Fixed Income Securities, Debt Markets and the Macro Economy

PV refers to the course textbook:
Pietro Veronesi, *Fixed Income Securities: Valuation, Risk, and Risk Management*, John Wiley & Sons, 2010.
All tables are also available in electronic format (i.e., in and Excel file) on Moodle.

Class 1 – Introduction and overview of debt markets

PV examples 1.1 and 1.2.

1. What are the four main types of securities in the U.S. Treasury market? Briefly describe the main features and discuss the main differences among them.

2. If you need to borrow short term from the money market and you hold one particular treasury security which is in high demand, which market will you go for and why?

3. **Example 1.1:** Suppose that a trader on September 18, 2007 (time t) wants to take a long position until a later time T on a given U.S. security, such as the 30-year Treasury bond. Let P_t denote the (invoice) price of the bond at time t. At time t, the trader buys the bond at the market price P_t and enters into a repurchase agreement with the repo dealer with a repo rate r and a haircut h.

   - Please draw a schematic representation of the repo transaction at time t and T.
   - What is the repo interest payment and return on capital for the trader?

4. **Example 1.2:** Now consider a trader who thinks a particular bond, such as the on-the-run 30-year Treasury bond, is overpriced and wants to take a bet that its price will decline in the future. If the trader does not have the bond to sell outright, then he or she can enter into a reverse repo with a repo dealer to obtain the bond to sell. More specifically, in a reverse repo, the trader essentially borrows the security from the dealer, sells it in the market and posts cash collateral with the dealer.

   - Please draw a schematic representation of the reverse repo transaction at time t and T.
   - What is the profit from the reverse repo transaction?

Solutions

1.

Type	Maturity	Coupon	Principal
Treasury Bills	4,13,26 and 52 weeks	None	Fixed
Treasury Notes	2, 5 and 10 years	Fixed, semi-annual	Fixed
Treasury Bonds	30 years	Fixed, semi-annual	Fixed
TIPS	5, 10 and 20 years	Fixed, semi-annual	Adjusted for inflation

2. The REPO market.

3. **Exercise 1.1.** See Excel Spread Sheet.

4. **Exercise 1.2.** See Excel Spread Sheet.

Class 2 – Basics of fixed income securities and market conventions

PV exercises 2.2; 2.5; 2.6 and 2.7.

1. **Exercise 2.2:** Compute the price, the yield and the continuously compounded yield for the following Treasury bills. For the 1-year Treasury bill also compute the semi-annually compounded yield.
   (a) 4-week with 3.48% discount (December 12, 2005)
   (b) 4-week with 0.13% discount (November 6, 2008)
   (c) 3-month with 4.93% discount (July 10, 2006)
   (d) 3-month with 4.76% discount (May 8, 2007)
   (e) 3-month with 0.48% discount (November 4, 2008)
   (f) 6-month with 4.72% discount (April 21, 2006)
   (g) 6-month with 4.75% discount (June 6, 2007)
   (h) 6-month with 0.89% discount (November 11, 2008)
   (i) 1-year with 1.73% discount (September 30, 2008)
   (j) 1-year with 1.19% discount (November 5, 2008)
2. **Exercise 2.5:** Consider a 10-year coupon bond paying 6% coupon rate.
   (a) What is its price if its yield to maturity is 6%? what if it is 5% or 7%?
   (b) Compute the price of the coupon bond for yields ranging from between 1% and 15%. Plot the resulting bond price versus the yield to maturity. What does the plot look like?
3. **Exercise 2.6:** Consider the data in Table 1 (Table 2.4 in PV). Consider two bonds, both with 7 years to maturity but with different coupon rates. Let the two coupon rates be 15% and 3%.
   (a) Compute the prices and the yields to maturity of these coupon bonds.
   (b) How do the yields to maturity compare to each other? If they are different, why are they different? Would the difference in yields imply that one is a better "buy" than the other?

Table 1: **Yield Curve on March 15, 2000**

Maturity	Yield	Maturity	Yield	Maturity	Yield
0.25	6.33%	2.75	6.86%	5.25	6.39%
0.50	6.49%	3.00	6.83%	5.50	6.31%
0.75	6.62%	3.25	6.80%	5.75	6.24%
1.00	6.71%	3.50	6.76%	6.00	6.15%
1.25	6.79%	3.75	6.72%	6.25	6.05%
1.50	6.84%	4.00	6.67%	6.50	5.94%
1.75	6.87%	4.25	6.62%	6.75	5.81%
2.00	6.88%	4.50	6.57%	7.00	5.67%
2.25	6.89%	4.75	6.51%	7.25	5.50%
2.50	6.88%	5.00	6.45%	7.50	5.31%

4. **Exercise 2.7:** Today is May 15, 2000.
   (a) Compute the discount curve $Z(0,T)$ for $T = 6$ month, 1 year, 1.5 years, and 2 years from the following data:
      - A 6-month zero coupon bond priced at $96.8 (issued 5/15/2000)
      - A 1-year note with 5.75% coupon priced at$99.56 (issued 5/15/1998)

- A 1.5 year note with 7.5% coupon price at \$100.86 (issued 11/15/1991)
- A 2 year note with 7.5% coupon priced at \$101.22 (issued 5/15/1992)

(b) Once you get the discount curve $Z(0, T)$ you take another look at the data and you find the following 1-year bonds:

- A 1-year note with 8% coupon period at \$101.13 (issued 5/15/1991)
- A 1-year note with 13.13% coupon period at \$106.00 (issued 4/2/1981)

Compute the prices for these bonds with the discounts you found. Are the prices the same as what the market says? Is there an arbitrage opportunity? Why?

Solutions

1. **Exercise 2.2.** See Excel Solutions.

2. **Exercise 2.5.** If the coupon rate is equal to the yield to maturity the bond is trading 'at par', the price of the bond will be equal to 100. If the yield to maturity is above the coupon rate the bond price will trade 'at a discount' (below 100) and if the yield to maturity is below the coupon rate the bond price will trade 'at a premium' (above 100). See the Excel Spread Sheet solutions for numerical answers.

3. **Exercise 2.6.** The price of the 15% coupon bond is 151.23 with a YTM of 5.94%. The price of the 3% coupon bond is 84.35 with a YTM of 5.74%. The YTM of the two bonds are similar even though one bond has a 5-times large coupon rate. However, this does not imply one bond is 'better' than another.

4. **Exercise 2.7.** The discount rates are (*i*) 0.968 (6 mn) , (*ii*) 0.941 (1 yr) , (*iii*) 0.903 (1.5 yr) , (*iv*) 0.874 (2.0yr) . The no-arbitrage prices of the two bonds are 101.707 and 106.603, respectively, which are different than the traded prices. There is an arbitrage opportunity.

Class 3 – Fixed income markets and interest rate risk management

PV exercises 3.2; 3.6 and 4.8 (a)(b).

1. **Exercise 3.2:** An investor is planning a \$100 million short-term investment and is going to choose among two different portfolios. This investor is seriously worried about interest rate volatility in the market. Compute the duration of the portfolios. Which one is more adequate for the investor's objective? Assume today is May 15, 2000, which means you may use the yield curve presented in Table 2 (Table 3.6 in PV).

   Portfolio A

   - 40% invested in $4\frac{1}{4}$-year bonds paying 5% semiannually
   - 25% invested in 7-year bonds paying 2.5% semiannually
   - 20% invested in $1\frac{3}{4}$-year floating rate bonds with a 30 basis point spread, paying 2.5% semiannually (Assume that the coupon applying to the next reset date has been set at $r_2(0) = 6.4\%$.)
   - 10% invested in 1-year zero coupon bonds
   - 5% invested in 2-year bonds paying 3% quarterly

   Portfolio B

   - 40% invested in 7-year bonds paying 10% semiannually
   - 25% invested in $4\frac{1}{4}$-year bonds paying 3% quarterly
   - 20% invested in 90-day zero coupon bonds
   - 10% invested in 2-year floating rate bonds with zero spread, paying semiannually
   - 5% invested in $1\frac{1}{2}$-year bonds paying 6% semiannually

Table 2: **Yield Curve on May 15, 2000**

Maturity	Yield	Maturity	Yield	Maturity	Yield
0.25	6.33%	2.75	6.86%	5.25	6.39%
0.50	6.49%	3.00	6.83%	5.50	6.31%
0.75	6.62%	3.25	6.80%	5.75	6.24%
1.00	6.71%	3.50	6.76%	6.00	6.15%
1.25	6.79%	3.75	6.72%	6.25	6.05%
1.50	6.84%	4.00	6.67%	6.50	5.94%
1.75	6.87%	4.25	6.62%	6.75	5.81%
2.00	6.88%	4.50	6.57%	7.00	5.67%
2.25	6.89%	4.75	6.51%	7.25	5.50%
2.50	6.88%	5.00	6.45%	7.50	5.31%

2. **Exercise 3.6:** Due to a series of unfortunate events, the investor in Exercise 3.2 just found out that he must raise \$50 million. The investor must sell (short) his longest tenure bonds to raise the \$50 million. In other words, for portfolio A the investor would spend the same on all securities except for the 7 year coupon bonds (paying 2.5% semiannually) from which the investor will short enough to get to \$50 million. For portfolio B the investor would spend the same on all other securities except for the 7-year coupon bonds (paying 10% semiannually) from which the investor will short enough to get to \$50 million.

   (a) How many bonds of each kind does the investor have to short?

   (b) What is the new dollar duration of each portfolio?

   (c) Does the conclusion arrived at in Exercise 3.2 stand?

Table 3: **Level, Slope and Curvature**

	3 month	6 month	1 year	2 year	3 year	5 year	7 year	10 year
β(Level)	1.0180	0.9509	0.9196	1.0344	1.0299	1.0180	1.0111	1.0180
β(Slope)	-0.2568	-0.3252	-0.4317	-0.3507	-0.1424	0.2432	0.5205	0.7432
β(Curvature)	-0.3284	-0.1404	0.0847	0.3228	0.3240	0.1716	-0.1058	-0.3284
R^2	99.65%	99.69%	98.88%	99.61%	99.77%	99.90%	99.73%	99.90%

3. **Exercise 4.8:** In this exercise you need to describe an immunization strategy for a portfolio, given the factors in Table 3 (Table 4.8 in PV). The term structure of interest rates at two dates are in Table 4 (Table 4.9 in PV).

   (a) You are standing at February 15, 1994 and you hold the following portfolio:

   - Long \$30 million of a 6-year inverse floater with the following quarterly coupon: coupon at $t = 20\% - r_4(t - 0.25)$, where $r_4(t)$ denotes the quarterly compounded, 3-month rate.
   - Long \$30 million of a 4-year floating rate bond with a 45 basis point spread paid semiannually
   - Short \$20 million of a 3-year coupon bond paying 4% semiannually

     (i) What is the total value of the portfolio?

     (ii) Compute the dollar duration of the portfolio.

   (b) You are worried about interest rate volatility. You decide to hedge your portfolio with the following bonds:

   - A 3-month zero coupon bond
   - A 6- year zero coupon bond

     (i) How much should you go short/long on these bonds in order to make the portfolio immune to interest rate changes?

     (ii) What is the total value of the portfolio now?

   [Hint: please refer to PV Chapter 2 for the pricing of the Inverse Floater.]

Table 4: **Two Term Structure of Interest Rates**

Maturity	02/15/94 c.c. yield	02/15/94 Z(t,T)	05/13/94 c.c. yield	05/13/94 Z(t,T)
0.25	3.53%	0.9912	4.13%	0.9897
0.50	3.56%	0.9824	4.74%	0.9766
0.75	3.77%	0.9721	5.07%	0.9627
1.00	3.82%	0.9625	5.19%	0.9495
1.25	3.97%	0.9516	5.49%	0.9337
1.50	4.14%	0.9398	5.64%	0.9189
1.75	4.23%	0.9287	5.89%	0.9020
2.00	4.43%	0.9151	6.04%	0.8862
2.25	4.53%	0.9031	6.13%	0.8712
2.50	4.57%	0.8921	6.23%	0.8558
2.75	4.71%	0.8786	6.31%	0.8406
3.00	4.76%	0.8670	6.39%	0.8255
3.25	4.89%	0.8531	6.42%	0.8117
3.50	4.98%	0.8400	6.52%	0.7959
3.75	5.07%	0.8268	6.61%	0.7805
4.00	5.13%	0.8145	6.66%	0.7663
4.25	5.18%	0.8023	6.71%	0.7519
4.50	5.26%	0.7893	6.73%	0.7387
4.75	5.31%	0.7770	6.77%	0.7251
5.00	5.38%	0.7641	6.83%	0.7106
5.25	5.42%	0.7525	6.86%	0.6977
5.50	5.43%	0.7418	6.89%	0.6846
5.75	5.49%	0.7293	6.93%	0.6713
6.00	5.53%	0.7176	6.88%	0.6619

Solutions

1. **Exercise 3.2.** Portfolio A has a duration of 3.40 while portfolio B has a duration of 3.34. If the investor is worried about interest rate volatility he should choose portfolio B.

2. **Exercise 3.6.** The investor would need to sell 25 bonds from portfolio A or 10 bonds from portfolio B. Portfolio A has now has a dollar duration of 19.36 while portfolio B has a dollar duration of 62.61. If the investor is worried about interest rate volatility he would now choose portfolio A.

3. **Exercise 4.8 (a)**

Security	Price, P	\$ (mn)	N (mn)	\$$D$	\$$D \times N$
6yr IF @ 20% - fl quart	\$146.48	\$30.00	0.205	1,140.28	233.541
4yr fl 45bps semi	\$101.62	\$30.00	0.295	53.54	15.805
3yr @ 4% semi	\$97.82	(\$20.00)	-0.204	279.16	-57.076
				Port. \$$D$	**192.27**

Exercise 4.8 (b)

For each bond in the portfolio compute the factor duration (see Fact 4.4):

	Price, P	D_1	D_2	weight, w	$D_1 \times w$	$D_2 \times w$
6yr IF						
coupon	\$174.72	4.27	1.01	1.19	5.09	1.21
float	(\$100.00)	0.25	-0.06	-0.68	-0.17	0.04
fix	\$71.76	6.09	2.29	0.49	2.98	1.12
total	\$146.48			1	**7.90**	**2.38**
4yr fl						
coupon	\$1.62	2.21	-0.37	0.02	0.04	-0.01
float	\$100.00	0.48	-0.16	0.98	0.47	-0.16
total	\$101.62			1	**0.50**	**-0.17**
3yr						
total	\$97.82	2.93	-0.43	1	**2.93**	**-0.43**

In the same fashion, compute the factor durations of the Long/Long/Short portfolio:

Security	Position	P	\$ mn	w	N (mn)	D_1	D_2	$w \times D_1$	$w \times D_2$
6yr IF @ 20% - fl quart	Long	\$146.48	\$30.00	0.75	0.205	7.90	2.38	5.93	1.78
4yr fl 45bps semi	Long	\$101.62	\$30.00	0.75	0.295	0.50	-0.17	0.38	-0.12
3yr @ 4% semi	Short	\$97.82	(\$20.00)	-0.5	-0.204	2.93	-0.43	-1.47	0.22
	Total		\$40.00	1				**4.84**	**1.87**

Obtain the factor durations for the short and long zero bonds:

	Portfolio	Short Bond	Long Bond
Price	\$40	\$98.24	\$71.76
D_1	4.84	0.48	6.09
D_2	1.87	-0.16	2.29

Use expression (4.29)–(4.30) to compute the weights in each zero bond:

	$P/P_i, i = \{S, L\}$	numerator	denominator	weight
	(A)	(B)	(C)	(A)×(B)/(C)
k_S	-0.41	-0.33	2.08	0.06
k_L	-0.56	-1.68	-2.08	-0.45

The investor has to establish the following hedge:

Hedge			
Position	Security	Amount	N
Long	0.5 zero	\$6.36	0.065
Short	6yr zero	(\$32.27)	-0.450

The value of the portfolio is: $40 + 6.36 - 32.27 = \$14.08\,\text{mn}$.

c. i. \$36.82

ii. \$13.03

iii. The hedge covered part of the portfolio losses. The change in the value of the portfolio is not just due to coupon payment since the term structure has shifted as well.

	Original	Now	Δ value
Unhedged port.	\$40.00	\$36.82	(\$3.18)
Hedge	(\$25.92)	(\$23.79)	\$2.13
Total	\$14.08	\$13.03	(\$1.05)

d. The hedge would have worked better if it was not for the passage of time and the coupon effect.

i. \$37.71

ii. \$14.26

	Original	Now	Δ value
Unhedged port.	\$40.00	\$37.71	(\$2.29)
Hedge	(\$25.92)	(\$23.45)	\$2.46
Total	\$14.08	\$14.26	\$0.17

Table 5: **The Term Structure of Interest Rates on May 15, 2000**

Maturity	c.c. yield	Maturity	c.c. yield	Maturity	c.c. yield
0.25	6.17%	2.75	6.78%	5.25	6.71%
0.50	6.52%	3.00	6.76%	5.50	6.63%
0.75	6.32%	3.25	6.77%	5.75	6.69%
1.00	6.71%	3.50	6.76%	6.00	6.62%
1.25	6.76%	3.75	6.63%	6.25	6.63%
1.50	6.79%	4.00	6.77%	6.50	6.61%
1.75	6.77%	4.25	6.77%	6.75	6.58%
2.00	6.72%	4.50	6.71%	7.00	6.57%
2.25	6.72%	4.75	6.66%		
2.50	6.79%	5.00	6.70%		

Class 4 – Interest rate derivatives

PV exercises 5.3; 5.11; 6.2 and 6.10.

1. **Exercise 5.3:** Today is May 15,2000 and continuously compounded term structure of interest rate is shown in Table 5 (Table 5.7 in PV). You are faced with the two investment strategies below. Is there an arbitrage opportunity?
   - Invest \$100 million in 2.5-year zero coupon bonds.
   - Invest \$100 million in a 1-year zero coupon bond and agree with the bank to invest the proceeds for the following 1.5 years at the quoted forward rate, which is: $f_2(1, 1, 2.5) = 7.56\%$ (assume also continuously compounded).

2. **Exercise 5.11:** As of December 2, 2008, the 30-year swap spread had been negative for a whole month. In particular, on that day, the 3-month repo rate was 0.5%, the LIBOR rate was 2.21%, the 30-year swap rate was 2.85%, and the semi-annually compounded yield-to-maturity of the 4.5% Treasury Bond maturing on May 15, 2038 was 3.18%.
   (a) Is there an arbitrage? Discuss the swap spread trade that you would set up to take advantage of these rates.
   (b) Assuming that the U.S. government is less likely to default than swap dealers, how can you rationalize these rates? What risks would setting up this trade involve? Discuss. (Recall that there was an ongoing credit crisis).

3. **Exercise 6.2:** On March 21, 2007 a firm enters into 100 90-day Euro dollar futures contracts (contract size is $\$1,000,000$). The quoted futures price on this day is : \$93.695. Over the life of the contracts, prices move as in Table 6 (Table 6.10 in PV).
   (a) What will be the daily P&L from futures for the quoted dates be?
   (b) What will the cumulative P&L for the quoted dates be?
   (c) You find out that 90-day Eurodollar futures are subject to the following requirements: Initial Margin: $\$1,485$ (per contract); Maintenance Margin: $\$1,110$ (per contract).
      (i) What will the cash flow of the contract be, assuming that the firm never takes money from the margin account?
      (ii) What will the cash flow of the contract be if the firm decides to take every profit from the contract instead of leaving it in the margin account?

Table 6: **Eurodollar Futures**

Date	Price	Date	Price	Date	Price	Date	Price
21-Mar-07	93.6350	7-May-07	93.1850	21-Jun-07	93.0950	7-Aug-07	93.1500
22-Mar-07	93.6850	8-May-07	93.1750	22-Jun-07	93.2100	8-Aug-07	93.2600
23-Mar-07	93.5700	9-May-07	93.2450	25-Jun-07	93.1900	9-Aug-07	93.2050
26-Mar-07	93.5000	10-May-07	93.0950	26-Jun-07	93.0800	10-Aug-07	93.2500
27-Mar-07	93.3200	11-May-07	92.8250	27-Jun-07	93.0900	13-Aug-07	93.3400
28-Mar-07	93.3300	14-May-07	92.9350	28-Jun-07	93.0150	14-Aug-07	93.3350
29-Mar-07	93.3300	15-May-07	92.8700	29-Jun-07	92.9450	15-Aug-07	93.3350
30-Mar-07	93.3750	16-May-07	92.8950	2-Jul-07	92.9650	16-Aug-07	93.3700
2-Apr-07	93.3400	17-May-07	93.0100	3-Jul-07	92.9750	17-Aug-07	93.4450
3-Apr-07	93.3550	18-May-07	93.0900	5-Jul-07	92.9300	20-Aug-07	93.3800
4-Apr-07	93.3800	21-May-07	93.1200	6-Jul-07	92.9500	21-Aug-07	93.4150
5-Apr-07	93.3400	22-May-07	93.1200	9-Jul-07	92.9950	22-Aug-07	93.4050
6-Apr-07	93.4550	23-May-07	93.0500	10-Jul-07	93.0350	23-Aug-07	93.4400
9-Apr-07	93.3950	24-May-07	92.9250	11-Jul-07	93.0250	24-Aug-07	93.3900
10-Apr-07	93.2450	25-May-07	92.9150	12-Jul-07	93.0650	27-Aug-07	93.3850
11-Apr-07	93.2400	29-May-07	92.8950	13-Jul-07	93.1200	28-Aug-07	93.4800
12-Apr-07	93.1750	30-May-07	92.8650	16-Jul-07	93.1850	29-Aug-07	93.6200
16-Apr-07	93.0600	31-May-07	93.0100	17-Jul-07	93.1850	30-Aug-07	93.5650
17-Apr-07	93.1700	1-Jun-07	93.0600	18-Jul-07	93.2650	31-Aug-07	93.5100
18-Apr-07	93.2650	4-Jun-07	93.1200	19-Jul-07	93.2650	4-Sep-07	93.3450
19-Apr-07	93.0800	5-Jun-07	93.1950	20-Jul-07	93.2600	5-Sep-07	93.3700
20-Apr-07	93.0600	6-Jun-07	93.2250	23-Jul-07	93.2950	6-Sep-07	93.5100
23-Apr-07	93.1150	7-Jun-07	93.1400	24-Jul-07	93.2900	7-Sep-07	93.5150
24-Apr-07	93.1000	8-Jun-07	93.0650	25-Jul-07	93.1900	10-Sep-07	93.4500
25-Apr-07	93.0200	11-Jun-07	93.1400	26-Jul-07	93.1700	11-Sep-07	93.5050
26-Apr-07	93.1350	12-Jun-07	93.1850	27-Jul-07	93.2350	13-Sep-07	93.4900
27-Apr-07	93.0150	13-Jun-07	93.1500	30-Jul-07	93.2300	14-Sep-07	93.6200
30-Apr-07	93.0250	14-Jun-07	93.1450	31-Jul-07	93.3100	17-Sep-07	93.5400
1-May-07	93.0850	15-Jun-07	93.0200	1-Aug-07	93.2650	18-Sep-07	93.4200
2-May-07	93.1350	18-Jun-07	92.9950	2-Aug-07	93.1900	19-Sep-07	93.4850
3-May-07	93.2500	19-Jun-07	93.0100	3-Aug-07	93.1400	20-Sep-07	93.4950
4-May-07	93.2500	20-Jun-07	93.0600	6-Aug-07	93.1600	21-Sep-07	93.5900

4. **Exercise 6.10:** Today is $t = 0$. You are given the following data:

   - The 6-month zero coupon bond is priced at $98.24
   - The 9-month zero coupon bond is priced at $97.21
   - Call option(European) on the 13 week Treasury bill with maturity in 6-months and strike price of $99.12 is priced at$0.2934
   - Put option (European) on the 13 week Treasury bill with maturity in 6-months and strike price of $99.12 is priced at $0.1044

   (a) Are the securities priced correctly?

   (b) Assume that someone tells you that she is 100% sure that the call option is priced correctly. Can you design a strategy to take advantage of the arbitrage opportunity, if there is one?

Solutions

1. **Exercise 5.3**

   Compute the forward discount factor:

$$F(0, 1, 2.5) = Z(0, 2.5)/Z(0, 1) = 0.9024 \tag{22}$$

   Obtain the forward rate consistent with the current yield curve:

$$f_2(0, 1, 2.5) = 2 \times \left[\frac{1}{F(0, 1, 2.5)^{\frac{1}{1.5\times 2}}} - 1 \right] = 6.96\% \tag{23}$$

   The current forward rate on the market is lower than the one offered by the bank at 7.56%. There is an arbitrage opportunity.

2. Question 5.11

   On that day, the net spread was strongly negative:

$$\begin{aligned} SS &= c - ytm = -0.33\% \\ LRS &= \text{LIBOR} - \text{repo} = 1.71\% \\ SS - LRS &= -2.04\% \end{aligned}$$

   You could envision the following strategy to exploit the large negative spread:

   1. Buy Treasury through a repo transaction: get coupon, pay repo
   2. Enter the floating-for-fixed swap: pay fixed, get LIBOR

3. **Exercise 6.2**

   See excel spreadsheet for numerical details.

   a. For each day, compute the P&L as:

$$\text{Daily P\&L} = \$1\text{mn} \times 0.25 \times \frac{P^{fut}(t+dt,T) - P^{fut}(t,T)}{100} \times 100 \text{ contracts}$$

   b. Total P&L = $\$1\text{mn} \times 0.25 \times (6.37\% - 6.41\%) \times 100$ contracts $= -\$11\,250$ mn

   c. In both situations i. and ii. the cash-flow is -$112.50 per contract.

4. **Exercise 6.10**

   a. The securities are not correctly priced, as the put-call parity is violated:

Put	$0.1044
Call	$0.2934
$P^{fwd}(0, 0.5, 0.75)$	98.96
K	99.12
$Z(0, 0.5) \times (P^{fwd} - K)$	-0.1590
Call from P/C parity	($0.0546)

   b. Strategy long call, short put, short forward gives a positive cash-flow of $0.3480 at no risk.

Table 7: **A One-Step Binomial Interest Rate Tree**

period ⇒	$i = 0$	$i = 1$
time(in years)⇒	$t = 0$	$t = 0.5$
	$r_0 = 2\%$	$r_{1,u} = 4\%$ with prob.$p = 1/2$
		$r_{1,d} = 1\%$ with prob.$1 - p = 1/2$

Table 8: **A One-Period Risk Neutral Interest Rate Tree**

period ⇒	$i = 0$	$i = 1$
time(in years)⇒	$t = 0$	$t = 0.5$
	$r_0 = 4\%$	$r_{1,u} = 6\%$ with prob.$p = 1/2$
		$r_{1,d} = 3\%$ with prob.$1 - p = 1/2$

Class 5 – One-step trees

PV exercises 9.1 and 9.3.

1. **Exercise 9.1:** Consider the interest rate tree in Table 7 (Table 9.9 in PV).
   (a) Compute the expected 6-month Treasury rate $E[r_1]$.
   (b) The 1-year Treasury bill is trading at $P_0(2) = 97.4845$. What is the (continuously compounded) forward rate for the periods $i = 1$ to $i = 2$? How does it compare with the expected rate computed in Part (a)? Explain.
   (c) Compute the market price of risk λ. Interpret.
   (d) Compute the risk neutral probability p^*. Interpret.

2. **Exercise 9.3:** Consider the tree in Table 8 (Table 9.10 in PV). You estimated the risk neutral probability to move up the tree to be $p^* = 1/2$.
   (a) Compute the value of the zero coupon bonds maturing at time $i = 1$ and at $i = 2$.
   (b) Compute the continuously compounded yields for both bonds.
   (c) Compute the value of an option with payoff Option Payoff at $1 = 100 \times max(r_1 - 4\%, 0)$
   (d) Set up the replicating portfolio that uses the bond prices determined in Part(a), that is able to replicate the option's payoff. Check that this portfolio in fact replicates the option.
   (e) Given the tree for the option, set up a replicating portfolio made of the short-term bond and the option that is able to replicate the prices of the long term bond at time 1, that is $P_{1,u}(2)$ and $P_{1,d}(2)$.

Solutions

1. **Question 9.1**

    - a. The expected return is equal to 2.5%.

    - b. The forward rate (continosly compounded) is equal to3.0954%. This is lower than the expected rate computed in Part (a). If we observe high forward rates it may be because of two possibilities: either market participants expect higher future interest rates; or they are strongly averse to risk, and thus the price of long term bonds is low today.

    - c. The market price of risk equals: $\lambda = -0.1980$. The high (negative) market price of risk, means that market participants have high risk aversion, which may explain the price of long term bonds today.

    - d. The risk neutral probability equals: $p^* = 0.7000$. The interpretation is the same as in Part (c).

2. **Question 9.3**

a. The value of the zero coupon bond maturing at $i = 1$ is 98.0199. The value of the zero coupon bond maturing at $i = 2$ is 95.8417.

b. The contiously compounded yield (y_i) for each bond is: $y_1 = 0.0400$ and $y_2 = 0.0425$.

c. The value of the option is $V_0 = 0.9802$.

d. The replicating portfolio holds (N_1) 1.3434 of the bond maturing at $i = 1$, and (N_2) -1.3637 of the bond maturing at $i = 2$. The following table summarizes this:

	Price	Position	Total
$P_z(0,1)$	98.0199	1.3434	131.6758
$P_z(0,2)$	95.8417	-1.3637	-130.6956
		Portfolio	0.9802

Of course the value of the replicating portfolio is the same as the option. When interest rates go up from 4% to 6% we have that:

	Price	Position	Total
$P_z(1,1)$	100.00	1.3434	134.34
$P_z(1,2)$	97.0446	-1.3637	-132.34
		Portfolio	2.00

When interest rates go down from 4% to 3% we have that:

	Price	Position	Total
$P_z(1,1)$	100.00	1.3434	134.34
$P_z(1,2)$	98.5112	-1.3637	-134.34
		Portfolio	0.00

Where both scenarios replicate the payoffs of the option.

e. Using the short-term bond and the option we have:

	Price	Position	Total
$P_z(0,1)$	98.0199	0.9851	95.5605
$P_z(0,2)$	0.9802	-0.7333	-0.7188
		Portfolio	95.8417

Where 95.8417 is, as seen before, the price of the bond.

Class 6 – Revision for midterm exam

Class 7 – Multi-step trees

Formative assignment: can be handed in for feedback at the beginning of the class
PV exercises 10.1 and 11.1 (a)(b).

1. **Exercise 10.1:** Using the past history of short-term interest rates, you estimated by regression the model $r_{t+dt} = \alpha + \beta r_t + u_{t+dt}$. Suppose that the parameter estimates generated the tree for interest rates in Table 9 (Table 10.16 in PV), where there is equal probability to move up or down the tree. Assume also for simplicity that each interval of time represents 1 year, that is, $\triangle = 1$. Finally, assume that the current zero coupon bond expiring at time $i = 2$ has a price equal to $Z_0(2) = 0.9$.

Table 9: **An Interest Rate Tree**

$i = 0$	$i = 1$	$i = 2$
$r_0 = 0.04$	$r_{1,u} = 0.07$	$r_{2,uu} = 0.1$
		$r_{2,ud} = 0.05$
	$r_{1,d} = 0.03$	$r_{2,du} = 0.05$
		$r_{2,dd} = 0.02$

   (a) How does the 2 year bond $Z_0(2)$ evolve? Compute $Z_{1,u}(2)$ and $Z_{1,d}(2)$, and draw the tree for the bond that expires at time $i = 2$ [recall the notation: $Z_{i,j}(k)$ is the bond at time i in node j with maturity date k.]

   (b) Use the calculation in Part(a) to compute the market price of risk λ embedded in the current 2-year zero coupon bond $Z_0(2)$.

   (c) Consider an option with maturity $T = 1$ to buy (at T=1) one unit of a 1-year zero coupon bond at the price of $K = 95$.

      (i) What is the market price of risk of this option? Why?

      (ii) Use your calculation is part i to compute the value of the option.

      (iii) Confirm your calculation by using the risk neutral approach.

   (d) Assume that the risk neutral probabilities computed above are constant over time. Compute the price of a 2-year European call option on a 1-year zero coupon bond, with strike price $K = 96$.

   (e) Compute the replicating portfolio that replicates the option payoff in Part(d). Check that the portfolio indeed replicates the payoff. Discuss the intuition.

2. **Exercise 11.1:** You have estimated the risk neutral model for the continuously compounded interest rate as in Table 10 (Table 11.21 in PV). There is equal probability to move up or down the tree and each interval time represents 1 year, that is, $\triangle = 1$.

Table 10: **An Interest Rate Tree**

$i = 0$	$i = 1$	$i = 2$
$r_0 = 0.04$	$r_{1,u} = 0.0868$	$r_{2,uu} = 0.1299$
		$r_{2,ud} = 0.0723$
	$r_{1,d} = 0.0268$	$r_{2,du} = 0.0723$
		$r_{2,dd} = 0.0147$

   (a) Compute the current ($i = 0$) zero coupon spot curve, for all possible maturities;

(b) Compute the price of a security that pays \$100 at time $i = 2$ if the interest rate at that time is 0.0723 and zero otherwise. That is,

$$CF(2) = \begin{cases} 100 & \text{if r(2) = 0.0723} \\ 0 & \text{otherwise} \end{cases}$$

Solutions

1. **Exercise 10.1**

   a. The bond evolves in the following way:

90.0000	93.2394	100.000
	97.0446	100.000
		100.000

   b. The market price of risk is: λ = -0.3709

   c. The anwers are the following:

      i. The market price of risk is: λ = -0.3709. Which is the same as the one computed from the bond, this is because the market price of risk is the same across securities since it measures the willingness of agents to hold risky assets. In other words it is a measure on the economic agents and not on the instruments themselves.

      ii. The price of the option is 0.2238.

      iii. The result holds the same.

   d. The price of the option is 0.0245.

   e. The option has the following structure over time:

0.0245	0.0000	0.0000
	0.2234	0.0000
		2.0199

   The positions on the short-term bond (N_1) and on the three-period bond (N_2) vary over time, in order to replicate the option structure, in the following way:

N_1		N_2	
-0.0244	0.0000	0.0287	0.0000
	-0.6632		0.6972

2. **Exercise 11.1**

- a. The zero coupon spot curve for all possible maturities is:
  - Z(0,1) = 0.9608
  - Z(0,2) = 0.9081
  - Z(0,3) = 0.8462
- b. The price of the security is: 45.4073.

Table 11: **Zero Coupon Bond Prices on January 8, 2002**

Maturity (years)	Price	Yield
0.5	99.13377	1.74
1	97.89252	2.13
1.5	96.14622	2.62
2	94.10114	3.04
2.5	91.71355	3.46
3	89.2258	3.8
3.5	86.8142	4.04
4	84.50158	4.21
4.5	82.18478	4.36
5	79.77181	4.52
5.5	77.4339	4.65

Class 8 – Risk neutral trees

We'll start with a simple BDT binomial tree model with 5 years, example 11.2 in PV. PV exercise 11.5.

1. **Example 11.2:** Consider the term structure of interest rates on January 8, 2002. The term structure of interest rates and the zero coupon bonds are given in Table 11 (Table 10.11 in PV). In the data the zero coupon bond expiring on date $k = 1$ is $P_0(1) = 99.1338$, implying $r_0 = 1.74\%$, which is the root of the tree. Fit the term structure of interest rates with simple BDT model with 5 years.

2. **Exercise 11.5:** Let today be November 3, 2008.

   (a) Use the LIBOR rate and the swap data on November 3, 2008 in Table 12 (Table 11.26 in PV) and fit the LIBOR curve.

   (b) From the LIBOR discount curve, fit the Ho-Lee model of the interest rates, with quarterly steps. You can use the LIBOR volatility reported in the text, or estimate the LIBOR volatility yourself. Data on LIBOR are available on the British Bankers Association Web site(www.baa.org.uk).

   (c) Compare risk neutral expected future interest rates to the continuously compounded interest rates. How does the difference depend on the assumed volatility of the interest rate? (Hint: for each assumed volatility of the interest rate, you need to refit the tree to make sure that the tree correctly reflects the forward rates. Do the exercise for 3 values of volatility).

   (d) Compute the value of 1-year, 2-year and 3-year cap. Compare your value with the one in the data, in Table 12 (Table 11.26 in PV).

   (e) Compute the value of a 5-year swap (the swap rate in Table 12 (Table 11.26 in PV)) with quarterly payments (i.e., assume that both floating and fixed payers pay at quarterly frequency). Is the value of the swap obtained from the tree what you would expect from first principles?

   (f) Use the swap tree computed in Part(e) to compute the value of 1-year, at-the-money swaption to enter into a 4-year swap.

Table 12: **Swap Rates and Cap Prices on November 3, 2008**

3 Month LIBOR(%)	2.8588	
Maturity	Swap Rates(%)	Cap Price (×100)
0.50	2.6486	0.0528
0.75	2.4929	0.1313
1.00	2.4320	0.2401
1.25	2.4491	0.3826
1.50	2.4938	0.5405
1.75	2.5561	0.7106
2.00	2.6260	0.8932
2.25	2.7252	1.1095
2.50	2.8630	1.3729
2.75	3.0108	1.6636
3.00	3.1400	1.9502
3.25	3.2471	2.2235
3.50	3.3474	2.4973
3.75	3.4408	2.7711
4.00	3.5270	3.0451
4.25	3.6076	3.3208
4.50	3.6835	3.5968
4.75	3.7531	3.8700
5.00	3.8150	4.1370

Solutions.

1. **Exercise 11.2** See Excel Spreadsheet.

2. **Exercise 11.5**

a. The zeros from the LIBOR curve are the following:

t	$P_z(0,t)$	t	$P_z(0,t)$	t	$P_z(0,t)$
0.25	99.2904	2.75	92.0454	5.25	81.4258
0.50	98.6053	3.00	90.9898	5.50	80.3754
0.75	98.1544	3.25	89.9522	5.75	79.3266
1.00	97.6061	3.50	88.8979	6.00	78.2843
1.25	96.9954	3.75	87.8326	6.25	77.2529
1.50	96.3399	4.00	86.7628	6.50	76.2365
1.75	95.6373	4.25	85.6875	6.75	75.2401
2.00	94.8950	4.50	84.6072	7.00	74.2689
2.25	94.0613	4.75	83.5326	7.25	73.3133
2.50	93.0947	5.00	82.4744	7.50	72.3608

b. The Ho-Lee model in the first 2 years ($i = 8$):

$i = 0$	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
2.85%	2.82%	2.96%	3.42%	4.08%	4.67%	5.28%	5.87%	6.67%
	2.04%	2.17%	2.63%	3.30%	3.89%	4.50%	5.08%	5.89%
		1.39%	1.85%	2.51%	3.11%	3.72%	4.30%	5.11%
			1.07%	1.73%	2.33%	2.93%	3.52%	4.32%
				0.95%	1.54%	2.15%	2.73%	3.54%
					0.76%	1.37%	1.95%	2.76%
						0.59%	1.17%	1.98%
							0.39%	1.19%
								0.41%

c. The following table compares the risk neutral expected future interest rate with the forward rate.

	$\sigma = 0.0078$		$\sigma = 0.05$		$\sigma = 0.001$	
t	$E^*[r_t]$	f(t-1,t)	$E^*[r_t]$	f(t-1,t)	$E^*[r_t]$	f(t-1,t)
0	2.849%	2.849%	2.849%	2.849%	2.849%	2.849%
0.25	2.430%	2.430%	2.438%	2.430%	2.430%	2.430%
0.5	2.174%	2.173%	2.204%	2.173%	2.173%	2.173%
0.75	2.243%	2.241%	2.311%	2.241%	2.241%	2.241%
1	2.514%	2.511%	2.636%	2.511%	2.511%	2.511%
1.25	2.717%	2.712%	2.908%	2.712%	2.712%	2.712%
1.5	2.935%	2.928%	3.209%	2.928%	2.928%	2.928%
1.75	3.126%	3.117%	3.499%	3.117%	3.117%	3.117%
2	3.542%	3.530%	4.029%	3.530%	3.530%	3.530%
2.25	4.147%	4.132%	4.764%	4.132%	4.132%	4.132%
2.5	4.553%	4.534%	5.315%	4.534%	4.534%	4.534%
2.75	4.637%	4.614%	5.558%	4.614%	4.614%	4.614%

Note that as volatility increases, so does the difference between the rates.

d. The value of the 1-year cap is 0.2374, the value of the 2-year cap is 0.6003, and the value of the 3-year cap is 1.3805. In general the model underestimates the price of the securities.

e. The value of the swap is zero, as expected.

f. The value of the swaption is: 1.9956.

Class 9 – American options

Formative assignment: can be handed in for feedback at the beginning of the class
PV exercises 12.1 (a) and 12.2.

1. **Exercise 12.1:** Suppose that you estimated the risk neutral tree for interest rates in Table 13 (Table 12.11 in PV), where there is equal risk neutral probability to move up or down the tree. Assume also for simplicity that each interval of time represents 1-year, that is, $\triangle = 1$.

Table 13: **An Interest Rate Tree**

$i = 0$	$i = 1$	$i = 2$
	$r_{1,u} = 5.5\%$	$r_{2,uu} = 6\%$
		$r_{2,ud} = 5\%$
$r_0 = 5\%$		
	$r_{1,d} = 4.7\%$	$r_{2,du} = 5\%$
		$r_{2,dd} = 4.5\%$

   (a) Compute the value of a non-callable and callable bonds, with principal= 100, maturity $i = 3$, and annual coupon rate = 5.25%. Which of the two bonds is more expensive for investors? Why?

2. **Exercise 12.2:** Consider again the risk neutral tree for interest rates in Table 13 (Table 12.11 in PV), where there is equal risk neutral probability to move up or down the tree.

   (a) Compute the tree for an American swaption with maturity $i = 3$ and strike rate $c = 5.25\%$.

   (b) Consider now a callable bond with maturity $i = 3$, principal= 100, and annual coupon rate = 5.25%.

   (c) An investor is long the callable bond priced in the previous exercise, and is worried about prepayment risk. Can you suggest how the investor can employ an American swaption to hedge against prepayment risk? Assume the investor is not worried about the interest rate risk.

   (d) Assume that the investor in the callable bond is instead worried about interest rate risk. Can you suggest a hedging strategy that would cover the investor against changes in the interest rates?

Solutions

1. **Exercise 12.1.** The value of the non-callable bond is 100.13 and the value of the callable bond is 99.74. The non-callable bond is more expensive because it doesn't have the prepayment risk that the callable bond has.

2. **Exercise 12.2.**

   - a. The value of the American swaption is 0.2101.
   - b. The value of the callable bond is 99.74.
   - c. In order to only hedge the prepayment risk you hedge the underlying option, instead of the callable bond itself. The hedging strategy is the following:

$i =$	0	1
Short-term bond (N1)	0.0077	0.0012
American swaption (N2)	-1.6144	-0.1252

     Note that there is no hedging strategy for node (1,1) since the option is retired at that point.

   - d. To hedge the interest rate risk (which includes prepayment risk) of the callable bond the investor can use the following strategy:

$i =$	0	1
Short-term bond (N1)	1.0525	1.0525
American swaption (N2)	-1.8073	-0.9418

     As in the previous part, there is no hedging strategy for node (1,1).

Class 10 – Basics of RMBS

PV exercises 8.2 and 8.6.

1. **Exercise 8.2:** Consider the following MBS pass through with principal \$300 million. The original mortgage pool has a WAM=360 months(30 years) and a WAC=7%. The pass through security pays a coupon equal to 6.5% Instead of the yield curve, you are given the following parameters from the extended Nelson Siegel model: $\theta_0 = 6,278.30$, $\theta_1 = -6,278.25$, $\theta_2 = -6,292.47$, $\theta_3 = -0.04387$, $\lambda_1 = 27,056.49$, and $\lambda_2 = 30.48$. That is, to compute the continuously compounded zero coupon yield with maturity T, the formula is $r(0,T) = \theta_0 + (\theta_1 + \theta_2)\frac{1-e^{-\frac{T}{\lambda_1}}}{\frac{T}{\lambda_1}} - \theta_2 e^{-\frac{T}{\lambda_1}} + \theta_3(\frac{1-e^{-\frac{T}{\lambda_2}}}{\frac{T}{\lambda_2}} - e^{-\frac{T}{\lambda_2}})$. The discount with maturity T is then $Z(0,T) = e^{-r(0,T)\times T}$.

   [The Nelson Siegel model postulates a parametric functional form for the discount factor $Z(0,T)$ as a function of maturity T and uses the current bond prices to estimate the parameters of this functional form. The discount factor is posited to be given by $Z(0,T) = e^{-r(0,T)\times T}$ where the continuously compounded yield with maturity T is given by $r(0,T) = \theta_0 + (\theta_1 + \theta_2)\frac{1-e^{-\frac{T}{\lambda}}}{\frac{T}{\lambda}} - \theta_2 e^{-\frac{T}{\lambda}}$, where $\theta_0, \theta_1, \theta_2$ and λ are parameters to be estimated from the current bond data. The Nelson Siegel model works well, but it lacks the flexibility to match term structures that are highly nonlinear. The economist Lars Svensson proposed an extension to the model, the extended Nelson Siegel model, which captures severe non-linearities in the shape of term structure of interest rates. Please see Chapter 2 for more details.]

   (a) What is the price of the pass through? Assume a constant PSA = 150%.

   (b) Compute the duration of this security assuming that the PSA remains constant at 150%.

   (c) Compute the effective duration of this security assuming that the PSA increases to 200% if the term structure shifts down by 50 basis points, while it decreases to 120% if the term structure shifts up by 50 basis points. Comment on any difference compared to your result in part (b).

   (d) Compute the effective convexity of this security under the same PSA assumptions as in part(c). Interpret your results.

2. **Exercise 8.6:** Consider the following MBS pass through with principal \$300 million. The original mortgage pool has a WAM = 360 months (30 years) and a WAC= 7.00%. The pass through security pays a coupon equal to 6.5%. Use the same spot rate $r(0,T) =$ as computed in 8.2 for your calculations. The pass through is divided into a Principal Only tranche and an Interest Only tranche.

   (a) What is the price of each tranche? Assume a constant PSA= 150%.

   (b) Compute the effective duration of the IO and PO tranches assuming that the PSA increases to 200% if the term structure shifts down by 50 basis points, while it decreases to 120% if the term structure shifts up by 50 basis points. Which tranche is more sensitive to interest rate movements? Which tranche is less sensitive?

   (c) Compute the effective convexity of the IO and PO tranches under the same PSA assumptions as in Part(b). Interpret your results.

   (d) If you decide to buy all the tranches, is this the same as holding the MBS pass through from 8.2 (e.g., Does it have the same price? Same duration?)

Solutions

1. **Exercise 8.2.** Follow the lines of Example 8.2 and Table 6.3. Plug in the NS parameters to compute the discount curve.

    - a. The price is 316.39 mn, and is above the 300 mn par value of the security.
    - b. Under the (unrealistic) assumption of constant PSA , you can apply the definition of duration in Chapter 3, and compute it in a standard way (see Section 3.2.3). The duration is 6.47.
    - c.,d. Compute the prices of the pass through under the different scenarios taking into account the parallel shift in the curve and the change in the PSA. Use definitions 8.1 and 8.2 to obtain the effective duration and convexity, respectively.

Prices under the three scenarios:	
P($dr = 0$bps, PSA=150%)	316.39
P($dr = +50$bps, PSA=120%)	306.68
P($dr = -50$bps, PSA=200%)	323.89

Effective Duration	5.44
Effective Convexity	-278.44

The effective duration is lower than the one obtained under the assumption that the change in rates does not affect the PSA. Standard duration overstates the sensitivity of the MBS price to changes in interest rates. In contrast to the Treasury bonds, the convexity of an MBS is negative (i.e. the value profile is concave with respect to interest rate changes). Therefore, convexity presents a source of risk to investors. This risk is associated with the prepayment option that homeowners have. Effectively, an MBS investor is short an American call option to the homeowners.

2. **Exercise 8.6**

a.	"Price"	Duration
Interest Only (IO)	127.56	5.54
Principal Only (PO)	188.82	7.09
Total	316.39	6.47

b.,c.	IO	PO
Effective Duration	-16.66	20.37
Effective Convexity	-1678.04	667.08

Prices under the three scenarios		
P(dr = 0bps, PSA=150%)	127.56	188.82
P(dr = +50bps, PSA=120%)	135.51	171.17
P(dr = −50bps, PSA=200%)	114.26	209.63

Value of the IO drops when interest rates decline and the PSA increases. The PO must increase dramatically, to counterbalance this effect and to ensure the higher total value of the pass through. Therefore, IO has a negative duration, while PO has a positive duration. In general, PO is a bullish instrument that benefits from falling interest rates. IO, instead, it a bearish security that benefits from raising interest rates. In the falling interest rate environment, the PO (IO) has a negative (positive) convexity. In the increasing rate environment, these properties reverse. In either case, the convexity, has a dampening effect in that it reduced the profits and limits the losses on each security.

d. Buying all tranches is equivalent to holding the MBS.

Class 11 – Federal funds rate

PV exercises 7.1 and 7.2.

1. **Exercise 7.1:** In this chapter we estimated two models to predict the Fed funds rate

$$
\begin{aligned}
r^{FF}(t) =& \alpha + \beta_1 r^{FF}(t-1) + \beta_2 X^{Pay}(t-1) + \beta_3 X^{Inf}(t-1) \\
r^{FF}(t) =& \alpha + \beta_1 r^{FF}(t-1) + \beta_2 X^{Pay}(t-1) + \beta_3 X^{Inf}(t-1) + \beta_4 X^{Fut}(t, t-1)
\end{aligned}
$$

   (a) Use the estimates in Table 14 (Table 7.6 in PV) to perform a one-month-ahead prediction of the Fed funds target rate using the data in Table 15 (Table 7.7 in PV). That is, for instance, using the entries on Dec-06, compute the predicted Fed funds rate on Jan-07. Similarly, using the data on Jan-07 to predict the Fed Funds rate on Feb-07. And so on. Perform the exercise using both models.

Table 14: **Parameter Estimates Of Model 1 and Model 2**

	α	β_1	β_2	β_3	β_4
Model 1	-0.000196	0.9594	0.0731	0.0326	
Model 2	-0.000112	0.1395	0.0242	0.0062	0.8454

   (b) The first column in Table 15 (Table 7.7 in PV) provides the actual ex-post Fed Funds target rate. Plot both models predictions of the Fed funds rate and the actual values. How close are the estimates?

   (c) Compute the sum of squared errors for both models. Based on this calculation, which model seems to be more accurate?

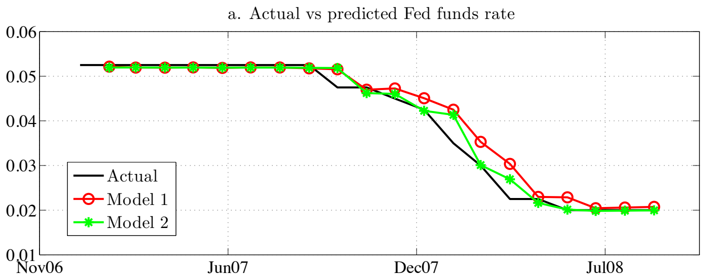
Table 15: **Fed Funds Target Prediction**

date	FF (t-1)	Payroll growth	Inflation	Futures
Dec-06	5.25%	0.08%	2.08%	5.24%
Jan-07	5.25%	0.07%	2.44%	5.25%
Feb-07	5.25%	0.13%	2.75%	5.25%
Mar-07	5.25%	0.06%	2.57%	5.25%
Apr-07	5.25%	0.11%	2.68%	5.25%
May-07	5.25%	0.10%	2.65%	5.25%
Jun-07	5.25%	0.07%	2.37%	5.25%
Jul-07	5.25%	0.00%	1.94%	5.25%
Aug-07	5.25%	0.08%	2.76%	5.25%
Sep-07	4.75%	0.12%	3.54%	4.66%
Oct-07	4.75%	0.07%	4.37%	4.63%
Nov-07	4.50%	0.01%	4.12%	4.22%
Dec-07	4.25%	-0.01%	4.40%	4.16%
Jan-08	3.50%	-0.05%	4.12%	2.96%
Feb-08	3.00%	-0.06%	4.00%	2.67%
Mar-08	2.25%	-0.01%	3.88%	2.17%
Apr-08	2.25%	-0.04%	4.08%	2.00%
May-08	2.00%	-0.05%	4.90%	2.01%
Jun-08	2.00%	-0.04%	5.52%	2.01%
Jul-08	2.00%	-0.06%	5.36%	2.02%
Aug-08	2.00%	-0.12%	4.94%	2.02%
Sep-08	2.00%			

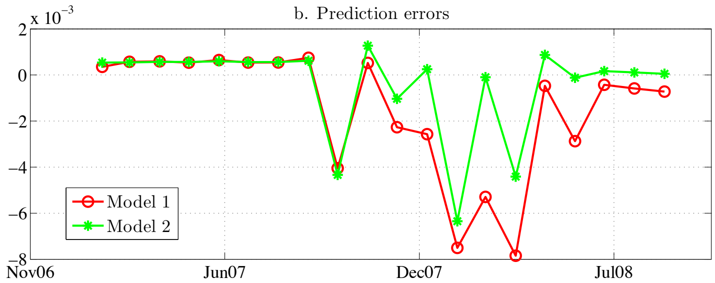
Solutions

1. **Exercise 7.1**

- a.



- b.



- c. Sum of squared errors is lower for the second model:

  Model 1: $SSR1 \times 104 = 1.8692$
  Model 2: $SSR2 \times 104 = 0.8644$.

Class 12 – Revision for final exam