

Closed Economy General Equilibrium, R

- To determine R, we use the steady-state stability condition that the mass of successful entrants equals the mass of exiting firms

$$[1 - G(\varphi^*)] \, M_e = \delta M$$

- Using this steady-state stability condition to substitute for $1 - G(\varphi^*)$ in the free entry condition (3), competitive entry implies that total payments to labor used in entry equal total firm profits:

$$L_e = M_e f_e = M\bar{\pi} = \Pi$$

- Total payments to labor used in production equal total revenue minus total firm profits:

$$L_p = R - M\bar{\pi} = R - \Pi$$

- Therefore, total revenue equals total labor payments and the labor market clears:

$$R = L = L_p + L_e$$

Closed Economy General Equilibrium, P

- To determine P in (6), we need to solve for $\tilde{\varphi}$ and M
- Having determined φ^*, $\tilde{\varphi}$ follows immediately from (5)
- The mass of firms can be determined from:

$$M = \frac{R}{\bar{r}} = \frac{L}{\sigma(\bar{\pi} + f)},$$

where $\bar{r}$ and $\bar{\pi}$ can be written as a function of φ^* and $\tilde{\varphi}$, which have both been determined:

$$\bar{r} = \int_{\varphi^*}^{\infty} r(\varphi) \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi = r(\tilde{\varphi}) = \left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1} \sigma f,$$

$$\bar{\pi} = \int_{\varphi^*}^{\infty} \pi(\varphi) \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi = \pi(\tilde{\varphi}) = \left[\left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1} - 1\right] f.$$

Open Economy Model

- Consider a world of symmetric countries
- Suppose that each country can trade with $n \geq 1$ other countries
- Choose the wage in *one* country as the numeraire, which with country symmetry implies $w = w^* = 1$
- To export, a firm must incur a fixed export cost of f_x units of labor
- In addition, exporters face iceberg variable costs of trade such that $\tau > 1$ units of each variety must be exported for 1 unit to arrive in the foreign country
- Observe that firms face the same elasticity of demand in both markets. Thus, export prices are a constant multiple of domestic prices due to the variable costs of trade:

$$p_x(\varphi) = \tau p_d(\varphi) = \frac{\tau}{\rho\varphi}$$

- Consumer optimization implies that export market revenue is a constant fraction of domestic market revenue:

$$r_x(\varphi) = \tau^{1-\sigma} r_d(\varphi) = \tau^{1-\sigma} R(P\rho\varphi)^{\sigma-1} \tag{7}$$

Firm Exporting Decision

- Total firm revenue depends on whether the firm exports:

$$r(\varphi) = \begin{cases} r_d(\varphi) & \text{not export} \\ r_d(\varphi) + n r_x(\varphi) = (1 + n\tau^{1-\sigma}) r_d(\varphi) & \text{export} \end{cases}$$

- Consumer love of variety and fixed production costs $\Rightarrow$ no firm will ever export without also serving the domestic market
- Therefore, we can separate the fixed production cost to domestic market and the fixed exporting cost to export market
  - When deciding whether to export, firms compare export market profits to the fixed exporting costs
- Given fixed exporting costs, there is an exporting cutoff productivity φ_x^* such that only firms with $\varphi \geq \varphi_x^*$ export:

$$r_x(\varphi_x^*) = \sigma f_x \tag{8}$$

Selection into Export Markets

- A large empirical literature, before and after Melitz (2003), finds evidence of selection into export markets
  - Only some firms export
  - Exporters are more productive than non-exporters
- From the relative revenues of firms with different productivities in the same market (1), and from relative revenue in the domestic and export markets (7), we have, respectively:

$$r_d(\varphi_x^*) = \left(\frac{\varphi_x^*}{\varphi^*}\right)^{\sigma-1} r_d(\varphi^*), \qquad r_x(\varphi_x^*) = \tau^{1-\sigma} r_d(\varphi_x^*).$$

- Using the zero-profit and exporting cutoff conditions, (2) and (8), we obtain the following relationship between the productivity cutoffs:

$$\varphi_x^* = \tau \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} \varphi^*, \tag{9}$$

where selection into export markets, $\varphi_x^* > \varphi^*$, requires $\tau^{\sigma-1} f_x > f$

Average Firm Revenue and Profits

- Average firm revenue and profits are now:

$$\bar{r} = r_d(\tilde{\varphi}) + \chi n r_x(\tilde{\varphi}_x), \qquad \bar{\pi} = \pi_d(\tilde{\varphi}) + \chi n \pi_x(\tilde{\varphi}_x),$$

where average revenue in each market is:

$$\bar{r}_d = r_d(\tilde{\varphi}) = \left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1} \sigma f, \qquad \bar{r}_x = r_x(\tilde{\varphi}_x) = \left(\frac{\tilde{\varphi}_x}{\varphi_x^*}\right)^{\sigma-1} \sigma f_x,$$

and average profits in each market are:

$$\bar{\pi}_d = \pi_d(\tilde{\varphi}) = \left[\left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1} - 1\right] f,$$

$$\bar{\pi}_x = \pi_x(\tilde{\varphi}_x) = \left[\left(\frac{\tilde{\varphi}_x}{\varphi_x^*}\right)^{\sigma-1} - 1\right] f_x$$

Free Entry

- The free entry condition in the open economy becomes:

$$v_e = [1 - G(\varphi^*)] \frac{[\bar{\pi}_d + \chi n \bar{\pi}_x]}{\delta} = f_e,$$

  where $[1 - G(\varphi^*)]$ is the probability of successful entry, $\bar{\pi}_d$ is expected domestic profits conditional on successful entry, $\chi \equiv [1 - G(\varphi_x^*)] / [1 - G(\varphi^*)]$ is the probability of exporting conditional on successful entry, and $\bar{\pi}_x$ is expected export profits conditional on exporting

- Using the relationship between variety revenues and the zero-profit and exporting cutoff conditions, the *FEC* becomes:

$$\begin{aligned} v_e = \frac{f}{\delta} \int_{\varphi^*}^{\infty} \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma-1} - 1 \right] g(\varphi) d\varphi \\ + \frac{f_x}{\delta} n \int_{\varphi_x^*}^{\infty} \left[\left(\frac{\varphi}{\varphi_x^*} \right)^{\sigma-1} - 1 \right] g(\varphi) d\varphi = f_e \end{aligned} \quad (10)$$

Aggregate Variables

- Define weighted average productivity for the export market:

$$\tilde{\varphi}_x = \left[\int_{\varphi_x^*}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1 - G(\varphi_x^*)} d\varphi \right]^{\frac{1}{\sigma-1}} . \tag{11}$$

- The price index P can be written as a function of the mass of firms that supply each market M_t and overall weighted average productivity $\tilde{\varphi}_t$:

$$P = M_t^{\frac{1}{1-\sigma}} p(\tilde{\varphi}_t) = M_t^{\frac{1}{1-\sigma}} \frac{1}{\rho \tilde{\varphi}_t},$$

$$\tilde{\varphi}_t = \left\{ \frac{1}{M_t} \left[M \tilde{\varphi}^{\sigma-1} + n M_x \left(\tau^{-1} \tilde{\varphi}_x \right)^{\sigma-1} \right] \right\}^{\frac{1}{\sigma-1}} ,$$

$$M_t = M + n M_x, \qquad M_x = \chi M$$

Open Economy General Equilibrium

- The open economy general equilibrium is referenced by the quadruple $\{\varphi^*, \varphi_x^*, P, R\}$
- All other endogenous variables can be written in terms of this quadruple
- The steady-state equilibrium is characterized by a constant mass of firms entering each period, M_e, constant masses of firms producing and exporting, M and M_x, and stationary *ex post* distributions of firm productivity in the domestic and export markets, $g(\varphi)/\left[1 - G(\varphi^*)\right]$ and $g(\varphi)/\left[1 - G(\varphi_x^*)\right]$
- To determine general equilibrium, use the recursive structure of the model
- Equilibrium φ^* can be determined from the free entry condition (10), substituting for φ_x^* using the relationship between the cutoffs (9)
- Having determined φ^*, φ_x^* follows from the relationship between the cutoffs (9)

Open Economy General Equilibrium, R

- To determine R, use the steady-state stability condition that the mass of successful entrants equals the mass of exiting firms

$$[1 - G(\varphi^*)]\, M_e = \delta M$$

- Using this steady-state stability condition to substitute for $1 - G(\varphi^*)$ in the free entry condition (3), competitive entry implies that total payments to labor used in entry equal total firm profits:

$$L_e = M_e f_e = M\left[\bar{\pi}_d + \chi n \bar{\pi}_x\right] = \Pi$$

- Total payments to labor used in production are:

$$L_p = R - M\left[\bar{\pi}_d + \chi n \bar{\pi}_x\right] = R - \Pi.$$

- Therefore total revenue equals total labor payments and the labor market clears:

$$R = L = L_p + L_e.$$

- Labor used in production includes fixed production, fixed exporting and variable production costs

Open Economy General Equilibrium, P

- To determine P, use the expressions for $\tilde{\varphi}_t$ and M_t above
- Having pinned down φ^* and φ_x^*, we can determine $\chi = [1 - G(\varphi_x^*)] \,/\, [1 - G(\varphi^*)]$, $\tilde{\varphi}$ and $\tilde{\varphi}_x$
- Having pinned down the probability of exporting and weighted average productivities, we can determine $\bar{r}$ and $\bar{\pi}$
- We can therefore also determine the mass of firms serving the domestic market and exporting

$$M = \frac{R}{\bar{r}} = \frac{L}{\sigma(\bar{\pi} + f + \chi n f_x)}, \qquad M_x = \chi M,$$

- Having pinned down M and M_x, we have determined M_t
- Having pinned down M_t, M, M_x and weighted average productivities, we have determined $\tilde{\varphi}_t$

- The open economy *FEC* provides a downward-sloping relationship between the productivity cutoffs φ^* and φ_x^*

$$v_e = \frac{f}{\delta} \int_{\varphi^*}^{\infty} \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma-1} - 1 \right] g(\varphi) d\varphi + \frac{f_x}{\delta} n \int_{\varphi_x^*}^{\infty} \left[\left(\frac{\varphi}{\varphi_x^*} \right)^{\sigma-1} - 1 \right] g(\varphi) d\varphi = f_e$$

- The closed economy *FEC* can be obtained by considering the case where trade costs become prohibitive and $\varphi_x^* \to \infty$

- Key result: The opening of trade raises the zero-profit productivity cutoff below which firms exit, φ^*.

The Effects of Trade (long run)

- The opening of trade leads to a rise in the zero profit cutoff productivity [see Figure 2]
- Low productivity firms, between φ_A^* and φ_I^*, exit
  - Increased exit by low productivity firms
- Intermediate productivity firms between φ_I^* and φ_{xI}^* sell domestically, but don't export
- Firms with productivities greater than φ_{xI}^* enter export markets
  - Selection into export markets
  - Expansion in revenue for exporting firms
- These changes in industry composition raise aggregate industry productivity
- As the zero profit cutoff productivity and average revenue rise:
  - Mass of domestically produced varieties falls: $M_I < M_A$
  - Total mass of varieties available for consumption typically rises: $(1 + n\chi)M_I > M_A$
  - Welfare necessarily rises due to aggregate productivity gains

Comparing firms, with and without trade

- Revenue

$$r_d(\varphi) < r_A(\varphi) < r_d(\varphi) + nr_x(\varphi)$$

  - all firms lose domestic sales
    - if firm doesn't export, it incurs a loss in total revenue
  - if firm exports, its revenue increases
- Profit
  - if firm doesn't export, profit falls for sure (since revenue, and then variable profit, falls)
  - if firm exports, there are 2 opposing forces:
    - revenue rises
    - it needs to incur the export fixed cost

$$\begin{aligned}\Delta\pi(\varphi) &= \pi(\varphi) - \pi_A(\varphi) = \frac{1}{\sigma}\left[r_d(\varphi) + nr_x(\varphi) - r_A(\varphi)\right] - nf_x \\ &= \varphi^{\sigma-1} f\left[\frac{1 + n\tau^{1-\tau}}{(\varphi^*)^{\sigma-1}} - \frac{1}{(\varphi_a^*)^{\sigma-1}}\right] - nf_x\end{aligned}$$

- $\frac{d\Delta\pi(\varphi)}{d\varphi} > 0$
- $\Delta\pi(\varphi_x^*) < 0$

Trade & survival of the fittest: channels

1. Product market competition (more, and tougher, competitors)
   - No (CES)
2. Competition for domestic resources (labor)
   - Yes: additional demand for resources to pay for f_x and for expansion of more productive firms; more entry due to higher potential returns $\Longrightarrow$ real wages rise and weak firms exit

Trade liberalization (rather than big bang)

- $n \uparrow$ (e.g. enlargement of a trading bloc):
  - $\varphi^* \uparrow$, $\varphi_x^* \uparrow$
  - Weak firms lose domestic market share; exporting firms expand
    * Market share and profits reallocated toward more efficient firms: aggregate productivity gains, $W \uparrow$
- $\tau \downarrow$
  - $\sim n \uparrow$, but $\varphi_x^* \downarrow$
- $f_x \downarrow$
  - Also similar to $n \uparrow$ and $\tau \downarrow$, except that in this case market share and profits of previous exporting firms does not change; they increase only for firms that start to export due to $f_x \downarrow$

What to learn?

- Goal of model: not just to explain firm selection into exporting, but also to explain industry restructuring, and the effects on industry productivity and welfare
- The opening of trade leads to reallocations of resources across firms within industries
  - Low productivity firms exit
  - Intermediate productivity surviving firms contract
  - High productivity surviving firms enter export markets and expand
  - Change in industry composition
- Improvements in aggregate industry productivity
- No change in firm productivity
- Selection into export markets but no feedback from exporting to firm productivity

Subsequent Research

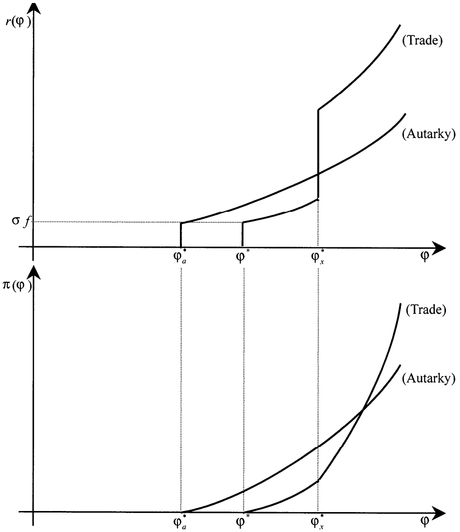
- Helpman, E., M. Melitz and S. Yeaple (2004). "Export Versus FDI with Heterogeneous Firms," *American Economic Review* 94, 300-316
  - Introduces both exports and FDI as alternative means of serving a foreign market
  - Introduces an outside sector to tractably characterize equilibrium with many asymmetric sectors
- Antras, P. and E. Helpman (2004). "Global Sourcing," *Journal of Political Economy* 112(3), 552-580
  - Combines the Melitz model with the Antras (2003) model of incomplete contracts and trade
- Bernard, A., S. Redding and P. Schott (2007). "Comparative Advantage and Heterogeneous Firms," *Review of Economic Studies* 73(1), 31-66
  - Incorporates the Melitz model into the framework of integrated equilibrium of Helpman and Krugman (1985)

Subsequent Research

- Chaney, T. (2008). "Distorted Gravity: the Intensive and Extensive Margins of International Trade," *American Economic Review* 98(4), 1707-1721
  - Provides a simplified static version of the Melitz model without ongoing firm entry and with an outside sector
  - Examines the model's implications for the extensive and intensive margins of international trade
- Bernard, A., P. Schott and S. Redding (2011). "Multi-product Firms and Trade Liberalization," *Quarterly Journal of Economics* 126(3), 1271-1318
  - Motivated by the empirical importance of multi-product firms, uses the Melitz (2003) framework to develop a general equilibrium model of multi-product firms
  - The model accounts for key observed features of the distribution of exports across firms, products and countries
  - Trade liberalization gives rise to measured within-firm productivity growth by inducing firms to focus on their core competencies

Melitz (2003)

Trade Reallocation



Back to Model