Summer School 2018 Midsession Examination

FM225

Fixed Income Securities, Debt Markets and the Macro Economy

Suitable for all candidates.

Instructions to candidates

Time Allowed: **2 hours**

Answer ALL questions. Marks are allocated as shown.

An electronic calculator may be used.

This exam is worth **50%** of your total grade.

Good luck!

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1. **One Step Binomial Tree. [30 marks]**

   The table below gives the term structure of continuously compounded interest rates and the associated one period interest rate tree. Assume that the probability of the interest rate to go up or down is equal to 50%. Assume that all bonds have a face value of 100.

Maturity (t, T)	Yield (cont. comp.) $r(t, T)$
0.25	3.16%
0.50	3.78%
0.75	3.89%
1.00	5.01%

$$
\begin{tikzcd}
 & r_{1,u} = 4.28\% \\
r_0 = 3.16\% \arrow[ur, "50\%"] \arrow[dr, "50\%"'] & \\
 & r_{1,d} = 2.19\%
\end{tikzcd}
$$

   (a) [5 marks] What is the price of a 6-month zero coupon bond at time 0 (i.e. $P_0(2)$ as per the notation used in the course)? Explain your calculations.

   (b) [5 marks] What is the price of the above bond after 3 months in the "up" and "down" states (i.e. $P_{1,u}(2)$ and $P_{1,d}(2)$ as per the notation used in the course)? Explain your calculations.

   (c) [10 marks] Compute the time 0 price of a 3-month European *second power* put option on the 6-month zero coupon bond. A second power put option is like a put option in which we use the second power of the underlying asset price rather than the price itself i.e. the intrinsic value of such an option at maturity is:

$$max\left(K - (P_T)^2, 0\right).$$

   where K denotes the strike price and P_T is the price of the underlying security at maturity of the option. Assume that the strike price is $K = 9801$. Explain your calculations and describe the quantities characterising the replicating portfolio.

   (d) [10 marks] Explain the concept of market price of risk and compute this quantity using the 6-month zero bond.

2. **Spot rates, forward rates, swap rates. [30 marks]**

The table below gives the current term structure of continuously compounded interest rates. Assume that all bonds have a face value of 100.

i	Maturity T_i	Yield (cont. comp.) $r(0,T_i)$
1	0.25	1.96%
2	0.50	2.16%
3	0.75	2.34%
4	1.00	2.58%

(a) [15 marks] Compute: the discount factors $Z(0,T_i)$; the continuously compounded forward rates $f(0,T_i-\Delta,T_i)$, where $\Delta = 0.25$; the swap rates $c(0.25)$, $c(0.50)$, $c(0.75)$ and $c(1.00)$ i.e. the swap rates for contracts with maturities from 0.25 to 1.00 [hint: what is the value of a swap at initiation?]. Explain your answers.

(b) [5 marks] Computes the time 0 prices of forward contracts for a zero-coupon bonds for delivery time T_i and maturity $T_i + .25$ i.e. compute $P_z^{fwd}(0,T_i,T_i+.25)$.

(c) [10 marks] Suppose a bank has promised to a firm to receive \$100m at time T_1, and return to the firm at time T_2 the \$100m plus the forward return rate between time T_1 and T_2, i.e. the bank promises to pay \$100m $\times\, e^{0.25\times f(0,T_1,T_2)}$. Explain how the bank can perfectly hedge this exposure (be careful in specifying the quantities involved in the trading strategy and verify that the strategy does deliver the promised hedging).

3. **Interest rate risk management, duration and convexity. [40 marks]**

The table below gives the quarterly discount factors. Use them when needed. Assume for all bonds that the notional value is 100.

Maturity (t, T)	Discount factor $Z(t, T)$
0.25	0.9840
0.50	0.9680
0.75	0.9520
1.00	0.9360
1.25	0.9190
1.50	0.9040
1.75	0.8880
2.00	0.8730

(a) [5 marks] Calculate the duration and convexity of a 2-year zero coupon bond.

(b) [4 marks] How do the duration and convexity of the a 2-year zero coupon bond evolve over time? [Plotting these quantities as a function of time might be useful]

(c) [8 marks] Calculate the duration and convexity of a 2-year fixed coupon bond paying 5% semiannually.

(d) [8 marks] Calculate the duration and convexity of a 2-year floating rate bond, with no spread, that pays semiannually. How do the duration and convexity of such a bond evolve over time?

(e) [5 marks] Calculate the duration of a 1.25-year floating rate bond that pays the floating rate plus 50 bps spread semiannually. You know that last quarter the semiannual rate was 6.4%. Given the maturity of the bond, you know that the next payment is due in three months.

(f) [10 marks] Explain succinctly the concept of *factor neutrality* and *factor based* duration hedging, stressing the advantages and challenges of this approach.