On the Critical Delays of Mobile Networks Under Lévy Walks and Lévy Flights

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***Abstract*—Delay-capacity tradeoffs for mobile networks have been analyzed through a number of research works. However, Lévy mobility known to closely capture human movement patterns has not been adopted in such work. Understanding the delay-capacity tradeoff for a network with Lévy mobility can provide important insights into understanding the performance of real mobile networks governed by human mobility. This paper analytically derives an important point in the delay-capacity tradeoff for Lévy mobility, known as the critical delay. The critical delay is the minimum delay required to achieve greater throughput than what conventional static networks can possibly achieve (i.e., $O(1/\sqrt{n})$ per node in a network with n nodes). The Lévy mobility includes Lévy flight and Lévy walk whose step-size distributions parametrized by $\alpha \in (0,2]$ are both heavy-tailed while their times taken for the same step size are different. Our proposed technique involves: 1) analyzing the joint spatio-temporal probability density function of a time-varying location of a node for Lévy flight, and 2) characterizing an embedded Markov process in Lévy walk, which is a semi-Markov process. The results indicate that in Lévy walk, there is a phase transition such that for $\alpha \in (0,1)$, the critical delay is always $\Theta(n^{\frac{1}{2}})$, and for $\alpha \in [1,2]$ it is $\Theta(n^{\frac{\alpha}{2}})$. In contrast, Lévy flight has the critical delay $\Theta(n^{\frac{\alpha}{2}})$ for $\alpha \in (0,2]$.**

***Index Terms*—Critical delay, delay-capacity tradeoff, human mobility, Lévy mobility, network performance scaling.**

Manuscript received February 01, 2012; revised October 29, 2012; accepted November 13, 2012; approved by IEEE/ACM TRANSACTIONS ON NETWORKING Editor D. Manjunath. Date of publication December 13, 2012; date of current version October 11, 2013. This work was supported in part by the National Science Foundation under Grants CNS-0910868 and CNS-1016216; the Korea Communications Commission (KCC), Korea, under the R&D program supervised by the Korea Communications Agency (KCA) (KCA-2012-11913-05004); and the Ulsan National Institute of Science and Technology (UNIST) under the 2012 Research Fund. A preliminary version of this paper was published in the Proceedings of the IEEE International Conference on Computer Communications (INFOCOM), Shanghai, China, April 10–15, 2011.

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Digital Object Identifier 10.1109/TNET.2012.2229717

I. INTRODUCTION

SINCE the seminal work by Gupta and Kumar [1] on the capacity of wireless networks, delay and throughput tradeoffs for wireless networks have been extensively studied for various mathematical techniques, scheduling algorithms, channel models, mobility models, and physical-layer techniques. The work by Grossglauser and Tse [2] showed that the per-node throughput remains constant ($\Theta(1)$) when node mobility is used for communication. This result is surprising because Gupta and Kumar [1] had previously shown that the per-node throughput ($O(1/\sqrt{n})$) in wireless networks with no mobility diminishes as the number of nodes n increases. This throughput gain is achieved at the cost of larger delays.

The amount of delay that a network needs to sacrifice to guarantee a given throughput has been studied under various mobility models [3]–[5]. In particular, Sharma *et al.* [6] studied the minimum delays required to achieve more per-node throughput than $\Theta(1/\sqrt{n})$[1] under various mobility models including i.i.d., random waypoint, random direction, and Brownian motion. This minimum delay is called *critical delay*. However, although the work provides a nice framework for studying delay-capacity scaling for wireless networks under a family of random walk models, the practical values of these mobility models are limited. While these models are simple enough for mathematical tractability, they do not reflect realistic mobility patterns commonly exhibited in real mobile networks.

Humans are a major factor in mobile networks since most mobile nodes or devices (smartphones and cars) are carried or driven by humans. Recent studies [7]–[9] on human mobility show that step-size distributions[2] are heavy-tailed, where a *step* is defined to be the straight-line trip of a moving object (e.g., particles or humans) from one location to another without a directional change or pause. Also, [7] reveals that the mobility of humans show larger mean squared displacement (MSD), which is characterized by the speed of diffusion, meaning that the movement of humans results in faster speed of diffusion than that of a random walk whose step size conforms to a Gaussian distribution. These findings from large-scale experimental data involving GPS traces and cell tower log traces of mobile phones that humans always carry provided statistical evidence that the human mobility patterns in reality closely resemble the mobility

[1]As [1] showed, $\Theta(1/\sqrt{n})$ is the maximum throughput that wireless networks relying on naive multihop transmissions can achieve without the help of node mobility.

[2]Step size is often referred to as flight length in some literature.

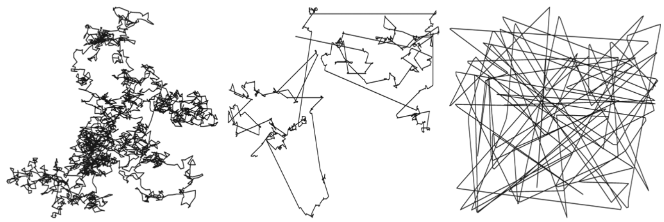


Fig. 1. Sample trajectories of (a) Brownian motion, (b) Lévy mobility, and (c) random waypoint.

patterns described by Lévy process [10] rather than other popular random processes such as Brownian motion and random waypoint.

Lévy mobility is a random walk mobility whose step-size distribution is parametrized by $\alpha \in (0, 2]$ and is heavy-tailed except in the extreme case of $\alpha = 2$. For $\alpha \in (0, 2)$, the distribution is well approximated by a power-law distribution $1/z^{1+\alpha}$, where z is a step size. For $\alpha = 2$, the step size conforms to Gaussian distribution.[3] Intuitively, such a random walk contains many short steps and a small yet significant number of exceptionally long steps. With different values of α, the movement patterns of Lévy mobility models are widely different. Smaller α induces a larger number of long steps. This type of mobility pattern is significantly different from Brownian motion and random waypoint as illustrated in Fig. 1. In the literature, there are two types of Lévy mobility models for classification: *Lévy flight* and *Lévy walk*. In Lévy flight, every step takes a *constant time* irrespective of its step size, and in Lévy walk, it takes a *constant velocity*. Lévy flight and Lévy walk can show the same pattern of traces, but their time durations taken to have such traces are essentially different. Intuitively, Lévy flight can be easily slotted, while Lévy walk is not.

Unfortunately, understanding tradeoffs between throughput and delay under Lévy mobility models is technically very challenging and underexplored. Unlike the other random walk models permitting mathematical tractability, the Lévy process is not very well understood mathematically despite significant studies on Lévy process in mathematics and physics. Thus, the conventional techniques [5], [6] used to study delay-capacity tradeoffs cannot be applied to Lévy mobility models, especially to Lévy walk, which has high spatio-temporal correlation. In more specific, since Lévy walk is not eligible for discretization for Markovian analysis, its mathematical characteristics such as joint spatio-temporal probability density function (PDF) are hardly known. Due to such a difficulty, analyzing Lévy walk is generally considered to be very challenging.

Our main contribution is to analytically derive important tradeoffs between delay and capacity for both Lévy mobility models. An important point in this tradeoff is the "critical delay," which is the minimum delay for a mobile network to obtain a larger throughput than $\Theta(1/\sqrt{n})$. Our technique involves: 1) analyzing the joint spatio-temporal PDF of a time-varying location of a node and the diffusion equation of the node for Lévy flight, and 2) characterizing an embedded Markov process inherent in Lévy walk, which is a semi-Markov process. Since a different value of α induces a different mobility pattern, it also induces a different critical delay. We summarize our main results in Table I.

[3]Lévy mobility becomes discrete approximation of the Brownian motion in the extreme case of $\alpha = 2$.

TABLE I
SUMMARIZATION OF MAIN RESULTS

Mobility	α	Critical Delay
Lévy walk	$\alpha \in (0,1)$	$\Theta(\sqrt{n})$
	$\alpha \in [1,2]$	$\Theta(n^{\alpha/2})$
Lévy flight	$\alpha \in (0,2]$	$\Theta(n^{\alpha/2})$

Given that many human mobility traces are shown to have values of α between 0.53 and 1.81 [7], according to our results, mobile networks assisted by human mobility have critical delays between $\Theta(n^{0.27})$ and $\Theta(n^{0.91})$. Note that our results give much more detailed prediction of the critical delay for such mobile networks depending on α, while Brownian motion and random waypoint always show $\Theta(n)$ and $\Theta(n^{0.5})$ for their critical delays [6].

The rest of this paper is organized as follows. We first overview a list of related work in Section II and introduce our system model in Section III. More details of Lévy mobility model parameterized by α are described in Section IV, and the critical delays under Lévy flight and Lévy walk are investigated in Sections V and VI, respectively. Finally, we provide a high-level interpretation of our main results in Section VII and concluding remarks in Section VIII.

II. Related Work

Gupta and Kumar [1] showed that the per-node throughput of random wireless networks with n static nodes scales as a function of $O(1/\sqrt{n})$ and proposed a scheme achieving $\Theta(1/\sqrt{n \log n})$. The result for static wireless networks was later enhanced to $\Theta(1/\sqrt{n})$ by exercising individual power control [11], [12]. Grossglauser and Tse [2] proved that a constant per-node throughput is achievable by using mobility when the nodes follow ergodic and stationary mobility models. This result disproved the conventional belief that node mobility can negatively impact network capacity as it causes connectivity breakup and channel quality degradation.

Many follow-up studies [3], [4], [13]–[17] have been devoted to understand, characterize, and exploit the tradeoffs between throughput and delay. Especially, the delay required to obtain the constant throughput $\Theta(1)$ has been later studied under various mobility models [4], [16]–[18]. The studies proved that the delay to obtain $\Theta(1)$ of per-node throughput becomes $\Theta(n)$ for most mobility models such as i.i.d. mobility, random direction, random waypoint, and Brownian motion models.

Another interesting question that has attracted researchers is what should be the minimum delay to achieve asymptotically higher throughput than $\Theta(1/\sqrt{n})$, the per-node throughput of static networks. This has been studied under the notion of critical delay [5], [6] for two families of random mobility models: *hybrid random walk* and *random direction*. The hybrid random walk model splits the network of size 1 with $n^{2\beta}$ cells and further splits a cell into $n^{1-2\beta}$ subcells for $\beta \in [0, 1/2]$. Then, a node moves to a random subcell of an adjacent cell in every unit time-slot. In this model, i.i.d. mobility corresponds to $\beta = 0$, and random walk mobility corresponds to $\beta = 1/2$. For any $\beta \in [0, 1/2]$, critical delay is proved to be $\Theta(n^{2\beta})$. The random direction model chooses a random direction within $[0, 2\pi]$ and moves to the selected direction with a distance of $n^{-\gamma}$ with a velocity $n^{-1/2}$ for $\gamma \in [0, 1/2]$. In this model, random waypoint and Brownian motion are represented with $\gamma = 0$ and $\gamma = 1/2$, respectively. The critical delay is proved to be $\Theta(n^{1/2+\gamma})$.

III. Model Description

A. System Model

We consider a wireless mobile network indexed by n, where in the nth network, n nodes are distributed uniformly on a completely wrapped-around square $\mathcal{S}(n)$ whose width and height scale as $\Theta(\sqrt{n})$, and the density is fixed to 1 with increasing n.[4] In this paper, we set the width and the height of the square $\mathcal{S}(n)$ as $\sqrt{n}$. Note that changing the values of the width and the height from $\sqrt{n}$ to any other ones in the class $\Theta(\sqrt{n})$ does not change the main results of this paper. We assume that all nodes are homogeneous in that each node generates data with the same intensity to a per-source destination. The packet generation process at each node is assumed to be independent of node mobility.

A source-to-destination packet can be delivered by either direct one-hop transmission or over multiple hops, say k hops, using relay nodes. We call it k-hop relay transmission. We assume that all nodes can serve as relay nodes for other source nodes and the nodes serving as relay nodes can only forward packets rather than replicating packets (not to overproduce the same packets in the network).

To model interference in wireless networks, we use the *protocol model* as in [1], under which nodes transmit packet successfully at a constant rate W bits/s, if and only if the following is met: Let $\boldsymbol{X}_i(t)\,(\in \mathbb{R}^2)$ denote the location of node $i\,(i = 1, \ldots, n)$ at time $t\,(\geq 0)$. For a transmitter i, a receiver j and every other node $k \neq i, j$ transmitting simultaneously

$$d(\boldsymbol{X}_k(t), \boldsymbol{X}_j(t)) \geq (1 + \Delta)\, d(\boldsymbol{X}_i(t), \boldsymbol{X}_j(t))$$

where $d(\boldsymbol{x}, \boldsymbol{y})$ denotes the Euclidean distance between locations $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^2$, and Δ is some positive number.

In the literature, there are more advanced interference models such as *physical model* [1] and *generalized physical model* [12]. Physical model determines the feasibility of communication based on a threshold value on the signal-to-interference-plus-noise ratio (SINR) of the intended communication, while generalized physical model further defines the data rate of the indented communication by the SINR. According to [12], which first introduced generalized physical model in the study of network performance scaling, the lower-bound capacities of a network under the protocol model and the generalized physical model are the same in their orders. Thus, we focus on a simpler interference model, protocol model, through this paper to avoid too much complications incurred from the interference model. We leave the application of our techniques developed for obtaining the critical delay to the generalized physical model as a future work.

[4]This model is often referred to as an extended network model. In another model, called a unit network model, the network area is fixed to 1, and the density increases as n, while the spacing and velocity of nodes scale as $\Theta(1/\sqrt{n})$.

A packet can be delivered through a scheduling scheme that consists of *replication* or *forwarding*. We assume that only source nodes replicate packets and all other relay nodes forward them. As the names imply, replication copies a packet, and the packet transmitter keeps the packet, whereas in forwarding, the transmitter does not keep the original packet after successful transmission. This selective replication and forwarding depending on the node type are often applied to suppress the overflow of redundant packets in the network. Packets are delivered in two ways: *neighbor capture* and *multihop capture*. In neighbor capture, using mobility, relay or source nodes are located within the communication range of the destination. In the multihop capture, a source establishes a multihop path to the destination and delivers the packets over the path. We assume a fluid packet model [19] so that the delivery can occur immediately even in the case of multihop capture because the transmission delay is negligible compared to the delay from node mobility. We denote by Π the class of all scheduling schemes conforming to the descriptions above.

B. Performance Metrics

The primary performance metric in many networking systems is per-node throughput measured by the long-term average of received packets aggregated over nodes:

Definition 1 (Per-Node Throughput): Let $\lambda_\pi(n)$ denote the per-node throughput in the nth network under a scheduling scheme $\pi \in \Pi$. It is then given by

$$\lambda_\pi(n) \triangleq \liminf_{t \longrightarrow \infty} \frac{1}{n} \sum_{i=1}^{n} \frac{\lambda_{\pi:i}(t)}{t}$$

where $\lambda_{\pi:i}(t)$ is the total number of bits received at a destination node i up to time t under π.[5]

Another important metric is average delay:

Definition 2 (Average Delay): Let $D_\pi(n)$ denote the average delay in the nth network under a scheduling scheme $\pi \in \Pi$. It is then given by

$$D_\pi(n) \triangleq \lim_{k \longrightarrow \infty} \frac{1}{n} \sum_{i=1}^{n} \frac{1}{k} \sum_{j=1}^{k} D_{\pi:(i,j)}$$

where $D_{\pi:(i,j)}$ is the individual packet delay that a packet j experiences to arrive at a destination node i from its source node under π.

We give special attention to the notion of critical delay, first introduced in [6]:

Definition 3 (Critical Delay): The critical delay in the nth network, denoted by $C_\Pi(n)$, is the minimum average delay that must be tolerated under a given mobility model to achieve a per-node throughput of $\omega(1/\sqrt{n})$, i.e.,

$$C_\Pi(n) \triangleq \inf_{\{\pi \in \Pi : \lambda_\pi(n) = \omega(1/\sqrt{n})\}} D_\pi(n).$$

Per-node throughput $\Theta(1/\sqrt{n})$ is achievable by a scheduling scheme in static multihop networks [1]. Since node mobility can increase per-node throughput at the cost of larger delay, the critical delay quantifies the amount of delay that a network

[5]For simplicity, we omit the subscript π in $\lambda_\pi(n)$ unless confusion arises.

should sacrifice to achieve the guaranteed "baseline" per-node throughput. It can be used as a simple yet useful metric for a mobility model, representing how sensitive the delay is to increase per-node throughput.[6]

By the definition of critical delay, computing critical delay initially requires understanding the scheduling algorithms and the immediate capture (i.e., starting communication immediately when two nodes come into their communication ranges) as shown in [5] and [6]. Fortunately, [5] and [6] identified that the steps to apply those understandings boil down to evaluating the first exit time of a node from a disc whose radius is judiciously determined by the size of a network, which scales up as the number of nodes in the network increases. The technique of investigating the first exit time of a node is applicable to any mobility patterns including Brownian motion, random waypoint, and random direction mobility. Thus, we borrow the technique and apply it to Lévy mobility so that we can purely focus on investigating the first exit time of a node conforming Lévy mobility, which involves nontrivial mathematical challenges. We sketch how critical delay is obtained from the first exit time based on the steps in [5] and [6]. Let $\mathcal{D}(n)$ denote a disc within the square $\mathcal{S}(n)$ whose radius scales as $\Theta(\sqrt{n})$. Critical delay can simply be regarded as the *maximum time duration that a node cannot exit from the disc $\mathcal{D}(n)$ with probability approaching 1 as n goes to ∞*. In our extended network model, the average distance from a source node to a destination node is $\Theta(\sqrt{n})$ when they are uniformly distributed on $\mathcal{S}(n)$. Therefore, if nodes travel up to a distance $O(\sqrt{n})$, for a certain time duration, the distance between a source or a relay and a destination still remains $\Theta(\sqrt{n})$ on average, which results in $O(1/\sqrt{n})$ per-node throughput (see Lemma 1). Thus, it is obvious that a network aiming at obtaining $\omega(1/\sqrt{n})$ per-node throughput must allow a delay that is no less than the maximum time duration that the first exit of a node from the disc $\mathcal{D}(n)$ does not occur with probability approaching 1. This insight can be formally described with the notion of the first exit time.

Definition 4 (First Exit Time): Let $\boldsymbol{X}_i(0) = \boldsymbol{x}$. The first exit time for a disc of a radius r, denoted by $T(r)$, is defined as

$$T(r) \triangleq \inf\{t \geq 0 : \boldsymbol{X}_i(t) \notin B(\boldsymbol{x}, r)\}$$

where $B(\boldsymbol{x}, r)$ denotes the set of points $\boldsymbol{y}$ in $\mathcal{S}(n)$ such that $d(\boldsymbol{x}, \boldsymbol{y}) \leq r$.

Without loss of generality, we set the radius of the disc $\mathcal{D}(n)$ as $c_d\sqrt{n}$ where c_d is a constant in the range $(0, 1/2)$. Then, critical delay $C_\Pi(n)$ can be obtained by

$$C_\Pi(n) = \sup\left\{t(n) : \lim_{n \to \infty} \mathrm{P}\{T(c_d\sqrt{n}) > t(n)\} = 1\right\}.$$

Lemma 1 ([1], [5]): Suppose that on average each packet is relayed over a total distance no less than $\Theta(\sqrt{n})$ in an extended network model. Then, the per-node throughput $\lambda(n)$ scales as $O(1/\sqrt{n})$.

[6]Analyzing the delay-capacity tradeoff for throughput in the range $\Theta(n^{-\eta})$ $(0 \leq \eta \leq 1/2)$ is beyond the scope of this paper. Please refer to [20] for an upper bound on the tradeoff under Lévy flight with $\alpha \in (0, 2]$.

IV. Mobility Models: Lévy Flight and Lévy Walk

In this section, we formally define *Lévy mobility model: Lévy flight and Lévy walk*.

Lévy flight and Lévy walk processes are treated separately in the literature [21]–[23]. Lévy flight takes a *constant time* for any step irrespective of its step size, whereas Lévy walk takes a *constant velocity* for every step. Thus, in Lévy walk, the time taken for each step is proportional to the step size. The distinction between Lévy flight and Lévy walk is often made based on the speeds of their actual processes. Lévy flight is a "fast" mobility model that can reach its next destination in a constant time no matter how far it is. In a similar context, Lévy walk falls under a "slow" mobility model. An experimental velocity model suggested as a function of step size in [7] verifies that a human mobility lies in between Lévy flight and Lévy walk. For convenience, we use Lévy mobility model to indicate both of Lévy flight and Lévy walk, unless explicitly stated.

Let Z be a random variable denoting the step size under Lévy mobility model. Then, Z is generated from a random variable $\dot{Z}$ having the Lévy α-stable distribution [24] by the relation $Z = |\dot{Z}|$. The PDF of $\dot{Z}$ is given by

$$f_{\dot{Z}}(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-izt}\varphi_{\dot{Z}}(t)\,\mathrm{d}t \tag{1}$$

where $\varphi_{\dot{Z}}(t) \triangleq \mathrm{E}[e^{it\dot{Z}}]$ is the characteristic function of $\dot{Z}$ and is given by $\varphi_{\dot{Z}}(t) = e^{-|ct|^\alpha}$. Here, $|c| > 0$ is a scale factor that is a measure of the width of the distribution, and $\alpha \in (0, 2]$ is a distribution parameter and specifies the shape (i.e., heavytailness) of the distribution. The step size Z for $\alpha \in (0, 1)$ has infinite mean and variance, while Z for $\alpha \in [1, 2)$ has finite mean but infinite variance. For $\alpha = 2$, the Lévy α-stable distribution reduces to a Gaussian distribution with zero mean and variance $\sigma^2 = 2c^2$, and consequently the step size Z has finite mean and variance.

Due to the complex form of the distribution, the Lévy α-stable distribution for $\alpha \in (0, 2)$ is often given as a power-law type of asymptotic form, closely approximating the tail part of the distribution [24]

$$f_{\dot{Z}}(z) \sim \frac{1}{|z|^{1+\alpha}}. \tag{2}$$

For mathematical tractability, in our analysis we use the asymptotic form (2) instead of the exact form (1) for $\alpha \in (0, 2)$ while using the exact form (1) for $\alpha = 2$. The form (2) is known to closely approximate (1), and several papers in mathematics and physics, e.g., [21] and [25], analyze Lévy mobility using form (2). For the range of Z, since we use the extended network model, the step size Z is assumed to have a lower bound at 1 and an upper bound at $\sqrt{n}$, i.e., $\mathrm{P}\{1 \leq Z \leq \sqrt{n}\} = 1$.[7] Thus, the complementary cumulative distribution function (CCDF) of Z becomes $\mathrm{P}\{Z > z\} = 1$ for $z < 1$ and $\mathrm{P}\{Z > z\} = 0$ for $z \geq \sqrt{n}$. For $z \in [1, \sqrt{n})$, we have

$$\mathrm{P}\{Z > z\} = \begin{cases} c(n) \cdot \left(\frac{1}{z^\alpha} - \frac{1}{(\sqrt{n})^\alpha}\right), & \text{for } \alpha \in (0, 2) \\ c(n) \cdot \left(\mathrm{erf}\left(\frac{\sqrt{n}}{\sqrt{2}\sigma}\right) - \mathrm{erf}\left(\frac{z}{\sqrt{2}\sigma}\right)\right), & \text{for } \alpha = 2. \end{cases} \tag{3}$$

[7]The bounds are chosen equivalently to the lower bound at $1/\sqrt{n}$ and the upper bound at 1 for the step size in the unit network model [6].

Here, $\mathrm{erf}(\cdot)$ is the error function defined as $\mathrm{erf}(x) \triangleq \frac{2}{\sqrt{\pi}}\int_0^x \exp(-t^2)\,\mathrm{d}t$, and $c(n)$ is defined as[8]

$$c(n) \triangleq \begin{cases} \left(1 - \frac{1}{(\sqrt{n})^{\alpha}}\right)^{-1}, & \text{for } \alpha \in (0,2) \\ \left(\mathrm{erf}\left(\frac{\sqrt{n}}{\sqrt{2}\sigma}\right) - \mathrm{erf}\left(\frac{1}{\sqrt{2}\sigma}\right)\right)^{-1}, & \text{for } \alpha = 2. \end{cases}$$

Note that as n goes to ∞, the CCDF $\mathrm{P}\{Z > z\}$ for $z \geq 1$ goes to $1/z^{\alpha}$ for $\alpha \in (0,2)$.

In our analysis, we use the following assumptions on the Lévy mobility model: (A1) the time taken for each step in the Lévy flight is set to 1; and (A2) the velocity taken for each step in Lévy walk is set to 1. Note that as long as these two metrics are constant, the scaling property of critical delay remains the same, which justifies our assumptions.

V. Critical Delay Analysis for Lévy Flight

In this section, we will show that the critical delay $C_{\Pi}(n)$ under Lévy flight with a distribution parameter $\alpha \in (0,2]$ scales as $\Theta(n^{\frac{\alpha}{2}})$ (Theorem 1). In Section V-A, we explain technical challenges and our approach for proving Theorem 1. In Section V-B, we prove Theorem 1 by showing that the upper bound on $C_{\Pi}(n)$ scales as $O(n^{\frac{\alpha}{2}})$ (Lemma 3), and that the lower bound on $C_{\Pi}(n)$ scales as $\Omega(n^{\frac{\alpha}{2}})$ (Lemma 4).

A. Technical Approach

We begin with deriving a relation between the first exit time of a two-dimensional random process and the one for its one-dimensional projected process. We then describe trapping phenomena in a diffusion process that have a direct connection to the first exit time of a one-dimensional random process.

It is clear from Definition 4 that the statistical proprieties of the first exit time do not depend on the choice of node index i. Thus, we omit the node index i in the rest of the paper. Denote $\boldsymbol{X}(t) = (X_x(t), X_y(t))$ and consider the projected processes $\{X_x(t)\}_{t\geq 0}$ and $\{X_y(t)\}_{t\geq 0}$ onto x-axis and y-axis, respectively. We define for the projected processes the first exit time similarly to Definition 4

$$T_x(r) \triangleq \inf\{t \geq 0 : |X_x(t) - X_x(0)| \geq r\}$$
$$T_y(r) \triangleq \inf\{t \geq 0 : |X_y(t) - X_y(0)| \geq r\}.$$

Since the event $\{|X_x(t) - X_x(0)| \geq r\}$ implies the event $\{d(\boldsymbol{X}(t), \boldsymbol{X}(0)) \geq r\}$, we obtain

$$\mathrm{P}\{T_x(r) \leq t\} \leq \mathrm{P}\{T(r) \leq t\}. \tag{4}$$

In addition, it is clear that

$$\begin{aligned}\mathrm{P}\{T(r) \leq t\} &\leq \mathrm{P}\left\{T_x\left(r/\sqrt{2}\right) \leq t \text{ or } T_y\left(r/\sqrt{2}\right) \leq t\right\} \\ &\leq 2\,\mathrm{P}\left\{T_x\left(r/\sqrt{2}\right) \leq t\right\}\end{aligned} \tag{5}$$

where the second inequality comes from the union bound and the symmetry of node motion. Combining (4) and (5), we have for all $t \geq 0$

$$\mathrm{P}\{T_x(r) \leq t\} \leq \mathrm{P}\{T(r) \leq t\} \leq 2\,\mathrm{P}\left\{T_x\left(r/\sqrt{2}\right) \leq t\right\}. \tag{6}$$

Our technical approach is mainly based on (6) and is to bound the first exit time distribution for two-dimensional Lévy flight by the one for the corresponding one-dimensional projected process $\{X_x(t)\}_{t\geq 0}$. We henceforth study the first exit time distribution for the process $\{X_x(t)\}_{t\geq 0}$.

The first exit time analysis for one-dimensional random processes has been intensively studied in physics and mathematics, e.g., [26]. Specifically, trapping phenomena (of a diffusing particle) in physics and their related theories have a direct connection to our first exit time problem as explained in the following. Consider a particle that diffuses in a finite interval $[0, 2r]$ $(\subset \mathbb{R})$ having trapping boundaries at $x = 0, 2r$. Let $L(t)$ $(\in \mathbb{R})$ be a random variable denoting the location of the particle at time t. The particle is assumed to be initially located at $L(0) = r$, and eventually it is trapped at either of both boundaries with probability 1. Upon the particle being trapped, it disappears in the interval. We call the state of the particle *survival state* until the particle is trapped and disappears. By convention, we let $L(t) \triangleq \varnothing$ if the particle is not in survival state at time t. If we assume $X_x(0) = L(0)\,(= r)$, then $X_x(t)$ and $L(t)$ for $t > 0$ are related as follows:

$$L(t) \overset{\mathrm{d}}{=} \begin{cases} X_x(t), & \text{if } t < T_x(r) \\ \varnothing, & \text{if } t \geq T_x(r) \end{cases} \tag{7}$$

where $\overset{\mathrm{d}}{=}$ denotes "equal in distribution." Hence, we have from (7) that

$$\mathrm{P}\{T_x(r) \leq t\} = \mathrm{P}\{L(t) = \varnothing\}. \tag{8}$$

That is, the survival time of a particle in the trapping model has the same distribution as the first exit time $T_x(r)$ of a node under Lévy flight.

The technical approach for analyzing the critical delay in the literature is as follows. In the case of Brownian motion, there are two general techniques in studying the critical delay. One is to discretize mobility and then apply a Markovian analysis [6]. The other is to use a continuous mobility model and solve a diffusion equation to obtain a joint spatio-temporal PDF of a time-varying location of a node [5]. The latter enables one to obtain the distribution of $L(t)$ whose spatial derivative is often referred to as *occupation probability*.[9] The occupation probability of Brownian motion can be decomposed to find the components constituting it. From this decomposition process, we find that there is a dominating term that characterizes the limiting behavior of the first exit time distribution.

In the case of Lévy flight, the joint spatio-temporal PDF has a similar form to that of Brownian motion. In addition, the occupation probabilities and the first exit time distributions for Brownian motion and Lévy flight have similar structures in the

[8]To be precise, $c(n)$ is also a function of α, i.e., $c(n) = c(n, \alpha)$. Since we focus on scaling properties with respect to n for a fixed α, we omit the argument α in $c(n, \alpha)$ for notational simplicity. By the same reason, in the rest of the paper, we emphasize only n in all variables that depend on both n and α.

[9]The occupation probability in a trapping model corresponds to the joint spatio-temporal PDF in a random walk model. The mathematical definition and the distinction between the occupation probability and the joint spatio-temporal PDF will be given in Section V-B.

aspect of the dominating terms. Hence, by identifying and characterizing the dominating term for Lévy flight, we can obtain the critical delay under Lévy flight.

B. Analysis

In this section, we provide the detailed result for the critical delay under Lévy flight. Our main result is derived by following three steps. 1) The occupation probability is obtained from the solution of a differential equation that governs the movement of a particle. 2) From the occupation probability, we obtain the survival probability (which will be defined later), which in turn yields the first exit time distribution. 3) By investigating the limiting behavior of the first exit time distribution, we can finally obtain the order of the critical delay.

Step 1: Let $P(x,t) \triangleq \frac{\mathrm{d}}{\mathrm{d}x}\mathrm{P}\{L(t) \le x\}$. Intuitively, $P(x,t)$ represents probability that the particle is located at x at time t. We call $P(x,t)$ the *occupation probability*, and it has the following properties.

- (P1) $\lim_{t\to\infty} P(x,t) = 0 \;\forall x \in \mathbb{R}$.
- (P2) $\int_0^{2r} P(x,0)\,\mathrm{d}x = \mathrm{P}\{L(0) = r\} = 1$.
- (P3) $\int_0^{2r} P(x,t)\,\mathrm{d}x \le 1 \;\forall t > 0$.
- (P4) $P(0,t) = P(2r,t) = 0 \;\forall t \ge 0$.
- (P5) Since $P(x,0)$ is a PDF having a support $\{r\}$, we have $P(x,0) = \delta_{x,r}$, where δ_{x_1,x_2} denotes the Kronecker delta which is defined to be 1 if $x_1 = x_2$, and 0 otherwise.

To be precise, $P(x,t)$ for $t > 0$ could not be a PDF due to (P3). However, the function obtained by normalizing $P(x,t)$ with the integral $\int_0^{2r} P(x,t)\,\mathrm{d}x$, denoted by $\bar{P}(x,t)$, becomes a PDF for a finite time t. We call $\bar{P}(x,t)$ the *joint spatio-temporal PDF* at location x and time t.

In the first step, we obtain the occupation probability $P(x,t)$ for the process $\{L(t)\}_{t\ge0}$. For this, we need to characterize the associated one-dimensional process $\{X_x(t)\}_{t\ge0}$. We first consider the case of $\alpha \in (0,2)$ and summarize the result in the following lemma.

Lemma 2: Suppose that $\{\boldsymbol{X}(t)\}_{t\ge0}$ is two-dimensional Lévy flight with a distribution parameter $\alpha \in (0,2)$. Then, as n goes to ∞, the projected process onto x-axis $\{X_x(t)\}_{t\ge0}$ approaches to one-dimensional Lévy flight having the same distribution parameter α. It holds for the process $\{X_y(t)\}_{t\ge0}$.

Proof: Let Z_i and θ_i $(i = 1,2,\ldots)$ be random variables denoting the ith step size and direction of the process $\{\boldsymbol{X}(t)\}_{t\ge0}$, respectively. Then, $\boldsymbol{X}(t)$ for $t = 1,2,\ldots$ can be expressed as

$$\begin{aligned}\boldsymbol{X}(t) &= (X_x(t), X_y(t))\\ &= \boldsymbol{X}(0) + \left(\sum_{i=1}^{t} Z_i\cos\theta_i,\, \sum_{i=1}^{t} Z_i\sin\theta_i\right). \end{aligned}\tag{9}$$

We will show that, as n goes to ∞, arbitrary step size of the projected processes (i.e., $Z_i|\cos\theta_i|$ and $Z_i|\sin\theta_i|$) has a power-law type CCDF with an exponent α, i.e., for $z \ge 1$

$$\begin{aligned}\lim_{n\to\infty}\mathrm{P}\{Z_i|\cos\theta_i| > z\} &= \lim_{n\to\infty}\mathrm{P}\{Z_i|\sin\theta_i| > z\}\\ &= \frac{c^\star}{z^\alpha}\end{aligned}\tag{10}$$

where $c^\star \triangleq \frac{2}{\pi}\int_0^{\frac{\pi}{2}}(\cos\vartheta)^\alpha\mathrm{d}\vartheta$. Since the projected processes take a constant time for every step irrespective of step size, the property in (10) proves the lemma.

Now we prove (10). By conditioning on the values of the random variable $\theta_i \sim \mathrm{Uniform}[0,2\pi]$, we can rewrite the CCDF of $Z_i|\cos\theta_i|$ as

$$\begin{aligned}\mathrm{P}\{Z_i|\cos\theta_i| > z\} &= \int_0^{2\pi}\mathrm{P}\{Z_i|\cos\theta_i| > z|\,\theta_i = \vartheta\}\mathrm{d}F_{\theta_i}(\vartheta)\\ &= \frac{1}{2\pi}\int_0^{2\pi}\mathrm{P}\{Z_i|\cos\vartheta| > z\}\mathrm{d}\vartheta\\ &= \frac{2}{\pi}\int_0^{\frac{\pi}{2}}\mathrm{P}\{Z_i\cos\vartheta > z\}\mathrm{d}\vartheta\end{aligned}\tag{11}$$

where the last two equalities come from the independence of the random variables Z_i and θ_i, and the symmetry of the function $|\cos\vartheta|$, respectively. Using (3), the probability $\mathrm{P}\{Z_i\cos\vartheta > z\}$ in (11) can be obtained for $\vartheta \in [0,\frac{\pi}{2}]$ as

$$\mathrm{P}\{Z_i\cos\vartheta > z\} = \begin{cases} c(n)\cdot\left(\left(\frac{\cos\vartheta}{z}\right)^\alpha - \left(\frac{1}{\sqrt{n}}\right)^\alpha\right), & \text{for } \vartheta \in \left[0, \cos^{-1}\left(\frac{z}{\sqrt{n}}\right)\right)\\ 0, & \text{for } \vartheta \in \left[\cos^{-1}\left(\frac{z}{\sqrt{n}}\right), \frac{\pi}{2}\right].\end{cases}$$

Hence, the CCDF $\mathrm{P}\{Z_i|\cos\theta_i| > z\}$ is given by

$$\begin{aligned}\mathrm{P}\{Z_i|\cos\theta_i| > z\} = \frac{2c(n)}{\pi z^\alpha}&\int_0^{\cos^{-1}\left(\frac{z}{\sqrt{n}}\right)}(\cos\vartheta)^\alpha\mathrm{d}\vartheta\\ &- \frac{2c(n)}{\pi(\sqrt{n})^\alpha}\cos^{-1}\left(\frac{z}{\sqrt{n}}\right).\end{aligned}\tag{12}$$

Noting $\lim_{n\to\infty} c(n) = 1$ and $\lim_{n\to\infty}\cos^{-1}\left(\frac{z}{\sqrt{n}}\right) = \frac{\pi}{2}$, we have from (12) that

$$\lim_{n\to\infty}\mathrm{P}\{Z_i|\cos\theta_i| > z\} = \frac{2}{\pi z^\alpha}\int_0^{\frac{\pi}{2}}(\cos\vartheta)^\alpha\mathrm{d}\vartheta = \frac{c^\star}{z^\alpha}.$$

Since $|\sin\theta_i| \stackrel{\mathrm{d}}{=} |\cos\theta_i|$ for $\theta_i \sim \mathrm{Uniform}[0,2\pi]$, we have

$$\mathrm{P}\{Z_i|\sin\theta_i| > z\} = \mathrm{P}\{Z_i|\cos\theta_i| > z\}$$

which completes the proof. □

Motivated by Lemma 2 and (7), we study the occupation probability for one-dimensional Lévy flight with $\alpha \in (0,2)$ in a finite interval $[0,2r]$ having trapping boundaries. For mathematical tractability, our study in this section assumes continuous limit where the scale factor $|c|$ in (1) approaches to zero. Then, the occupation probability $P(x,t)$ for $\alpha \in (0,2)$ is governed by the following fractional Fokker–Planck equation [23, Eq. (22)], [27, Eq. (28)]:

$$\frac{\partial P(x,t)}{\partial t} = F\frac{\partial^\alpha P(x,t)}{\partial|x|^\alpha}\tag{13}$$

where $F\,(= F_\alpha > 0)$ is a generalized diffusion coefficient and $\frac{\partial^\alpha}{\partial|x|^\alpha}$ is the Riesz–Feller derivative of fractional order α [28]. We next consider the case of $\alpha = 2$. In this case, as the scale factor $|c|$ approaches to zero, the two-dimensional Lévy flight converges to a Wiener process that mathematically models a continuous movement of Brownian motion. Since one-dimensional projected process of two-dimensional Brownian motion is also Brownian motion [5], the occupation probability for $\alpha = 2$ is governed by the normal diffusion equation where the spatial derivative of order α with $\alpha \in (0,2)$ in (13) is replaced by the second-order derivative with $\alpha = 2$ [26]. Therefore, with continuous limit, the occupation probability $P(x,t)$ for $\alpha \in (0,2]$ can be described by the differential equation in (13). Through Appendix A, we show that the order of the critical delay under Lévy flight does not change with continuous limit.

Applying the standard method of separation of variables gives the solution of (13) as follows:

$$P(x,t) = \sum_{i=1}^{\infty} h_i \psi_i(x) \exp\left(\lambda_i F t\right). \tag{14}$$

Here, h_i $(i = 1, 2, \ldots)$ are determined from the initial condition $P(x,0) = \delta_{x,r}$ [as shown in (P5)] and are given by $h_i = \psi_i(r)$. The functions $\psi_i(x)$ and the constants λ_i can be obtained from the solutions of the problem $\mathfrak{D}[\psi_i(x)] = \lambda_i \psi_i(x)$ for the operator $\mathfrak{D} \triangleq \frac{\mathrm{d}^\alpha}{\mathrm{d}|x|^\alpha}$, and are called eigenfunctions and eigenvalues of $\mathfrak{D}$, respectively. Without loss of generality, we assume that λ_i are arranged as $|\lambda_1| < |\lambda_2| < \cdots$.

Step 2: Let $S(t) \triangleq \mathrm{P}\{L(t) \neq \varnothing\}$. Intuitively, $S(t)$ represents probability that the particle has not hit any trapping boundary by time t. We call $S(t)$ the *survival probability*. The survival probability can be obtained from the occupation probability $P(x,t)$ by $S(t) = \int_0^{2r} P(x,t)\,\mathrm{d}x$. Thus, from (14), the survival probability is given by

$$S(t) = \sum_{i=1}^{\infty} \psi_i(r) \int_0^{2r} \psi_i(x)\,\mathrm{d}x \exp\left(\lambda_i F t\right). \tag{15}$$

The first exit time distribution $\mathrm{P}\{T_x(r) \le t\}$ can be obtained from the survival probability $S(t)$ through the following relation:

$$\mathrm{P}\{T_x(r) \le t\} = \mathrm{P}\{L(t) = \varnothing\} = 1 - S(t). \tag{16}$$

Here, the first equality comes from (8), and the second equality comes from the definition of $S(t)$. By combining (15) and (16), we obtain the first exit time distribution in terms of the eigenfunctions $\psi_i(x)$ and the eigenvalues λ_i as follows:

$$\mathrm{P}\{T_x(r) \le t\} = 1 - \sum_{i=1}^{\infty} \psi_i(r) \int_0^{2r} \psi_i(x)\mathrm{d}x \exp\left(\lambda_i F t\right). \tag{17}$$

For $\alpha = 2$, the eigenfunctions and the eigenvalues in (17) can be obtained from the boundary conditions $P(0,t) = P(2r,t) = 0$ $\forall t \ge 0$ [as shown in (P4)] and are given by $\psi_i(x) = \sqrt{\frac{1}{r}} \sin\left(\frac{i\pi x}{2r}\right)$ and $\lambda_i = -\left(\frac{i\pi}{2r}\right)^2$, respectively [26]. For $\alpha \in (0,2)$, Gitterman [27] provided a solution of (13) whose eigenfunctions and eigenvalues are given by $\psi_i(x) = \sqrt{\frac{1}{r}} \sin\left(\frac{i\pi x}{2r}\right)$ and $\lambda_i = -\left(\frac{i\pi}{2r}\right)^\alpha$, respectively. Thus, under Lévy flight with $\alpha \in (0,2]$, the first exit time distribution can be expressed as an infinite series of exponential functions as follows:

$$\mathrm{P}\{T_x(r) \le t\} = 1 - \sum_{i=1}^{\infty} \eta_i \exp\left(-\frac{\rho_i}{r^\alpha} t\right) \tag{18}$$

where $\eta_i \triangleq \frac{2\{1-\cos(i\pi)\}}{i\pi} \sin\left(\frac{i\pi}{2}\right)$ and $\rho_i \triangleq F(\frac{i\pi}{2})^\alpha$. In Appendix B, we show that the tail distribution $\mathrm{P}\{T_x(r) > t\}$ behaves asymptotically for $t \to \infty$ as

$$\lim_{t\to\infty} \frac{\mathrm{P}\{T_x(r) > t\}}{\frac{4}{\pi} \exp\left(-F\left(\frac{\pi}{2r}\right)^\alpha t\right)} = 1. \tag{19}$$

That is, the probability $\mathrm{P}\{T_x(r) > t\}$ under Lévy flight decays exponentially with rate $F(\frac{\pi}{2r})^\alpha$, as t goes to ∞.

As will be shown later in the proof of Lemmas 3 and 4, the smallest (i.e., dominant) decay rate in the exponential functions in (14) (i.e., $|\lambda_1|$) determines the limiting behavior of the first exit time distribution. That is, the smallest decay rate characterizes the critical delay under Lévy flight. The solutions in [26] and [27] show that the dominant decay rate $|\lambda_1|$ scales as $\Theta(r^{-\alpha})$ for $\alpha \in (0,2]$.

Step 3: We are now ready to derive the main result of this section. By using the closed-form expression for $\mathrm{P}\{T_x(r) \le t\}$ in (18), we investigate the order of the critical delay, stated in Lemmas 3 and 4.

Lemma 3 (Upper Bound For Lévy Flight): Suppose that under Lévy flight with a distribution parameter $\alpha \in (0,2]$, the time $t \triangleq \hat{t}(n)$ in $\mathrm{P}\{T(c_d\sqrt{n}) > t\}$ scales as $\Theta(n^{\frac{\alpha}{2}+\epsilon})$ for an arbitrary $\epsilon > 0$. Then, we have

$$\lim_{n\to\infty} \mathrm{P}\{T(c_d\sqrt{n}) > \hat{t}(n)\} = 0$$

which shows that the critical delay $C_\Pi(n)$ under Lévy flight scales as $O(n^{\frac{\alpha}{2}})$.

Proof: We will prove this lemma by showing that $\lim_{n\to\infty} \mathrm{P}\{T_x(c_d\sqrt{n}) \le \hat{t}(n)\} = 1$. Then, by substituting $r = c_d\sqrt{n}$ and $t = \hat{t}(n)$ into (6) and taking a limit to n, we obtain

$$\begin{aligned} 1 &= \lim_{n\to\infty} \mathrm{P}\{T_x(c_d\sqrt{n}) \le \hat{t}(n)\} \\ &\le \lim_{n\to\infty} \mathrm{P}\{T(c_d\sqrt{n}) \le \hat{t}(n)\}. \end{aligned}$$

That is, $\lim_{n\to\infty} \mathrm{P}\{T(c_d\sqrt{n}) \le \hat{t}(n)\} = 1$, or equivalently, $\lim_{n\to\infty} \mathrm{P}\{T(c_d\sqrt{n}) > \hat{t}(n)\} = 0$, which proves the lemma.

First, consider the case of $\alpha = 2$. We substitute $r = c_d\sqrt{n}$ and $t = \hat{t}(n)$ into (18). Then, the series on the right-hand side of (18) becomes a function of n, and (for notational convenience) we let

$$\begin{aligned} \mathrm{P}\{T_x(c_d\sqrt{n}) \le \hat{t}(n)\} &= 1 - \sum_{i=1}^{\infty} \eta_i \exp\left(-\frac{\rho_i}{(c_d)^2 n} \hat{t}(n)\right) \\ &\triangleq 1 - \hat{S}(n). \end{aligned}$$

We now need to take a limit to $\hat{S}(n)$. To validate the interchange of the order of limit and summation, we will show that there exists a constant $\hat{n} \in \mathbb{N}$ such that the infinite series $\hat{S}(n)$ converges uniformly on $\hat{\mathcal{D}} \triangleq [\hat{n}, \infty)$.[10] The uniform convergence will be shown by using the well-known Weierstrass M test [29].

Since $\hat{t}(n) = \Theta(n^{1+\epsilon})$, there exist constants $\hat{n} \in \mathbb{N}$ and $\hat{c} > 0$ such that

$$\hat{t}(n) \geq \hat{c}n^{1+\epsilon} \qquad \text{for all } n \geq \hat{n}. \tag{20}$$

Let $\hat{m} \triangleq F(\pi/2c_d)^2\hat{c}(\hat{n})^\epsilon\ (> 0)$. Then, the ith function of the series $\hat{S}(n)$ is bounded by a constant $\hat{M}_i \triangleq \frac{4}{\pi}\{\exp(-\hat{m})\}^i$ for all $n \geq \hat{n}$ as follows:

$$\begin{aligned}\left|\eta_i \exp\left(-\frac{\rho_i}{(c_d)^2 n}\hat{t}(n)\right)\right| &\leq \frac{4}{\pi}\exp\left(-\frac{\rho_i}{(c_d)^2 n}\hat{c}n^{1+\epsilon}\right)\\ &\leq \frac{4}{\pi}\exp\left(-\frac{Fi(\pi)^2}{4(c_d)^2}\hat{c}(\hat{n})^\epsilon\right)\\ &= \hat{M}_i.\end{aligned}$$

Here, the first inequality comes from the bounds $|\eta_i| \leq \frac{4}{\pi}\ \forall i \in \mathbb{N}$ and (20), and the second inequality comes from the bounds $i^2 \geq i\ \forall i \in \mathbb{N}$ and $n^\epsilon \geq (\hat{n})^\epsilon\ \forall n \geq \hat{n}$. Note that the series $\sum_{i=1}^\infty \hat{M}_i$ converges since it is a geometric series with a common ratio $\exp(-\hat{m}) \in (0, 1)$. Since the target of the functions is a complete normed vector space, the infinite series $\hat{S}(n)$ converges uniformly on $\hat{\mathcal{D}}$. Consequently, we can interchange the order of limit and summation, and we have

$$\begin{aligned}&\lim_{n\to\infty} \mathrm{P}\{T_x(c_d\sqrt{n}) \leq \hat{t}(n)\}\\ &\quad= 1 - \lim_{n\to\infty}\hat{S}(n)\\ &\quad= 1 - \sum_{i=1}^\infty \eta_i \lim_{n\to\infty}\exp\left(-\frac{\rho_i}{(c_d)^2 n}\hat{t}(n)\right).\end{aligned}$$

Since $\hat{t}(n) = \Theta(n^{1+\epsilon})$, we furthermore have

$$\lim_{n\to\infty}\exp\left(-\frac{\rho_i}{(c_d)^2 n}\hat{t}(n)\right) = 0$$

which gives $\lim_{n\to\infty} \mathrm{P}\{T_x(c_d\sqrt{n}) \leq \hat{t}(n)\} = 1$. This completes the proof for $\alpha = 2$.

Next, consider the case of $\alpha \in (0, 2)$. Similarly to the proof for $\alpha = 2$, we can prove this case by substituting $r = c_d\sqrt{n}$ and $t = \hat{t}(n)$ into (18) and showing that

$$\lim_{n\to\infty} \mathrm{P}\{T_x(c_d\sqrt{n}) \leq \hat{t}(n)\} = 1. \tag{21}$$

Since the dominant decay rate $|\lambda_1|$ scales as $\Theta(r^{-\alpha}) = \Theta(n^{-\frac{\alpha}{2}})$, by using approaches in the proof for $\alpha = 2$, we can show (21). Due to similarities, we omit the details. □

Lemma 4 (Lower Bound for Lévy Flight): Suppose that under Lévy flight with a distribution parameter $\alpha \in (0, 2]$, the time $t \triangleq \tilde{t}(n)$ in $\mathrm{P}\{T(c_d\sqrt{n}) > t\}$ scales as $\Theta(n^{\frac{\alpha}{2}-\epsilon})$ for an arbitrary $\epsilon > 0$. Then, we have

$$\lim_{n\to\infty} \mathrm{P}\{T(c_d\sqrt{n}) > \tilde{t}(n)\} = 1$$

which shows that the critical delay $C_\Pi(n)$ under Lévy flight scales as $\Omega(n^{\frac{\alpha}{2}})$.

Proof: We will prove this lemma by showing that $\lim_{n\to\infty} \mathrm{P}\{T_x(c_d\sqrt{n}/\sqrt{2}) \leq \tilde{t}(n)\} = 0$. Then, by substituting $r = c_d\sqrt{n}$ and $t = \tilde{t}(n)$ into (6) and taking a limit to n, we obtain

$$\begin{aligned}&\lim_{n\to\infty} \mathrm{P}\{T(c_d\sqrt{n}) \leq \tilde{t}(n)\}\\ &\quad\leq 2\lim_{n\to\infty} \mathrm{P}\left\{T_x\left(c_d\sqrt{n}/\sqrt{2}\right) \leq \tilde{t}(n)\right\}\\ &\quad= 0.\end{aligned}$$

That is, $\lim_{n\to\infty} \mathrm{P}\{T(c_d\sqrt{n}) \leq \tilde{t}(n)\} = 0$, or equivalently, $\lim_{n\to\infty} \mathrm{P}\{T(c_d\sqrt{n}) > \tilde{t}(n)\} = 1$, which proves the lemma.

First, consider the case of $\alpha = 2$. We substitute $r = c_d\sqrt{n}/\sqrt{2}$ and $t = \tilde{t}(n)$ into (18). Then, the series on the right-hand side of (18) becomes a function of n, and analogously to the proof of Lemma 3, we let

$$\begin{aligned}&\mathrm{P}\left\{T_x\left(c_d\sqrt{n}/\sqrt{2}\right) \leq \tilde{t}(n)\right\}\\ &\quad= 1 - \sum_{i=1}^\infty \eta_i\exp\left(-\frac{2\rho_i}{(c_d)^2 n}\tilde{t}(n)\right)\\ &\quad\triangleq 1 - \tilde{S}(n).\end{aligned}$$

Similarly to the proof of Lemma 3, we will show that there exists a constant $\tilde{n} \in \mathbb{N}$ such that the infinite series $\tilde{S}(n)$ converges uniformly on $\tilde{\mathcal{D}} \triangleq [\tilde{n}, \infty)$.

Since $\tilde{t}(n) = \Theta(n^{1-\epsilon})$, there exist constants $\tilde{n} \in \mathbb{N}$ and $\tilde{c} > 0$ such that

$$\tilde{t}(n) \geq \tilde{c}n^{1-\epsilon} \qquad \text{for all } n \geq \tilde{n}. \tag{22}$$

For a technical purpose for showing the uniform convergence, we restrict the domain of n as $\tilde{\mathcal{D}}_d \triangleq [\tilde{n}, d]$ for an arbitrary $d \geq \tilde{n}$. Let $\tilde{m} \triangleq F(\pi/\sqrt{2}c_d)^2\tilde{c}d^{-\epsilon}$. Then, the ith function of the series $\tilde{S}(n)$ is bounded by a constant $\tilde{M}_i \triangleq \frac{4}{\pi}\{\exp(-\tilde{m})\}^i$ for all $n \in \tilde{\mathcal{D}}_d$ as follows:

$$\begin{aligned}\left|\eta_i \exp\left(-\frac{2\rho_i}{(c_d)^2 n}\tilde{t}(n)\right)\right| &\leq \frac{4}{\pi}\exp\left(-\frac{2\rho_i}{(c_d)^2 n}\tilde{c}n^{1-\epsilon}\right)\\ &\leq \frac{4}{\pi}\exp\left(-\frac{Fi(\pi)^2}{2(c_d)^2}\tilde{c}d^{-\epsilon}\right)\\ &= \tilde{M}_i.\end{aligned}$$

Here, the first inequality comes from the bounds $|\eta_i| \leq \frac{4}{\pi}\ \forall i \in \mathbb{N}$ and (22), and the second inequality comes from the bounds $i^2 \geq i\ \forall i \in \mathbb{N}$ and $n^{-\epsilon} \geq d^{-\epsilon}\ \forall n \in \tilde{\mathcal{D}}_d$. Note that the series $\sum_{i=1}^\infty \tilde{M}_i$ converges since it is a geometric series with a common ratio $\exp(-\tilde{m}) \in (0, 1)$. Hence, the infinite series $\tilde{S}(n)$ converges uniformly on $\tilde{\mathcal{D}}_d$. Since d is arbitrary, we get uniform convergence on $\tilde{\mathcal{D}}$. Consequently, we can interchange the order of limit and summation, and we have

$$\begin{aligned}&\lim_{n\to\infty} \mathrm{P}\left\{T_x\left(c_d\sqrt{n}/\sqrt{2}\right) \leq \tilde{t}(n)\right\}\\ &\quad= 1 - \lim_{n\to\infty}\tilde{S}(n)\\ &\quad= 1 - \sum_{i=1}^\infty \eta_i \lim_{n\to\infty}\exp\left(-\frac{2\rho_i}{(c_d)^2 n}\tilde{t}(n)\right).\end{aligned}$$

Since $\tilde{t}(n) = \Theta(n^{1-\epsilon})$, we furthermore have

$$\lim_{n\to\infty}\exp\left(-\frac{2\rho_i}{(c_d)^2 n}\tilde{t}(n)\right) = 1$$

[10] $\mathbb{N}$ denotes a set of positive integers.

which gives

$$\lim_{n\to\infty} \mathrm{P}\left\{T_x\left(c_d\sqrt{n}/\sqrt{2}\right) \le \tilde{t}(n)\right\} = 1 - \sum_{i=1}^{\infty} \eta_i.$$

Note from (18) that $\mathrm{P}\{T_x(c_d\sqrt{n}/\sqrt{2}) \le 0\} = 1 - \sum_{i=1}^{\infty} \eta_i$. In addition, it is obvious that $\mathrm{P}\{T_x(c_d\sqrt{n}/\sqrt{2}) \le 0\} = 0$. Therefore, we have $\lim_{n\to\infty} \mathrm{P}\{T_x(c_d\sqrt{n}/\sqrt{2}) \le \tilde{t}(n)\} = 0$. This completes the proof for $\alpha = 2$.

Next, consider the case of $\alpha \in (0,2)$. Similarly to the proof for $\alpha = 2$, we can prove this case by substituting $r = c_d\sqrt{n}/\sqrt{2}$ and $t = \tilde{t}(n)$ into (18) and showing that

$$\lim_{n\to\infty} \mathrm{P}\left\{T_x\left(c_d\sqrt{n}/\sqrt{2}\right) \le \tilde{t}(n)\right\} = 0. \tag{23}$$

Since the dominant decay rate $|\lambda_1|$ scales as $\Theta(r^{-\alpha}) = \Theta(n^{-\frac{\alpha}{2}})$, by using approaches in the proof for $\alpha = 2$, we can show (23). Due to similarities, we omit the details. □

Combining Lemmas 3 and 4 yields the following theorem.

Theorem 1: The critical delay $C_\Pi(n)$ under Lévy flight with a distribution parameter $\alpha \in (0,2]$ scales as $\Theta(n^{\frac{\alpha}{2}})$.

Remark 1: The main idea behind the proof of Lemmas 3 and 4 was that the smallest decay rate in the exponential functions in (18) (i.e., $\frac{\rho_1}{r^\alpha}$) determines the limiting behavior of the first exit time distribution. That is, the smallest decay rate characterizes the critical delay under Lévy flight.

Remark 2: Overall, the framework of this paper is to exploit the relation between the critical delay and the first exit time of a mobile node as in [5] and [6]. This framework is general in that it is applicable to any stationary mobility models. Hence, if one can have a formula for the first exit time distribution of a mobile node, then this framework (especially our technique in Step 3) can be used to analyze the critical delay. Moreover, if one can have either the occupation probability or the survival probability, then direct application of our framework in Steps 1–3 in sequence can give the order of the critical delay.

VI. Critical Delay Analysis for Lévy Walk

In this section, we will show that the critical delay $C_\Pi(n)$ under Lévy walk with a distribution parameter α scales as $\Theta(n^{\frac{1}{2}})$ for $\alpha \in (0,1)$ and $\Theta(n^{\frac{\alpha}{2}})$ for $\alpha \in [1,2]$ (Theorem 2). In Section VI-A, we explain technical challenges and our approach for proving Theorem 2. In Section VI-B, we prove Theorem 2 by showing that the upper bound on $C_\Pi(n)$ scales as $O(n^{\frac{1}{2}})$ for $\alpha \in (0,1)$ and $O(n^{\frac{\alpha}{2}})$ for $\alpha \in [1,2]$ (Lemma 6), and that the lower bound on $C_\Pi(n)$ scales as $\Omega(n^{\frac{1}{2}})$ for $\alpha \in (0,1)$ and $\Omega(n^{\frac{\alpha}{2}})$ for $\alpha \in [1,2]$ (Lemma 7).

A. Technical Approach

We first explain the technical challenges that preclude the use of our technique for Lévy flight as well as other conventional techniques. We next explain our technical approach to deal with these challenges. The technical challenges are twofold and are mainly inherent in the Lévy walk nature.

1) We begin with the description of differences between Lévy flight and Lévy walk from a modeling perspective. Let t_i $(i = 1,2,\ldots)$ denote the time instant when the ith step begins. We take the time t_i as the embedded point of the process $\{\boldsymbol{X}(t)\}_{t\ge 0}$, and focus on the corresponding embedded process $\{\boldsymbol{E}_i\}_{i\in\mathbb{N}} \triangleq \{\boldsymbol{X}(t_i)\}_{i\in\mathbb{N}}$. Under both Lévy mobility models, at each embedded point t_i, the destination of the next step of the ith step (i.e., $\boldsymbol{E}_{i+1}$) is chosen independently of the past locations at time $t < t_i$ and depends only on the current location at time $t = t_i$. That is, the embedded process $\{\boldsymbol{E}_i\}_{i\in\mathbb{N}}$ satisfies the following Markov property:
$$\begin{aligned}\mathrm{P}\{&\boldsymbol{E}_{i+1} = \boldsymbol{x}_{i+1} \,|\, \boldsymbol{E}_j = \boldsymbol{x}_j, j = 1,\ldots,i\}\\ &= \mathrm{P}\{\boldsymbol{E}_{i+1} = \boldsymbol{x}_{i+1} \,|\, \boldsymbol{E}_i = \boldsymbol{x}_i\}.\end{aligned}$$
Thus, under both Lévy mobility models, the process $\{\boldsymbol{X}(t_i)\}_{i\in\mathbb{N}}$ becomes a discrete-time Markov chain. However, the fact that the embedded point t_i is chosen in a different way for Lévy flight and Lévy walk incurs the key challenge. In the case of Lévy flight, it is chosen deterministically as $t_i = i - 1$. Therefore, Lévy flight is a discrete-time Markov process. However, in the case of the Lévy walk, the embedded point is chosen stochastically and is correlated with step size as follows: $t_i = \sum_{j=1}^{i-1} Z_j$ (where Z_j is a random variable denoting the jth step size). Therefore, the Lévy walk is a semi-Markov process [21] whose embedded process becomes Lévy flight.
2) The proof of Lemma 2 also shows that, for a given two-dimensional Lévy walk, its one-dimensional projected processes also have a power-law type of step-size distribution. However, the velocity of the projected processes is not a constant for every step, which implies that neither of one-dimensional projected processes of two-dimensional Lévy walk can be one-dimensional Lévy walk.
Consequently, the technique used for Lévy flight in this paper is not applicable because it requires decoupling of space and time. In addition, the occupation probability $P(x,t)$ is not available and the derivation is not mathematically tractable.
To cope with these technical challenges, we propose a different approach based on a stochastic analysis technique characterizing the embedded Markov process of a semi-Markov process. Specifically, our approach is to derive a relation between the first exit time under Lévy flight (i.e., embedded Markov process) and that under Lévy walk (i.e., semi-Markov process). From this relation, our technique derives a tight upper bound for the critical delay. Then, by combining the upper bound and a lower bound for the critical delay inferred from our analytical result of Lévy flight in Section V, we can provide the exact order of the critical delay under Lévy walk.

B. Analysis

Let $N(n)$ be a random variable denoting the number of steps occurred until $t \le T(c_d\sqrt{n})$. Then

$$T(c_d\sqrt{n}) = \begin{cases} c_d\sqrt{n}, & \text{if } N(n) = 1\\ \sum\limits_{i=1}^{N(n)-1} Z_i + \bar{Z}_{N(n)}, & \text{if } N(n) \ge 2 \end{cases} \tag{24}$$

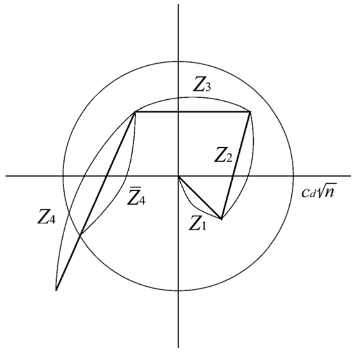


Fig. 2. Example of the random variables $N(n)\,(=4)$, Z_i and $\bar{Z}_{N(n)}$.

where $\bar{Z}_{N(n)}$ is a random variable denoting the moving distance during the $N(n)$th step until exiting the disc $\mathcal{D}(n)$ (see Fig. 2). Note that $\bar{Z}_{N(n)}$ is not identically distributed with Z_i, and we have

$$\bar{Z}_{N(n)} < 2c_d\sqrt{n} \qquad \text{with probability } 1. \tag{25}$$

The random variable $N(n)$ is closely related to the first exit time under Lévy flight, denoted by $T_{\rm LF}(c_d\sqrt{n})$, as follows:

$$N(n) \overset{\rm d}{=} \lceil T_{\rm LF}(c_d\sqrt{n}) \rceil \tag{26}$$

where $\lceil x \rceil$ denotes the smallest integer larger than or equal to x. In Lemma 5, we derive the order of $\mathrm{E}[N(n)]$, which will be used to study the critical delay under Lévy walk.

Lemma 5: $\mathrm{E}[N(n)]$ scales as $\Theta(n^{\frac{\alpha}{2}})$ for $\alpha \in (0,2]$.

Proof: From Lemma 3, we have $\lim_{n\to\infty} \mathrm{P}\{T_{\rm LF}(c_d\sqrt{n}) \le \hat{t}(n)\} = 1$ when $\hat{t}(n) = \Theta(n^{\frac{\alpha}{2}+\hat{\epsilon}})$ for $\alpha \in (0,2]$ and an arbitrary $\hat{\epsilon} > 0$. Hence, we have

$$\mathrm{E}[T_{\rm LF}(c_d\sqrt{n})] = O(n^{\frac{\alpha}{2}+\hat{\epsilon}}). \tag{27}$$

From Lemma 4, we have $\lim_{n\to\infty} \mathrm{P}\{T_{\rm LF}(c_d\sqrt{n}) > \tilde{t}(n)\} = 1$ when $\tilde{t}(n) = \Theta(n^{\frac{\alpha}{2}-\tilde{\epsilon}})$ for $\alpha \in (0,2]$ and an arbitrary $\tilde{\epsilon} > 0$. Thus, we have

$$\mathrm{E}[T_{\rm LF}(c_d\sqrt{n})] = \Omega(n^{\frac{\alpha}{2}-\tilde{\epsilon}}). \tag{28}$$

By choosing $\hat{\epsilon}$ and $\tilde{\epsilon}$ arbitrarily small, from (27) and (28), we have

$$\mathrm{E}[T_{\rm LF}(c_d\sqrt{n})] = \Theta(n^{\frac{\alpha}{2}}) \qquad \forall \alpha \in (0,2]. \tag{29}$$

Note from (26) that

$$\mathrm{E}[T_{\rm LF}(c_d\sqrt{n})] \le \mathrm{E}[N(n)] \le \mathrm{E}[T_{\rm LF}(c_d\sqrt{n})] + 1 \tag{30}$$

which shows that the order of $\mathrm{E}[N(n)]$ is the same as that of $\mathrm{E}[T_{\rm LF}(c_d\sqrt{n})]$. Therefore, combining (29) and (30) yields the lemma. $\square$

With the help of Lemma 5, we can derive an upper bound for the critical delay under Lévy walk.

Lemma 6 (Upper Bound for Lévy Walk): Suppose that under Lévy walk with a distribution parameter α, the time $t \triangleq \hat{t}(n)$ in $\mathrm{P}\{T(c_d\sqrt{n}) > t\}$ scales as $\Theta(n^{\frac{1}{2}+\epsilon_1})$ for an arbitrary $\epsilon_1 > 0$ and $\alpha \in (0,1)$, and $\Theta(n^{\frac{\alpha}{2}+\epsilon_2})$ for an arbitrary $\epsilon_2 > 0$ and $\alpha \in [1,2]$. Then, we have

$$\lim_{n\to\infty} \mathrm{P}\{T(c_d\sqrt{n}) > \hat{t}(n)\} = 0$$

which shows that the critical delay $C_\Pi(n)$ under Lévy walk scales as $O(n^{\frac{1}{2}})$ for $\alpha \in (0,1)$ and $O(n^{\frac{\alpha}{2}})$ for $\alpha \in [1,2]$.

Proof: Using Markov's inequality [30], we have

$$\mathrm{P}\{T(c_d\sqrt{n}) > \hat{t}(n)\} \le \frac{\mathrm{E}[T(c_d\sqrt{n})]}{\hat{t}(n)}. \tag{31}$$

We calculate the expectation $\mathrm{E}[T(c_d\sqrt{n})]$ on the right-hand side of (31) by conditioning on the values of $N(n)$ as

$$\begin{aligned}\mathrm{E}[T(c_d\sqrt{n})] &= \mathrm{E}\big[\mathrm{E}[T(c_d\sqrt{n})|N(n)]\big] \\ &= \sum_{k=1}^{\infty} \mathrm{E}[T(c_d\sqrt{n}) \,|\, N(n)=k] \cdot \mathrm{P}\{N(n)=k\}.\end{aligned} \tag{32}$$

From (24), we have for $k=1$,

$$\mathrm{E}[T(c_d\sqrt{n}) \,|\, N(n)=k] = c_d\sqrt{n}. \tag{33}$$

In addition, from (24), we have for $k = 2,3,\ldots$

$$\begin{aligned}&\mathrm{E}[T(c_d\sqrt{n}) \,|\, N(n)=k] \\ &\quad= \mathrm{E}\left[\sum_{i=1}^{N(n)-1} Z_i + \bar{Z}_{N(n)} \,\middle|\, N(n)=k\right] \\ &\quad= \sum_{i=1}^{k-1} \mathrm{E}[Z_i \,|\, N(n)=k] + \mathrm{E}[\bar{Z}_k \,|\, N(n)=k].\end{aligned} \tag{34}$$

The random variables $Z_i\,(i=1,\ldots,k-1)$ and $\bar{Z}_k$ in (34) are correlated with the random variable $N(n)\,(=k)$, whereas the random variables $Z_i\,(i=k+1,k+2,\ldots)$ are independent of $N(n)\,(=k)$. Specifically, for $i=1,\ldots,k-1$, the step size Z_i should be less than the diameter of the disc $\mathcal{D}(n)$ (i.e., $2c_d\sqrt{n}$). In addition, the truncated step size $\bar{Z}_k$ should satisfy the inequality in (25). Hence, the conditional expectations on the right-hand side of (34) are bounded as follows:

$$\begin{aligned}\mathrm{E}[Z_i|N(n)=k] &\le \mathrm{E}[Z|Z \le 2c_d\sqrt{n}] \\ \mathrm{E}[\bar{Z}_k|N(n)=k] &\le 2c_d\sqrt{n}\end{aligned} \tag{35}$$

where Z denotes the generic random variable for Z_i.[11] Combining (32)–(35), we obtain an upper bound for $\mathrm{E}[T(c_d\sqrt{n})]$ as follows:

$$\begin{aligned}&\mathrm{E}[T(c_d\sqrt{n})] \\ &\quad\le c_d\sqrt{n}\cdot\mathrm{P}\{N(n)=1\} + 2c_d\sqrt{n}\sum_{k=2}^{\infty}\mathrm{P}\{N(n)=k\} \\ &\qquad + \mathrm{E}[Z|Z \le 2c_d\sqrt{n}]\sum_{k=2}^{\infty}(k-1)\cdot\mathrm{P}\{N(n)=k\}\end{aligned}$$

[11] $\mathrm{E}[Z_i \,|\, N(n)=k] = \mathrm{E}[Z]$ for $i=k+1,k+2,\ldots$.

$$\leq 2c_d\sqrt{n}\sum_{k=1}^{\infty}\mathrm{P}\{N(n)=k\} + \mathrm{E}[Z|Z\leq 2c_d\sqrt{n}]\sum_{k=1}^{\infty}k\cdot\mathrm{P}\{N(n)=k\}$$
$$= 2c_d\sqrt{n}+\mathrm{E}[Z|Z\leq 2c_d\sqrt{n}]\cdot\mathrm{E}[N(n)]. \tag{36}$$

Using (3), we can calculate the conditional expectation $\mathrm{E}[Z|Z \leq 2c_d\sqrt{n}]$ in (36), and it scales for each $\alpha \in (0,2]$ as follows:

$$\mathrm{E}[Z|Z\leq 2c_d\sqrt{n}] = \begin{cases} \frac{\alpha}{1-\alpha}\frac{(2c_d\sqrt{n})^{1-\alpha}-1}{1-(2c_d\sqrt{n})^{-\alpha}}, & \text{for } \alpha\in(0,1)\\ \frac{\log(2c_d\sqrt{n})}{1-(2c_d\sqrt{n})^{-1}}, & \text{for } \alpha=1\\ \frac{\alpha}{\alpha-1}\frac{1-(2c_d\sqrt{n})^{1-\alpha}}{1-(2c_d\sqrt{n})^{-\alpha}}, & \text{for } \alpha\in(1,2)\\ \frac{\sqrt{2}\sigma}{\sqrt{\pi}}\frac{\exp\left(\frac{-1}{2\sigma^2}\right)-\exp\left(-2(c_d)^2n/\sigma^2\right)}{\mathrm{erf}\left(c_d\sqrt{2n}/\sigma\right)-\mathrm{erf}\left(1/\sqrt{2}\sigma\right)}, & \text{for } \alpha=2 \end{cases} = \begin{cases}\Theta(n^{(1-\alpha)/2}), & \text{for } \alpha\in(0,1)\\ \Theta(\log(n)), & \text{for } \alpha=1\\ \Theta(n^0), & \text{for } \alpha\in(1,2].\end{cases}$$

Since $\mathrm{E}[N(n)]$ scales as $\Theta(n^{\frac{\alpha}{2}})$ by Lemma 5, the term on the right-hand side of (36) scales as

$$2c_d\sqrt{n}+\mathrm{E}[Z|Z\leq 2c_d\sqrt{n}]\cdot\mathrm{E}[N(n)] = \begin{cases}\Theta(n^{\frac{1}{2}}), & \text{for } \alpha\in(0,1)\\ \Theta(n^{\frac{1}{2}}\log(n)), & \text{for } \alpha=1\\ \Theta(n^{\frac{\alpha}{2}}), & \text{for } \alpha\in(1,2].\end{cases} \tag{37}$$

Thus, we have from (36) and (37) the following:

$$\lim_{n\to\infty}\frac{\mathrm{E}[T(c_d\sqrt{n})]}{\hat{t}(n)} \leq \lim_{n\to\infty}\frac{2c_d\sqrt{n}+\mathrm{E}[Z|Z\leq 2c_d\sqrt{n}]\cdot\mathrm{E}[N(n)]}{\hat{t}(n)} = 0.$$

Therefore, from (31), we have

$$\lim_{n\to\infty}\mathrm{P}\{T(c_d\sqrt{n})>\hat{t}(n)\}\leq\lim_{n\to\infty}\frac{\mathrm{E}[T(c_d\sqrt{n})]}{\hat{t}(n)}\leq 0$$

i.e., $\lim_{n\to\infty}\mathrm{P}\{T(c_d\sqrt{n}) > \hat{t}(n)\} = 0$. This completes the proof. □

Lemma 7 (Lower Bound for Lévy Walk): Suppose that under Lévy walk with a distribution parameter α, the time $t \triangleq \tilde{t}(n)$ in $\mathrm{P}\{T(c_d\sqrt{n}) > t\}$ scales as $\Theta(n^{\frac{1}{2}-\epsilon_1})$ for an arbitrary $\epsilon_1 > 0$ and $\alpha \in (0,1)$, and $\Theta(n^{\frac{\alpha}{2}-\epsilon_2})$ for an arbitrary $\epsilon_2 > 0$ and $\alpha \in (1,2]$. Then, we have

$$\lim_{n\to\infty}\mathrm{P}\{T(c_d\sqrt{n})>\tilde{t}(n)\}=1$$

which shows that the critical delay $C_\Pi(n)$ under Lévy walk scales as $\Omega(n^{\frac{1}{2}})$ for $\alpha \in (0,1)$ and $\Omega(n^{\frac{\alpha}{2}})$ for $\alpha \in [1,2]$.

Proof: We will prove this lemma by showing for each of the cases of $\alpha \in (0,1)$ and $\alpha \in [1,2]$ that

$$\lim_{n\to\infty}\mathrm{P}\{T(c_d\sqrt{n})\leq\tilde{t}(n)\}=0.$$

We first consider the case of $\alpha \in (0,1)$. Since a Lévy walker moves with a constant velocity $v = 1$, it takes at least $c_d\sqrt{n}$ time to exit from the disc $\mathcal{D}(n)$. Thus, it is obvious that

$$\mathrm{P}\{T(c_d\sqrt{n})<c_d\sqrt{n}\}=0. \tag{38}$$

Since $\tilde{t}(n) = \Theta(n^{\frac{1}{2}-\epsilon_1})$, there exists a constant $\tilde{n} \in \mathbb{N}$ such that $\tilde{t}(n) < c_d\sqrt{n}$ for $n \geq \tilde{n}$. Hence, we have for $n \geq \tilde{n}$

$$\mathrm{P}\{T(c_d\sqrt{n})\leq\tilde{t}(n)\}\leq\mathrm{P}\{T(c_d\sqrt{n})<c_d\sqrt{n}\}. \tag{39}$$

Combining (38) and (39) and then taking limits, we have

$$\lim_{n\to\infty}\mathrm{P}\{T(c_d\sqrt{n})\leq\tilde{t}(n)\}\leq\lim_{n\to\infty}\mathrm{P}\{T(c_d\sqrt{n})<c_d\sqrt{n}\} = 0,$$

i.e., $\lim_{n\to\infty}\mathrm{P}\{T(c_d\sqrt{n}) \leq \tilde{t}(n)\} = 0$. We have proved the lemma in the case of $\alpha \in (0,1)$.

We next consider the case of $\alpha \in [1,2]$. In the following, we use the notations $T_{\mathrm{LF}}(\cdot)$ and $T_{\mathrm{LW}}(\cdot)$ to distinguish the first exit times between Lévy flight and Lévy walk. We will show based on (24) and (26) that for $t \geq 0$

$$\mathrm{P}\{T_{\mathrm{LW}}(c_d\sqrt{n})\leq t\}\leq\mathrm{P}\{T_{\mathrm{LF}}(c_d\sqrt{n})\leq t+1\}. \tag{40}$$

From (24), if $N(n) = 1$, then $T_{\mathrm{LW}}(c_d\sqrt{n}) = c_d\sqrt{n} > 0 = N(n) - 1$. In addition, if $N(n) \geq 2$, then $T_{\mathrm{LW}}(c_d\sqrt{n}) = \sum_{i=1}^{N(n)-1} Z_i + \hat{Z}_N > \sum_{i=1}^{N(n)-1} Z_i \geq N(n) - 1$, where the last inequality comes from the assumption that the step size Z has a lower bound at 1 (given in Section IV). Combining above two cases gives

$$T_{\mathrm{LW}}(c_d\sqrt{n})\geq N(n)-1.$$

From (26), we obtain $N(n) \overset{\mathrm{d}}{=} \lceil T_{\mathrm{LF}}(c_d\sqrt{n})\rceil \geq T_{\mathrm{LF}}(c_d\sqrt{n})$. Thus, we have with probability 1

$$T_{\mathrm{LW}}(c_d\sqrt{n})\geq T_{\mathrm{LF}}(c_d\sqrt{n})-1.$$

This proves (40). Substituting $t = \tilde{t}(n)$ into (40), we obtain

$$\mathrm{P}\{T_{\mathrm{LW}}(c_d\sqrt{n})\leq\tilde{t}(n)\}\leq\mathrm{P}\{T_{\mathrm{LF}}(c_d\sqrt{n})\leq\tilde{t}(n)+1\}. \tag{41}$$

For $\tilde{t}(n)$ scaling as $\Theta(n^{\frac{\alpha}{2}-\epsilon_2})$, $\tilde{t}(n)+1$ also scales as $\Theta(n^{\frac{\alpha}{2}-\epsilon_2})$. Consequently, by Lemma 4, the probability on the right-hand side of (41) becomes in the limit

$$\lim_{n\to\infty}\mathrm{P}\{T_{\mathrm{LF}}(c_d\sqrt{n})\leq\tilde{t}(n)+1\}=0.$$

Therefore, from (41), we have $\lim_{n\to\infty}\mathrm{P}\{T_{\mathrm{LW}}(c_d\sqrt{n}) \leq \tilde{t}(n)\} = 0$, which proves the lemma in the case of $\alpha \in [1,2]$. This completes the proof. □

Combining Lemmas 6 and 7 yields the following theorem.

Theorem 2: The critical delay $C_\Pi(n)$ under Lévy walk with a distribution parameter α scales as $\Theta(n^{\frac{1}{2}})$ for $\alpha \in (0,1)$ and $\Theta(n^{\frac{\alpha}{2}})$ for $\alpha \in [1,2]$.

VII. Discussion

We summarize the high-level interpretations of this paper. Fig. 3. shows the critical delays under Lévy walk and Lévy flight, parameterized by α. Lévy flight shows that the critical

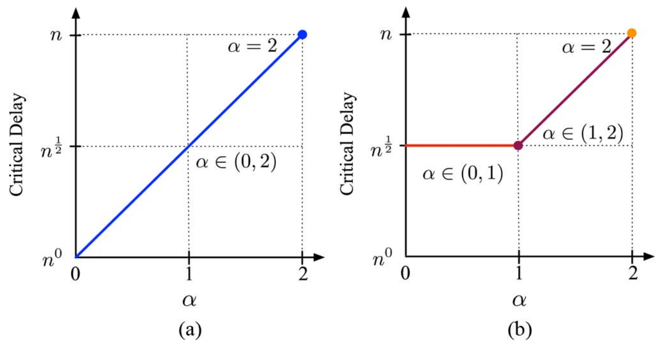


Fig. 3. Critical delays under (a) Lévy flight and (b) Lévy walk for different α.

TABLE II
EXPERIMENTAL α VALUES FOR DIFFERENT SITES PRESENTED IN [7]

Site	α	Site	α
KAIST	0.53	New York City	1.62
NCSU	1.27	Disney World	1.20
		State fair	1.81

delay proportionally increases with α. However, in the case of the Lévy walk, we can find a phase transition such that when $\alpha \in (0, 1)$, the critical delay is constantly $\Theta(n^{\frac{1}{2}})$ and shifts to the proportional increasing phase when $\alpha \in [1, 2]$. Two different scaling regions are essentially related to the fact that the mean step size of Lévy walk for $\alpha \in (0, 1)$ is infinite but finite for $\alpha \in [1, 2]$. In contrast to Lévy walk, the travel time independence of step size in Lévy flight leads to continuous scaling over α. Note that for $\alpha = 2$ (i.e., Brownian motion) our result coincides with that in [6] which also studied the critical delay under Brownian motion.

By using values of α from experimental measurements from [7], we can see how network delay scales with human mobility in practice. To give an insight to the readers, we show α values measured from five different sites in Table II presented in [7] with a flight extraction method, "rectangle."[12] We see that critical delays for human mobility range from $\Theta(n^{0.27})$ to $\Theta(n^{0.91})$. Human mobility mainly has $\alpha > 1$, in which case a longer delay than $\Theta(\sqrt{n})$ is needed. This implies that it may be hard to design a low-delay protocol for mobile networks under human mobility.

Our contribution is not restricted to the mathematical derivation of delay scaling for new mobility models. We provided techniques that connect the diffusion equation of a continuous time random walk to the delay scaling as well as that analyze the delay scaling of semi-Markovian movements. We expect that our techniques can be further developed to the analysis of other detailed performance metrics such as contact time distribution and the generalized delay-capacity tradeoff for various levels of per-node throughput.

Future work includes investigation of throughput and delay scaling for mobile networks with heterogeneous and collective node mobilities. In addition to the recent research topics on "per-node throughput scaling" under inhomogeneous spatial node distributions (i.e., Cox process, Neyman–Scott process, Matérn cluster process, and Thomas process), e.g., [31] and [32], our paper can be an important step to the study of delay scaling under such heterogeneous networks. There is an insight from [8] that in human-assisted networks, the actual delays might be even shorter. This is because human mobility is not completely random: People tend to visit the same locations and regularly meet a group of people every day. Although their mobility can be characterized by heavy-tail distributions, these regularities in daily mobility significantly facilitate routing of packets among people (as long as they are socially connected). Therefore, there remains a possibility of designing a low-delay protocol for mobile networks under heterogeneous human mobility by judiciously utilizing these social factors.

[12]We do not present α values from other extraction methods in [7] that intentionally exclude some detailed motions of real traces. To capture specific behaviors of humans, one can borrow those α values.

VIII. CONCLUSION

We have presented Lévy mobility models consisting of Lévy flight and Lévy walk parameterized by α and studied the critical delay under both mobility models. Lévy mobility is known as a realistic human mobility so that the critical delay we provided here can be essential in designing an architecture and protocols of a wireless mobile network. The insight that the critical delay scales as $\Theta(n^{\frac{\alpha}{2}})$ for Lévy mobility models in the range of $\alpha \in [1, 2]$ is especially important because it is anticipating that the delay of mobile networks with human mobility (e.g., smartphone networks, pocket switched networks) could be quite high in practice, considering the α values measured in real traces. The insight tells that mobile networks operated by human mobility patterns may need to prepare an alternative path for delay-sensitive data as well as even for delay-tolerable data whose tolerance level is limited.

APPENDIX A
CRITICAL DELAY ANALYSIS FOR LÉVY FLIGHT WITHOUT CONTINUOUS LIMIT

In Section V, we have studied the critical delay under Lévy flight using continuous limit. By following the technique in [6], we can study the critical delay without continuous limit (i.e., with a nonzero scale factor $|c|$) and can derive a lower bound for the critical delay under Lévy flight. Lemma 8 summarizes the result.

Lemma 8: With a nonzero scale factor $|c|$, the critical delay $C_\Pi(n)$ under Lévy flight with a distribution parameter $\alpha \in (0, 2]$ scales as $\Omega(n^{\frac{\alpha}{2}})$.

Before proving the lemma, we give a remark. The scaling property of the critical delay with continuous limit (shown in Theorem 1) works as an upper bound for the one without continuous limit. Hence, the result in Lemma 8 shows that our analysis in Section V gives the tightest upper bound, which justifies our technique using continuous limit. We now give the proof of Lemma 8.

Proof: Similarly to the proof of Lemma 4, we will prove this lemma by showing that

$$\lim_{n\to\infty} \mathrm{P}\left\{T_x\left(c_d\sqrt{n}/\sqrt{2}\right) \le \tilde{t}(n)\right\} = 0 \qquad (42)$$

where $\tilde{t}(n) = \Theta(n^{\frac{\alpha}{2}-\epsilon})$ for an arbitrary $\epsilon > 0$ and $\alpha \in (0,2]$. Then, from (6), we obtain $\lim_{n\to\infty} \mathrm{P}\{T(c_d\sqrt{n}) \le \tilde{t}(n)\} \le 2\lim_{n\to\infty} \mathrm{P}\{T_x(c_d\sqrt{n}/\sqrt{2}) \le \tilde{t}(n)\} = 0$. That is, we have $\lim_{n\to\infty} \mathrm{P}\{T(c_d\sqrt{n}) \le \tilde{t}(n)\} = 0$, or equivalently, $\lim_{n\to\infty} \mathrm{P}\{T(c_d\sqrt{n}) > \tilde{t}(n)\} = 1$, which shows that the critical delay $C_\Pi(n)$ scales as $\Omega(n^{\frac{\alpha}{2}})$.

Let $n \in \mathbb{N}$ be a fixed natural number. In the following, we will derive an upper bound on the first exit time distribution for the nth network. Without loss of generality, we assume $\mathbf{X}(0) = (0,0)$. Then, from (9), $X_x(t)$ for $t = 1, 2, \ldots$ can be expressed as

$$X_x(t) = \sum_{i=1}^{t} Z_i \cos\theta_i. \tag{43}$$

Note that $Z_i \cos\theta_i$ $(i = 1, \ldots, t)$ in (43) are i.i.d. (independent and identically distributed) random variables. Let $Z\cos\theta$ be the generic random variable for $Z_i\cos\theta_i$. Then, for each *fixed* $n \in \mathbb{N}$ (i.e., in the fixed nth network), $Z\cos\theta$ takes values in the *bounded* interval $[-\sqrt{n}, \sqrt{n}]$ since $\mathrm{P}\{1 \le Z \le \sqrt{n}\} = 1$ in our extended network model (see footnote 7 in Section IV). By the independence of random variables Z and θ, the mean of $Z\cos\theta$ is given by $\mathrm{E}[Z\cos\theta] = \mathrm{E}[Z]\mathrm{E}[\cos\theta] = 0$. Thus, for each $n \in \mathbb{N}$, $X_x(t)$ in (43) becomes a sum of i.i.d. bounded random variables having zero mean, and therefore we can apply Hoeffding's inequality [6, Lemma 8] to the sum $X_x(t)$. A straightforward application of the inequality gives an upper bound on $\mathrm{P}\{X_x(t) \ge r/\sqrt{2}\}$ for the nth network as follows:

$$\mathrm{P}\left\{X_x(t) \ge r/\sqrt{2}\right\} \le \exp\left(\frac{-r^2}{8\mathrm{Var}[X_x(t)]}\right).$$

Here, $\mathrm{Var}[X_x(t)]$ denotes the variance of $X_x(t)$ and is obtained by $\mathrm{Var}[X_x(t)] = t\mathrm{E}[(Z\cos\theta)^2]$ from the i.i.d. property of $Z_i\cos\theta_i$. By the symmetry of node motion, we further have

$$\mathrm{P}\left\{|X_x(t)| \ge r/\sqrt{2}\right\} \le 2\exp\left(-\frac{r^2}{8t\mathrm{E}[(Z\cos\theta)^2]}\right). \tag{44}$$

Due to (A1), the event $\{T_x(r/\sqrt{2}) \le k\}$ for $k = 1, 2, \ldots$ implies the event $\bigcup_{t=1}^{k}\{|X_x(t)| \ge r/\sqrt{2}\}$. Hence, we have

$$\begin{aligned}\mathrm{P}\left\{T_x\left(r/\sqrt{2}\right) \le k\right\} &\le \sum_{t=1}^{k} \mathrm{P}\left\{|X_x(t)| \ge r/\sqrt{2}\right\} \\ &\le 2\sum_{t=1}^{k} \exp\left(-\frac{r^2}{8t\mathrm{E}[(Z\cos\theta)^2]}\right) \\ &\le 2k\exp\left(-\frac{r^2}{8k\mathrm{E}[(Z\cos\theta)^2]}\right) \end{aligned}\tag{45}$$

where the second inequality comes from (44). Substituting $r = c_d\sqrt{n}$ and $k = \tilde{t}(n)$ into (45), we have

$$\begin{aligned}\mathrm{P}&\left\{T_x\left(c_d\sqrt{n}/\sqrt{2}\right) \le \tilde{t}(n)\right\} \\ &\le 2\tilde{t}(n)\exp\left(-\frac{(c_d)^2 n}{8\tilde{t}(n)\mathrm{E}[(Z\cos\theta)^2]}\right). \end{aligned}\tag{46}$$

In the following, we will derive a bound for $\mathrm{E}[(Z\cos\theta)^2]$. Since $\mathrm{E}[(Z\cos\theta)^2] = \mathrm{E}[(Z|\cos\theta|)^2]$, we have

$$\mathrm{E}[(Z\cos\theta)^2] = \int_0^{\sqrt{n}} z^2 \mathrm{d}F_{Z|\cos\theta|}(z). \tag{47}$$

We first consider the case of $\alpha \in (0,2)$. From the CCDF of $Z|\cos\theta|$ given for $z \ge 1$ in (12), we have for $z \ge 1$,

$$\begin{aligned}\frac{\mathrm{d}F_{Z|\cos\theta|}(z)}{\mathrm{d}z} &= -\frac{\mathrm{d}}{\mathrm{d}z}\mathrm{P}\{Z|\cos\theta| > z\} \\ &= \frac{2\alpha c(n)}{\pi z^{\alpha+1}}\int_0^{\cos^{-1}\left(\frac{z}{\sqrt{n}}\right)} (\cos\vartheta)^\alpha \mathrm{d}\vartheta \\ &\le \frac{\alpha c^\star c(n)}{z^{\alpha+1}}.\end{aligned}$$

Thus, the integral on the right-hand side of (47) is bounded above by

$$\begin{aligned}&\int_0^{\sqrt{n}} z^2 \mathrm{d}F_{Z|\cos\theta|}(z) \\ &= \int_0^{1} z^2 \mathrm{d}F_{Z|\cos\theta|}(z) + \int_1^{\sqrt{n}} z^2 \mathrm{d}F_{Z|\cos\theta|}(z) \\ &\le \mathrm{P}\{0 \le Z|\cos\theta| \le 1\} + \int_1^{\sqrt{n}} z^2 \frac{\alpha c^\star c(n)}{z^{\alpha+1}}\mathrm{d}z \\ &= \mathrm{P}\{0 \le Z|\cos\theta| \le 1\} + \frac{\alpha c^\star c(n)}{2-\alpha}(n^{1-\frac{\alpha}{2}} - 1)\end{aligned}$$

from which we have

$$\mathrm{E}[(Z\cos\theta)^2] = O(n^{1-\frac{\alpha}{2}}) \qquad \text{for } \alpha \in (0,2). \tag{48}$$

We next consider the case of $\alpha = 2$. By following the approach in the case of $\alpha \in (0,2)$, we have

$$\mathrm{E}[(Z\cos\theta)^2] = O(n^{1-\frac{\alpha}{2}}) \qquad \text{for } \alpha = 2. \tag{49}$$

Combining (48) and (49) gives $\mathrm{E}[(Z\cos\theta)^2] = O(n^{1-\frac{\alpha}{2}})$ for $\alpha \in (0,2]$. Hence, there exist constants $\bar{n} \in \mathbb{N}$ and $\bar{c} > 0$ such that

$$\mathrm{E}[(Z\cos\theta)^2] \le \bar{c}n^{1-\frac{\alpha}{2}} \qquad \text{for all } n \ge \bar{n}. \tag{50}$$

In addition, since $\tilde{t}(n) = \Theta(n^{\frac{\alpha}{2}-\epsilon})$, there exist constants $\tilde{n} \in \mathbb{N}$ and $\tilde{c} > 0$ such that

$$\tilde{t}(n) \le \tilde{c}n^{\frac{\alpha}{2}-\epsilon} \qquad \text{for all } n \ge \tilde{n}. \tag{51}$$

By (50) and (51), the term on the right-hand side of (46) is further bounded by

$$\begin{aligned}&2\tilde{t}(n)\exp\left(-\frac{(c_d)^2 n}{8\tilde{t}(n)\mathrm{E}[(Z\cos\theta)^2]}\right) \\ &\qquad \le 2\tilde{c}n^{\frac{\alpha}{2}-\epsilon}\exp\left(-\frac{(c_d)^2 n^\epsilon}{8\tilde{c}\bar{c}}\right).\end{aligned}\tag{52}$$

By L'Hôspital's rule, (52) becomes in the limit as

$$\begin{aligned}&\lim_{n\to\infty} 2\tilde{t}(n)\exp\left(-\frac{(c_d)^2 n}{8\tilde{t}(n)\mathrm{E}[(Z\cos\theta)^2]}\right) \\ &\qquad \le \lim_{n\to\infty} 2\tilde{c}n^{\frac{\alpha}{2}-\epsilon}\exp\left(-\frac{(c_d)^2 n^\epsilon}{8\tilde{c}\bar{c}}\right) = 0.\end{aligned}\tag{53}$$

Combining (46) and (53) proves (42). $\square$

APPENDIX B
PROOF OF (19)

From (18), it is straightforward to show that

$$\lim_{t\to\infty} \frac{\mathrm{P}\{T_x(r) > t\}}{\frac{4}{\pi}\exp\left(-F(\frac{\pi}{2r})^{\alpha} t\right)} = 1 + \frac{\pi}{4}\lim_{t\to\infty}\sum_{i=2}^{\infty}\eta_i \xi_i(t) \tag{54}$$

where $\xi_i(t) \triangleq \exp(-F(\frac{\pi}{2r})^{\alpha}(i^{\alpha}-1)t)$. To guarantee the interchange of the order of limit and summation, we will use Abel's test [29] as follows. First, we have $\xi_{i+1}(t) \le \xi_i(t)$ for each $t \ge 0$. In addition, $|\xi_i(t)|$ is bounded by 1 for all $t \ge 0$ and all i. Since $\sum_{i=1}^{\infty}\eta_i = 1 - \mathrm{P}\{T_x(r) \le 0\} = 1$ from (18), we have $\sum_{i=2}^{\infty}\eta_i = \sum_{i=1}^{\infty}\eta_i - \eta_1 = 1 - \eta_1 < \infty$. Hence, the sum in (54) converges uniformly on $[0,\infty)$, and we can interchange the order of limit and summation as follows:

$$\lim_{t\to\infty} \frac{\mathrm{P}\{T_x(r) > t\}}{\frac{4}{\pi}\exp\left(-F(\frac{\pi}{2r})^{\alpha} t\right)} = 1 + \frac{\pi}{4}\sum_{i=2}^{\infty}\eta_i \lim_{t\to\infty}\xi_i(t) = 1.$$

This completes the proof. □

REFERENCES

[1] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 388–404, Mar. 2000.

[2] M. Grossglauser and D. N. C. Tse, "Mobility increases the capacity of ad hoc wireless networks," *IEEE/ACM Trans. Netw.*, vol. 10, no. 4, pp. 477–486, Aug. 2002.

[3] S. Toumpis and A. Goldsmith, "Large wireless networks under fading, mobility, and delay constraints," in *Proc. IEEE INFOCOM*, 2004, vol. 1, pp. 609–619.

[4] M. Neely and E. Modiano, "Capacity and delay tradeoffs for ad-hoc mobile networks," *IEEE Trans. Inf. Theory*, vol. 51, no. 6, pp. 1917–1937, Jun. 2005.

[5] X. Lin, G. Sharma, R. R. Mazumdar, and N. B. Shroff, "Degenerate delay-capacity tradeoffs in ad-hoc networks with Brownian mobility," *IEEE Trans. Inf. Theory*, vol. 52, no. 6, pp. 2777–2784, Jun. 2006.

[6] G. Sharma, R. Mazumdar, and N. B. Shroff, "Delay and capacity tradeoffs in mobile ad hoc networks: A global perspective," *IEEE/ACM Trans. Netw.*, vol. 15, no. 5, pp. 981–992, Oct. 2007.

[7] I. Rhee, M. Shin, S. Hong, K. Lee, and S. Chong, "On the Levy-walk nature of human mobility," in *Proc. IEEE INFOCOM*, 2008, pp. 924–932.

[8] K. Lee, S. Hong, S. Kim, I. Rhee, and S. Chong, "SLAW: A new mobility model for human walks," in *Proc. IEEE INFOCOM*, 2009, pp. 855–863.

[9] M. C. Gonzalez, C. A. Hidalgo, and A.-L. Barabasi, "Understanding individual human mobility patterns," *Nature*, vol. 453, pp. 779–782, Jun. 2008.

[10] M. F. Shlesinger, J. Klafter, and Y. M. Wong, "Random walks with infinite spatial and temporal moments," *J. Statist. Phys.*, vol. 27, no. 3, pp. 499–512, Mar. 1982.

[11] M. Franceschetti, O. Dousse, D. Tse, and P. Thiran, "Closing the gap in the capacity of wireless networks via percolation theory," *IEEE Trans. Inf. Theory*, vol. 53, no. 3, pp. 1009–1018, Mar. 2007.

[12] A. Agarwal and P. R. Kumar, "Capacity bounds for ad hoc and hybrid wireless networks," *Comput. Commun. Rev.*, vol. 34, no. 3, pp. 71–81, 2004.

[13] N. Bansal and Z. Liu, "Capacity, delay and mobility in wireless ad-hoc networks," in *Proc. IEEE INFOCOM*, 2003, vol. 2, pp. 1553–1563.

[14] E. Perevalov and R. Blum, "Delay limited capacity of ad hoc networks: Asymptotically optimal transmission and relaying strategy," in *Proc. IEEE INFOCOM*, 2003, vol. 2, pp. 1575–1582.

[15] A. Tsirigos and Z. J. Haas, "Multipath routing in the presence of frequent topological changes," *IEEE Commun. Mag.*, vol. 39, no. 11, pp. 132–138, Nov. 2001.

[16] G. Sharma and R. Mazumdar, "Scaling laws for capacity and delay in wireless ad hoc networks with random mobility," in *Proc. IEEE ICC*, 2004, vol. 7, pp. 3869–3873.

[17] X. Lin and N. B. Shroff, "The fundamental capacity-delay tradeoff in large mobile ad hoc networks," in *Proc. 3rd Annu. Mediterr. Ad Hoc Netw. Workshop*, 2004, pp. 1–13.

[18] M. Neely and E. Modiano, "Dynamic power allocation and routing for satellite and wireless networks with time varying channels," Ph.D. dissertation, Massachusetts Institute of Technology, Cambridge, MA, 2004.

[19] A. E. Gamal, J. Mammen, B. Prabhakar, and D. Shah, "Optimal throughput-delay scaling in wireless networks: Part I: The fluid model," *IEEE Trans. Inf. Theory*, vol. 52, no. 6, pp. 2568–2592, Jun. 2006.

[20] Y. Kim, K. Lee, N. B. Shroff, I. Rhee, and S. Chong, "On the generalized delay-capacity tradeoff of mobile networks With Lévy flight mobility The Ohio State University, Columbus, OH, 2012 [Online]. Available: arXiv: http://arxiv.org/abs/1207.1514

[21] P. M. Drysdale and P. A. Robinson, "Lévy random walks in finite systems," *Phys. Rev. E*, vol. 58, no. 5, pp. 5382–5394, 1998.

[22] R. Metzler and J. Klafter, "The random walk's guide to anomalous diffusion: A fractional dynamics approach," *Phys. Rep.*, vol. 339, no. 1, pp. 1–77, 2000.

[23] A. V. Chechkin, V. Y. Gonchar, J. Klafter, and R. Metzler, "Fundamentals of Lévy flight processes," *Adv. Chem. Phys.*, vol. 133, pp. 439–496, 2006.

[24] J. P. Nolan, "Stable distributions—Models for heavy tailed data," 2012 [Online]. Available: academic2.american.edu/~jpnolan

[25] M. Ferraro and L. Zaninetti, "Mean number of visits to sites in levy flights," *Phys. Rev. E*, vol. 73, no. 5, p. 057102, May 2006.

[26] S. Redner, *A Guide to First-Passage Processes*. New York: Cambridge Univ. Press, 2001.

[27] M. Gitterman, "Mean first passage time for anomalous diffusion," *Phys. Rev. E*, vol. 62, no. 5, pp. 6065–6070, 2000.

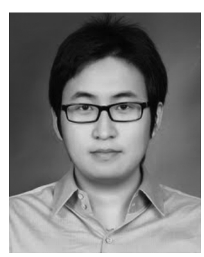
[28] I. Podlubny, *Fractional Differential Equations*. London, U.K.: Academic, 1999.

[29] J. E. Marsden and M. J. Hoffman, *Elementary Classical Analysis*. New York: Freeman, 1993.

[30] S. M. Ross, *Stochastic Processes*. New York: Wiley, 1996.

[31] M. Garetto, P. Giaccone, and E. Leonardi, "Capacity scaling in ad hoc networks with heterogeneous mobile nodes: The super-critical regime," *IEEE/ACM Trans. Netw.*, vol. 17, no. 5, pp. 1522–1535, Oct. 2009.

[32] G. Alfano, M. Garetto, E. Leonardi, and V. Martina, "Capacity scaling of wireless networks with inhomogeneous node density: Lower bounds," *IEEE/ACM Trans. Netw.*, vol. 18, no. 5, pp. 1624–1636, Oct. 2010.



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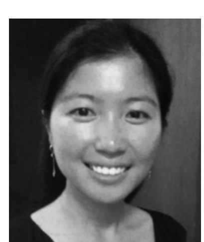

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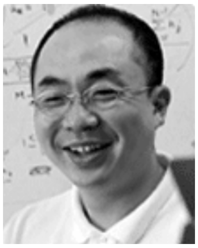

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