Determinants of economic growth: Insights from demographics, labor markets, and international trade

FEM11149 - Introduction to Data Science

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Introduction

Economic growth is commonly defined as the sustained increase in real Gross Domestic Product (GDP), reflecting a country's ability to expand production, improve living standards, and maintain long-term development. Forecasts of Gross Domestic Product (GDP) growth are crucial for policymakers, researchers, and international organizations, as they inform fiscal policy, trade negotiations, and borrowing decisions. For instance, governments may finance higher expenditure through debt, but the sustainability of such debt ultimately depends on the economy's future growth potential. Classical models, such as Solow's growth theory, explain how long-term economic growth can be achieved through technological advancements, labor force growth, and capital accumulation. The Demographic Transition Model (DTM) describes how declining mortality and fertility rates affect population structures, which in turn impact economic growth. Meanwhile, the open-economy theory highlights international trade as a potential driver of growth. Identifying the determinants of GDP growth is important for testing existing theoretical predictions, as well as for designing effective policies that foster sustainable development and macroeconomic stability.

Building on this motivation, the central research question guiding this analysis is: **To what extent can economic and demographic variables, such as trade balance, unemployment, and population characteristics, be used to predict GDP growth?** Investigating this question is significant because it clarifies the structural factors that shape economic performance and provides a foundation for more accurate forecasting. A better understanding of these relationships enables policymakers to anticipate risks, develop sustainable fiscal strategies, and allocate resources more efficiently. It may also aid in reducing misallocation of resources, where funds are directed toward ineffective or low-impact policies rather than those that promote sustainable, long-term growth.

Data

The analysis is based on the data obtained from the *World Bank's Global Jobs Indicators Database* and *Balance of Payments statistics*. The dataset consists of 73 variables that capture information on widely used macroeconomic and demographic indicators from 150 countries. The target variable is GDP growth, a measure of economic performance. The data includes explanatory variables like trade balance, unemployment, net trade in goods and services, labor force participation, and population characteristics. Most of these are expressed as percentages or ratios, ensuring consistency and reducing the need for additional transformations prior to analysis. For example, the trade balance is measured as a share of GDP, while unemployment is expressed as a percentage of the labor force.

Simple descriptive statistics illustrate the variation across economies. On average, the annual GDP growth rate is 2.6%, but country-level outcomes vary widely, ranging from severe contractions of over –34.3% to strong expansions of more than 13%. The average unemployment rate is 8.1%, with some countries experiencing very low rates (close to 0.2%) while others face levels above 27%. Population growth is generally positive, averaging at 1.4%, but ranges from slight declines (–1.9%) to rapid expansions exceeding 5% per year.

By combining macroeconomic indicators with demographic and labor market measures, the dataset offers a broad overview of the potential drivers of GDP growth. Its cross-country coverage enables the identification of structural patterns and relationships that can inform both policy and strategic decision-making. While the dataset does not capture every possible determinant of growth, it offers a sufficiently comprehensive view to support meaningful insights into the how different factors contribute to growth performance across countries.

Methods

Regression analysis is a widely used statistical technique for examining how a dependent variable changes based on one or more explanatory variables. This is particularly useful in economic contexts, where the goal is often to predict the quantitative impact of multiple structural and demographic factors on a country's performance. The standard approach, Ordinary Least Squares (OLS), estimates coefficients by minimizing the squared differences between observed and predicted values. In its general form, an OLS regression can be expressed as $y_i = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_k x_{ik} + \varepsilon_i$, where y_i is the dependent variable, x_{ij} are the explanatory variables, β_j are coefficients, and ε_i is the error term. OLS relies on assumptions such as linearity of relationships, independence of errors, homoscedasticity (constant variance of errors), and the absence of perfect multicollinearity among explanatory variables. In practice, economic and demographic data often challenge these assumptions, leading to weakened interpretability and predictive accuracy. For example, variables such as unemployment, labor participation, and population growth may be correlated, leading to multicollinearity that inflates standard errors and undermines the reliability of coefficient estimates.

Penalized regression methods aim to solve this issue by adding a penalty term to the absolute size of the coefficient estimates, shrinking them towards zero. The LASSO regression (Least Absolute Shrinkage and Selection Operator) in particular uses an L_1 penalty, which stabilizes estimation and performs variable selection by setting some coefficients exactly to zero instead of fitting all available predictors. Mathematically, LASSO minimizes the sum of squared errors plus a penalty term, expressed as $\min_\beta \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 + \lambda \sum_{j=1}^{p} |\beta_j|$, where λ controls the strength of the penalty. Larger values of λ produce simpler models with fewer predictors, while smaller values allow more variables to remain in the model. Ridge regression applies an L_2 penalty, minimizing the sum of squared residuals plus $\lambda \sum_j \beta_j^2$, which also shrinks the coefficients towards zero but does not remove them entirely. This makes Ridge useful when dealing with many correlated predictors, as it distributes the effect across them instead of excluding variables. Elastic Net addresses this by blending the L_1 and L_2 penalties into one objective, $\min_\beta \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 + \lambda \left[\alpha \sum_{j=1}^{p} |\beta_j| + (1-\alpha) \sum_{j=1}^{p} \beta_j^2\right]$, where $\alpha \in [0, 1]$ controls the balance between LASSO and Ridge. This flexibility makes Elastic Net particularly effective when predictors are highly correlated, as it can select groups of variables together while still maintaining shrinkage for stability.

For model diagnostics, the main concern is predictive power rather than just achieving the best in-sample fit. While the coefficient of determination (R^2) provides an indication of how much of the variation in the dependent variable is explained, a high R^2 does not necessarily indicate strong predictive accuracy. To avoid overfitting, penalized regressions rely on cross-validation, which evaluates how well the model generalizes to unseen data. The performance of each model is typically assessed using a loss function, most commonly the mean squared error (MSE) or the mean absolute error (MAE). Based on this evaluation, two values of the penalty parameter λ are often reported: λ_{min}, which minimizes the chosen prediction error, and λ_{1se}, which selects a simpler model where its error is within one standard error of the minimum.

While λ_{min} achieves the lowest estimated prediction error, it can sometimes result in overfitting by tailoring

the model too closely to the specific folds of the cross-validation procedure. This can reduce stability and limit the model's ability to generalize to new data, effectively capturing both noise and signal. The "1-SE rule" mitigates this risk by selecting the largest λ within one standard error of the minimum error. This approach yields a simpler and more parsimonious model, trading a very small loss in accuracy for improved robustness and generalizability. In practice, the stability of cross-validation also depends on how the folds are generated. Setting a random seed (e.g., using `set.seed()` in R) ensures that results are reproducible, since different random splits of the data can otherwise lead to slightly different values of λ_{min} and λ_{1se}. Using different seeds can therefore yield different models, especially when predictors are highly correlated, which highlights the importance of interpreting the results with caution and relying on methods that improve stability, such as Elastic Net. For this reason, results should not be viewed as exact, but rather as indicators of broader economic patterns.

Together, LASSO and Elastic Net provide a powerful framework for analyzing high-dimensional economic data. LASSO highlights the most relevant drivers of the dependent variable, while Elastic Net improves stability when predictors are correlated. By applying these methods, it becomes possible to balance interpretability and predictive accuracy, ensuring that the resulting models are both theoretically informative and practically useful.

Results

To assess the determinants of GDP growth, we estimated an OLS regression model using key economic and demographic variables that we deemed relevant through economic theory and statistical methods (like stepwise and variance inflation factors), with and without the inclusion of net trade in goods and services. Table 5 (in Appendix A) presents the results of these two specifications. The comparison shows that adding net trade does not improve the model: the coefficient on net trade is effectively zero and not statistically significant, and the explanatory power of the model even falls slightly when it is included (Adjusted R^2 decreases from 0.137 to 0.131). This suggests that net trade does not contribute additional explanatory value for GDP growth in this dataset. Instead, factors such as life expectancy, employment, and GDP per capita remain the more relevant predictors of growth.

Adding nonlinear and interaction terms does affect the model - refer to table 6 in Appendix A - to a limited extent. The adjusted R^2 increases slightly compared to the baseline specification (from 0.131 without nonlinearities to 0.137 with them), indicating a modest improvement in explanatory power. Importantly, the squared GDP per capita term becomes statistically significant, suggesting a convergence effect: countries with higher income levels grow more slowly, but the nonlinear term implies diminishing returns as economies develop. Similarly, the export value index and life expectancy remain strong predictors of growth.

On the other hand, the interaction term between unemployment and labor force participation does not reach significance, nor does squared population growth, meaning they add little explanatory value. The residual standard error and overall fit remain close to the baseline model, which shows that while nonlinear terms enrich the interpretation of certain variables (especially GDP per capita), they do not drastically improve model performance.

Overall, the models perform reasonably well, but the diagnostic plots highlight some issues that should be taken into account. The residuals are generally centered around zero, yet a few unusual observations appear as outliers, suggesting that some countries may exert more influence on the results than others. Among the diagnostics, the Scale–Location plot (Figure X) is particularly important because it shows that the variance of the errors changes with fitted values, a sign of heteroskedasticity. This means the model explains some parts of the data more consistently than others, which can make the reported standard errors less reliable. To address this, we rely on robust standard errors and check for influential points to ensure that no single country drives the findings. With these adjustments, the results remain reliable, but they should be interpreted as indicating general patterns rather than precise predictions. **need to include the plot and make it proper**

To test the robustness of the models, the models were re-estimated the model using LASSO, the results

showed that only a small number of variables consistently matter for explaining GDP growth. At a moderate penalty, LASSO kept factors like employment, life expectancy, unemployment, exports, and GDP per capita, while shrinking most of the other variables to zero. This means those extra variables don't add much predictive power. When the penalty is made stronger, the model drops everything and reduces to just the average growth rate, showing that the data do not strongly support keeping many predictors. In comparison to the OLS model, which included every variable, LASSO highlights the ones that really drive the results and removes the noise. In practice, this makes the model simpler, less prone to overfitting, and more focused on the variables that actually matter.

Standardizing predictors before applying LASSO is important because the penalty is applied to the scale of the coefficients. When variables are standardized, the changes are on a comparable scale, so the penalty is lower and the model can retain meaningful predictors. Without standardization, variables measured in larger units are penalized more heavily, which can cause all coefficients to shrink to zero regardless of their predictive power. As shown in Table 1, without standardization being applied, most coefficients go to zero, while with standardization some predictors remain in the model. This demonstrates that standardization ensures the penalty works fairly across variables.

While the LASSO results are informative, they were estimated on a reduced set of predictors to illustrate the role of variable selection and standardization. To further improve upon these baseline models, we now employ the full dataset and introduce additional forms of regularization. In particular, ridge regression and elastic net allow us to balance shrinkage and selection in different ways, potentially yielding models that are both more stable and more accurate in prediction.

Cross-validation identified an optimal penalty of .min = 0.1827, which minimizes prediction error, and a more restrictive value of .1se = 2, which favors a simpler model. At .min, the LASSO retains a broad set of predictors, including demographic variables such as adolescent fertility rates, population structure, and life expectancy, along with labor market indicators like employment in industry, labor force participation, and unemployment. Institutional and economic measures, such as the export value index, tax-related indicators, and credit information, also remain in the model. These results suggest that a wide range of socioeconomic factors can contribute to explaining GDP growth when the model is allowed more flexibility. However, at .1se, the penalty is stronger, and all coefficients shrink to zero. This sharp reduction illustrates why relying solely on .min can sometimes lead to overfitting: the model captures more noise in an attempt to minimize error. The 1-SE rule mitigates this risk by favoring a more parsimonious model that sacrifices a small amount of predictive accuracy in exchange for greater robustness. Economically, this highlights that although factors like human capital (life expectancy, employment patterns) and trade (exports) appear relevant under weaker penalization, their effects are not strong or consistent enough to survive stricter regularization in our dataset.

For Elastic Net, cross-validation selected a mixing parameter of = 0.4, which indicates that the best-performing model combines both LASSO (variable selection) and Ridge (shrinkage) elements, with a slightly stronger tilt toward Ridge. The optimal penalty values were .min = 0.3163, minimizing the mean squared error, and .1se = 2.2320, which yields a more parsimonious model. Compared to LASSO, Elastic Net provides greater stability by shrinking correlated predictors together rather than arbitrarily selecting only one of them. This is particularly relevant in a dataset like ours, where many economic and demographic indicators are strongly correlated (e.g., labor market variables, population measures). In practice, the results suggest that while LASSO highlights individual variables as potential drivers of GDP growth, Elastic Net balances this by keeping groups of related predictors, making it less sensitive to data noise and more reliable for generalization. Thus, Elastic Net serves as a compromise between the sparsity of LASSO and the stability of Ridge regression, offering a more robust framework for capturing the complex relationships in the data.

Table 1: Optimal tuning parameters for LASSO and Elastic Net

Method	Metric	Value
LASSO	λ.min	0.1827
LASSO	λ.1se	2.0000
Elastic Net	α	0.4000

Method	Metric	Value
Elastic Net	λ.min	0.3163
Elastic Net	λ.1se	2.2320

Table 2: Test Performance (RMSE, MAE, R^2, SD_y)

Model	RMSE	MAE	R2	SD_y
Baseline (mean of train)	6.570	2.925	-0.053	6.486
Lasso (lambda.min)	6.584	3.002	-0.057	6.486
Lasso (lambda.1se)	6.570	2.925	-0.053	6.486
Elastic Net =0.4 (lambda.min)	**6.552**	**3.057**	**-0.047**	**6.486**
Elastic Net =0.4 (lambda.1se)	6.570	2.925	-0.053	6.486

At first glance, it might seem enough to run a regular regression and simply remove the variables that don't look significant. But this approach has some important problems. Standard regression struggles when variables are highly related to each other, or when there are many variables compared to the number of observations. In those cases, the estimates can jump around a lot depending on small changes in the data, which makes the model unreliable.

Penalized regression solves this by adding a "penalty" that discourages the model from becoming too complex. This keeps the estimates more stable and helps the model focus on the variables that really matter. It also avoids the risk of overfitting, which can happen if we keep testing and removing variables until only a few remain — that kind of trial-and-error tends to fit the current dataset too closely, but performs poorly when applied to new data.

Another key advantage is that penalized regression can automatically deal with situations where traditional regression would fail entirely — for example, when there are more variables than countries in the dataset, or when variables are so correlated that standard regression cannot separate their effects. In these situations, methods like LASSO or Elastic Net still produce workable, interpretable models.

In short, penalized regression is not just a more complicated way of doing regression — it's a safer and more reliable approach that helps us avoid misleading conclusions. Instead of giving us a model that only looks good on paper, it provides results that are more likely to hold up in practice, which is essential for making sound policy and business decisions.

Additionally, we wanted to check whether some countries tend to grow faster than most others. To make that precise, we defined a binary target **Growing more** (1 if annual GDP growth $>$ **2.7%**, 0 otherwise) and fit logistic regression with a **Ridge** penalty, comparing the cross-validated choices λ_{min} and λ_{1se}. As reported in **Table 3**, the λ_{1se} model outperforms λ_{min} on the test set—higher **Accuracy** (0.525 vs 0.475) and, more importantly, lower **LogLoss** (0.6871 vs 0.8054) and **Brier** (0.2471 vs 0.2774), with **AUC** also improving (0.563 vs 0.464).

Table 3: Ridge (logistic) — Test Metrics

Model	Accuracy	LogLoss	Brier	AUC
Ridge (lambda.min)	0.475	0.8054	0.2774	0.464
Ridge (lambda.1se)	0.525	0.6871	0.2471	0.563

When we repeat the analysis with a different train–test split (**Table 4**), performance shifts: in **Run 1**, λ_{min} yields better **LogLoss**, **Brier**, and **AUC** (0.6553, 0.2321, 0.619), while λ_{1se} slightly improves **Accuracy** (0.575 vs 0.525). In **Run 2**, the pattern returns to what we saw in **Table 3**, with λ_{1se} leading across metrics.

This split-to-split variability—visible in **Table 4**—is exactly what we would expect with a modest sample and a threshold near many observations. Averaging the two runs in **Table 4**, λ_{1se} offers better **Accuracy** and calibration (lower **LogLoss**/**Brier**), whereas λ_{min} retains a slight **AUC** advantage. Taken together with **Table 3**, we favor the more regularized λ_{1se} model for its stability and better-calibrated probabilities.

If we could add data, we would first prioritize **more countries (rows)** to reduce variance and stabilize results; then we would add **relevant** predictors (institutions, investment, demographics, trade exposure, external shocks) to capture more of the true drivers. Predicting **continuous GDP growth** is preferable when magnitudes matter for budgeting and scenario analysis, whereas predicting **Growing more** is more useful for threshold-based decisions where an actionable probability—"will this country exceed 2.7%?"—is exactly what is needed.

Table 4: Ridge (logistic) — Test Metrics (different seeds)

	Run 1				Run 2			
Model	Accuracy	LogLoss	Brier	AUC	Accuracy	LogLoss	Brier	AUC
Ridge (lambda.min)	0.525	0.6553	0.2321	0.619	0.475	0.8054	0.2774	0.464
Ridge (lambda.1se)	0.575	0.6821	0.2445	0.500	0.525	0.6871	0.2471	0.563

Appendix

Appendix A

Table 7: Comparison of LASSO Coefficients With and Without Standardization

	No.Standardization	Standardization
(Intercept)	2.609816	-6.626631
`Fertility rate, total (births per woman)`	0.000000	0.000000
`Employers, total (% of total employment) (modeled ILO estimate)`	0.000000	-0.398982
`Labor force participation rate, total (% of total population ages 15+) (modeled ILO estimate)`	0.000000	0.002144
`Life expectancy at birth, male (years)`	0.000000	0.164192
`Unemployment, total (% of total labor force) (modeled ILO estimate)`	0.000000	-0.061668
`Export value index (2000 = 100)`	0.000000	0.001245
`Population growth (annual %)`	0.000000	0.000000
`Population growth (annual %)^2`	0.000000	0.000000
`GDP per capita, PPP (constant 2011 international $)`	0.000000	-0.000045
`GDP per capita, PPP (constant 2011 international $)^2`	0.000000	0.000000
`Net trade in goods and services (BoP, current US$)`	0.000000	0.000000
`Wage and salaried workers, total (% of total employment) (modeled ILO estimate)`	0.000000	-0.000340
`Unemp x LFP`	0.000000	0.000000

Table 5: Comparison of OLS Models: With vs. Without Net Trade

	Dependent variable:	
	GDP growth (annual %)	
	With Net Trade	Without Net Trade
	(1)	(2)
Fertility rate	−0.140	−0.136
	(0.643)	(0.639)
Employers (%)	−0.477***	−0.477***
	(0.123)	(0.122)
Labor force part. (%)	0.010	0.010
	(0.040)	(0.040)
Life expectancy (male)	0.261***	0.262***
	(0.093)	(0.093)
Unemployment (%)	−0.066	−0.066
	(0.064)	(0.064)
Export value index	0.002*	0.002*
	(0.001)	(0.001)
Population growth (%)	0.202	0.196
	(0.479)	(0.474)
GDP per capita (PPP)	−0.0001**	−0.0001**
	(0.00003)	(0.00003)
Net trade (BoP, US$)	0.000	
	(0.000)	
Wage & salaried workers (%)	−0.010	−0.010
	(0.026)	(0.026)
Intercept	−12.514	−12.574
	(8.524)	(8.475)
Observations	150	150
R^2	0.189	0.189
Adjusted R^2	0.131	0.137
Residual Std. Error	4.250 (df = 139)	4.235 (df = 140)
F Statistic	3.242*** (df = 10; 139)	3.627*** (df = 9; 140)
Note:		*p<0.1; **p<0.05; ***p<0.01

Table 6: OLS Regression with Nonlinear and Interaction Effects

	Dependent variable:
	GDP growth (annual %)
Fertility rate	−0.127
	(0.651)
Employers (%)	−0.500***
	(0.123)
Labor force participation (%)	−0.009
	(0.058)
Life expectancy (male)	0.303***
	(0.096)
Unemployment (%)	−0.262
	(0.342)
Export value index	0.002**
	(0.001)
Population growth (%)	0.011
	(0.658)
Population growth $(\%)^2$	0.049
	(0.186)
GDP per capita (PPP)	−0.0002**
	(0.0001)
GDP per capita $(\text{PPP})^2$	0.003*
	(0.001)
Wage & salaried workers (%)	0.007
	(0.028)
Unemp × LFP	0.003
	(0.006)
Constant	−13.929
	(8.753)
Observations	150
R^2	0.206
Adjusted R^2	0.137
F Statistic	2.967*** (df = 12; 137)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 8: Variable selection across LASSO and Elastic Net

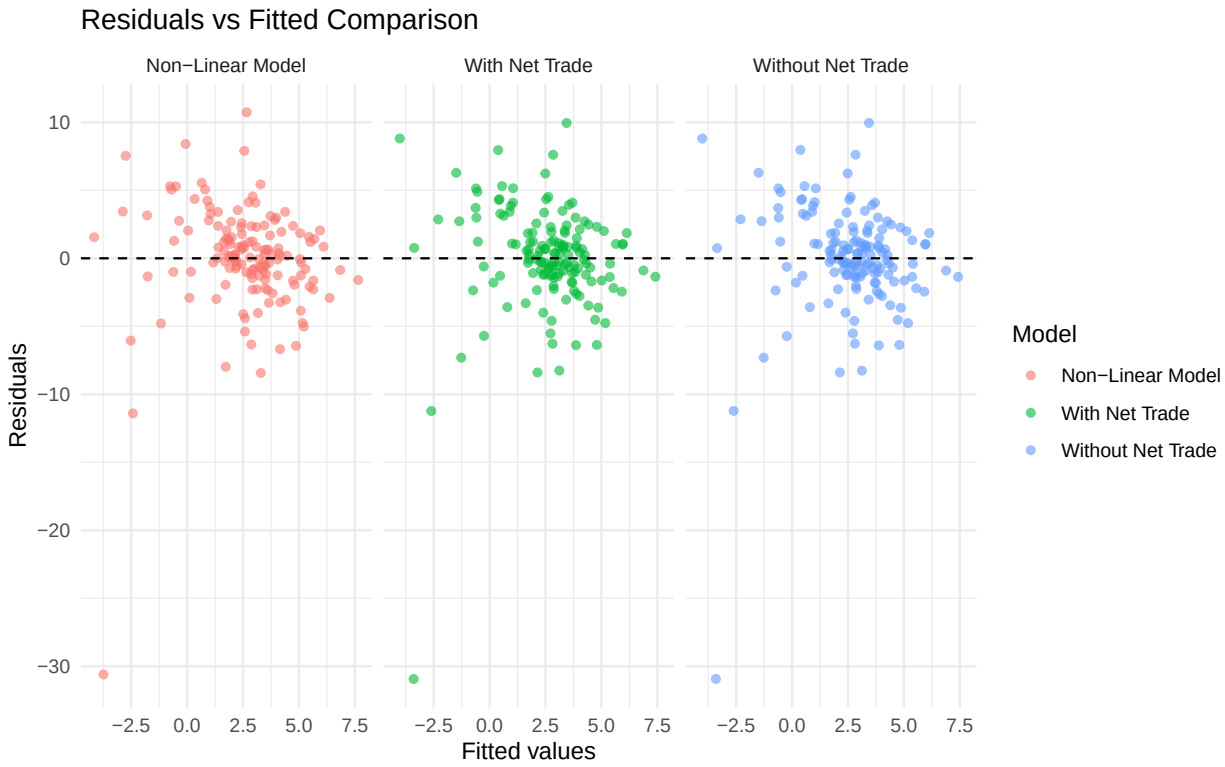
Variable	L_min	L_1se	ENet_min	ENet_1se
(Intercept)	T	T	T	T
Adolescent fertility rate (births per 1,000 women ages 15-19)	T	F	T	F
Contributing family workers, female (% of female employment) (modeled ILO estimate)	T	F	T	F
Depth of credit information index (0=low to 8=high)	T	F	T	F
Employers, male (% of male employment) (modeled ILO estimate)	T	F	T	F
Employment in industry, male (% of male employment) (modeled ILO estimate)	T	F	T	F
Export value index (2000 = 100)	T	F	T	F
GDP per capita, PPP (constant 2011 international $)	T	F	T	F
Labor force participation rate, female (% of female population ages 15+) (modeled ILO estimate)	T	F	T	F
Labor force participation rate, male (% of male population ages 15+) (modeled ILO estimate)	T	F	T	F
Life expectancy at birth, male (years)	T	F	T	F
Own-account workers, total (% of male employment) (modeled ILO estimate)	T	F	T	F
Population ages 15-64, total	T	F	T	F
Population ages 65 and above (% of total)	T	F	T	F
Rural population	T	F	T	F
Tax payments (number)	T	F	T	F
Time required to start a business (days)	T	F	T	F
Time to prepare and pay taxes (hours)	T	F	T	F
Unemployment, male (% of male labor force) (modeled ILO estimate)	T	F	T	F
Unemployment, youth male (% of male labor force ages 15-24) (modeled ILO estimate)	T	F	T	F
Access to electricity (% of population)	F	F	T	F
Contributing family workers, total (% of total employment) (modeled ILO estimate)	F	F	T	F
Employers, female (% of female employment) (modeled ILO estimate)	F	F	T	F
Employment in industry (% of total employment) (modeled ILO estimate)	F	F	T	F
Export volume index (2000 = 100)	F	F	T	F
Labor force, total	F	F	T	F
Own-account workers, male (% of male employment) (modeled ILO estimate)	F	F	T	F
Time required to enforce a contract (days)	F	F	T	F

Table 9: Variables Selected under .min vs .1se

Variable	Coef_min	Coef_1se
(Intercept)	-1.7794971	3.001157
Adolescent fertility rate (births per 1,000 women ages 15-19)	-0.0039180	NA
Contributing family workers, female (% of female employment) (modeled ILO estimate)	-0.0067494	NA

Variable	Coef_min	Coef_1se
Depth of credit information index (0=low to 8=high)	0.0269231	NA
Employers, male (% of male employment) (modeled ILO estimate)	-0.1109801	NA
Employment in industry, male (% of male employment) (modeled ILO estimate)	0.0328390	NA
Export value index (2000 = 100)	0.0013346	NA
GDP per capita, PPP (constant 2011 international $)	-0.0000529	NA
Labor force participation rate, female (% of female population ages 15+) (modeled ILO estimate)	-0.0413561	NA
Labor force participation rate, male (% of male population ages 15+) (modeled ILO estimate)	0.0076160	NA
Life expectancy at birth, male (years)	0.1067299	NA
Own-account workers, total (% of male employment) (modeled ILO estimate)	0.0079536	NA
Population ages 15-64, total	0.0000000	NA
Population ages 65 and above (% of total)	-0.0107295	NA
Rural population	0.0000000	NA
Tax payments (number)	0.0261170	NA
Time required to start a business (days)	-0.0061819	NA
Time to prepare and pay taxes (hours)	-0.0031672	NA
Unemployment, male (% of male labor force) (modeled ILO estimate)	-0.0353272	NA
Unemployment, youth male (% of male labor force ages 15-24) (modeled ILO estimate)	-0.0287594	NA

Appendix B


Residuals vs Fitted Comparison
Non−Linear Model
With Net Trade
Without Net Trade
Residuals
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0
−10
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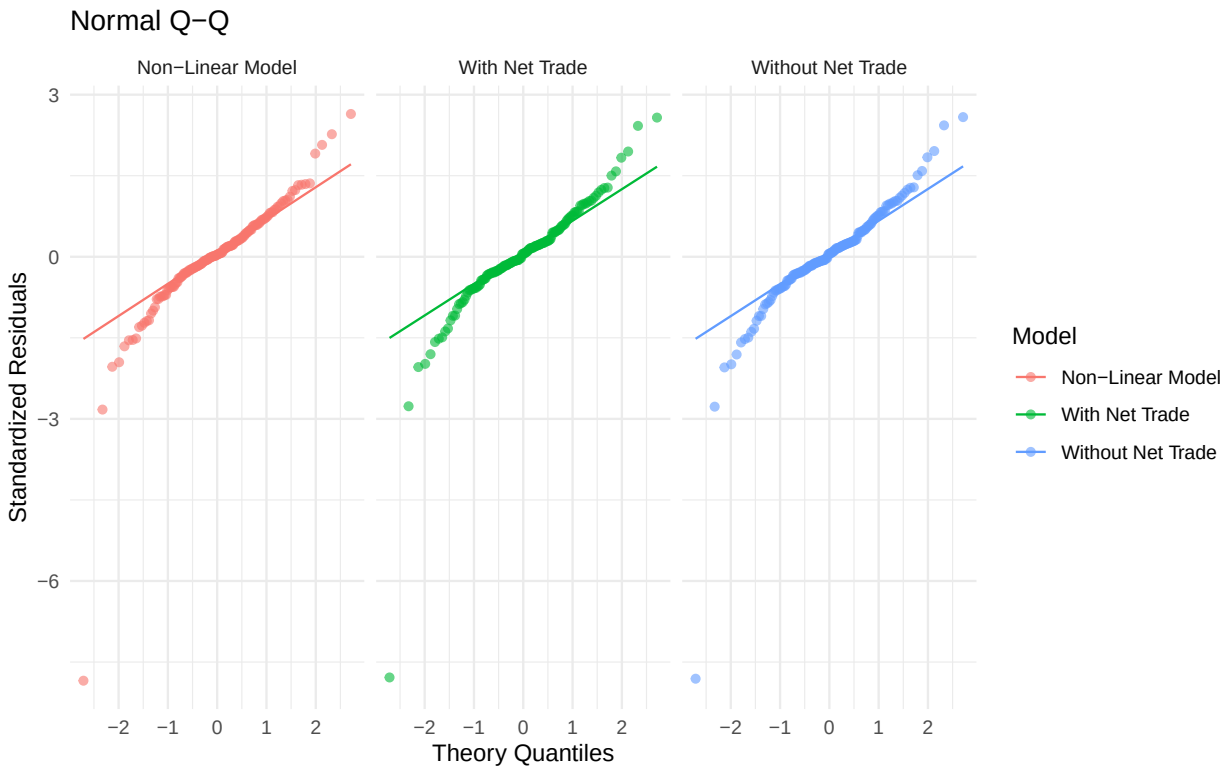
−30
−2.5
0.0
2.5
5.0
7.5
Fitted values
Model
Non−Linear Model
With Net Trade
Without Net Trade


Normal Q−Q
Non−Linear Model
With Net Trade
Without Net Trade
Standardized Residuals
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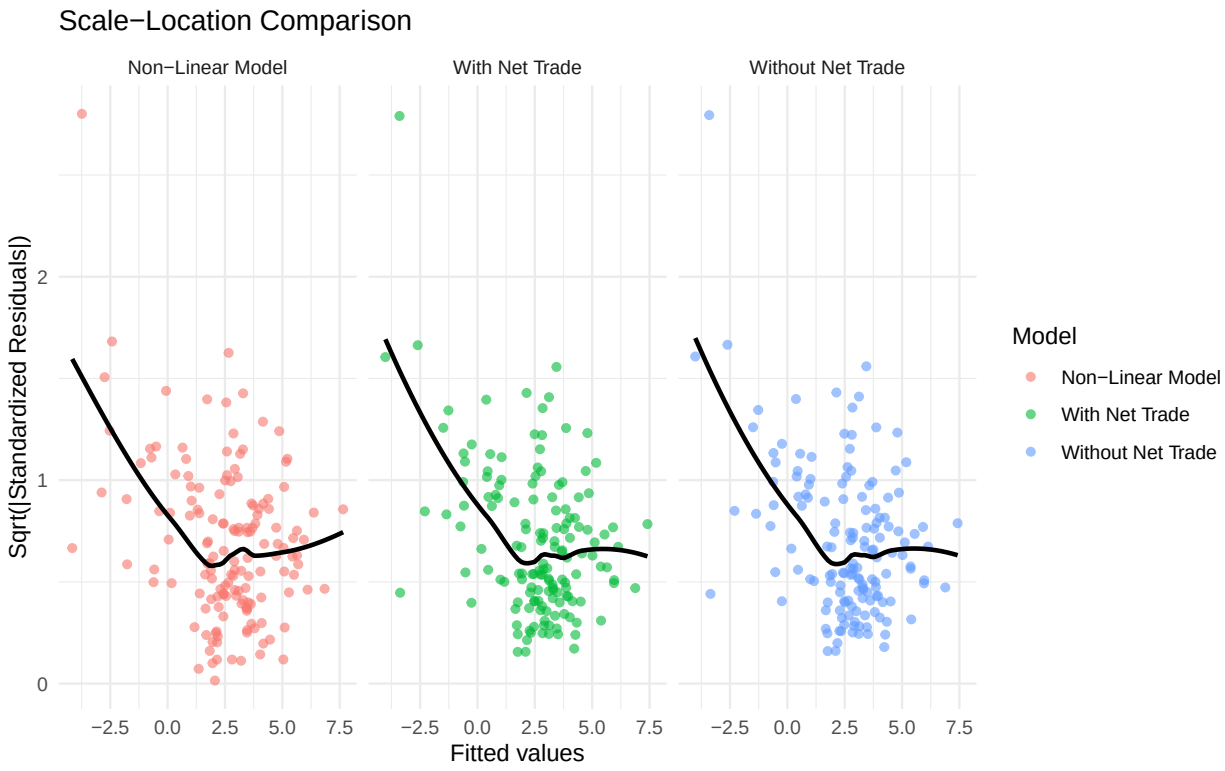
Non−Linear Model
With Net Trade
Without Net Trade



Appendix C

```
# Data Splitting
set.seed(254646); n1<-110; idx<-sample(nrow(df), n1)  # draw n1 rows
data_clean<-df; dfm_train<-data_clean[idx,]; dfm_test<-data_clean[-idx,]  # train/test
Xvars_mat_train<-model.matrix(`GDP growth (annual %)`~.-Country, data=dfm_train)[,-1]  # drop intercept
Yvar_train<-dfm_train$`GDP growth (annual %)`  # target train
Xvars_mat_test<-model.matrix(`GDP growth (annual %)`~.-Country, data=dfm_test)[,-1]   # drop intercept
Yvar_test<-dfm_test$`GDP growth (annual %)`    # target test

# OLS Regressions without trade and with trade
model <- lm(`GDP growth (annual %)`~`Fertility rate, total (births per woman)`+`Employers, total (% of total employment) (modele
+`Labor force participation rate, total (% of total population ages 15+) (modeled ILO estimate)` +`Life expectancy at birth, mal
+`Unemployment, total (% of total labor force) (modeled ILO estimate)`+`Export value index (2000 = 100)`+`Population growth (ann
+`GDP per capita, PPP (constant 2011 international $)`+`Net trade in goods and services (BoP, current US$)`
+`Wage and salaried workers, total (% of total employment) (modeled ILO estimate)`,data = df,singular.ok = FALSE)
model_without <- update(model, . ~ . - `Net trade in goods and services (BoP, current US$)`, singular.ok = FALSE)

# OLS Regression with non linear and interaction effects
df2<-within(df,{`GDP per capita, PPP (constant 2011 international $)^2`<-(`GDP per capita, PPP (constant 2011 international $)`)
`Population growth (annual %)^2`<-(`Population growth (annual %)`^2)
`Unemp x LFP`<-`Unemployment, total (% of total labor force) (modeled ILO estimate)`*`Labor force participation rate, total (% o
model_ext <- lm(`GDP growth (annual %)`~`Fertility rate, total (births per woman)`+`Employers, total (% of total employment) (mo
+`Labor force participation rate, total (% of total population ages 15+) (modeled ILO estimate)`+`Life expectancy at birth, male
+`Unemployment, total (% of total labor force) (modeled ILO estimate)`+`Export value index (2000 = 100)`
+`Population growth (annual %)`+`Population growth (annual %)^2` # nonlinear
+`GDP per capita, PPP (constant 2011 international $)`+`GDP per capita, PPP (constant 2011 international $)^2` # convergence
+`Wage and salaried workers, total (% of total employment) (modeled ILO estimate)`+`Unemp x LFP`,data = df2,singular.ok = FALSE)

models <-list("With Net Trade"=model,"Without Net Trade"=model_without,"Non-Linear Model"=model_ext) #putting in list
df_models <-lapply(names(models),function(name){augment(models[[name]]) %>% mutate(Model=name)}) %>% bind_rows() #tidy dataframe
gg_resfit <-ggplot(df_models, aes(.fitted, .resid, color = Model)) + geom_point(alpha=0.6)+
geom_hline(yintercept=0,linetype="dashed") + facet_wrap(~Model)+labs(title="Residuals vs Fitted Comparison",
```

```
x="Fitted values",y="Residuals")+theme_minimal() #Residuals vs Fitted plot
gg_qq <- ggplot(df_models,aes(sample=.std.resid,color=Model))+stat_qq(alpha=0.6)+stat_qq_line()+facet_wrap(~Model)+
labs(title="Normal Q-Q",x ="Theory Quantiles",y="Standardized Residuals")+theme_minimal() # Normal Q-Q plot
gg_scale <- ggplot(df_models, aes(.fitted, sqrt(abs(.std.resid)),color=Model))+geom_point(alpha = 0.6)+
geom_smooth(se = FALSE, color = "black")+facet_wrap(~Model)+labs(title = "Scale-Location Comparison",
x="Fitted values",y="Sqrt(|Standardized Residuals|)")+theme_minimal() #Scale-Location plot

# LASSO Regression
X_lasso <- model.matrix(`GDP growth (annual %)`~`Fertility rate, total (births per woman)`+`Employers, total (% of total employm
`Labor force participation rate, total (% of total population ages 15+) (modeled ILO estimate)`+`Life expectancy at birth, male
`Unemployment, total (% of total labor force) (modeled ILO estimate)`+`Export value index (2000 = 100)`+
`Population growth (annual %)`+`Population growth (annual %)^2`+`GDP per capita, PPP (constant 2011 international $)`+
`GDP per capita, PPP (constant 2011 international $)^2`+`Net trade in goods and services (BoP, current US$)`+
`Wage and salaried workers, total (% of total employment) (modeled ILO estimate)`+`Unemp x LFP`, # Interaction
data = df2)[, -1] # drop intercept
y_lasso <- df2$`GDP growth (annual %)`
#Cross-validated LASSO
set.seed(2354245);cv_lasso_nl_int<-cv.glmnet(X_lasso,y_lasso,alpha = 1,nfolds = 10,standardize = TRUE)
cv_lasso_nl_int$lambda.min;cv_lasso_nl_int$lambda.1se
coef(cv_lasso_nl_int, s = "lambda.min");coef(cv_lasso_nl_int, s = "lambda.1se")

set.seed(2134);cv_no_std <- cv.glmnet(X_lasso, y_lasso, alpha = 1, nfolds = 10, standardize = FALSE)
coef(cv_no_std, s = "lambda.min") # LASSO without standardization
set.seed(2134);cv_std <- cv.glmnet(X_lasso, y_lasso, alpha = 1, nfolds = 10, standardize = TRUE)
coef(cv_std, s = "lambda.min") # LASSO with standardization (default in glmnet)

coef_no_std<-as.matrix(coef(cv_no_std,s="lambda.min"));coef_std<-as.matrix(coef(cv_std,s="lambda.min"))
comparison <- data.frame(Variable = rownames(coef_no_std),`No Standardization` = round(coef_no_std[,1], 6),
Standardization = round(coef_std[,1], 6)) # Combining into a comparison table

# LASSO Grid search for lambda
set.seed(254646);lambda_grid<-seq(0.001,2,length.out=100)
cv_lasso<-cv.glmnet(Xvars_mat_train,Yvar_train,family="gaussian",alpha=1,lambda=lambda_grid,
nfolds = 10,type.measure="mse",standardize=TRUE)
best_lambda_min<-cv_lasso$lambda.min; best_lambda_1se<-cv_lasso$lambda.1se

coef_min<-as.matrix(coef(cv_lasso,s="lambda.min"));coef_1se<-as.matrix(coef(cv_lasso,s="lambda.1se"))
sel_min<-coef_min[coef_min[,1] !=0, ,drop=FALSE] #Get selected variables from extracted coefficients
sel_1se<-coef_1se[coef_1se[,1] != 0, ,drop=FALSE] #non-zero coefficients
df_min<-data.frame(Variable=rownames(sel_min),Coef_min=sel_min[,1]) #Create data frames
df_1se<-data.frame(Variable=rownames(sel_1se),Coef_1se=sel_1se[,1])
comparative_tbl<-merge(df_min,df_1se,by="Variable",all=TRUE)

rmse<-function(y,p) sqrt(mean((y-p)^2)); mae<-function(y,p) mean(abs(y-p))
r2<-function(y,p) 1-sum((y-p)^2)/sum((y-mean(y))^2) # metrics functions
row_metrics<-function(name,y,p){data.frame(Model=name,RMSE=rmse(y,p),
MAE=mae(y,p),R2=r2(y,p),SD_y=sd(y))}
nz_coefs<-function(cv_obj,s){cf<-as.matrix(coef(cv_obj,s=s))
out<-data.frame(Variable=rownames(cf),Coef=cf[,1],row.names=NULL);subset(out,Coef!=0)}
# Lasso Predictions
pred_lasso_min <- as.numeric(predict(cv_lasso, newx = Xvars_mat_test, s = "lambda.min"))
pred_lasso_1se <- as.numeric(predict(cv_lasso, newx = Xvars_mat_test, s = "lambda.1se"))
met_lasso_min  <- row_metrics("Lasso (lambda.min)",  Yvar_test, pred_lasso_min)
met_lasso_1se  <- row_metrics("Lasso (lambda.1se)",  Yvar_test, pred_lasso_1se)
sel_lasso_min  <- nz_coefs(cv_lasso, "lambda.min"); sel_lasso_1se  <- nz_coefs(cv_lasso, "lambda.1se")
pred_base <- rep(mean(Yvar_train), length(Yvar_test)) # Baseline (mean)
met_base  <- row_metrics("Baseline (mean of train)", Yvar_test, pred_base)

# ELASTIC NET, grid search for alpha and training the model
set.seed(2546); alpha_grid<-seq(0.1,0.9,by=0.1);
cv_list<-lapply(alpha_grid,function(a) cv.glmnet(Xvars_mat_train,Yvar_train,family="gaussian",
alpha=a,nfolds=10,type.measure="mse",standardize=TRUE)); cv_mins<-sapply(cv_list, function(cv) min(cv$cvm)); best_i<-which.min(c
best_cv <- cv_list[[best_i]];cat("\nElastic Net - best alpha:",best_a,"| lambda.min:",signif(best_cv$lambda.min,4), "| lambda.1s

lam_tbl<-data.frame(Method=c("LASSO","LASSO"),Metric=c("$ $.min","$ $.1se"),
Value=c(signif(best_lambda_min,4),signif(best_lambda_1se,4))) # existing lasso table
elastic_tbl<-data.frame(Method=c("Elastic Net","Elastic Net","Elastic Net"),Metric=c("$ $","$ $.min","$ $.1se"),
```

```r
Value=c(best_a,signif(best_cv$lambda.min,4),signif(best_cv$lambda.1se,4))) #adding elastic net results
lam_tbl <- rbind(lam_tbl, elastic_tbl) #combine
knitr::kable(lam_tbl, caption = "Optimal tuning parameters for LASSO and Elastic Net") #display

# Elastic Net Predictions.
pred_en_min<-as.numeric(predict(best_cv, newx = Xvars_mat_test, s = "lambda.min"))
pred_en_1se<-as.numeric(predict(best_cv, newx = Xvars_mat_test, s = "lambda.1se"))
met_en_min<-row_metrics(paste0("Elastic Net  =", best_a, " (lambda.min)"), Yvar_test, pred_en_min)
met_en_1se<-row_metrics(paste0("Elastic Net  =", best_a, " (lambda.1se)"), Yvar_test, pred_en_1se)
sel_en_min<-nz_coefs(best_cv, "lambda.min"); sel_en_1se<-nz_coefs(best_cv, "lambda.1se")

metrics_all<-rbind(met_base, met_lasso_min, met_lasso_1se, met_en_min, met_en_1se) #metric comparison
metrics_tbl<-metrics_all%>%dplyr::select(Model, RMSE, MAE, R2, SD_y) %>%
mutate(across(where(is.numeric),~round(.,3))); best_rmse_idx<-which.min(metrics_all$RMSE)
kable(metrics_tbl,caption="Test Performance (RMSE, MAE, $R^2$, $SD_y$)")%>%
kable_styling(full_width=FALSE,position="center",bootstrap_options=c("striped","hover","condensed"))%>%
row_spec(best_rmse_idx, bold = TRUE)

top_k<-15 #Best features used
top_lasso_min <- head(sel_lasso_min[order(abs(sel_lasso_min$Coef), decreasing = TRUE), ], top_k)
top_lasso_1se <- head(sel_lasso_1se[order(abs(sel_lasso_1se$Coef), decreasing = TRUE), ], top_k)
top_en_min    <- head(sel_en_min[order(abs(sel_en_min$Coef),   decreasing = TRUE), ], top_k)
top_en_1se    <- head(sel_en_1se[order(abs(sel_en_1se$Coef),   decreasing = TRUE), ], top_k)
count_nz <- function(df) sum(df$Variable != "(Intercept)") # amount of variables selected in each model
cat("\n Amount of Variables -> Lasso min:", count_nz(sel_lasso_min),"| Lasso 1se:", count_nz(sel_lasso_1se),
   "| ENet min:", count_nz(sel_en_min), "| ENet 1se:", count_nz(sel_en_1se), "\n")
vars_lasso_min <-sel_lasso_min$Variable; vars_lasso_1se<-sel_lasso_1se$Variable
vars_en_min<-sel_en_min$Variable; vars_en_1se<-sel_en_1se$Variable # extract variable names
all_vars<-unique(c(vars_lasso_min,vars_lasso_1se,vars_en_min,vars_en_1se)) # combined into 1 master vector
comparative_df<-data.frame(Variable=all_vars,L_min=ifelse(all_vars%in%vars_lasso_min,"T","F"),
L_1se=ifelse(all_vars %in% vars_lasso_1se,"T","F"),ENet_min=ifelse(all_vars %in% vars_en_min,"T","F"),
ENet_1se = ifelse(all_vars %in% vars_en_1se,"T","F"))

# Ridge and Logistic
df_logistic<-data_clean[ , !(names(data_clean) %in% "Country")]
df_logistic$GrowingMore <- as.integer(df_logistic$`GDP growth (annual %)` > 2.7)
set.seed(1111); train_index<-sample.int(nrow(df_logistic),110) # Data Splitting (Seed One)
train_set<-df_logistic[train_index, ]; test_set  <- df_logistic[-train_index, ]
y_train<-train_set$GrowingMore; y_test<-test_set$GrowingMore
X_train <- model.matrix(GrowingMore ~ . -`GDP growth (annual %)`, data=train_set)[, -1]
X_test  <- model.matrix(GrowingMore ~ . - `GDP growth (annual %)`, data=test_set )[,-1]

cv_ridge<-cv.glmnet(X_train,y_train,family="binomial",alpha=0,nfolds=10,type.measure="deviance",standardize=TRUE)
lam_min<-cv_ridge$lambda.min; lam_1se<-cv_ridge$lambda.1se
cat("lambda.min =",signif(lam_min,4)," | lambda.1se =",signif(lam_1se,4), "\n")
# Prediction on Test Using both variables.
p_min <- as.numeric(predict(cv_ridge, newx = X_test, s = "lambda.min", type = "response"))
p_1se <- as.numeric(predict(cv_ridge, newx = X_test, s = "lambda.1se", type = "response"))
pred_min <- ifelse(p_min >= 0.5, 1, 0); pred_1se <- ifelse(p_1se >= 0.5, 1, 0) # Classify at 0.5

logloss<-function(y,p){p<-pmin(pmax(p,1e-15),1-1e-15);-mean(y*log(p)+(1-y)*log(1-p))}
brier<-function(y,p) mean((p-y)^2) # Metrics for performance
acc_min<-mean(pred_min==y_test); acc_1se<-mean(pred_1se==y_test); ll_min<-logloss(y_test,p_min)
ll_1se<-logloss(y_test,p_1se); br_min<-brier(y_test,p_min); br_1se<-brier(y_test,p_1se)
get_auc<-function(y,p){if(requireNamespace("pROC",quietly=TRUE))as.numeric(pROC::auc(y,p))else NA_real_}
auc_min<-get_auc(y_test,p_min); auc_1se<-get_auc(y_test,p_1se) # optional AUC
results<-data.frame(Model=c("Ridge (lambda.min)","Ridge (lambda.1se)"),Accuracy=c(acc_min,acc_1se),
LogLoss=c(ll_min,ll_1se),Brier=c(br_min,br_1se),AUC=c(auc_min,auc_1se)); print(results,row.names=FALSE)

results_tbl<-results%>%transmute(Model,Accuracy=round(Accuracy,4),LogLoss=round(LogLoss,4),
Brier=round(Brier,4),AUC=ifelse(is.na(AUC),NA,round(AUC,3)))
kable(results_tbl,caption="Ridge (logistic) - Test Metrics")%>%
kable_styling(full_width=FALSE,position="center",bootstrap_options = c("striped","hover","condensed"))

# Final Model Coefficients, and example showing 10 largest (by |coef|) at lambda.1se
coef_min<-coef(cv_ridge,s="lambda.min"); coef_1se<-coef(cv_ridge,s="lambda.1se")
ix<-order(abs(as.numeric(coef_1se)),decreasing = TRUE); print(coef_1se[ix[1:10], ,drop=FALSE])
```

```r
set.seed(9999); train_index<-sample.int(nrow(df_logistic),110) # Data Splitting (Seed Two)
train_set<-df_logistic[train_index, ]; test_set<-df_logistic[-train_index, ]
y_train<-train_set$GrowingMore; y_test<-test_set$GrowingMore
X_train<-model.matrix(GrowingMore ~ . -`GDP growth (annual %)`, data=train_set)[, -1]
X_test<-model.matrix(GrowingMore ~ . - `GDP growth (annual %)`, data=test_set )[,-1]

# Ridge and Elastic Net.
cv_ridge<-cv.glmnet(X_train,y_train,family="binomial",alpha=0,nfolds = 10,type.measure="deviance",standardize=TRUE)
lam_min<-cv_ridge$lambda.min; lam_1se<-cv_ridge$lambda.1se
cat("lambda.min =", signif(lam_min,4), " | lambda.1se =", signif(lam_1se,4), "\n")
# Prediction on Test Using both variables.
p_min <- as.numeric(predict(cv_ridge, newx = X_test, s = "lambda.min", type = "response"))
p_1se <- as.numeric(predict(cv_ridge, newx = X_test, s = "lambda.1se", type = "response"))
pred_min<-ifelse(p_min>=0.5,1,0); pred_1se<-ifelse(p_1se>=0.5,1,0) # Classify at 0.5

logloss<-function(y,p){p<-pmin(pmax(p,1e-15),1-1e-15); -mean(y*log(p)+(1-y)*log(1-p))} # Metrics for performance
brier<-function(y,p) mean((p-y)^2); acc_min<-mean(pred_min==y_test); acc_1se<-mean(pred_1se==y_test)
ll_min<-logloss(y_test,p_min); ll_1se<-logloss(y_test,p_1se); br_min<-brier(y_test,p_min); br_1se<-brier(y_test,p_1se)
get_auc<-function(y,p){if(requireNamespace("pROC",quietly=TRUE))as.numeric(pROC::auc(y, p))else NA_real_}
auc_min<-get_auc(y_test,p_min); auc_1se<-get_auc(y_test,p_1se) # optional AUC
results1<-data.frame(Model=c("Ridge (lambda.min)","Ridge (lambda.1se)"),Accuracy=c(acc_min,acc_1se),LogLoss=c(ll_min,ll_1se),
Brier=c(br_min,br_1se),AUC=c(auc_min,auc_1se)); print(results, row.names = FALSE)
results_tbl_1<-results1%>%transmute(Model,Accuracy=round(Accuracy,4),LogLoss=round(LogLoss,4),Brier=round(Brier,4),
AUC=ifelse(is.na(AUC),NA,round(AUC,3))); kable(results_tbl_1,caption="Ridge (logistic) - Test Metrics") %>%
kable_styling(full_width = FALSE, position = "center", bootstrap_options = c("striped","hover","condensed"))

coef_min<-coef(cv_ridge,s="lambda.min"); coef_1se<-coef(cv_ridge,s="lambda.1se")
ix<-order(abs(as.numeric(coef_1se)),decreasing=TRUE); print(coef_1se[ix[1:10], ,drop=FALSE])

tblA<-results_tbl_1%>% select(Model,Accuracy,LogLoss,Brier,AUC)%>% mutate(across(-Model,~round(.,4)))
tblB<-results_tbl%>% select(Model,Accuracy,LogLoss,Brier,AUC)%>% mutate(across(-Model,~round(.,4)))
wide<-inner_join(tblA,tblB,by="Model",suffix=c("_run1","_run2"))
kable(wide,col.names=c("Model","Accuracy","LogLoss","Brier","AUC","Accuracy","LogLoss","Brier","AUC"),
caption="Ridge (logistic) - Test Metrics (different seeds)") %>% kable_styling(full_width=FALSE,position="center",
bootstrap_options=c("striped","hover","condensed")) %>% add_header_above(c(" " =1, "Run 1" =4, "Run 2" =4))

kable(comparison[ , -1],caption="Comparison of LASSO Coefficients With and Without Standardization") # Pretty table
knitr::kable(comparative_df,caption="Variable selection across LASSO and Elastic Net") # Display nicely
knitr::kable(comparative_tbl, caption = "Variables Selected under .min vs .1se") # Show the table

print(gg_resfit); print(gg_qq); print(gg_scale)
```