FM225: Fixed Income Securities, Debt Markets and the Macro Economy

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Class 12: FSE Revision

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One-Step Trees

- 3 recipes to price a security/derivative
  - Replication (one period! Without intermediate CF!)

$$N_1 \times 100 + N_2 \times P_{1,u}(2) = V_{1,u}(1)$$

$$N_1 \times 100 + N_2 \times P_{1,d}(2) = V_{1,d}(1)$$

  - Risk premium & market price of risk

$$\lambda_0 = \frac{e^{-r_0 \times \Delta} \times \mathbb{E}[P_1(2)] - P_0(2)}{P_{1,u}(2) - P_{1,d}(2)} = \frac{e^{-r_0 \times \Delta} \times \mathbb{E}[V_1(2)] - V_0(2)}{V_{1,u}(2) - V_{1,d}(2)}$$

  - Risk-neutral pricing

$$p^* = \frac{e^{r_0 \times \Delta} P_0(2) - P_{1,d}(2)}{P_{1,u}(2) - P_{1,d}(2)}$$

Multi-Step Trees

- Check tree structures first! (parameters, combining tree?)
- Risk-neutral probability is not always 0.5. It should be larger than the real probability (risk-averse, bad states)
- Coupon bond, **cap, floor, swap,** European option

$$CF_{i,j}(i+1) = \Delta \times N \times \max[r_n(i,j) - r_k, 0]$$

$$CF_{i,j}(i+1) = \Delta \times N \times \max[r_k - r_n(i,j), 0]$$

$$CF_{i,j}(i+1) = \Delta \times N \times [r_n(i,j) - c]$$

New Product Pricing

- Step 1: Understand the payoff (reference rate should have the same compounding frequency as the payment frequency)

$$r_n(i,j) = n \times (e^{r_{i,j} \times \Delta} - 1)$$

- Step 2: Draw a CF tree
- Step 3: Draw a pricing tree

$$P_{i,j} = e^{-r_{i,j} \times \Delta} \times \left[\frac{1}{2} P_{i+1,j} + \frac{1}{2} P_{i+1,j+1} + CF_{i,j}(i+1) \right]$$

Important Notes

- FV (Notional) = 100
- Compounding frequency & discount factors
- Time intervals ($n = 1/\Delta$) !!!
- Price notations
- CF time (one period after)
- No dynamic replication, no American option, no swaption, no callable bond (but negative convexity!!!)

Forward Rates & Risk-Neutral Expectation

- Forward rates are not equal to the expectation of future interest rates $f(0, 0.5, 1) > \mathbb{E}[r_1]$. High forward rates may be because of two possibilities:
  - Market participants expect higher future interest rates;
  - Market participants are strongly averse to risk, and thus the price of long-term bonds is low today.
- The forward rate is not even equal to the risk neutral expected future interest rate $\mathbb{E}^*[r_1]$ because of non-linearity.
  - They are very close;
  - The divergence between rates is due to a missing convexity adjustment.

Comparison between HL & BDT Models

- Ho-Lee Model:
  - gives non-zero probability to negative interest rates, and small probability to high interest rates.
  - approximately normal distribution in the limit
- BDT Model:
  - gives essentially zero probability to interest rates below 1%, but it assigns a much higher probability to high interest rates.
  - generates an asymmetric, positively skewed distribution of interest rates, that looks like a log-normal distribution.
- Make sure you understand the general process of calibration (conceptual questions)

Risk-Neutral Trees

- The only purpose of risk-neutral tree is to compute the the **price** of the interest rate securities through no-arbitrage law.
  - Risk aversion is embedded in the level of θ_i
  - Derivative security prices are sensitive to the interest rate distributional difference: BDT model performs badly in the low interest rate environment.

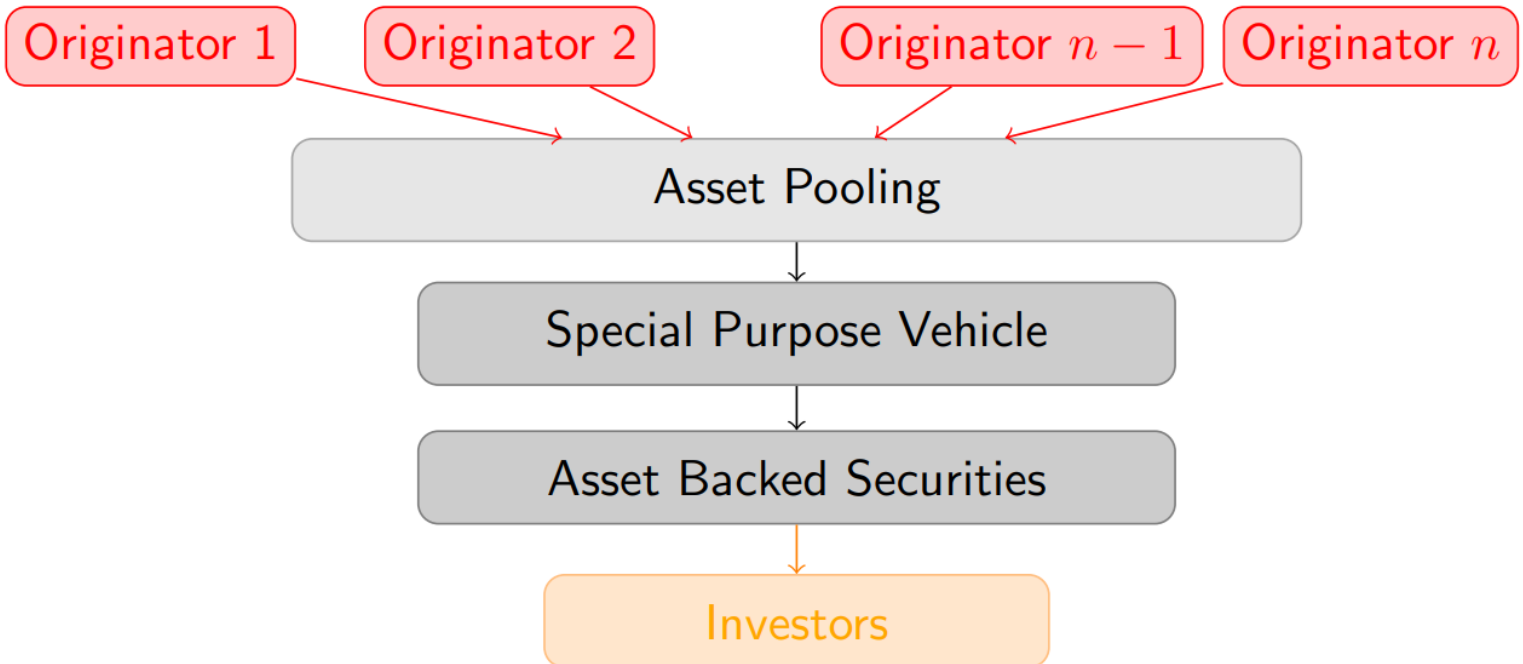
US Treasury Debt Securities

- Treasury Bills, Notes, Bonds
- **TIPS**
  - Fixed interest rate; Principal adjusted by changes in CPI (3-month lag)
  - Protection against both inflation and deflation (only adjusted when inflation)
  - Show credibility by Treasury to keep inflation low
  - Difference between nominal and real bonds represents expected inflation and inflation risk premium.
- **STRIPS**
  - Created by dealer firm as long-term "treasury ZCB";
  - Split into coupon ZCB and principal ZCB; reconstitution if STRIPS are cheap.

US Treasury derivatives

- US Treasury Futures
  - Very liquid (large scale of open interest)
  - Cash settled for T-bill futures
  - Deliver notes/bonds for T-note/bond futures
  - Quoted on bank discount yield basis for T-bill futures
- Eurodollar Futures
  - Reference rate: 3-month LIBOR

Securitization



Security's Name	Collateral Asset
Residential Mortgage-Backed Security (RMBS)	Residential mortgages with similar characteristics
Commercial Mortgage-Backed Security (CMBS)	Commercial mortgages with similar characteristics
Asset-Backed Securities (ABS)	Receivables, such as auto loans, credit cards, and so on
Collateralized Debt Obligations (CDO)	Investment and high yield corporate bonds, other structured products, CDS
Collateralized Loan Obligations (CLO)	Corporate loans

Benefit & Risk Considerations

- Benefit:
  - Diversification
  - Off-balance sheet financing
  - Liquidity
  - Credit enhancing
- Risk consideration:
  - Default Risk: agency MBS is default free because it is backed by the government and it has more restrictions on the assets.
  - Prepayment Risk: receive CF too early compared to the expected life of the mortgage (especially when interest rate declines).

Mortgage Valuation

- Mortgage coupon payment & valuation:

$$L = \sum_{i=1}^{30\times 12} \frac{C}{(1+\frac{\bar{r}_{12}^m}{12})^i}$$

$$\Rightarrow C = \frac{L}{\sum_{i=1}^{30\times 12} A^i} \text{ where } A = \frac{1}{1+\frac{\bar{r}_{12}^m}{12}}$$

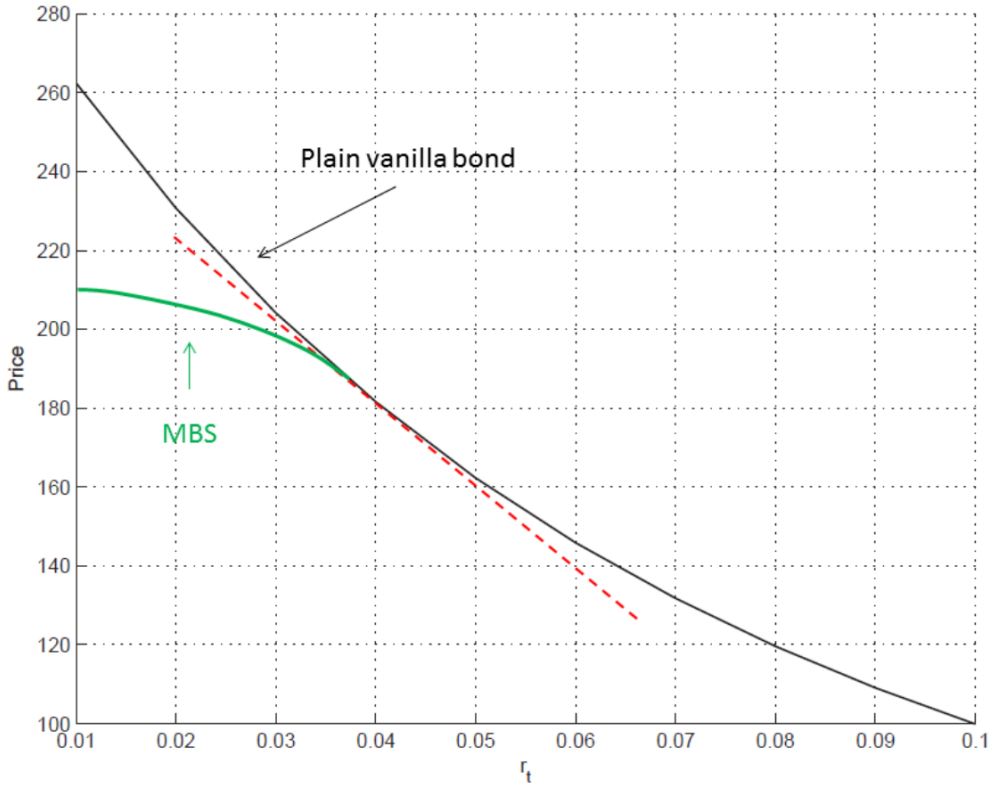
- Interest and Principal paid each period:

$$I_t = \frac{\bar{r}_{12}^m}{12} \times L_t$$

$$L_t^{paid} = C - I_t$$

$$L_{t+1} = L_t - L_t^{paid}$$

Negative Convexity (Conceptual Questions)



- Point 1: Interest rate ↓, prepayment ↑
- Point 2: Prepayment ↑, Duration ↓, negative convexity
- Point 3: Hedge the prepayment option

Sensitivity of MBS (IO, PO, PT)

- Effective Duration:

$$D = -\frac{1}{P}\frac{P(+x \text{ bps}) - P(-x \text{ bps})}{2x \text{ bps}}$$

- Effective Convexity:

$$C = \frac{1}{P}\frac{P(+x \text{ bps}) + P(-x \text{ bps}) - 2P}{(x \text{ bps})^2}$$

RMBS Market Players

- Agency Market:
  - Ginnie Mae (GNMA), Fannie Mae (FNMA), Freddie Mac (FHLMC)
- Non-agency Market (conceptual question):
  - Private Label Markets for mortgages which do NOT satisfy the requirement of government-sponsored agencies

Monetary Policy (Conceptual Questions)

- Open Market Operations
- Discount Window (primary, secondary, seasonal)
- Reserve Requirements (10% of deposit, interest)
- Federal Funds Rate:
  - Interest rate at which depository institutions lend balances at the Federal Reserve to other depository institutions overnight;
  - Fed sets target rate at FOMC meetings and pursue the goal through monetary policies.

Monetary Policy Challenges

- Credibility
- Key economic info/data is only available with a lag
- State of the economy need to be estimated
- Unclear about the economy response
- Other factors influence:
  - Fiscal Policies (tax & government spending)
  - Forward-looking financial market
  - Unpredictable aggregated demand and supply

Credit Crisis

Points to review:

- Monetary policy goals (price, economy, employment, etc.)
- Conventional & Unconventional Tools (credit easing, quantitative easing) - pay attention to the unconventional tools!

Final Exam & End of Session 2

- Office Hour:
  - LSE Library LG2 9:50am - 12:00pm
- Final Exam:
  - Pay attention to what Cameron said on Thursday!
  - LSE ID, Exam Room, Question numbers, Answer partial steps
  - Try to use a pen instead of a pencil
- End of Session 2:
  - Check the end of session activities on your email
  - Provide Feedback !!!