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A simulated annealing algorithm for first train transfer problem in urban railway networks

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ABSTRACT

Passengers often have to transfer between different subway lines to reach their destinations. Time coordination of first trains between feeder and connecting lines plays an important role in reducing passenger transfer waiting time. This paper addresses the first train synchronization problem, and proposes a first train coordination model which aims at minimizing total passenger transfer waiting time. Taking into account the specification of the first train problem, we use mixed-integer variables to enable the correct calculation of the waiting time for the "first available" train at each transfer station. In addition, we develop a simulated annealing algorithm to deal with a case study of the Beijing subway network. Results indicate that the proposed approach reduces the passenger waiting time from 705.1 min of the original first train timetable to 567.42 min of the scheduled one.

1. Introduction

The development of public transportation is an increasingly popular topic in metropolises. Facing road congestion, car parking and air pollution problems, travelers are more likely to select public railway transit if they are provided with good services at their disposal, e.g., comfort, punctuality, security, regularity, and economy. It is the goal of operators to design such an urban railway transit to ensure the quality of the transportation service. The train timetabling problem (TTP) in urban railway networks is to find an effective train schedule that regulates train arrival and departure times at stations. Introduced by Lusby et al. [1], TTP is one of the most time consuming, important and difficult tasks in passenger railway planning and management. Moreover, we can see in Fig. 1 that the timetabling is the basis of track allocation, train routing (in stations), rolling stock scheduling and crew scheduling problems. To develop a timetable, several inputs are necessary (see Table 1), e.g., demand by time of day, time for first and last trips and running times. As a result, the outputs of the whole timetable are trip arrival and departure times.

Fig. 2 illustrates the topology of a simple subway network. Trains are dispatched at a regular headway of each line, from the original station to the final station. Trains stop at intermediate stations as well as transfer stations to let passengers get on, get off and exchange. However, the first train transfer problem is usually critical if it is not paid much attention to. Herein, the first train indicates the first operating train in each line every day. Taking the Beijing subway in Fig. 2 for instance, four first trains depart from vehicle depots in two bi-directional lines (Line 1 and Line 10). The first train running in the **down** train direction of **Line 1** (L1D) arrives at GuoMao station at 5:05 am, and the first train in the **down** train direction of

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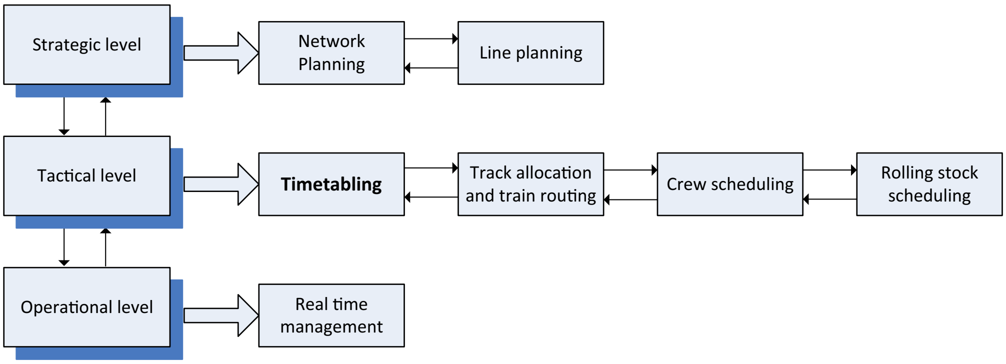

Fig. 1. Hierarchy of decision problems in the railway planning and management [1].

Table 1
Transit planning process [9].

Necessary inputs	Planning activity	Outputs
Demand by time of day Times for first and last trips Running times	Timetable development	Trip departure times Trip arrival times

Station	First feeder train		Transfer	First connecting train	Waiting
GuoMao	L1D→L10D	5:05:00	4.5 min	5:48:00	38 min 30 sec
	L1D→L10U	5:05:00	4.5 min	6:13:00	63 min 30 sec
GongZhuFen	L1U→L10D	5:17:00	3 min	6:30:00	70 min
	L1D→L10D	5:36:00	3 min	6:30:00	51 min

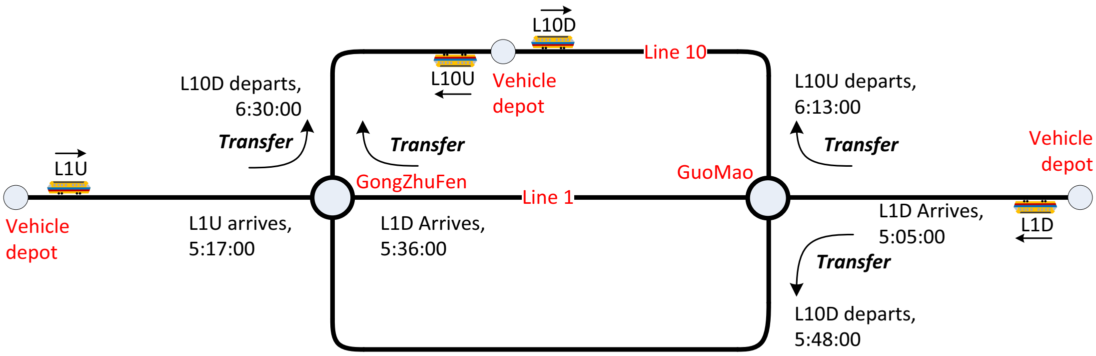

Fig. 2. A snapshot of the first train timetable in Beijing subway.

Line 10 (L10D) departs at 5:48 am. It takes passengers 4.5 min to transfer between line 1 and line 10. As a result, the transfer waiting time from L1D to L10D is approximately 38.5 min that makes passengers wait too long. In addition, the transfer waiting times for passengers from L1D to L10U at GuoMao, from L1U to L10D at GongZhuFen, and from L1D to L10D at GongZhuFen reach 63.5 min, 70 min and 51 min, respectively. It is no exaggeration to say that long waiting time discourages passengers from riding urban railway transit.

Fig. 3 shows two different situations base on the arrival/departure times of the first feeder train and the first connecting train; the arrows above the timeline represent arrival times of trains at one transfer station on line l, while the arrows below capture departure times of trains at the transfer station on line l'. Case A is under the scheduled first train timetable and case B is not, which means the first train transfer time of A is less than that of B. Both A and B adopt a fixed headway timetable. As seen in case A and case B, passengers transfer from train 1 to train 1′, from train 1′ to train 2, from train 2 to train 2′, etc. For the first two groups of transfers, we have the following equation,

$$(transfer + wait_{1'}^{1}) + (-dwell_{1'} + transfer + wait_{2}^{1'} - dwell_{2}) = Headway_{l}, \tag{1}$$

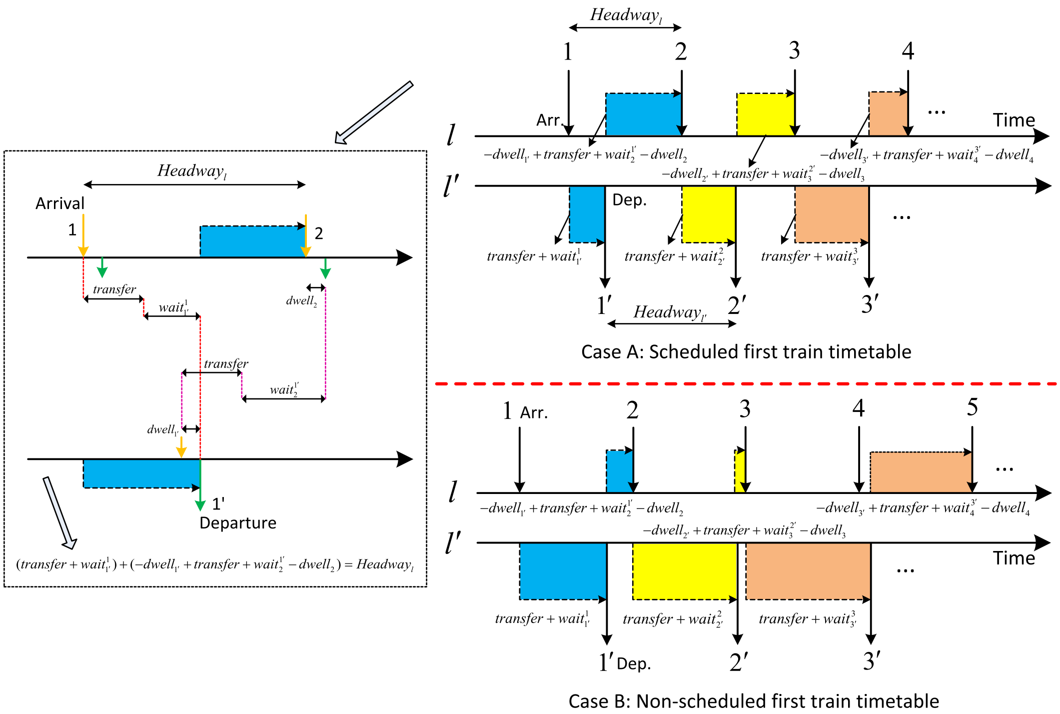

Fig. 3. Illustration the importance of the first train transfer problem.

where *transfer* represents the period for any passenger to get off the feeder train and walk to the connecting train; $wait_v^u$ represents the transfer waiting time for passengers who transfer from train u to train v; $dwell_u$ represents the dwell time of train u at the transfer station; and $Headway_l$ represents the headway on line l.

Considering several groups of transfers, we have Eq. (2).

$$(transfer + wait_{1'}^{1}) + (-dwell_{1'} + transfer + wait_{2}^{1'} - dwell_2) + \cdots + (transfer + wait_{y'}^{x}) + (-dwell_{y'} + transfer + wait_{z}^{y'} - dwell_z) = y' \cdot Headway_l. \tag{2}$$

In Eqs. (1) and (2), constants *transfer* and $Headway_l$ are fixed. The total transfer waiting time is only affected by $dwell_u$, where u represents any station on lines l and l'. If $dwell_u$ is set as a constant, then the overall transfer waiting time is also fixed.

Therefore, this paper concentrates on the first train timetabling problem in subway networks to determine a synchronized first train timetable, which will not increase the overall waiting time for the total transfers.

2. Literature review

Timetabling is one of the most attractive topics in the railway operation literature. Among plenty of literature, the train synchronization problem is a general topic. Mohring et al. [2] found that passengers often viewed their waiting time to be twice of what it actually was. Thus, minimization of the transfer waiting time has become a major objective in many studies for the timetable synchronization problem, such as Kwan and Chang [3], Wong et al. [4], Shafahi and Khani [5], Liu and Kozan [6], Ibarra-Rojas and Rios-Solis [7], etc.

Many different situations, models and methods have been studied in the domain of the timetabling. To provide an overview of the approaches used in the literature, we select some studies classified in Table 2 according to three aspects. The main differentiation factor is the objective. The optimization algorithm or method is another criterion. Lastly, we list the highlights of each paper.

Some unique objectives considered in the timetabling include minimizing makespan [8], minimizing the maximum duration between consecutive train arriving at a station [9], minimizing the cost of train delay and energy consumption [10,11], minimizing the number of trains [12], maximizing company profits [13,14], and minimizing train travel time [15].

Generally, timetables could be divided into two categories, a periodical timetable and a non-periodical timetable. The periodical timetable has been used widely from the very beginning, e.g., London 1863, Paris 1900, Berlin 1902, and

Table 2
Classification of some papers dealing with timetable synchronization problem.

Literatures	Model/objective	Algorithm/method	Highlights
Daduna and Voss [37]	Minimizing the transfer waiting time	Tabu search	Consider additional decision variables like dwell times and run times to the dimension of the problem to make more flexible synchronization
Liebchen and Möhring [38]	Maximizing the number of "frequency meets"	MIP model and CPLEX solver	Raise the question of systematically looking for short fundamental cycle bases, in order to keep the search space for a MIP-solver small
Chang and Chung [39]	Minimizing train running time, dwell time, headway and passenger travelling time; Minimizing the shift of the timetable when reschedule it	Genetic algorithm	Provide a mechanism that can be the common basis of generating a pre-planned timetable and a re-scheduled timetable
Wong et al. [4]	Minimizing the total passenger transfer waiting time	MIP model and CPLEX solver	Use binary variables to represent the waiting times to the "next available" train at the interchange stations
Kwan and Chang [3]	Modeling the synchronization problem into a dual objective problem, including passenger dissatisfaction index and timetable deviation index	Genetic algorithm	Formulate a measure for synchronization by means of a total passenger dissatisfaction index and the impact of such synchronization on the original timetable
Shafahi and Khani [5]	Minimizing the passenger waiting time at transfer stations	Genetic algorithm	Consider the extra stopping time of vehicles at transfer stations so that transfers can be better performed
Ibarra-Rojas and Rioa-Solis [7]	Maximizing the number of synchronizations	MIP model and branch and bound algorithm	Solve two main problems at some specific nodes of the network: avoiding bus bunching of different lines and allowing passenger transfer
Heydar et al. [19]	Minimizing the length of the dispatching cycle and the total stopping time	MIP model and CPLEX solver	Restate the problem from Bergmann [40] using improved notation, to modify Bergmann's model by adding a second objective and deleting unnecessary variables
Bocewicz [41]	Modeling the robust formulation of multimodal transportation processes	Declarative modeling approach and constraint programming techniques	Provided a framework to address the needs for transportation networks robustness with the consideration of capacity and demand requirements

Beijing 1965. Some studies on the periodical timetable include Odijk [16], Peeters and Kroon [12], Kroon and Peeters [17], Liebchen [18], Shafahi and Khani [5], Heydar et al. [19], Kang et al. [8], etc.

On the other hand, the non-periodical timetable that adjusts the headway of trains is inclined to meet passengers' travel demands. Some recent studies on this area include Carey and Lockwood [20], Higgins et al. [21], Caprara et al. [13], Zhou and Zhong [22], Caprara et al. [23], Carey and Crawford [24], Zhou and Zhong [25], D'Ariano et al. [26], Burdett and Kozan [27], Liu and Kozan [28], Burdett and Kozan [8], Cacchiani et al. [29], Castillo et al. [15], Liu and Kozan [6], Harrod [30], Narayanaswami and Rangaraj [31], etc. The non-periodical timetables are more practical for real-time scheduling or rescheduling.

Although there are many works on timetabling/scheduling problem, few of them concerns with the first train timetabling can be found. In addition, previous models are inadequate to solve the first train problems such as train dispatching time, train coordination/synchronization, passenger transfer, and passenger waiting time. Table 3 lists the characteristics of the first train, the last train and the rush hour trains, which contain passenger flow, train capacity utilization, passenger transfer efficiency and train operation constraints. As seen, the first train, the last train and the rush hour trains represent three different typical periods, and different models are required to describe the corresponding problems.

Table 3
Comparison of the first train, the last train and rush-hour trains.

Aspects	First train	Last train [42]	Rush-hour trains
Passenger flow	Low volume, passenger flow increases gradually	Low volume, passenger flow decreases gradually	High volume
Train capacity	Generally enough	Generally enough except unexpected passenger flow	Lack train capacity in peak hours. Passengers are forced to wait the following trains
Transfer problem	Have connecting trains, and consider transfer waiting time	May not have connecting trains; also consider transfer waiting time	Have connecting trains, and consider transfer waiting time
Constraints	Dispatch time, safety, operations, and equipment	Dispatch time, safety, operations, and equipment	Headway, safety, operations, and equipment

The contributions of this paper can be summarized as follows. First, by analyzing the first train timetable synchronization problem, a first train coordination model, which minimizes the total passenger transfer waiting time, is proposed. Second, the complexity of the problem is mainly due to the use of a large set of mixed-integer variables; each enables the correct representation of the waiting time to the "first available" train. Third, by using simulated annealing to deal with the case study of the Beijing subway network, our method reduces the total passenger waiting time from 705.1 min to 567.42 min.

3. First train transfer model

We present the first-train coordination model that aims at minimizing the total passenger transfer waiting time by adjusting the first train dispatching time, the running time, and the station dwell time of each line. Subway lines start to service at different times. Therefore, it brings a problem that some early transfer passengers from the feeder train have to wait for the posterior connecting train for a long time. To solve the long waiting time problem, we propose the first train model as follows.

3.1. Notation

The necessary **parameters** are:

L: the set of lines in the network, $l \in L$, $L = \{l | l = 1, 2, \ldots, n\}$, where n is the total number of lines in the urban railway network.
$S(l)$: the set of stations in line l, $s_l^q \in S(l)$, $S(l) = \{s_l^1, s_l^2, \ldots, s_l^{m_l}\}$, where $s_l^{m_l}$ is the last station in line l. Accordingly, s_l^{q-1} will be the station preceding s_l^q on line l. Note that notation s_l^q will be replaced with s for simplification wherever necessary in this paper.
$T_{earliest}$: the earliest operation time of trains provided by the Metro corporation.
T_{latest}: the latest starting operation time of trains provided by the Metro corporation.
$T^R_{\min lk(k-1)}$: the minimum running time of the first train from station s_l^{k-1} to station s_l^k (on line l).
$T^R_{\max lk(k-1)}$: the maximum running time of the first train from station s_l^{k-1} to station s_l^k (on line l).
$T^{Dw}_{\min}$: the minimum station dwell time of the first train.
$T^{Dw}_{\max}$: the maximum station dwell time of the first train.
$T^{Tra}_{sll'}$: the passenger transfer (walking) time from lines l to l' at station s. We assume $T^{Tra}_{sl'l} = T^{Tra}_{sll'}$.
T^H_l: the headway of line l.

The **decision variables** of the proposed model are:

$t^R_{lk(k-1)}$: the first train running time from station s_l^{k-1} to station s_l^k (on line l).
t^{Dw}_{ls}: the first train dwell time at station s in line l.
t^A_{sl}: the first train arrival time at station s in line l.
t^D_{sl}: the first train departure time from station s in line l.
t^0_l: the dispatch time of the first train from vehicle depot in line l.
$t^w_{sll'}$: the passenger transfer waiting time from line l to line l' at station s.

We present the problem in Fig. 2 in a time domain. Trains are dispatched periodically according to a cyclic timetable depicted in Fig. 4. Earlier feeder passengers may wait quite a long time to the connecting train. In addition, feeder passengers will accumulate at the platform if they are not carried smoothly. Consequently, management of passenger flow at the station becomes difficult. With the consideration of the above situations and the potential problems, we deal with the first train coordination problem in this paper, aiming at minimizing the passenger transfer waiting time by synchronizing the first trains in subway networks.

3.2. Basic assumptions

In real operations, various situations occur randomly. For example, passengers move at different speeds, and not all passengers catch their intended transfers. Some passengers prefer having seats and waiting for the following trains, but on the other hand some passengers will board the crowded train as soon as it arrives. Because our model is a planning one, we put forward the following assumptions to simplify the model formulations.

Assumption 1. Passenger transfer time between two lines at a certain transfer station is fixed. As mentioned above, it may take different periods of time for passengers to finish transfers due to their ages, packages, walking speeds, etc. We collected the real transfer time in the Beijing subway network by a number of experiments and data processing.

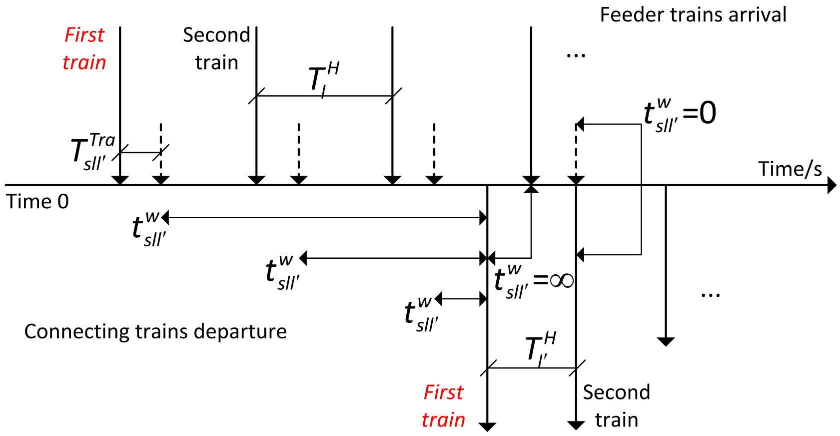

Fig. 4. Illustration of the first train transfer problem at a particular station.

Assumption 2. At each station, we assume that passengers will board the first arriving train to reduce their waiting times. In rush hours, passengers may give up the first connecting train and wait for the following trains for having a seat because they appreciate being able to sit during the trip [32]. Nevertheless, according to the real data from the Beijing subway, there are not many passengers on the first train.

Assumption 3. Trains after the first train running in each line adopt a fixed headway because the periodic timetable not only has the largest capacity but also reduces the complexity of the model. It should be noted that the headway of each line is obtained from the Beijing subway.

3.3. Constraints

Fig. 5 shows two lines, feeder line l and connecting line l', crossing at a transfer station. The first train departs from the original station, passes a number of ordinary stations, and finally arrives at the end station. During the trip, any train dwells at each station for some time to wait for passengers alighting and boarding. In particular, two lines cross at the transfer station, and passengers complete transferring from line l to line l' or vice versa. The following constraints ensure trains operate smoothly.

For any line $l \in L$, constraints (3) and (4) track the arrival time t_{sl}^A and the departure time t_{sl}^D of the first train at/from station $s_l^q \in S(l)$, where t_l^o represents the dispatch time of the first train from the vehicle depot in line l, $\sum_{k=2}^{q} t_{lk(k-1)}^R$ accumulates the total running time to station s_l^q, and $\sum_{k=1}^{q} t_{lk}^{Dw}$ adds up the total dwell time from the first station to station s_l^q.

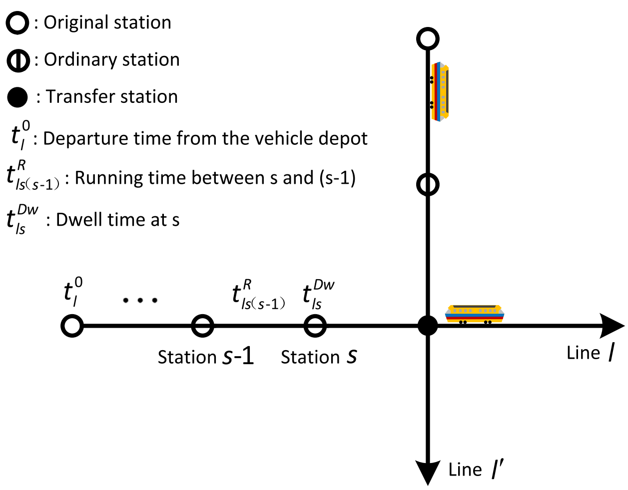

Fig. 5. Illustration of train operation.

$$t_{sl}^{A} = t_{l}^{o} + \sum_{k=2}^{q} t_{lk(k-1)}^{R} + \sum_{k=1}^{q-1} t_{lk}^{Dw}, \tag{3}$$

$$t_{sl}^{A} = t_{l}^{o} + \sum_{k=2}^{q} t_{lk(k-1)}^{R} + \sum_{k=1}^{q} t_{lk}^{Dw}. \tag{4}$$

With constraints (3) and (4), we can capture the appropriate passenger transfer waiting time by using constraint (5), where $T_{sll'}^{Tra}$ is the transfer time between lines l and l' at s. Here, the transfer waiting time equals the difference between the connecting train departure time $(h \cdot T_{l'}^{H} + t_{sl'}^{D})$ and the passenger arrival time $(t_{sl}^{A} + T_{sll'}^{Tra})$.

$$t_{sll'}^{w} = h \cdot T_{l'}^{H} + t_{sl'}^{D} - (t_{sl}^{A} + T_{sll'}^{Tra}). \tag{5}$$

For $t_{sl'}^{D} - (t_{sl}^{A} + T_{sll'}^{Tra}) \geqslant 0$, passengers get off the first feeder train in line l, and wait for the first connecting train in line l'. As to the situation of $t_{sl'}^{D} - (t_{sl}^{A} + T_{sll'}^{Tra}) < 0$, passengers miss the first connecting train and will board the following connecting trains in line l'. Here, h captures the number of trains that has passed when the first feeder passengers arrive at the connecting platform. The value of h is 0 if and only if the feeder line l and the connecting one are perfectly synchronized, in other words passengers from the first feeder train transfer to the first connecting one.

For all lines $l \in L$, $l' \in L$, and stations $s \in S(l)$, $s \in S(l')$, the following constrains should be satisfied.

Train dispatch time limits: The first-train dispatching time should be limited within an interval $[T_{earliest}, T_{latest}]$ to ensure the subway transportation on time. Without this constraint, the first train may start the service too early or too late.

$$T_{earliest} \leqslant t_{l}^{o} \leqslant T_{latest}, \tag{6}$$

$$T_{earliest} \leqslant t_{l'}^{o} \leqslant T_{latest}. \tag{7}$$

Running time limits: There are upper and lower bounds on running time in any segment. The running time between stations s_l^{k-1} and s_l^{k} of line l should be controlled within $[T_{\min lk(k-1)}^{R}, T_{\max lk(k-1)}^{R}]$ to ensure punctuality.

$$T_{\min lk(k-1)}^{R} \leqslant t_{lk(k-1)}^{R} \leqslant T_{\max lk(k-1)}^{R}, \tag{8}$$

$$T_{\min l'k(k-1)}^{R} \leqslant t_{l'k(k-1)}^{R} \leqslant T_{\max l'k(k-1)}^{R}. \tag{9}$$

Dwell time limits: The dwell time that can be adjusted at each station should be restricted within the minimum and maximum values. During this period, passengers finish their transfer jobs.

$$T_{\min}^{Dw} \leqslant t_{ls}^{Dw} \leqslant T_{\max}^{Dw}, \tag{10}$$

$$T_{\min}^{Dw} \leqslant t_{l's}^{Dw} \leqslant T_{\max}^{Dw}. \tag{11}$$

3.4. Model objective

The objective function in (12) is to minimize the total passenger transfer waiting time.

$$\min \ W = \sum_{s \in S(l) \cap S(l')} \sum_{l \in L} \sum_{l' \in L} t_{sll'}^{w}. \tag{12}$$

As mentioned, the non-negative terms, $t_{sll'}^{w}$, capture the appropriate waiting times for the corresponding group of transfers.

4. Solution algorithm

For scheduling problems, researchers usually use a very large number of different techniques, including exact and approximation algorithms. Simulated annealing (SA) has been adopted widely to solve engineering problems, e.g., train platform problem [33], transit network optimization problem [34], and bottleneck routing problem at railway stations [35], etc.

The SA starts from an initial solution at a high temperature, and makes a number of changes according to annealing schedules. For any two iterations, there are two objective values marked as f_{new} and f_{old}, and the difference between the objective values $(\Delta f = f_{new} - f_{old})$ is calculated. If $\Delta f \leqslant 0$, then the new solution is accepted with probability $\rho = 1$. Otherwise, it is accepted with a small probability ρ, $\rho = \exp(-\Delta f / T)$, where T is the current annealing temperature [36]. As the cooling proceeds to the set frozen point, the algorithm terminates.

One of the most significant tasks is called solution generation (annealing schedule). In this paper, three vectors $(t_1^R, t_2^R, \ldots, t_\varphi^R)$, $(t_1^{Dw}, t_2^{Dw}, \ldots, t_\chi^{Dw})$ and $(t_1^0, t_2^0, \ldots, t_\theta^0)$ represent first train segment running times, station dwell times and departure times from the vehicle depots, respectively. Therefore, variables including t_{sl}^{A}, $t_{sl'}^{D}$, and $t_{sll'}^{w}$ can be obtained by constraints 1–3. We first input original values of the three vectors as an initial solution. Then, another feasible solution is created by a vector modifying algorithm. For example, the running time t_i^R is modified by $t_i^R \times \mu$, where μ is a random value. Three principles are adopted in order to prevent the new solution infeasibility.

(1) If $t_i^R \times \mu$ is larger than the upper bound, then $t_i^R \times \mu$ will be replaced by the upper bound value.
(2) If $t_i^R \times \mu$ is smaller than the lower bound, then $t_i^R \times \mu$ will be replaced by the lower bound value.
(3) If $t_i^R \times \mu$ meets the constraints, then accept the new solution.

The vector modifying algorithm is given as follows.

Vector Modifying Algorithm
Input: Initial values of segment running time vector $(t_1^R,\ t_2^R, \ldots, t_\varphi^R)$
Produce a random number π in $[1, \varphi]$, where π is chosen with a uniform distribution and $\pi \in [1, \varphi]$.
Element switch, $t_\pi^R \times \mu$, where μ is a random number and is chosen with a uniform distribution in $[-M, M]$ (note that M is a positive large number).
If $(t_\pi^R \times \mu > t_{\max ls(s-1)}^R)$, then $t_\pi^R \times \mu = t_{\max ls(s-1)}^R$;
Else if $(t_\pi^R \times \mu < t_{\min ls(s-1)}^R)$, then $t_\pi^R \times \mu = t_{\min ls(s-1)}^R$;
Else $t_\pi^R = t_\pi^R \times \mu$;
End.
Output: Modified values of segment running time vector.

The **Vector Modifying Algorithm** is also applicable to vectors $(t_1^{Dw}, t_2^{Dw}, \ldots, t_\chi^{Dw})$ and $(t_1^0, t_2^0, \ldots, t_\phi^0)$. A new solution is created after modifying three vectors. Thus, the **Solution Generation Algorithm** is designed as follows.

Solution Generation Algorithm
For (i = 1 to 3) do
Conduct Vector Modifying Algorithm to $(t_1^R,\ t_2^R, \ldots, t_\varphi^R)$;
Conduct Vector Modifying Algorithm to $(t_1^{Dw}, t_2^{Dw}, \ldots, t_\chi^{Dw})$;
Conduct Vector Modifying Algorithm to $(t_1^0, t_2^0, \ldots, t_\phi^0)$;
End For.
New solution ← Constraints (1) and (2)

According to the Vector Modifying Algorithm and the Solution Generation Algorithm, the detailed SA algorithm is described as follows.

Step 1: Initialization
1.1. Set the initial parameters: initial temperature $T_0 = 100$, lowest temperature $\tau = 0.1$, cooling coefficient $\omega = 0.98$ and Markov length $ML = 50$. Input the original first train timetable.
1.2. Let $T = T_0$. Generate a feasible solution fs_0 according to the **Solution Generation Algorithm** and **Vector Modifying Algorithm**. Calculate the objective function value $f^{(0)}$, and display it.

$$f^{(0)} = \sum_{s \in S(l), s \in S(l')} \sum_{l \in L} \sum_{l' \in L} t_{sll'}^{w(0)},$$

1.3. Set the outer iteration $\eta = 0$.
Step 2: *For the current T, perform 2.1–2.4*
2.1. Set the inner iteration $\kappa = 1$.
2.2. Let $\eta = \eta + 1$. Obtain a new feasible solution fs_η through **Solution Generation Algorithm**. Calculate the objective function value $f^{(\eta)}$.
2.3. Calculate $\Delta f = f^{(\eta)} - f^{(\eta-1)}$. If $\Delta f \leqslant 0$, fs_η replaces $fs_{\eta-1}$, and display $f^{(\eta)}$. If not let fs_η replace $fs_{\eta-1}$ with the probability ρ, where $\rho = \exp(-\Delta f / T)$ and display $f^{(\eta)}$.
2.4. If $\kappa = ML$, go to *Step 3*; otherwise $\kappa = \kappa + 1$, and return to 2.2.
Step 3: *Stop or not*
3.1. If $T \leqslant 0.1$, stop. Otherwise $T = T \times 0.98$, and return to *Step 2*.

5. Discussion with a small sample

We design a simple network shown in Fig. 6, which has eight lines, four transfer stations, eight ordinary stations and eight downtown directions (with the number of transfer passengers), to verify the effectiveness of the model. The original parameters of the first train timetable are given in Table 4. All trains depart at time 0, and the transfer time between each pair of lines is 2 min. Note that the running times and the dwell times can be adjusted in the range of ±20% of the original values.

Using the proposed model and the SA, we obtain a scheduled first train timetable depicted in Table 5. The running times and the dwell times have different ranges of increases or decreases compared with those in Table 4. In addition, the train dispatching times are also changed. As seen, the first trains in lines 1, 2, 3, 5, 7 and 8 have been delayed in the departure.

In Table 6, "Upper" and "Lower" mean 120% and 80% of timetable parameters respectively. As seen, the technical speed and the travelling speed have improved to 58.33 km/h and 53.94 km/h. Moreover, an encouraging improvement is the passenger waiting time, from 326 min to 50.14 min. A first-train binary variable (FBV, $x_{sll'}$) is introduced to count how many pairs of lines are perfectly synchronized at transfer stations. The binary variable $x_{sll'}$ equals 1 if and only if the feeder line l and the connecting one l' are synchronized at station$s$. As seen, the FBV improves from 0 to 3 directions.

We further analyze eight downtown directions in Table 7. Seven directions have been reduced the transfer waiting time in large proportions except direction L6–L3. It stems from the fact that the passenger flow of L6–L3 is the smallest. Besides, transfer results in Table 7 show that the scheduled timetable achieves the synchronization of the first trains in most directions, e.g., L6–L1, L8–L2, L7–L2, L5–L3, and L8–L4.

6. Case study

In this section, we illustrate the quality of the first train timetable and the SA algorithm using real cases from the Beijing subway. As illustrated in Fig. 7, the Beijing subway network mainly consists of 18 one-way lines (9 two-way lines, up train and down train directions, respectively) and 31 transfer stations. To the first-train transfers, passengers mainly come from suburban areas to urban districts. For example, in the red box of Fig. 7, two transfer directions, which are the up direction of line 10 to the down direction of line 5 and the down direction of line 10 to the down direction of line 5, are selected as the key synchronization directions. Table 8 lists the locations of vehicle depots as well as the up and down train directions of each line.

Table 9 illustrates 83 transfer directions in the network and gives the original first train timetable. Each row indicates the transfer direction, the first feeder-train arrival time, the passenger transfer time, the first connecting-train arrival time, the connecting-train dwell time, the transfer waiting time and the FBV. In particular, the headways of line 1, 2, 4, 5, 10 and 13 are 5 min. The others are 10 min.

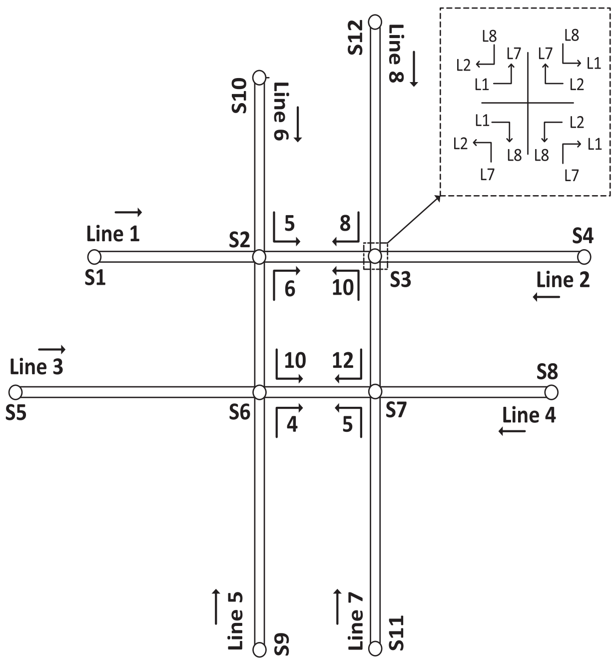

Fig. 6. A simple network for computation, a value represents the number of transfer passengers.

Table 4
Original first train dispatch time, running time and dwell time of each line (min).

Line	Dispatch time	Running time in segment 1	Dwell time	Running time in segment 2	Dwell time	Running time in segment 3
1	0	8	1	5	1	10
2	0	10	1	5	1	8
3	0	10	1	5	1	8
4	0	8	1	5	1	10
5	0	12	1	5	1	8
6	0	8	1	5	1	12
7	0	12	1	5	1	10
8	0	10	1	5	1	12

Table 5
Optimized train running time in sections and dwell time at transfer stations (min).

Line	Dispatching time	Running time in section 1	Dwell time	Running time in section 1	Dwell time	Running time in section 3
1	1	6.74	1.06	4	0.8	8
2	2	8.94	0.92	4	0.8	6.4
3	2	9.83	0.8	4	0.8	6.4
4	0	6.42	0.84	4	0.8	8
5	0.5	10.1	1.19	4	0.8	6.4
6	0	6.6	1.18	5.86	0.8	9.6
7	3	11.29	1.12	4.39	0.8	8
8	1	8.8	0.99	4.43	0.8	11.8

Table 6
Comparisons of different first train timetables.

Method	Operational parameters		Technical parameters		Waiting time (min)	FBV
	Total running time (min)	Total dwell time (min)	Technical speed (km/h)	Traveling speed (km/h)		
Original	196	16	50	46.23	326	0
Upper	163	13.3	60.12	55.58	300.8	0
Lower	235.2	19.2	41.67	38.53	441.2	0
Scheduled	168	14.5	58.33	53.94	50.14	3

Table 7
Transfer waiting time in two timetables (min).

Direction	Original timetable				Scheduled timetable			
	Feeder	Walking	Connecting	Waiting	Feeder	Walking	Connecting	Waiting
L6–L1	8	2	9	9	6.6	2	8.8	0.2
L5–L1	18	2	9	9	15.79	2	8.8	1.01
L8–L2	10	2	11	9	9.8	2	11.86	0.06
L7–L2	18	2	11	1	19.8	2	11.86	0.06
L6–L3	14	2	11	5	13.64	2	12.63	6.99
L5–L3	13	2	11	6	10.6	2	12.63	0.03
L8–L4	16	2	9	1	15.22	2	7.26	0.04
L7–L4	12	2	9	5	14.29	2	7.26	0.97

6.1. Optimized results of Beijing subway

We find that several aspects in Table 9 can be improved. To begin with, passengers wait for a long time when they transfer at many stations. Furthermore, synchronization of the first feeder train and the first connecting train stays at a low level. Therefore, passengers of many directions in Table 9 need to wait longer than normal practice (more than a period of headway). Finally, as introduced, the FBV improves the carrying capacity if the first trains coordinate at transfer stations. Consequently, the following results in Table 10 are compared with those in Table 9 in the above aspects.

6.1.1. Passenger waiting time

It is one of the most critical issues for passengers to wait for a long time when riding the first train. Generally, a complete trip consists of travel time, transfer time and waiting time. The prior two indicators cannot be reduced by a large percent. However, the waiting times of different timetables differ considerably. Thus, it is significantly important to minimize the

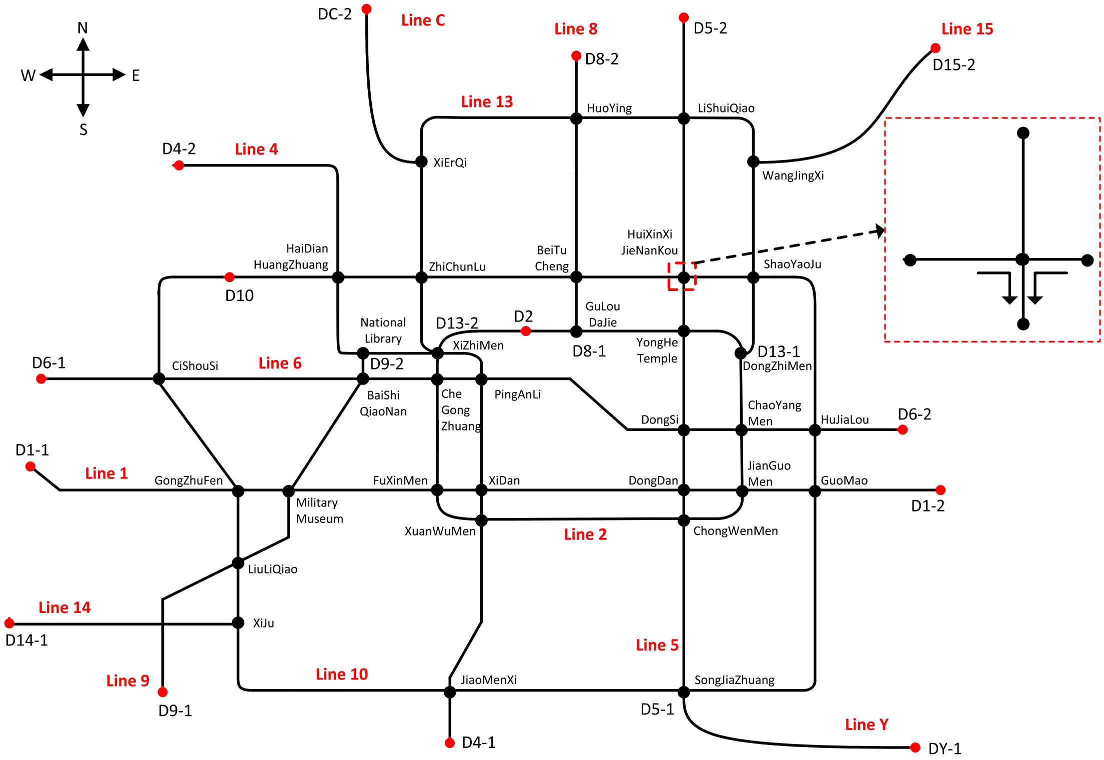

Fig. 7. Map of the Beijing subway (2014).

Table 8
Vehicle depots and train operation directions.

Line	Direction	Depot	Operation	Direction	Depot	Operation
L1	Up	D1-1	D1-1 → GongZhuFen	Down	D1-2	D1-2 → GuoMao
L2	Up	D2	D2 → GuLouDaJie	Down	D2	D2 → XiZhiMen
L4	Up	D4-1	D4-1 → JiaoMenXi	Down	D4-2	D4-2 → HaiDianHuangZhuang
L5	Up	D5-1	D5-1 → ChongWenMen	Down	D5-2	D5-2 → LiShuiQiao
L6	Up	D6-1	D6-1 → CiShouSi	Down	D6-2	D6-2 → HuJiaLou
L8	Up	D8-1	D8-1 → BeiTuCheng	Down	D8-2	D8-2 → HuoYing
L9	Up	D9-1	D9-1 → LiuLiQiao	Down	D9-2	D9-2 → BaiShiQiaoNan
L10	Up	D10	D10 → HaiDianHuangZhuang	Down	D10	D10 → CiShouSi
L13	Up	D13-1	D13-1 → ShaoYaoJu	Down	D13-2	D13-2 → ZhiChunLu
L14	Up	D14-1	D14-1 → XiJu	–	–	–
L15	–	–	–	Down	D15-2	D15-2 → WangJingXi
LC	–	–	–	Down	DC-2	DC-2 → XiErQi
LY	Up	DY-1	DY-1 → SongJiaZhuang	–	–	–

passenger waiting time and improve the transportation service. Compared to the current timetable, the total waiting time of the network with the optimized timetable decreases from 705.1 min to 567.42 min, improving by 19.53%.

6.1.2. Synchronization

Sometimes, passengers miss the connecting train by a few minutes. Furthermore, they even see the train closing the door and leaving. Train synchronization means the connecting train is present when passengers arrive at the platform. That is, the transfer waiting time is 0 min. If the new timetable increases the number of synchronized directions, it is an encouraging improvement. Only 1 direction out of 83 total transfer directions in Table 9 synchronizes, and the number of the synchronization improves to 6 in the new timetable.

6.1.3. Long waiting time

Passengers do not like waiting too long when they transfer. Generally, the waiting time is limited to a period of headway if passengers just miss the connecting train. However, for the first-train transfers, passengers may wait much longer. Thus,

Table 9
Original first train timetable.

Station	Transfer	Feeder Arr.	Walk (s)	Connecting Arr.	Dwell (s)	Waiting	FBV
FuXingMen	1.L1U–L2U	4:59:47	90	5:12:30	60	**12′13″**	1
	2.L1U–L2D	4:59:47	90	5:26:00	30	**25′13″**	1
XiDan	3.L4U–L1U	5:06:54	300	5:02:32	30	4′52″	0
	4.L4D–L1U	5:13:05	300	5:02:32	30	3″	0
DongDan	5.L1D–L5U	5:06:15	180	5:34:15	50	**25′50″**	1
	6.L5U–L1D	5:34:15	180	5:06:15	40	4′40″	0
	7.L5D–L1D	5:44:45	180	5:06:15	40	4′10″	0
JianGuoMen	8.L1D–L2U	5:03:45	90	5:13:15	45	**8′45″**	1
	9.L1D–L2D	5:03:45	90	5:21:30	45	**17′**	1
	10.L2D–L1D	5:21:30	90	5:03:45	45	1′30″	0
GuoMao	11.L10U–L1D	5:49:07	260	4:59:30	40	1′33″	0
	12.L10D–L1D	6:06:40	260	4:59:30	40	4′10″	0
XiZhiMen	13.L4D–L2U	5:03:00	120	5:29:30	60	**25′30″**	1
	14.L4D–L2D	5:03:00	120	5:10:00	30	**5′30″**	1
	15.L13U–L2U	5:32:07	360	5:29:30	60	2′27″	0
	16.L13U–L2D	5:32:07	360	5:10:00	30	2′13″	0
	17.L13U–L4D	5:32:07	420	5:03:00	40	4′33″	0
YongHe Temple	18.L5D–L2U	5:35:00	180	5:23:30	45	1′15″	0
	19.L5D–L2D	5:35:00	180	5:10:45	30	3′15″	0
	20.L2D–L5D	5:10:45	180	5:19:00	50	**6′5″**	1
DongZhiMen	21.L13D–L2U	5:25:15	240	5:20:00	60	1′45″	0
	22.L13D–L2D	5:25:15	240	5:14:45	30	1′	0
ChongWenMen	23.L5U–L2U	5:32:00	180	5:23:45	30	4′15″	0
	24.L5U–L2D	5:32:00	180	5:14:45	30	15″	0
XuanWuMen	25.L4U–L2U	5:05:09	240	5:17:00	60	**8′51″**	1
	26.L4U–L2D	5:05:09	240	5:21:15	60	**13′6″**	1
HaiDianHuang	27.L10U–L4D	6:37:52	240	4:51:00	40	4′48″	0
Zhuang	28.L10D–L4D	5:17:25	240	4:51:00	40	15″	0
BeiTuCheng	29.L10U–L8D	6:25:37	240	6:09:15	60	38″	0
	30.L10D–L8D	5:30:10	240	6:09:15	60	**36′5″**	1
HuiXinXiJie	31.L10U–L5D	6:21:37	90	5:28:30	30	43″	0
NanKou	32.L10D–L5D	5:34:10	90	5:28:30	30	3′20″	0
LiuShuiQiao	33.L13U–L5D	5:44:07	210	5:15:00	50	3′13″	0
	34.L13D–L5D	4:51:30	210	5:15:00	50	**20′50″**	1
	35.L5D–L13U	5:15:00	210	5:44:07	40	**26′17″**	1
	36.L5D–L13D	5:15:00	210	4:51:30	40	3′40″	0
HuoYing	37.L8D–L13U	5:47:15	210	4:48:07	30	3′2″	0
	38.L8D–L13D	5:47:15	210	4:33:00	40	2′55″	0
	39.L13U–L8D	4:48:07	210	5:47:15	30	**56′8″**	1
	40.L13D–L8D	4:33:00	210	5:47:15	30	**71′15″**	1
ZhiChunLu	41.L10U–L13U	6:34:07	260	5:12:37	30	4′40″	0
	42.L10D–L13U	5:21:40	260	5:12:37	30	2′7″	0
ShaoYaoJu	43.L10U–L13D	6:18:37	230	5:05:15	30	3′48″	0
	44.L10D–L13D	5:36:55	230	5:05:15	30	30″	0
GuLou	45.L8D–L2U	6:00:45	180	5:27:30	60	4′45″	0
DaJie	46.L8D–L2D	6:00:45	180	5:06:45	30	3′30″	0
	47.L10U–L5U	5:42:07	180	5:20:00	60	53″	0
SongJia	48.L10D–L5U	5:43:40	180	5:20:00	60	4′20″	0
Zhuang	49.LYU–L10U	5:42:07	180	5:42:07	30	2′30″	0
	50.LYU–L10D	5:42:07	180	5:43:40	30	4′3″	0
JiaoMenXi	51.L10U–L4U	5:34:07	180	4:53:09	30	1′32″	0
	52.L10D–L4U	5:51:40	180	4:53:09	30	3′59″	0
	53.L4U–L10U	5:12:00	180	5:34:07	30	**19′37″**	1
	54.L4U–L10D	5:12:00	180	5:51:40	30	**37′10″**	1
XiJu	55.L14U–L10U	6:04:00	180	5:16:07	30	4′37″	0
	56.L14U–L10D	6:04:00	180	6:07:40	30	**1′10″**	1
LiuLiQiao	57.L10U–L9U	5:14:07	180	5:52:24	20	**35′37″**	1
	58.L10D–L9U	6:10:40	180	5:52:24	20	9′4″	0
GongZhuFen	59.L10U–L1U	5:08:07	180	5:00:47	60	40″	0
	60.L10D–L1U	6:16:40	180	5:00:47	60	2′7″	0
CiShouSi	61.L10U–L6U	5:03:07	180	5:38:00	30	**32′23″**	1
	62.L10D–L6U	6:21:40	180	5:38:00	30	3′50″	0
BaiShi	63.L9U–L6U	5:35:24	180	5:43:30	30	**5′36″**	1
QiaoNan	64.L9D–L6U	6:14:25	180	5:43:30	30	6′35″	0
CheGong	65.L6U–L2U	5:48:30	180	5:08:30	30	2′30″	0
Zhuang	66.L6U–L2D	5:48:30	180	5:30:45	30	4′45″	0
PingAnLi	67.L6U–L4D	5:51:00	180	5:07:00	40	3′40″	0
	68.L6D–L4D	6:06:07	180	5:07:00	40	3′33″	0

Table 9 (*continued*)

Station	Transfer	Feeder Arr.	Walk (s)	Connecting Arr.	Dwell (s)	Waiting	FBV
DongSi	69.L6D–L5U	5:57:52	180	5:38:00	30	2′38″	0
	70.L6D–L5D	5:57:52	180	5:40:30	30	8″	0
	71.L5U–L6D	5:38:00	180	5:57:52	30	**17′22″**	1
	72.L5D–L6D	5:40:30	180	5:57:52	30	**14′52″**	1
ChaoYangMen	73.L6D–L2U	5:55:07	180	5:15:45	60	3′38″	0
	74.L6D–L2D	5:55:07	180	5:18:30	30	53″	0
HuJiaLou	75.L6D–L10U	5:49:52	180	6:05:37	20	**13′5″**	1
	76.L6D–L10D	5:49:52	180	5:49:40	20	2′8″	0
	77.L10U–L6D	6:05:37	180	5:49:52	30	1′45″	0
	78.L10D–L6D	5:49:40	180	5:49:52	30	7′42″	0
Library	79.L9U–L4D	5:37:24	180	4:57:45	40	3′1″	0
Military Museum	80.L9U–L1U	5:31:24	180	5:02:47	30	3′53″	0
	81.L9D–L1U	6:19:25	180	5:02:47	30	52″	0
WangJingXi	82.L15D–L13D	5:49:00	180	5:01:00	30	30″	0
XiErQi	83.LCD–L13U	5:45:00	180	4:58:07	30	0″	0

Bold means passengers from the first feeder train can transfer to the first connecting train.

Table 10
Optimized first train timetable.

Station	Transfer	Feeder Arr.	Walk (s)	Connecting Arr.	Dwell (s)	Waiting time	FBV
FuXingMen	1.L1U–L2U	5:14:00	90	5:17:00	60	**2′30″**	1
	2.L1U–L2D	5:14:00	90	5:26:00	30	**11′**	1
XiDan	3.L4U–L1U	5:10:45	300	5:16:45	30	**1′30″**	1
	4.L4D–L1U	5:21:05	300	5:16:45	30	1′10″	0
DongDan	5.L1D–L5U	5:09:45	180	5:34:15	50	**22′20″**	1
	6.L5U–L1D	5:34:15	180	5:09:45	40	3′10″	0
	7.L5D–L1D	5:36:45	180	5:09:45	40	40″	0
JianGuoMen	8.L1D–L2U	5:07:15	90	5:17:45	45	**9′45″**	1
	9.L1D–L2D	5:07:15	90	5:21:30	45	**13′30″**	1
	10.L2D–L1D	5:21:30	90	5:07:15	45	0″	0
GuoMao	11.L10U–L1D	5:45:00	260	5:03:00	40	420″	0
	12.L10D–L1D	5:58:00	260	5:03:00	40	1′20″	0
XiZhiMen	13.L4D–L2U	5:11:00	120	5:34:00	60	**22′**	1
	14.L4D–L2D	5:11:00	120	5:10:00	30	2′30″	0
	15.L13U–L2U	5:44:00	360	5:34:00	60	0″	0
	16.L13U–L2D	5:44:00	360	5:10:00	30	30″	0
	17.L13U–L4D	5:44:00	420	5:11:00	40	40″	0
YongHe Temple	18.L5D–L2U	5:27:00	180	5:28:00	45	3′45″	0
	19.L5D–L2D	5:27:00	180	5:10:45	30	1′15″	0
	20.L2D–L5D	5:10:45	180	5:27:00	50	**14′5″**	1
DongZhiMen	21.L13D–L2U	5:39:15	240	5:24:30	60	2′15″	0
	22.L13D–L2D	5:39:15	240	5:14:45	30	2′	0
ChongWenMen	23.L5U–L2U	5:32:00	180	5:28:15	30	3′45″	0
	24.L5U–L2D	5:32:00	180	5:14:45	30	15″	0
XuanWuMen	25.L4U–L2U	5:09:00	240	5:21:30	60	**9′30″**	1
	26.L4U–L2D	5:09:00	240	5:21:15	60	**9′15″**	1
HaiDianHuang	27.L10U–L4D	6:33:45	240	4:59:00	40	1′55″	0
Zhuang	28.L10D–L4D	5:08:45	240	4:59:00	40	1′55″	0
BeiTuCheng	29.L10U–L8D	6:21:30	240	5:42:30	60	8′	0
	30.L10D–L8D	5:21:30	240	5:42:30	60	**18′**	1
HuiXinXiJie	31.L10U–L5D	6:17:30	90	5:20:30	30	2′	0
NanKou	32.L10D–L5D	5:25:30	90	5:20:30	30	4′	0
LiuShuiQiao	33.L13U–L5D	5:56:00	210	5:07:00	50	3′20″	0
	34.L13D–L5D	5:05:30	210	5:07:00	50	3′50″	0
	35.L5D–L13U	5:07:00	210	5:56:00	40	**45′10″**	1
	36.L5D–L13D	5:07:00	210	5:05:30	40	40″	0
HuoYing	37.L8D–L13U	5:20:30	210	5:00:00	30	1′40″	0
	38.L8D–L13D	5:20:30	210	4:47:00	40	3′40″	0
	39.L13U–L8D	5:00:00	210	5:20:30	30	**17′30″**	1
	40.L13D–L8D	4:47:00	210	5:20:30	30	**30′30″**	1
ZhiChunLu	41.L10U–L13U	6:30:00	260	5:24:30	30	40″	0
	42.L10D–L13U	5:13:00	260	5:24:30	30	**7′40″**	1
ShaoYaoJu	43.L10U–L13D	6:14:30	230	5:19:15	30	1′55″	0
	44.L10D–L13D	5:28:15	230	5:19:15	30	3′10″	0
GuLou	45.L8D–L2U	5:34:00	180	5:32:00	60	1′	0
DaJie	46.L8D–L2D	5:34:00	180	5:06:45	30	15″	0

(*continued on next page*)

Table 10 (*continued*)

Station	Transfer	Feeder Arr.	Walk (s)	Connecting Arr.	Dwell (s)	Waiting time	FBV
	47.L10U–L5U	5:38:00	180	5:20:00	60	0″	0
SongJia	48.L10D–L5U	5:35:00	180	5:20:00	60	3″	0
Zhuang	49.LYU–L10U	5:54:45	180	5:38:00	30	45″	0
	50.LYU–L10D	5:54:45	180	5:35:00	30	2′45″	0
JiaoMenXi	51.L10U–L4U	5:30:00	180	4:57:00	30	4′30″	0
	52.L10D–L4U	5:43:00	180	4:57:00	30	1′30″	0
	53.LDU–L10U	4:57:00	180	5:30:00	30	**30′30″**	1
	54.LDU–L10D	4:57:00	180	5:43:00	30	**43′30″**	1
XiJu	55.L14U–L10U	5:49:00	180	5:12:00	30	30″	0
	56.L14U–L10D	5:49:00	180	5:59:00	30	**7′30″**	1
LiuLiQiao	57.L10U–L9U	5:10:00	180	6:04:00	20	**51′20″**	1
	58.L10D–L9U	6:02:00	180	6:04:00	20	9′20″	0
GongZhuFen	59.L10U–L1U	5:04:00	180	5:15:00	60	**9′**	1
	60.L10D–L1U	6:08:00	180	5:15:00	60	0″	0
CiShouSi	61.L10U–L6U	4:59:00	180	5:26:00	30	**24′30″**	1
	62.L10D–L6U	6:13:00	180	5:26:00	30	30″	0
BaiShi	63.L9U–L6U	5:47:00	180	5:31:30	30	2′	0
QiaoNan	64.L9D–L6U	6:01:00	180	5:31:30	30	8′	0
CheGong	65.L6U–L2U	5:36:30	180	5:13:00	30	4′	0
Zhuang	66.L6U–L2D	5:36:30	180	5:30:45	30	1′45″	0
PingAnLi	67.L6U–L4D	5:39:00	180	5:15:00	40	3′40″	0
	68.L6D–L4D	5:52:00	180	5:15:00	40	40″	0
DongSi	69.L6D–L5U	5:43:45	180	5:38:00	30	1′45″	0
	70.L6D–L5D	5:43:45	180	5:32:30	30	1′15″	0
	71.L5U–L6D	5:38:00	180	5:43:45	30	**3′15″**	1
	72.L5D–L6D	5:32:30	180	5:43:45	30	**8′15″**	1
ChaoYangMen	73.L6D–L2U	5:41:00	180	5:20:15	60	2′15″	0
	74.L6D–L2D	5:41:00	180	5:18:30	30	0″	0
HuJiaLou	75.L6D–L10U	5:35:45	180	6:01:30	20	**23′5″**	1
	76.L6D–L10D	5:35:45	180	5:41:00	20	**2′35″**	1
	77.L10U–L6D	6:01:30	180	5:35:45	30	1′45″	0
	78.L10D–L6D	5:41:00	180	5:35:45	30	2′15″	0
Library	79.L9U–L4D	5:49:00	180	5:05:45	40	4′25″	0
Military Museum	80.L9U–L1U	5:43:00	180	5:17:00	30	1′30″	0
	81.L9D–L1U	6:06:00	180	5:17:00	30	3′30″	0
WangJingXi	82.L15D–L13D	5:48:30	180	5:15:00	30	0″	0
XiErQi	83.LCD–L13U	5:45:37	180	5:10:00	30	2′30″	0

Bold means passengers from the first feeder train can transfer to the first connecting train.

we count the cases of long waiting times in two timetables. In Table 9, there are 22 directions with totally 534.33 min. In Table 10, there are 21 directions with 431.17 min. Obviously, the optimized timetable gives better results.

6.1.4. FBV

As defined, the FBV reflects the coordination of the first feeder train and the first connecting train. If two trains coordinate, then passengers can directly transfer to the first connecting train without waiting for the following trains. Sometimes the first feeder train may arrive much earlier than the first connecting train. Thus, passengers are advised to ride the later feeder trains to avoid long waiting time. In real operations, some passengers need to catch the first train to save time, e.g., catching a plane and commuting. The FBV guarantees that early passengers on the first feeder train can directly transfer to the first connecting train. As shown in Table 11, for the first-train service the FBV increases to 25 directions from the current 24 directions.

6.2. First train departure time of each line

As seen in Table 12, the optimized timetable modifies the first train departure time from each vehicle depot. Positive values represent delayed departure times and negative values indicate trains leaving in advance of the original timetable. For example, line 1U delays for 14′13″; line 2U departs earlier about 4′30″. Lines 2D, 5U, 8U, 14D, 15U, LYD, and LCU do not change significantly. Fig. 8 intuitively depicts the shift of the first-train departure times. As seen, this timetable brings forward the transportation service 26 min 45 s.

6.3. SA convergence test

We conduct a convergence test of objective values demonstrated in Fig. 9. To reach the best found solution of 567.42 min (34045.2 s in the *y*-axis of Fig. 9) for the objective function, 85,600 iterations were performed within 3.6 h. Indeed, the objective values stop changing after 43,000 iterations approximately. It indicates that the designed SA converges steadily.

Table 11
Comparison of original timetable and scheduled timetable.

Project	Original	Scheduled	Improvement
Waiting time	705.1 min	567.42 min	137.68 min
Synchronization	1	6	5
Long waiting	534.33 min	431.17 min	103.16 min
FBV	24	25	1

Table 12
Adjustment of the first train departure time on each line.

Line	1U	1D	2U	2D	4U	4D	5U	5D	6U	6D
Measure	14′13″	3′30″	−4′30″	0	3′51″	8′	0	−8′	−12′	−14′7″
Line	8U	8D	9U	9D	10U	10D	13U	13D	14U	14D
Measure	0	−26′45″	11′36″	−13′25″	−4′7″	−8′40″	11′53″	14′	−15′	0
Line	15U	15D	LYU	LYD	LCU	LCD	–	–	–	–
Measure	0	−30″	12′38″	0	0	37″	–	–	–	–

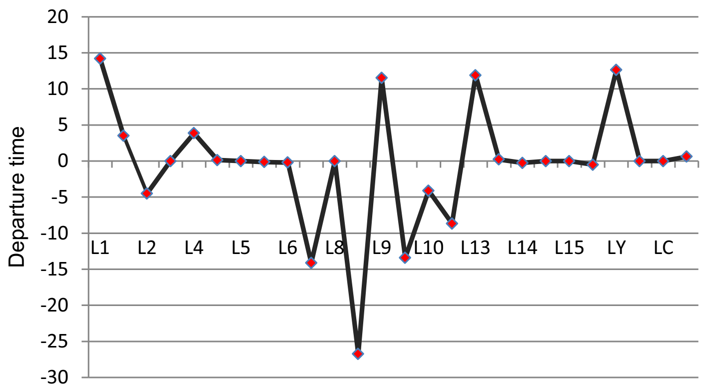

Fig. 8. Shift of the first train departure time of each line (up train and down train directions).

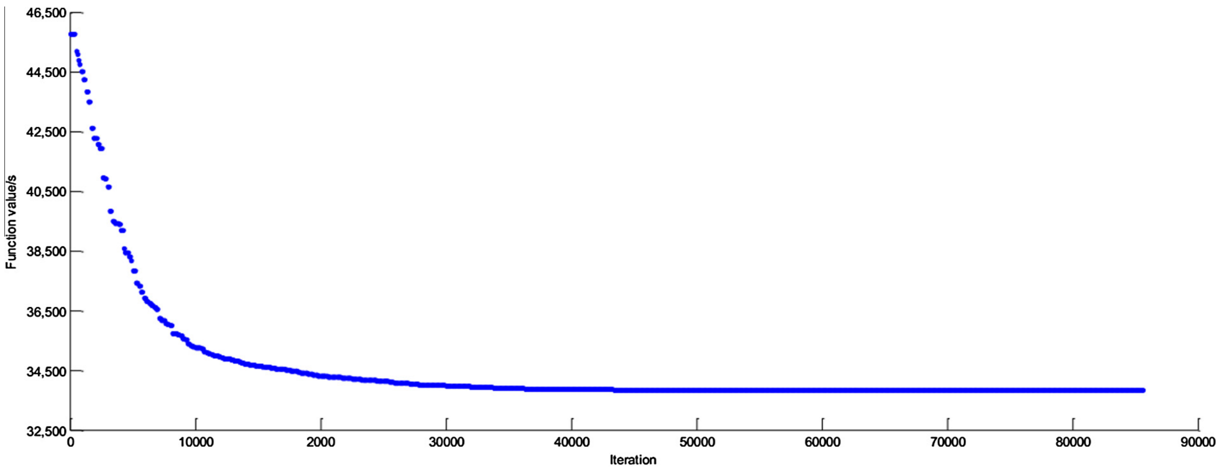

Fig. 9. Convergence test.

In addition, the performance of the SA is compared with the branch-and-bound (B&B) algorithm. The experiment is conducted in MATLAB 2012 in a 4×2.5 GHz CPU and 4 GB of RAM personal computer. The time to obtain the first feasible solution by the SA is only 0.15 s, compared to 12.6 min using the B&B method. After almost 12-h computation (from 21:00 to

9:00), the B&B returned with an objective value 637.58 min (38,255 s). Therefore, the performance of the SA is satisfactory in the aspects of the solution time and quality.

7. Conclusions

This paper addresses the first train synchronization problem, and proposes a first train coordination model which aims at minimizing the total passenger transfer waiting time. It is common for passengers of the first train to wait for a long time when they transfer. In the Beijing subway, there are 22 directions that passengers should wait for more than one headway time, with totally 534.33 min. The optimized first-train timetable reduces to 431.17 min. In addition, the passenger waiting time of the 83 transfer directions decreases from 705.1 min to 567.42 min.

In this paper, a simulated annealing algorithm, which is very efficient to find solutions, is designed to solve the problem. However, improvements to the algorithm efficiency, designing hybrid algorithms or developing an adaptive method can be considered in the future study. Finally, modeling the start-stop additional time of train operations at stations will make the scheduling more practical. These extensions need to be considered in greater depth.

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