Modelación de trayectorias de dispersión meteorológicas

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Contexto

- Polutantes
- Predicción
- Enfoque Euleriano v/s Lagrangiano
- Eficiencia computacional

- Conservación de masa

Modelo

$$dW_t = a_w(Z_t, W_t, t)dt + b_w(Z_t, W_t, t)dB_t$$
$$dZ_t = W_t dt$$

inferencia estadística
+ hipótesis

$$dW_t = \left(\frac{W_t}{\tau_w} + \frac{1}{2}(1 + \frac{W_t^2}{\sigma_w^2})\frac{\partial \sigma_w^2}{\partial z}\right)dt + \left(\frac{2\sigma_w^2}{\tau_w}\right)^{\frac{1}{2}} dB_t,$$
$$dZ_t = W_t dt$$

Lema de Ito

$$\Omega_t = \frac{W_t}{\sigma_w}$$

$$d\Omega_t = \left(-\frac{\Omega_t}{\tau_w} + \frac{\partial \sigma_w}{\partial z}\right)dt + \left(\frac{2}{\tau_w}\right)^{\frac{1}{2}} dB_t\ ,\ \ \Omega_0 \sim N(0,1),$$
$$dZ_t = \Omega_t \sigma_w dt\ ,\ \ Z_0 \sim N(z_0, \sigma_z^2)$$

Implementación

Esquema de Euler:

$$\Omega_n = \Omega_{n-1} + b(\Omega_{n-1}, \tau_{n-1}, \sigma_{n-1})\Delta t + s(\tau_{n-1})\Delta B$$
$$Z_n = Z_{n-1} + \Omega_{n-1}\sigma_{n-1}\Delta t$$

Implementación

Esquema de Milstein:

$$\Omega_n = \Omega_{n-1} + b(\Omega_{n-1}, \tau_{n-1}, \sigma_{n-1})\Delta t + s(\tau_{n-1})\Delta B + \frac{1}{2} s(\tau_{n-1}) s'(\tau_{n-1})[\Delta B^2 - \Delta t]$$

$$Z_n = Z_{n-1} + \Omega_{n-1}\sigma_{n-1}\Delta t$$

Implementación

Esquema de HON-SRKII:

$$
\begin{aligned}
\Omega_\mu &= \Omega_{n-1} + b(\Omega_{n-1}, \tau_{n-1}, \sigma_{n-1})\Delta t + s(\tau_{n-1})\Delta B \\
Z_\mu &= Z_{n-1} + \Omega_{n-1}\sigma_{n-1}\Delta t \\
\Omega_n &= \Omega_{n-1} + \tfrac{1}{2}(b(\Omega_{n-1}, \tau_{n-1}, \sigma_{n-1}) + b(\Omega_\mu, \tau_\mu, \sigma_\mu))\Delta t + s(\tau_{n-1})\Delta B \\
Z_n &= Z_{n-1} + \tfrac{1}{2}(\Omega_{n-1}\sigma_{n-1} + \Omega_\mu\sigma_\mu)\Delta t
\end{aligned}
$$

Implementación

Esquema de Leggraup:

$$\Delta_n \sim \mathcal{N}(0,1)$$

$$R_{n-1} = e^{\frac{-\Delta t}{\tau_{n-1}}}$$

$$\Omega_n = R_{n-1}\Omega_{n-1} + \sigma'_{n-1}\tau_{n-1}(1 - R_{n-1}) + (1 - R^2_{n-1})^{1/2}\Delta_{n-1}$$

$$Z_n = Z_{n-1} + \sigma_{n-1}\Omega_{n-1}\Delta_{n-1}$$

Implementación 2

Comportamiento inestable $\sigma_w(z) = 0.5(1+z)$ $\tau_w(z) = cte$

Comportamiento estable $\sigma_w(z) = 1.3(1+z)$ $\tau_w(z) = \frac{0.1z^{4/5}}{\sigma_w}$

Comportamiento neutral $\sigma_w(z) = 1.3e^{\frac{-2z}{\epsilon}}$ $\tau_w(z) = \frac{z}{2\sigma_w(1+15z/\epsilon)}$

X10

x1 unidad de tiempo

unidad de altura x1

¿Diferencias entre modelos?

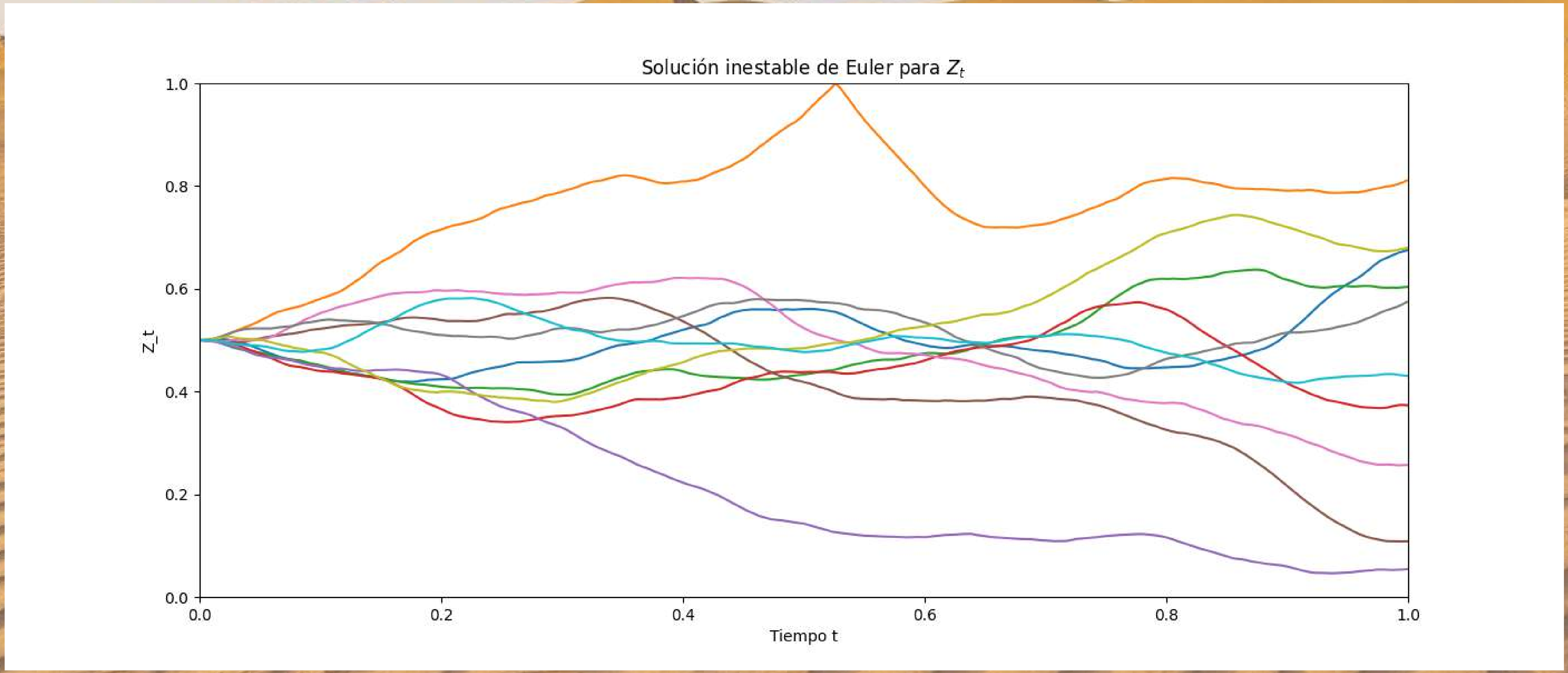
Diferencias entre posiciones finales por perfil (sin considerar Leggraup)

$$\sum_{k=1}^{N} |Z_i[k,T] - Z_j[k,T]|$$

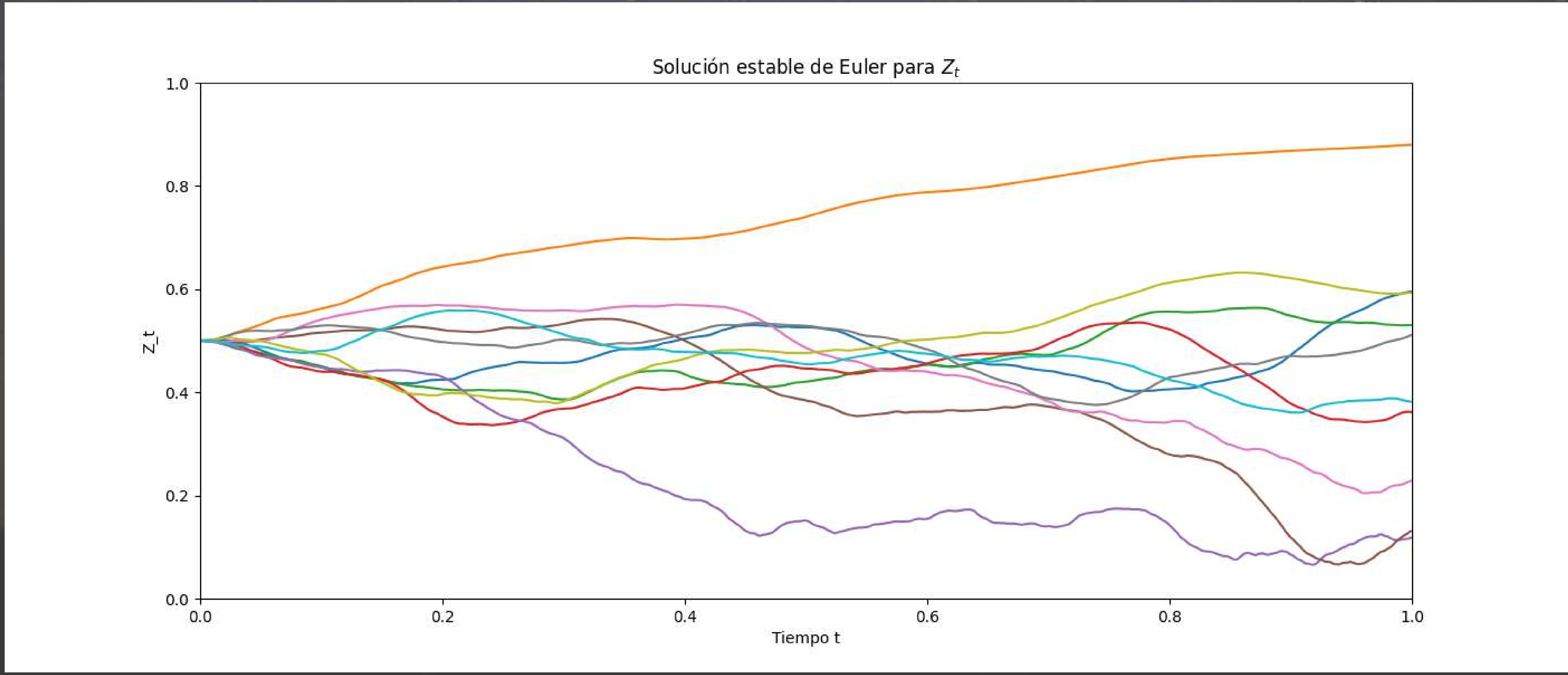
para i, j modelos
Euler, Milstein, H🎅N-SKRII

Inestable	Estable	Neutral
0.005155799200001462	0.007933398856934878	0.002410909905237093
0.04330068093297118	0.006485248213453171	0.0017152932854221925
0.044547458482343635	0.01320440622631501	0.0033884149099342176

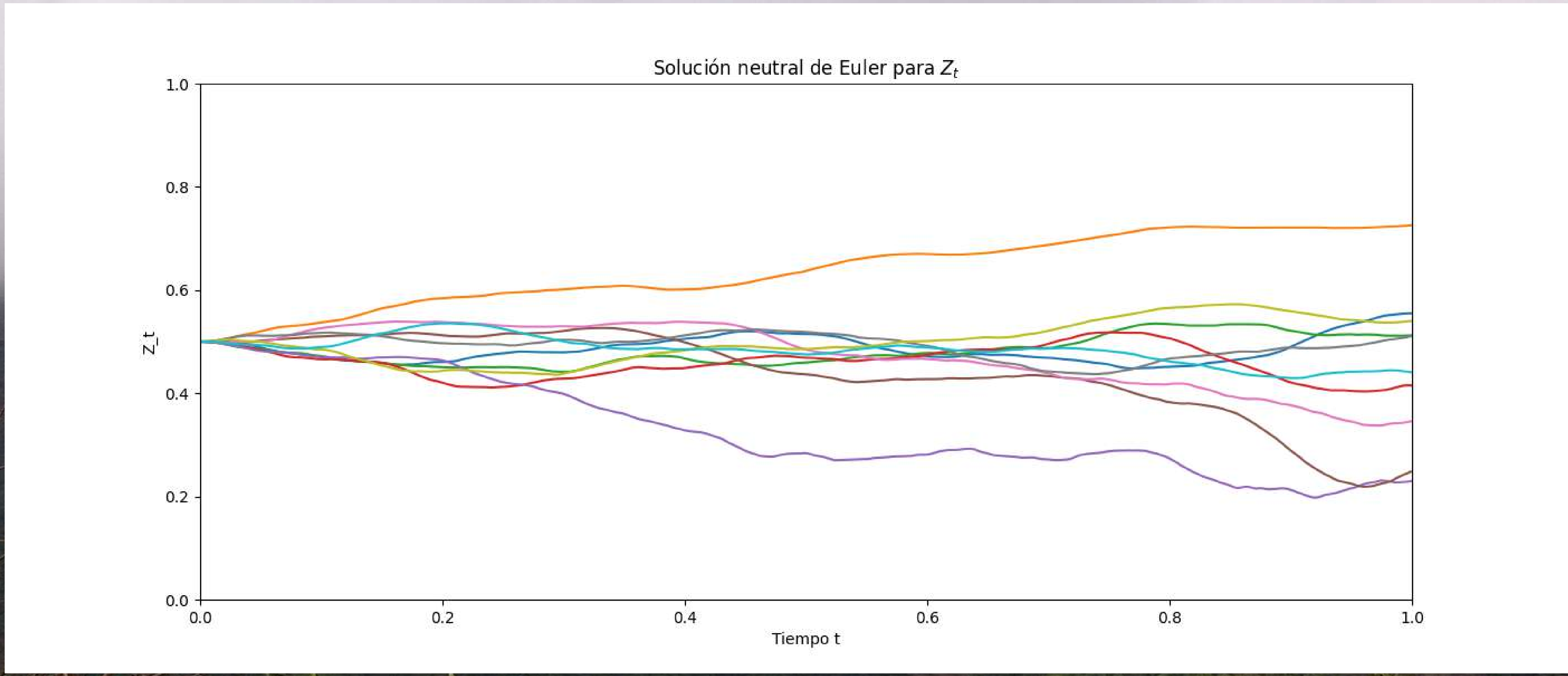
Posición caso inestable



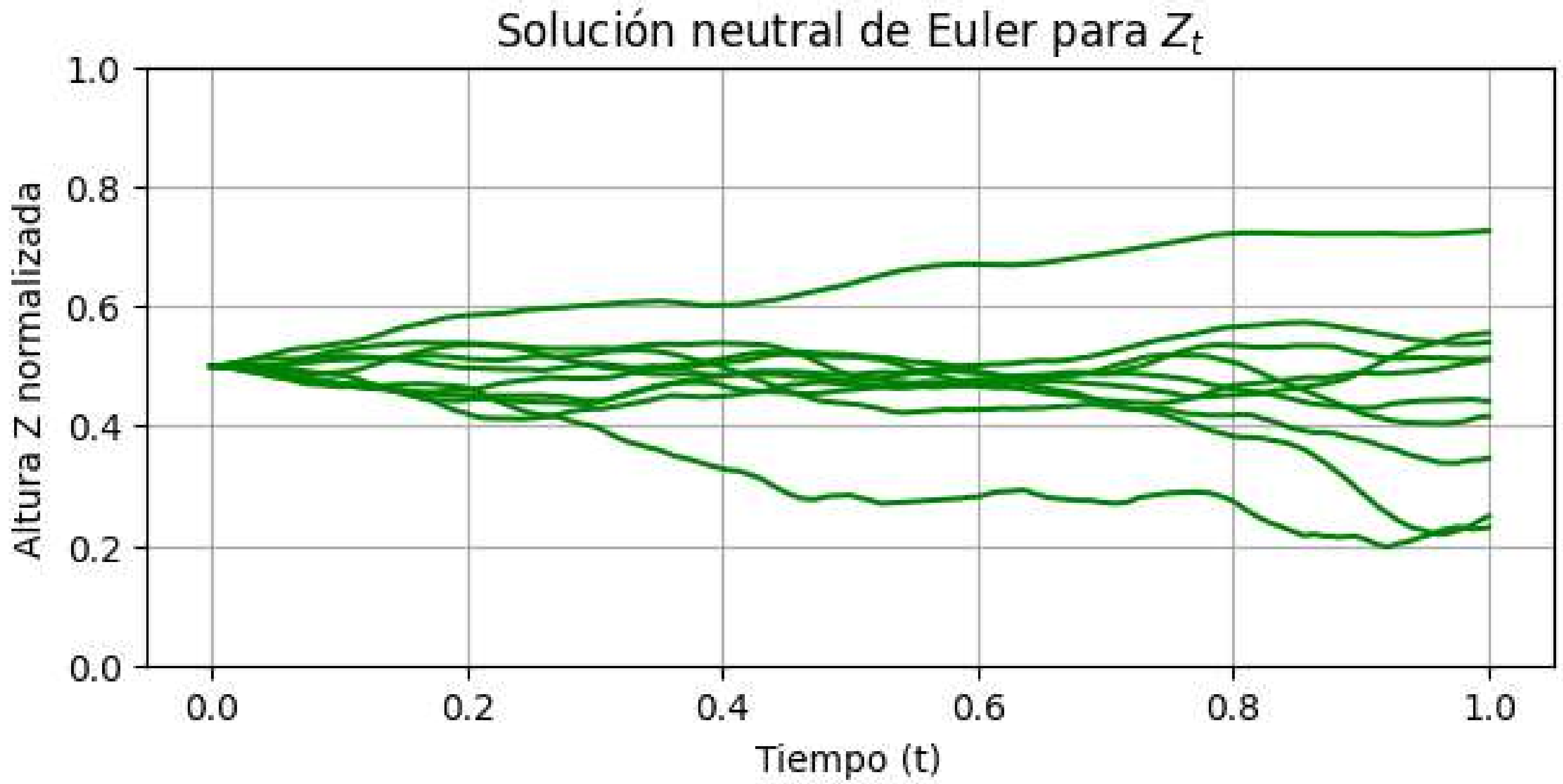
Posición caso estable

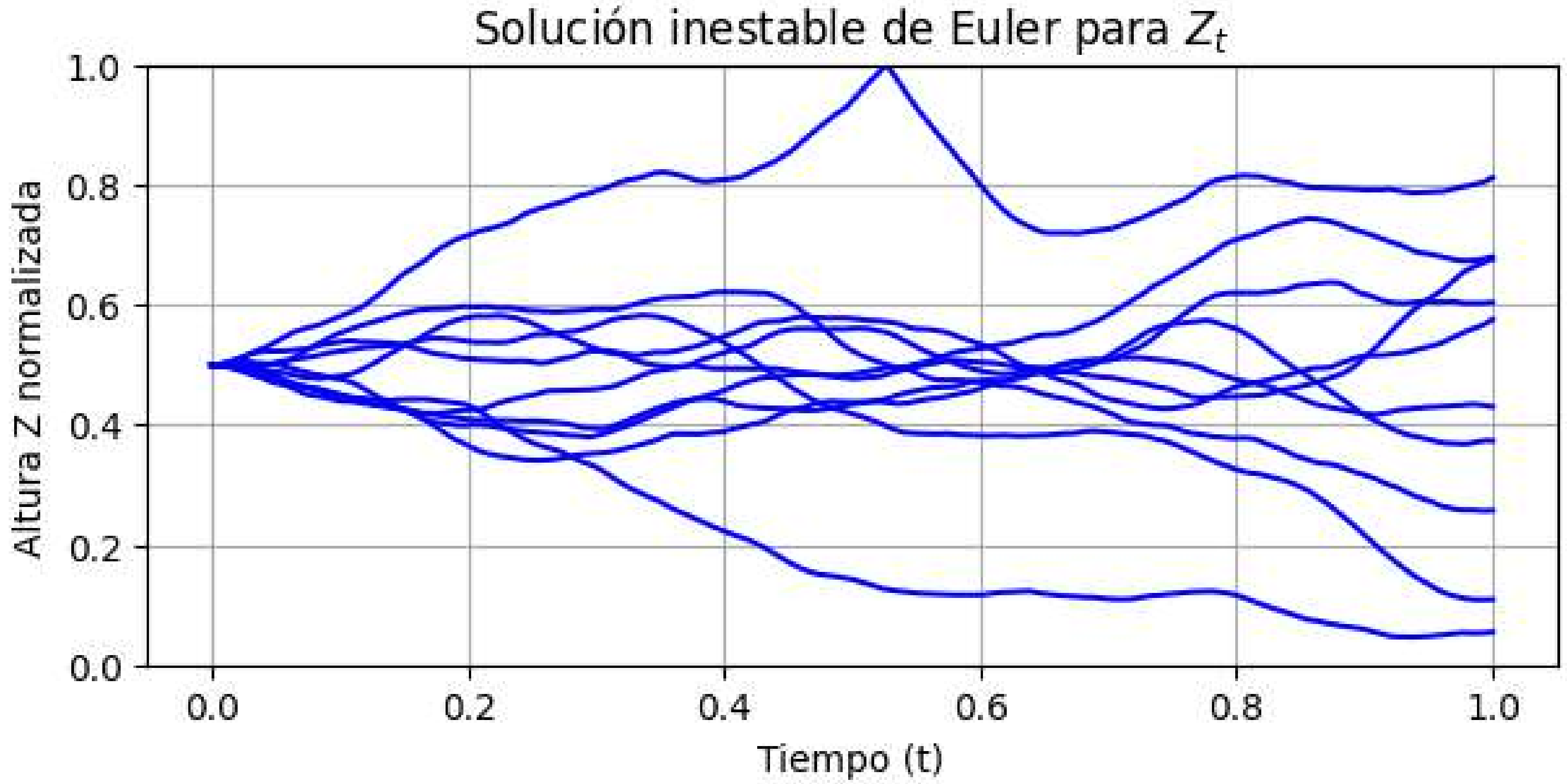


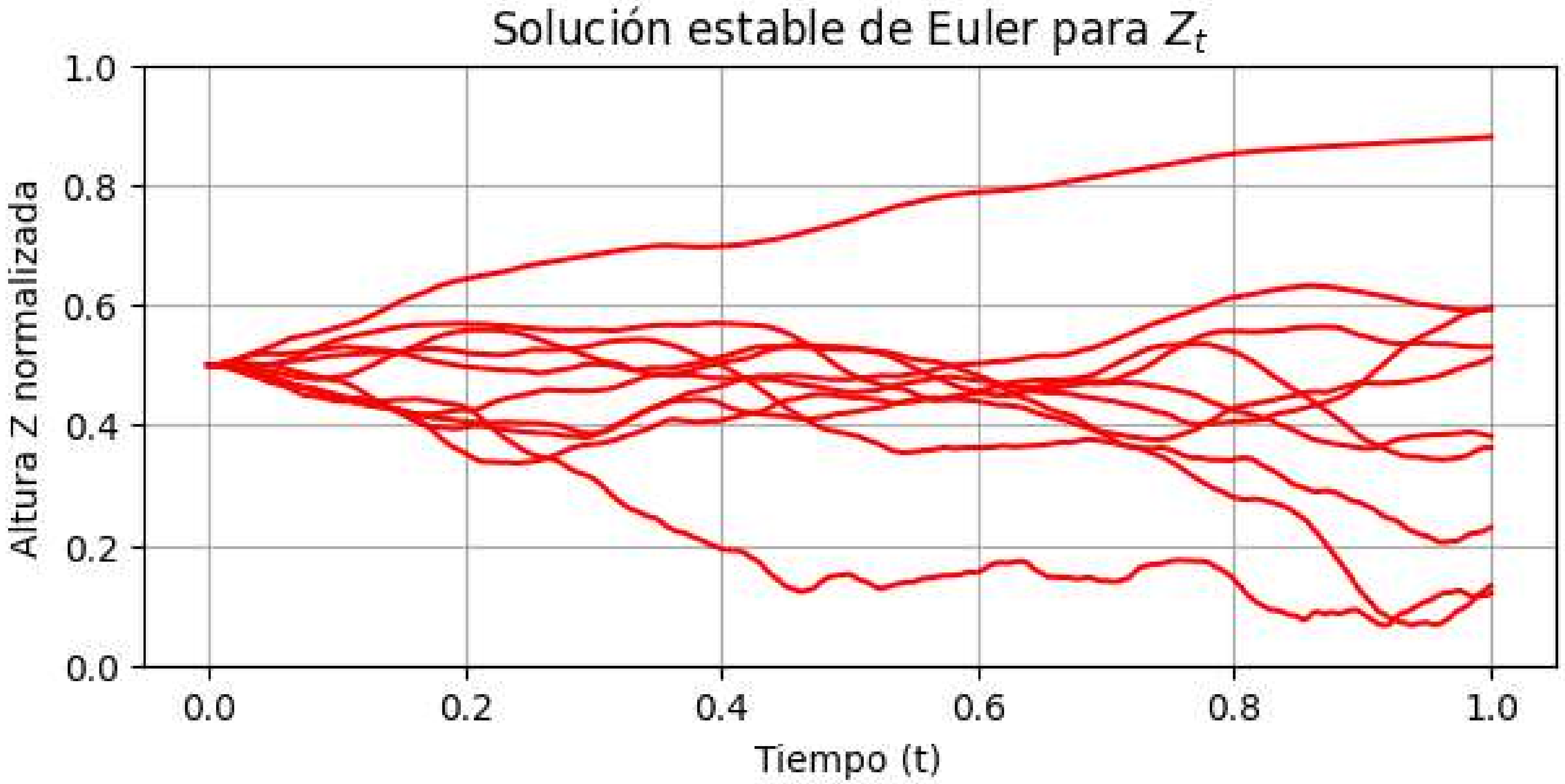
Posición caso neutral



¿Hay diferencias?





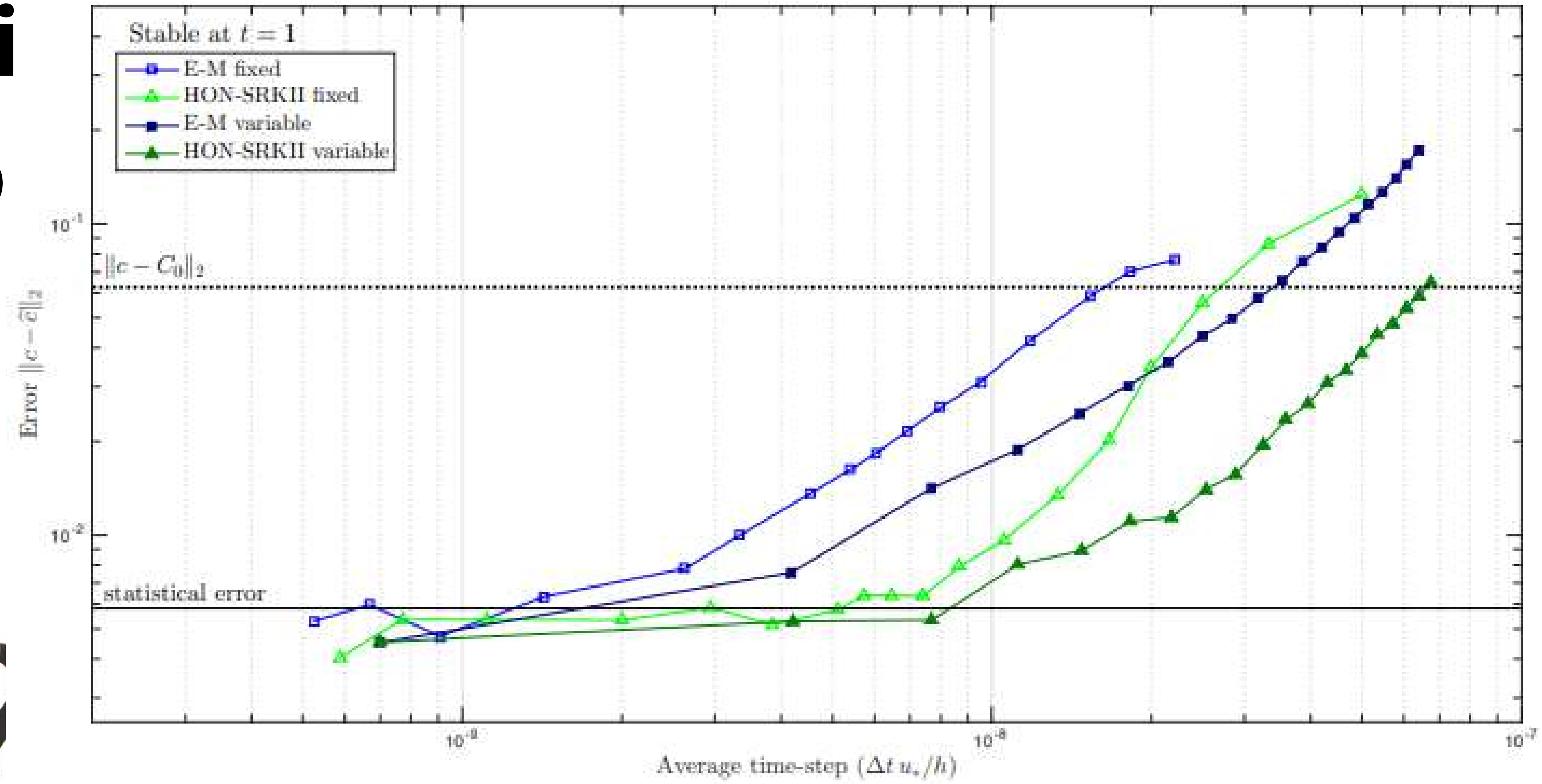


¿Cómo podríamos mejorar?

-método para saber si el modelo es correcto con FPE

-variar parámetros

-Leggraup



Referencias

[1] Nurul Huda, M. (2016). Stochastic trajectory modelling of atmospheric dispersion. University College London.

Gracias por su atención

Felices fiestas 🎅🏻