Problem set 1: Melitz (2003)

Consider the Melitz (2003) model. Suppose that productivity φ is Pareto distributed with the following density function:

$$g(\varphi) = k\varphi_{\min}^{k}\varphi^{-(k+1)}, \quad \varphi_{\min} > 0, \quad k > \sigma - 1,$$

where σ is the elasticity of substitution between varieties. The corresponding cumulative distribution function is:

$$G(\varphi) = 1 - \left(\frac{\varphi_{\min}}{\varphi}\right)^{k}.$$

Note the following properties of the Pareto distribution:

$$\text{Mean} = \frac{k}{k-1}\varphi_{\min}, \quad \text{Variance} = \frac{k\varphi_{\min}^{2}}{(k-1)^{2}(k-2)}, \quad \text{Coeff. Variation} = \frac{\varphi_{\min}}{(k-1)(k-2)},$$

where $k > 1$ is required for a finite mean and $k > 2$ is required for a finite variance.

(a) Solve for the closed economy values of φ^{*}, P, R.

(b) Suppose that two symmetric economies are opened to trade and fixed and variable trade costs are sufficiently high to induce selection into export markets. Solve for the open economy values of φ^{*}, φ_{x}^{*}, P, R.

(c) Explain intuitively the impact of the opening of trade on each closed economy? Explain the economic intuition underlying the exit of low productivity firm and the reallocation of resources across firms within industries.

(d) Suppose that the two closed economies were opened to free trade with zero trade costs ($\tau = 1$, $f_{x} = 0$). How (if at all) would this change the impact of the opening of trade on each closed economy? Explain the economic intuition for your answer.