Why Invoicing Currency Heterogeneity Matters: A Sufficient Statistics Approach*

Nicolas Hommel and Michael Jenuwine

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Abstract

This paper uses a new theoretical framework and administrative data from France to analyze exchange rate shock transmission in open economies with heterogeneous firms. In our framework, a small set of sufficient statistics captures how firm heterogeneity, and in particular currency invoicing heterogeneity, impacts the propagation of exchange rate shocks into sectoral prices, productivity, and markups. One moment, the covariance between firm markups and foreign currency invoicing, summarizes the impact of invoicing heterogeneity. In the data, we document that this covariance is positive: higher markup firms are significantly more likely to invoice in dollars and destination market currencies. We embed our framework into a three-country, open economy New Keynesian model, and find that firm heterogeneity leads to quantitatively large productivity and factor demand movements in export sectors, and amplifies the term of trade effect of exchange rate movements. JEL codes: E31, F31, F41, F42

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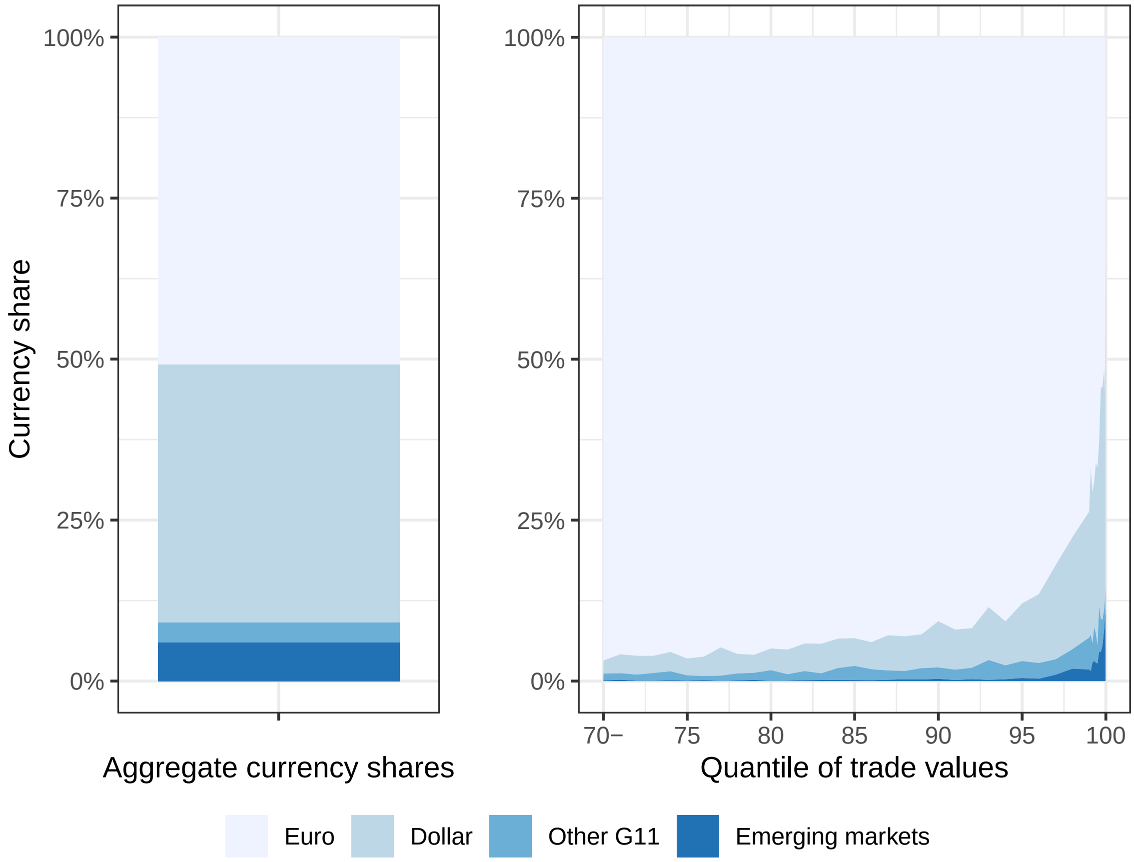
1 Introduction

Exchange rate movements impact countries' trade deficits and terms of trade. This fact is salient to policymakers, as evidenced by frequent accusations of currency manipulation between trading countries. The currency in which firms invoice trade contracts is an inconspicuous but crucial determinant of how exchange rate shocks affect the real economy (Magee, 1973; Gopinath et al., 2010). Empirically, large firms are one of the main drivers of international shock transmission (Cravino and Levchenko, 2016; di Giovanni et al., 2018, 2024). These very firms also account for the overwhelming majority of foreign currency invoicing, most importantly dollar invoicing (Amiti et al., 2022; Corsetti et al., 2022). Does firm heterogeneity modify the international transmission of exchange rate shocks? Are shocks to the dollar exchange rate special partly because large firms tend to use dollar prices to export, while small firms tend to use their home currency? Conventional models in open economy macroeconomics, with a single sector of homogeneous firms, are not designed to answer these questions.

This paper combines a new theoretical framework with an administrative dataset from France to analyze firm heterogeneity and exchange rate transmission in global trade networks. Our model permits firms within a sector to differ arbitrarily in their currency of invoicing, productivity, markup, pass-through of marginal costs, and price rigidity. We provide nonparametric formulas for the responses of sectoral prices, productivity, and markups following a small exchange rate shock. These formulas depend on a few sector-specific sufficient statistics. Notably, the covariance between foreign currency invoicing use and markups summarizes how currency invoicing heterogeneity impacts sectoral aggregates. This is because the distribution of invoicing determines which firms' relative prices move after an exchange rate shock, and the distribution of markups captures how relative price movements change misallocation. Our framework is flexible enough to capture realistic firm-level heterogeneity and tractable enough to be embedded in dynamic open economy New Keynesian models or general equilibrium models with international trade networks.

We measure the importance of invoicing heterogeneity using an administrative dataset covering the universe of French export transactions to destinations outside the eurozone. France is an ideal case study because the euro is a regional currency which coexists with the dominant currency, the dollar. This generates meaningful heterogeneity in invoicing (Figure 1). We document two new facts. First, there is substantial variation in invoicing currency shares across French exporting sectors. These sectoral currency shares systematically determine which exchange rate movements are passed through to sectoral prices. Second, within a sector, firms using foreign currencies tend to charge higher markups. This holds both at the firm level and at the product-market level. It implies that the main

Figure 1: Currency composition of French exports in 2018



Note. The left panel breaks down French exports to countries outside the Eurozone by currency. The right panel breaks down French exports by currency for each firm size centile. We sort firms into size bins according to export values, and compute the currency composition of imports and export for each firm. We report the equal-weighted average currency shares for each bin.

moment governing the macroeconomic impact of invoicing heterogeneity, the covariance between foreign currency invoicing and markups, is positive.

We quantify the importance of firm heterogeneity by embedding our theoretical framework into a canonical open economy New Keynesian model (Galí and Monacelli, 2005; Gopinath et al., 2020), building on Baqaee et al. (2023a). Our model differs from the three-equation New Keynesian benchmark in two ways. First, firm heterogeneity modifies the sectoral Phillips curves due to the interaction of strategic complementarities in price setting with nominal rigidities, including invoicing. Second, nominal aggregate shocks, in particular exchange rate shocks, give rise to endogenous dynamics in sectoral productivity. Calibrating the model nonparametrically to the French data, we find that firm heterogeneity leads to sizable endogenous productivity dynamics in the export sector due to nominal exchange rates movements. Fluctuations in sectoral productivity lead to fluctuations in factor demand. In particular, we find that firm heterogeneity significantly amplifies movements in labor demand in exporting sectors.

In our model, exchange rate movements generate dynamics in sectoral productivity when markups are heterogeneous and systematically covary with foreign currency invoicing. Misallocation arises from markup dispersion because firms' marginal products are not equalized. Empirically, low-markup firms invoice use their domestic currency to export, while high-markup firms also use dollars and the destination market currencies. A domestic currency appreciation will increase the relative prices of low-markup firms, reallocating resources away from them toward high-markup firms. This decreases misallocation by reducing dispersion in marginal products.

Currency invoicing heterogeneity also has implications for sectoral inflation dynamics. Firms' pricing decisions depend on competitors' prices through the price index, as in Kimball (1995). Competitors' prices may be rigid in different currencies, which generates non-trivial interactions between microeconomic pricing decisions and sectoral aggregates. For example, suppose exporters setting prices in their home currency face more elastic demand than exporters setting prices in the destination market currency. If the home currency appreciates with respect to the destination market currency, the relative price of exporters using their home currency increases and consumers substitute away from their products. This effect is amplified because those firms are precisely the ones facing more elastic demand. In response, competitors will strategically adjust their prices by more than if the demand elasticities were the same across firms.

Related literature. This article contributes to four strands of literature: currency invoicing, exchange rate pass-through, international monetary economics, and firm heterogeneity in international macroeconomics. The empirical literature on currency invoicing starts immediately after the fall of the Bretton–Woods system with Grassman (1973) and Page (1981) which document that firms located in the same country trade using different currencies, and that firms using foreign currencies tend to be larger. Subsequent work, starting with Goldberg and Tille (2008), use transaction-level data to confirm these findings and document new facts about currency invoicing. At the aggregate level, Boz et al. (2022) compile the currency shares for many countries starting in 1990 and show how currency shares determine exchange rate pass-through in the cross-section of countries. Recent work has emphasized that currency choice is an active firm decision reflecting inputs' exposure to currency risk, competition, and firm size (Amiti et al., 2022; Corsetti et al., 2022). We contribute to this literature in two ways. First, we build a framework that can accommodate the rich heterogeneity documented in the data. We show how firm heterogeneity affects the response of macroeconomic aggregates to exchange rates. Second, we document that the key moment summarizing the macroeconomic importance of invoicing currency heterogeneity, the covariance between markups and foreign currency invoicing, is positive.

The exchange rate pass-through literature begins with Krugman (1987) and Dornbusch (1987), who show how strategic complementarities can generate pricing-to-market. Recent empirical work highlights the importance of invoicing as an additional source of pricing-to-market (Gopinath et al., 2010; Fitzgerald and Haller, 2013; Chen et al., 2021). In particular, the fact that prices are sticky in their currency of denomination is robust and holds across countries and datasets (Gopinath and Itskhoki, 2022). Recently, the literature has explored heterogeneity in pass-through across firms (Berman et al., 2012; Amiti et al., 2019). We contribute to this literature by providing sufficient statistics capturing how each dimension of firm heterogeneity contributes to exchange rate pass-through. Our framework is related to that of Amiti et al. (2019), who show how heterogeneous marginal cost shocks pass-through to sectoral prices in a wide class of oligopolistic models with flexible prices. Their work differs from ours in that we study the propagation of a common exchange rate shock in the presence of nominal rigidities and heterogeneous invoicing. We show that, away from the flexible price limit, common exchange rate shocks are incompletely passed-through to sectoral prices.

The literature on international monetary economics has typically examined settings where firms homogeneously conduct producer currency pricing (Friedman, 1953), local currency pricing (Devereux and Engel, 2003), or dominant currency pricing (Gopinath et al., 2020; Egorov and Mukhin, 2023). The importance of the invoicing currency assumption was first pointed out by Magee (1973), and literature has shown that monetary policy has drastically different implications depending on the currency in which prices are sticky. Our work contributes to this literature by providing a framework to analyze the implications of arbitrary invoicing paradigms, including variation in currencies used at the firm-level. Importantly, we show theoretically and empirically that country-level currency shares do not characterize the transmission of exchange rate shocks. This is because there variation in currency invoicing within country across sectors, and within sector across firms.

Finally, this article is related to the growing literature on the implications of firm heterogeneity for monetary policy (Cravino, 2017; Meier and Reinelt, 2022; Baqaee et al., 2023a). We build on the work of Baqaee et al. (2023a), who demonstrate that in the presence of firm heterogeneity, a monetary tightening reallocates resources from high markup firms to low markup firms, increasing misallocation and flattening the Phillips curve. Our work extends theirs to open economy settings, where invoicing plays a key role in shaping price rigidities. Cravino (2017) shows that when currency use is heterogeneous across firms, exchange rate shocks impact misallocation. Unlike the other papers mentioned above, he models an open economy and allows for parametric heterogeneity in invoicing, markups, and pass-through. Our analysis differs in two important ways. First, we show how invoicing heterogeneity impacts not only allocative efficiency but also prices and markups. Second, we provide closed form nonparametric formulas that isolate the effect of each dimension of firm heterogeneity on sectoral aggregates.

Outline. The paper is organized as follows. In Section 2, we introduce a partial equilibrium framework to study how firm heterogeneity impacts exchange rate transmission to sectoral aggregates. In Section 3 we describe our dataset and document the importance of invoicing heterogeneity across sectors and across firms. In Section 4 we embed our framework into a dynamic open economy New Keynesian model, which we calibrate to the French data in Section 5. Section 6 presents and discusses the quantitative results of the model, and Section 7 concludes.

2 Conceptual framework

In this section, we introduce a tractable partial equilibrium framework to analyze firm heterogeneity in open economies with invoicing. Our framework builds on Baqaee et al. (2023a), which we extend to open economies. We consider an initial flexible price equilibrium and study local changes in sectoral aggregates following an exchange rate shock. In our model, changes in exchange rates are not neutral due to nominal rigidities.

2.1 Initial flexible price equilibrium

We now describe the flexible price equilibrium. We first define production and the three sectoral aggregates we study: prices, markups, and productivity. We then define firms' pricing strategies.

Production. We focus on a single sector j producing a sectoral good in the domestic country and exporting it to a destination country. The sectoral good combines differentiated goods $i \in [0, 1]$ using homothetic direct implicit additive (HDIA) technology, following Matsuyama and Ushchev (2017). Formally, the quantity of aggregate good produced Y_j is implicitly defined by

$$\int_0^1 \Phi_i \left(\frac{y_i}{Y_j} \right) \mathrm{d}i = 1, \tag{1}$$

where Φ_i is a good-specific function assumed to be increasing and strictly concave. This specification nests most aggregation systems used in international macroeconomics, including constant elasticity of substitution demand ($\Phi_i(y) = y^{\frac{\sigma-1}{\sigma}}$) and Kimball (1995) systems ($\Phi_i = \Phi$).

Cost minimization by the sectoral aggregator implies that the residual demand for each variety is

$$\frac{p_i}{\mathcal{P}_j} = \Phi_i'\left(\frac{y_i}{Y_j}\right), \tag{2}$$

where $\mathcal{P}_j$ is a price aggregator defined below. All prices are expressed in the destination currency. From Equation (2), the price elasticity of demand for differentiated good $i \in [0,1]$ is

$$\sigma_i\left(\frac{y_i}{Y_j}\right) = -\frac{\Phi_i'\left(\frac{y_i}{Y_j}\right)}{\Phi_i''\left(\frac{y_i}{Y_j}\right)\frac{y_i}{Y_j}}. \tag{3}$$

The demand elasticity depends on residual demand, and its curvature is allowed to vary across differentiated goods. As shown by Baqaee et al. (2023a), allowing for these two sources of variation is crucial to match the level of cross-sectional heterogeneity in pass-through rates documented by Berman et al. (2012) and Amiti et al. (2019).

Each differentiated good $i \in [0,1]$ is produced by an intermediate firm. Intermediates firms share the same constant returns technology F_j up to a Hicks-neutral productivity shifter A_i. We define the sectoral marginal cost as

$$M_j = \min_{x_{j1},\ldots,x_{jK}} \sum_{k=1}^{K} m_{jk}x_{jk} \quad \text{s.t.} \quad F_j(x_{j1},\ldots,x_{jK}) = 1,$$

where $x_{j1},\ldots,x_{jK}$ are inputs used for production and $m_{j1},\ldots,m_{jK}$ the respective prices for the inputs, expressed in the destination currency. The marginal cost of the firm producing good i is thus $M_i = M_j/A_i$.

Sectoral aggregation. We study three sectoral aggregates: prices, markups, and productivity. The *ideal* price index P_j is the unit cost of the sectoral good

$$P_j = \min_{\{y_i\}} \int_0^1 y_i p_i \mathrm{d}i, \quad \text{s.t.} \quad \int_0^1 \Phi_i\left(\frac{y_i}{Y_j}\right)\mathrm{d}i = 1. \tag{4}$$

To the first order, changes in the ideal price index coincide with changes in the producer price index measured by statistical agencies. We focus on this price aggregate because a large body of empirical work has studied exchange rate pass-through into the producer price index. Although this distinction is unimportant to this work, the ideal price index differs from the aggregator $\mathcal{P}_j$ that appears in the residual demand Equation (2) and is defined by

$$\mathcal{P}_j = \frac{P_j}{\int_0^1 \Phi_i'\left(\frac{y_i}{Y_j}\right)\frac{y_i}{Y_j}\mathrm{d}i}. \tag{5}$$

Turning to sectoral markups, we follow Baqaee and Farhi (2019) and define

$$\overline{\mu}_j = \left(\int_0^1 \frac{\lambda_i}{\mu_i} \mathrm{d}i \right)^{-1}, \tag{6}$$

where $\lambda_j = p_i y_i / P_j Y_j$ is the sales share of firm i.

Finally, sectoral productivity is

$$A_j = \frac{\overline{\mu}_j M_j}{P_j}. \tag{7}$$

Letting X_{jk} be the total quantity of input k purchased by firms in the sector, we can equivalently define productivity from

$$Y_j = A_j F_j(X_{j1}, \dots, X_{jK}). \tag{8}$$

This shows that production aggregates well in our economy, provided we keep track of the endogenous sectoral productivity A_j. Defining sectoral productivity as in Equation (7) is important when there are input-output linkages. Indeed, in an inefficient economy with input-output linkages, productivity measures such as output per hour or the Solow residual can detect changes in efficiency even when the underlying allocation does not change. Equation (7) defines a corrected Solow residual that correctly captures sectoral allocative efficiency (Baqaee and Farhi, 2019).

Pricing. Differentiated goods producers set prices monopolistically, which gives rise to the familiar Lerner formula for the desired price in the destination currency

$$\widetilde{p}_i \left(\frac{y_i}{Y_j} \right) = \widetilde{\mu}_i \left(\frac{y_i}{Y_j} \right) \times \frac{M_j}{A_i}, \quad \text{with} \quad \widetilde{\mu}_i \left(\frac{y_i}{Y_j} \right) = \frac{\sigma_i(y_i/Y_j)}{\sigma_i(y_i/Y_j) - 1}.$$

We denote the desired price and markup with a tilde to emphasize that they are set flexibly, without any nominal rigidity.

An important component of our model is the desired pass-through rate ρ_i, defined as the elasticity of desired prices to a change in marginal cost

$$\rho_i = \frac{\mathrm{d} \log \widetilde{p}_i}{\mathrm{d} \log M_i}.$$

Desired pass-through reflects both changes in costs and changes in desired markups. Given that markups are a function of demand elasticities, this adjustment is controlled by the superelasticity of demand.

Flexible price equilibrium. Differentiated goods producers set prices to maximize profits, taking input prices and residual demand curves as given. The sectoral aggregator minimizes costs, taking differentiated goods' prices as given. The initial exchange rate from the domestic currency to the destination currency is normalized to $E_j = 1$.

Notation. Given two firm-level variables x_i and y_i for industry j, we denote

$$\mathbf{E}_y[x_i] = \frac{\int_0^1 x_i y_i \mathrm{d}i}{\int_0^1 y_i \mathrm{d}i}.$$

We define other moments in an analogous manner.

2.2 Exchange rate shock and nominal rigidities

We now introduce the exchange rate shock we consider along with the nominal rigidities generating monetary nonneutrality.

Exchange rate shock. The timing is as follows.

1. At $t = 0$, the sector is in its flexible price equilibrium.
2. At $t = \frac{1}{2}$, there is a shock to the bilateral exchange between the domestic and destination countries E_j which also affects the sectoral marginal cost M_j.
3. At $t = 1$, firms with flexible prices adjust their prices optimally, while firms with sticky prices see their prices move mechanically with the exchange rate, depending on the invoicing currency used.

We let $\Delta \log E_j$ be the change in exchange rate and given a variable X_j, we define $\mathrm{d} \log X_j = \frac{\partial \log X_j}{\partial \log E_j} \times \Delta \log E_j$.

It is worth emphasizing that we do not explicitly model how marginal costs respond to changes in exchange rates. Our results are comparative statics, taking changes in exchange rates and marginal costs as given. Nevertheless, our framework is flexible enough that it can easily be embedded in models where both exchange rates and marginal costs are endogenous. Indeed, in Section 4, we show how to extend a canonical open economy new Keynesian model to incorporate firm heterogeneity.

Nominal rigidities. Following the shock, exchange rates and marginal costs have changed. As in Calvo (1983), some firms can exogenously adjust their prices while others cannot. We assume that prices invoiced in the domestic currency move one for one with the bilateral exchange rate, while prices invoiced in the destination currency do not move at all. This implies that for a firm unable to adjust its price

$$\mathrm{d} \log p_i = \iota_i \mathrm{d} \log E_j, \tag{9}$$

where ι_i is an indicator variable taking the value one if prices are invoiced in the domestic currency (PCP) and zero otherwise (LCP). For expositional convenience, we assume that firms either use the domestic currency or the destination currency. We will relax this assumption in the general equilibrium model used for counterfactuals.

We do not restrict the distribution of invoicing currency use, nor do we take a stance on what generates it. However, we make two critical assumptions. First, as stated above, prices are sticky in their currency of invoicing. This is in line with a large body of evidence (Gopinath and Itskhoki, 2022), as well as our own empirical findings. Second, firms do not change their currency of invoicing following the exchange rate shock. In Section 3.4, we show that this assumption is strongly supported by the French data: the share of exports invoiced in euros is very persistent both at firm and market levels. Corsetti et al. (2022) document similar persistence in invoicing for the United Kingdom, even after the significant devaluation of the pound sterling which followed the Brexit referendum.[1]

2.3 Firm-level exchange rate transmission

We now characterize how firms transmit exchange rate shocks. Changes in desired prices reflect changes in marginal cost and desired markups

$$\mathrm{d}\log \widetilde{p}_i = \mathrm{d}\log M_j + \mathrm{d}\log \widetilde{\mu}_i$$

Note that the change in marginal cost is the same for all firms in the sector. Firms may want to update their markups because they are now located at a different point on their residual demand curve. Specifically,

$$\mathrm{d}\log \widetilde{\mu}_i = \frac{1-\rho_i}{\rho_i}(\mathrm{d}\log \mathcal{P}_j - \mathrm{d}\log \widetilde{p}_i).$$

The term in parentheses captures changes in local competition and the coefficient $(1-\rho_i)/\rho_i$ reflects the curvature of markups, which may vary across firms. Combining the two equations and solving for desired prices yields

$$\mathrm{d}\log \widetilde{p}_i = \rho_i \mathrm{d}\log M_j + (1-\rho_i)\mathrm{d}\log \mathcal{P}_j. \tag{10}$$

The change in desired price is a weighted average of the change in marginal cost and changes in local competition, captured by the substitution price index, where the weight given to each component depends on the desired pass-through ρ_i.

For firms that cannot update, prices move mechanically depending on the

[1] Appendix E shows how switching costs provide a microfoundation for invoicing stickiness in a broad class of endogenous invoicing models.

currency of invoicing, as described in Equation (9). Therefore, in expectation,

$$\mathrm{d}\log p_i = \delta_i \mathrm{d}\log \widetilde{p}_i + (1-\delta_i)\iota_i \mathrm{d}\log E_j,$$

where δ_i is the Calvo price flexibility parameter, which may vary by firm. Substituting in Equation (10) describing desired price changes yields

$$\mathrm{d}\log p_i = \underbrace{\delta_i \rho_i \mathrm{d}\log M_j}_{\text{Marginal cost}} + \underbrace{\delta_i (1-\rho_i) \mathrm{d}\log \mathcal{P}_j}_{\text{Strategic complementarities}} + \underbrace{(1-\delta_i)\iota_i \mathrm{d}\log E_j}_{\text{Invoicing}}. \tag{11}$$

This expression summarizes how firms transmit exchange rate shocks. Exchange rate pass-through at the microeconomic level reflects flexible adjustments due to changes in costs and strategic complementarities, as well as mechanical adjustments due to nominal rigidity and invoicing. Equation (11) also illustrates how pricing-to-market naturally arises in our model. Indeed, two firms with the same desired pass-through and price stickiness facing the same shock may exhibit different exchange rate pass-through due to differences in local competition (Krugman, 1987; Dornbusch, 1987) and invoicing (Gopinath et al., 2010).

2.4 Sectoral prices

We are now ready to characterize the transmission of exchange rates into sectoral prices. By Shephard's lemma, the change in the ideal price index is given by the sales-weighted average of firm-level microeconomic pass-through

$$\mathrm{d}\log P_j = \mathbf{E}_{\lambda_j}[\mathrm{d}\log p_i].$$

Of course, since microeconomic exchange rate pass-through depends on the aggregator $\mathcal{P}_j$, this is an equilibrium relationship. Our first result gives the equilibrium change in the ideal price index, and generalizes the expressions of (Baqaee, Farhi and Sangani, 2023a) to open economies with invoicing.

Proposition 1. *Exchange rate pass-through to the sectoral ideal price index is*

$$\begin{aligned}\mathrm{d}\log P_j = &\underbrace{\kappa_M \mathrm{d}\log M_j}_{\textit{Marginal cost}} + \underbrace{\kappa_\iota \mathbf{E}_{\lambda_j}[(1-\delta_i)]\mathbf{E}_{\lambda_j}[\iota_i]\mathrm{d}\log E_j}_{\textit{Aggregate currency shares}} \\ &+ \underbrace{\kappa_\iota \operatorname{Cov}_{\lambda_j}(1-\delta_i, \iota_i)\mathrm{d}\log E_j}_{\textit{Heterogeneous price stickiness}} + \underbrace{\kappa_\sigma \operatorname{Cov}_{\lambda_j(1-\delta_j)}(\varsigma_i, \iota_i)\,\mathrm{d}\log E_j}_{\textit{Real rigidities due to heterogeneous invoicing}},\end{aligned} \tag{12}$$

where $\varsigma_i = \sigma_i / \mathbf{E}_{\lambda_j}[\sigma_i]$ is the scaled demand elasticity. The coefficients κ, which are defined in the appendix, depend only on $\mathbf{E}_{\lambda_j}[\delta_i]$, $\mathbf{E}_{\lambda_j \delta_j}[\rho_i]$, $\operatorname{Cov}_{\lambda_j}(\varsigma_i, \delta_i)$, and $\operatorname{Cov}_{\lambda_j \delta_j}(\varsigma_i, \rho_i)$.

Proposition 1 not only gives changes in sectoral prices as a function of the exchange rate shock and microeconomic fundamentals, but also provides an interpretable decomposition of exchange rate pass-through. The first term is the average pass-through due to the common change in marginal cost, and the second is the average pass-through due to invoicing. Without firm heterogeneity, this fully characterizes price changes. In the presence of firm heterogeneity, two additional terms enter. The third term captures potential correlation between foreign currency invoicing and price setting rigidities. The fourth term captures strategic complementarities arising from heterogeneous invoicing.

An implication of Proposition 1 is that given $\mathrm{d}\log E_j$ and the induced $\mathrm{d}\log M_j$, we can compute aggregate exchange rate pass-through using a small number of observable first and second moments of microeconomic primitives without needing to observe their full distributions. In this sense, we provide sufficient statistics to characterize the propogation of exchange rate shocks in open economies. We now present specific examples to build intuition.

Flexible prices or sticky prices. We first discuss the two limiting cases where firm heterogeneity is irrelevant for macroeconomic aggregates: fully flexible prices and fully sticky prices. Outside of these knife-edge cases, firm heterogeneity modifies price aggregation. First, when prices are fully flexible, meaning $\delta_i = 1$, pass-through is complete in the sense that

$$\mathrm{d}\log P_j = \mathrm{d}\log M_j.$$

In the absence of nominal rigidities, firms are always at their desired price. Individually, each firms passes-through the common cost shock and adjusts its markup depending on local competition. In aggregate, those two effects exactly add up to the common cost shock. This is consistent with Amiti et al. (2019), who show that pass-through is complete when prices are flexible and firms are hit by the same common shock.

Second, when prices are fully sticky, meaning $\delta_i = 0$, pass-through only reflects the aggregate currency invoicing shares

$$\mathrm{d}\log P_j = \mathbf{E}_{\lambda_j}[\iota_i]\mathrm{d}\log E_j.$$

This formula captures Magee's (1973) analysis of the short-run "currency-contract period," during which nominal contracts set prices in advance in a given currency. Different invoicing shares lead to radically different effects of exchange rate movements. In one extreme, all firms write contracts in their domestic currency ($\iota_i = 1$), and prices move one for one with the exchange rate. In the other extreme, all firms write contracts in the currency of the export destination ($\iota_i = 0$), and exchange rates do not affect prices.

Constant elasticity of substitution. Next, we consider the CES case in which markups are uniform, desired pass through is complete ($\rho_i = 1$) and strategic complementarities are shut down. In this case, we have

$$\mathrm{d}\log P_j = \underbrace{\mathbf{E}_{\lambda_j}[\delta_i]\mathrm{d}\log M_j}_{\text{Marginal cost}} + \underbrace{\mathbf{E}_{\lambda_j}[1-\delta_i]\mathbf{E}_{\lambda_j}[\iota_i]\mathrm{d}\log E_j}_{\text{Aggregate currency shares}} + \underbrace{\mathrm{Cov}_{\lambda_j}(1-\delta_i, \iota_i)\mathrm{d}\log E_j}_{\text{Heterogeneous price stickiness}}.$$

Even in this simple example, aggregate invoicing currency shares alone are not enough to compute aggregate pass-through. The heterogeneous price stickiness term must be accounted for. The intuition is that when firms using the producer currency update their prices less frequently than firms using the destination currency, the invoicing component of ERPT becomes larger. This channel is similar in spirit to the force emphasized by Carvalho and Nechio (2011), but heterogeneity in our setting is within sector rather than across sectors. Heterogeneity in price stickiness across firms within a sector is neither necessary nor sufficient to generate heterogeneity across sectors.

Homogeneous price stickiness. Finally, consider the case where price stickiness is the same across firms. Proposition 1 then simplfies to

$$\mathrm{d}\log P_j = \underbrace{\kappa_M \mathrm{d}\log M_j}_{\text{Marginal costs}} + \underbrace{\kappa_\iota(1-\delta_j)\mathbf{E}_{\lambda_j}[\iota_i]\mathrm{d}\log E_j}_{\text{Aggregate currency shares}} + \underbrace{\kappa_\sigma \,\mathrm{Cov}_{\lambda_j}(\varsigma_i, \iota_i)\,\mathrm{d}\log E_j}_{\text{Real rigidities due to heterogeneous invoicing}}.$$

This expression highlights how invoicing heterogeneity also enters through strategic complementarities in pricing. The intuition is that following a home currency depreciation, relative prices of firms using the home currency will decline. If those firms also tend to face more elastic demand, consumers will substitute toward their products relatively more because expenditure switching is proportional to the price-elasticity of demand. Due to the presence of strategic complementarities, other firms then decrease their prices by more than they would have in the absence of heterogeneity.

Relation to the literature. Our Proposition 1 relates to Proposition 3 of Amiti et al. (2019), which characterizes ERPT in a wide class of flexible-price models while allowing for idiosyncratic exposure to cost shocks. Our results differ in that we focus on common cost shocks in the presence of price stickiness and heterogeneous invoicing. Crucially, price stickiness gives rise to new channels for real and nominal rigidities that reflect heterogeneity in microeconomic primitives. Our results also relate to those of Baqaee et al. (2023a), who study common marginal cost shocks in closed economies. We extend their results to an open economy with exchange rate shocks and heterogeneous invoicing.

2.5 Sectoral productivity

We now describe the effects of exchange rate movements on sectoral productivity. The idea that exchange rate movements affect allocative efficiency follows from the classic price theory insight that monopolies distort prices and create misallocation. Indeed, misallocation increases when exchange rate shocks reallocate resources from high-markup firms to low-markup firms.

In our model, resource reallocation operates entirely through the expenditure-switching channel that follows changes in relative prices. Even though the same exchange rate shock hits all firms, they may update their prices differently due to heterogeneity in invoicing, pass-through, and price stickiness. This reallocates resources across firms, impacting sectoral productivity. The following proposition formalizes this reasoning.

Proposition 2. *Changes in sectoral productivity are given by*

$$\frac{\mathrm{d}\log A_j}{\mathrm{d}\log E_j} = \underbrace{\left[\theta_\delta \operatorname{Cov}_{\lambda_j}(\varsigma_i, \delta_i) + \theta_\rho \operatorname{Cov}_{\lambda_j \delta_j}(\varsigma_i, \rho_i)\right]}_{\text{Price stickiness and pass-through heterogeneity}} \times \underbrace{\left(\frac{\mathrm{d}\log M_j}{\mathrm{d}\log E_j} - \mathbf{E}_{\lambda_j}[(1-\delta_i)\iota_i]\right)}_{\text{Aggregate shock}}$$
$$+ \underbrace{\theta_\iota \operatorname{Cov}_{\lambda_j(1-\delta_j)}(\varsigma_i, \iota_i)}_{\text{Invoicing heterogeneity}}, \tag{13}$$

where $\varsigma_i = \sigma_i / \mathbf{E}_{\lambda_j}[\sigma_i]$ *is the scaled demand elasticity. The coefficients* θ*, defined in the appendix, depend only on* $\overline{\mu}_j$*,* $\mathbf{E}_{\lambda_j}[\delta_i]$*,* $\mathbf{E}_{\lambda_j\delta_j}[\rho_i]$*,* $\operatorname{Cov}_{\lambda_j}(\varsigma_i, \delta_i)$*, and* $\operatorname{Cov}_{\lambda_j\delta_j}(\varsigma_i, \rho_i)$.

In Equation (13), we decompose changes in sectoral efficiency. Reallocation is partly driven by heterogeneity in price stickiness and pass-through. This should come as no surprise, as it corresponds to Proposition 1 in Baqaee et al. (2023a). In particular, in the case of domestic production in which the producer and destination countries are the same, our expressions coincide with theirs.

There are also two new effects, which are specific to open economies and are the focus of this paper. First, invoicing directly modifies the scale of the aggregate shock. This is because, in open economies, prices move mechanically with the exchange rate of their invoicing currency. These mechanical movements enter additively with marginal cost in the aggregate shock. Second, there is a new term which captures the direct effect of invoicing heterogeneity on reallocation. It is determined by the covariance between invoicing currency and demand elasticity. In response to an exchange rate depreciation, the firms with prices sticky in the producer currency will see their relative prices in the destination currency decrease. This decrease in relative prices will induce more demand for those firms' goods, and reallocate resources towards them. If these exporting firms are also those with low demand elasticities and high markups ($\operatorname{Cov}_{\lambda_j(1-\delta_j)}(\varsigma_i, \iota_i) < 0$), the resource reallocation is toward firms that were inefficiently small in the initial equilibrium and sectoral productivity increases.

Relation to the literature. Little attention has been paid to the implications of invoicing for allocative efficiency relative to its implications for pass-through. One notable exception is Cravino (2017), which shows that invoicing generates heterogeneous price rigidities that affect allocative efficiency through markups in a model with parametric variation in markups, pass-through, and invoicing. This effect is captured in the last term of Equation (13). We add to this work by providing an analytical decomposition of changes in allocative efficiency that illustrates the interaction between invoicing, pass-through, and price stickiness given arbitrary firm-level heterogeneity in those dimensions.

Markups. Changes in aggregate markups obtain from changes in prices and productivity since $\overline{\mu}_j = P_j A_j / M_j$.

Corollary 1. *Changes in sectoral markups are given by*

$$\begin{aligned} \mathrm{d}\log\overline{\mu}_j &= -(1-\kappa_M)\mathrm{d}\log M_j + \mathbf{E}_{\lambda_j}[(1-\delta_i)\iota_i]\kappa_\iota \mathrm{d}\log M_j \\ &+ \left[\theta_\rho \operatorname{Cov}_{\lambda_j\delta_j}(\varsigma_i,\rho_i) + \theta_\delta \operatorname{Cov}_{\lambda_j}(\varsigma_i,\delta_i)\right]\left(\mathrm{d}\log M_j - \mathbf{E}\left[(1-\delta_i)\iota_i\right]\mathrm{d}\log E_j\right) \\ &+ (\kappa_\sigma+\theta_\iota)\operatorname{Cov}_{\lambda_j\delta_j}(\varsigma_i,\iota_i)\,\mathrm{d}\log E_j. \end{aligned} \tag{14}$$

This equation describes how firm heterogeneity modifies markup dynamics. As before, the effects of firm heterogeneity for exchange rate transmission wash-out in the flexible price limit ($\delta_i = 1$), where pass-through is complete and sectoral markups are constant. Equation (14) is in line with recent work by Burstein et al. (2020), who document that sectoral market structure, including demand elasticity and pass-through, is a crucial determinant of aggregate markup dynamics. Our results further suggest that, in the presence of nominal rigidities, exchange rate shocks lead to different markup dynamics than traditional cost shocks.

Output. Our partial equilibrium model does not pin down the level of output or changes in the level of output. Nevertheless, we can gain further insight on the implications of our results using that $Y_j = A_j F_j(X_{j1}, \ldots, X_{jK})$, where X_{jk} is the quantity of input k used by sector j. Log-linearizing it yields

$$\mathrm{d}\log Y_j = \mathrm{d}\log A_j + \mathrm{d}\log F_j(X_{j1},\ldots,X_{jK}) = \underbrace{\mathrm{d}\log A_j}_{\text{Reallocation}} + \underbrace{\sum_{k=1}^{K}\widetilde{\Omega}_{jk}\mathrm{d}\log X_{jk}}_{\text{Input usage}},$$

where $\widetilde{\Omega}_{jk}$ is the share of input k in total variable costs. Changes in output either reflect changes in input usage or in productivity due to reallocation. The evolution of input usage depends on factor prices, which are determined in general equilibrium. In Section 4, we give an example of general equilibrium structure that fully pins down the evolution of output.

3 Empirical analysis

In this section, we use detailed firm-level data to validate that there is significant variation in pass-through across industries that can be systematically linked to invoicing currency, and document a systematic association between invoicing currency and markups. As discussed above, the covariance between foreign currency invoicing and markups is the main moments that captures the effects of invoicing heterogeneity on sectoral aggregation.

3.1 Data

Customs data. We use detailed import and export transaction data from the French customs administration covering 2011 to 2020. The data are aggregated at the monthly frequency, and each entry corresponds to a unique firm identifier, destination, product, and invoicing currency when the trading partner is outside the eurozone. Products are available at the eight-digit level of the combined nomenclature. For each entry, we observe the trade value in euros, the quantity traded, the quantity unit of measurement, and the transaction weight in kilograms.

The data are nearly exhaustive for trade with partners outside the eurozone. When trading within the eurozone, firms only must report import and export information if their overall trade over the past calendar year exceeds €460,000. We limit our analysis to outside eurozone trade because invoicing data is not collecting for within-eurozone transactions, and it is likely that nearly all of that trade would be conducted in euros which limits the role of invoicing heterogeneity. We refer the reader to Bergounhon et al. (2018) for a detailed description of this dataset.

Firm characteristics. We take firm characteristics from balance sheet data collected by the French fiscal administration on all French firms. We merge it with our trade panel using a unique firm administrative identifier. For some very large firms, the fiscal administration consolidates legal entities into a larger economic entity. In that case, we use a crosswalk between economic and legal identifiers provided by the French statistical office to ensure that our dataset is complete. Appendix B contains a detailed description of the merging procedure, as well as the exact definitions of variables used.

3.2 Sectoral exchange rate pass-through

Proposition 1 predicts that exchange rate pass-through (ERPT) to industry prices should depend on invoicing. To test this implication, we rely on cross-sectional variation in invoicing across export destinations and industries (Figure 2). We show that changes in aggregate prices in sector-destinations where contracts are invoiced in euros tend to reflect the euro exchange rate, while changes in export prices in sector-destinations where contracts are invoiced in dollars tend to reflect the dollar exchange rate. To the best of our knowledge, the link between prices and invoicing at the industry level is novel. We run a standard exchange rate pass-through regression, as in Gopinath et al. (2020)

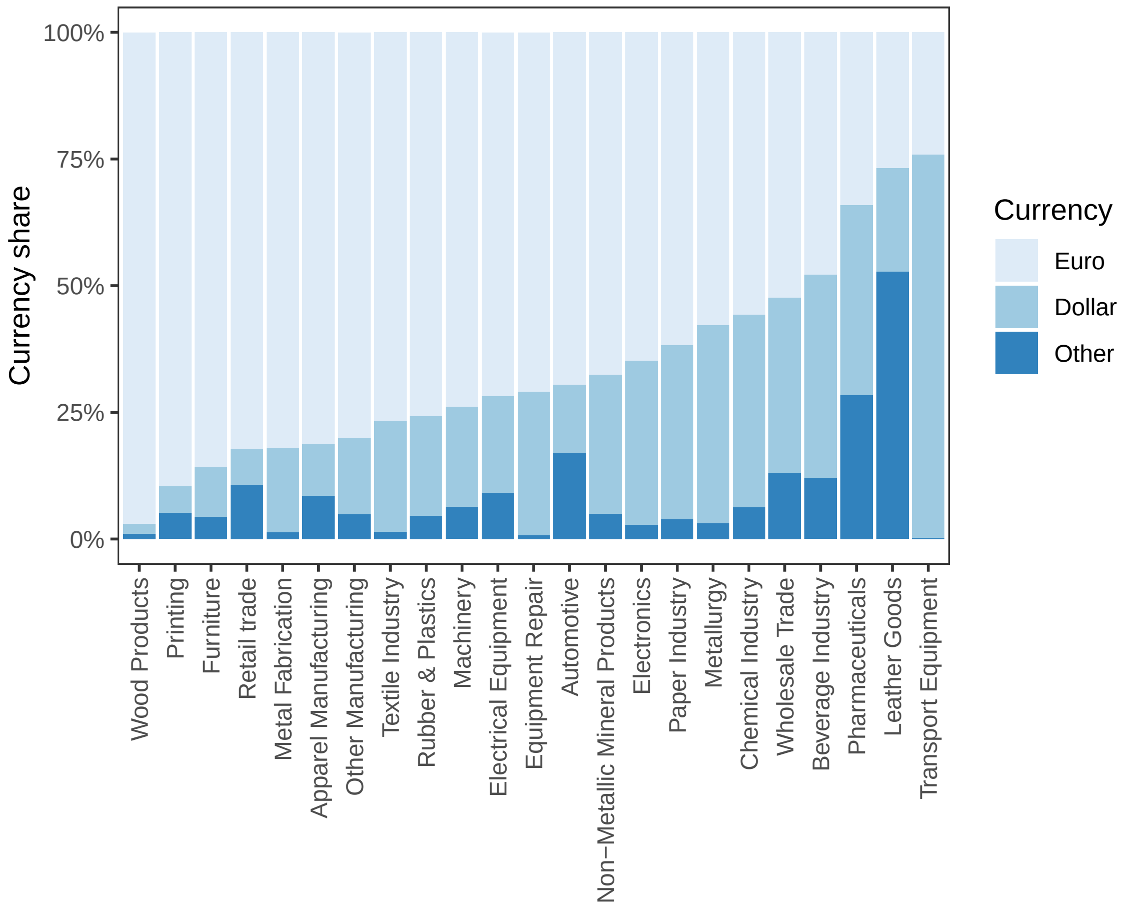
$$\Delta_h \log P_{sdt} = \alpha_{st} + \alpha_{dt} + \beta_h^{\$} \times \Delta_h \log E_{dt}^{\$} \times S_{sdt}^{\$} + \beta_h^{€} \times \Delta_h \log E_{dt}^{€} \times S_{sdt}^{€} + v_{hsdt}. \tag{15}$$

On the left-hand side, the dependent variable $\Delta_h \log P_{sdt}$ is the change between time t and $t-h$ of the price index of firms in sector s exporting to destination d. We construct it from the customs data as a trade-weighted average of price changes at the product and destination level (see Appendix B). On the right-hand side, α_{dt} is a destination and time fixed effect, α_{st} is a sector and time fixed effect, $E_{dt}^{\$}$ is the dollar to destination exchange rate, $E_{dt}^{€}$ is the euro to destination exchange rate, $S_{sdt}^{\$}$ and $S_{sdt}^{€}$ denote the corresponding currency shares.

Table 1 shows the results from estimating Equation (15) with different sets of weights and fixed effects. Column (1) shows that we cannot reject the hypothesis that pass-through of the euro exchange rate is complete over one year at conventional levels. However, this hides considerable heterogeneity, as shown in Column (2). Pass-through is only complete when the invoicing share is close to one. Once we control for dollar invoicing in Column (3), the coefficient on the euro exchange rate becomes smaller. Instead, we find that pass-through of the euro exchange rate is complete when the euro share is one and that pass-through of the dollar exchange rate is sizable the dollar share is one. When both shares are zero, price indices are unresponsive to either exchange rate.

These results align with our sectoral aggregation theory and demonstrate that the short-run "currency contract" view of Magee (1973) applies not only across countries (Gopinath et al., 2020), but also within country across destination and sector pairs. This has two important implications. First, observing aggregate currency shares at the level of a country is not sufficient to characterize pass-through empirically. Indeed, there is significant heterogeneity in currency shares across industries, as shown in Figure 2. Second, heterogeneity in invoicing shares across sectors implies that it is important to take input-output linkages into account. Indeed, the same exchange rate shock will be passed-through differently across sectors, and will therefore propagate through production linkages differently.

Figure 2: Invoicing shares across industries



Note. This figure shows the currency shares of ex-EU exports for two-digit manufacturing sectors in France. We omit some sectors due to statistical confidentiality requirements.

3.3 Market power and foreign currency invoicing

The above findings reveal substantial pass-through heterogeneity across sectors, reflecting heterogeneity in sectoral invoicing currency shares. However, our theory predicts a role for within-sector heterogeneity as well. As discussed in Section 2 the covariance between foreign currency invoicing and demand elasticities, is the key statistic that determines the macroeconomic effect of invoicing heterogeneity. Given that demand elasticities are a simple monotonic transformation of markups in our model, we focus on the covariance between foreign currency invoicing and markups in our empirical exercise. We show below that this covariance is positive: firms that trade in foreign currencies also tend to charge higher markups, both at the product-market level and at the firm level.

Invoicing and markups at the market level. Our first test leverages the fact that we observe unit values, a proxy for prices, at a highly disaggregated

Table 1: Exchange rate pass-through to industry prices

Dependent Variable:	$\Delta \log P_{sdt}$							
Model:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Variables								
$\Delta \log E_{dt}$	0.917	0.550	0.327		0.805	0.547	0.285	
	(0.048)	(0.082)	(0.088)		(0.074)	(0.113)	(0.059)	
$\Delta \log E_{dt} \times S_{sdt-4}$		0.519	0.717	0.713		0.488	0.705	0.629
		(0.094)	(0.104)	(0.099)		(0.177)	(0.124)	(0.132)
$\Delta \log E^{\$}_{dt} \times S^{\$}_{sdt-4}$			0.399	0.417			0.545	0.521
			(0.150)	(0.122)			(0.132)	(0.184)
Fixed-effects								
Year-Quarter	Yes	Yes	Yes	–	Yes	Yes	Yes	–
Destination-Sector	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year-Quarter-Destination	–	–	–	Yes	–	–	–	Yes
Fit statistics								
Destination-Sector	1,791	1,791	1,791	1,791	1,791	1,791	1,791	1,791
Observations	35,115	35,115	35,115	35,115	35,115	35,115	35,115	35,115
R^2	0.309	0.310	0.311	0.370	0.345	0.348	0.349	0.492
Within R^2	0.044	0.046	0.047	0.002	0.059	0.063	0.066	0.007
Weights	Equal	Equal	Equal	Equal	Exports	Exports	Exports	Exports

Note. This table reports the results from a linear regression of changes in price indices on exchange rates and invoicing. Price indices are constructed at the two-digit sector and destination level. Standard errors are clustered by sector and destination. All specifications also include two lags of the regressors.

level. This allows us to test whether firms that invoice using destination market or vehicle currencies also charge higher prices. Specifically,

$$\log p^{\ell}_{ikt,u} = \alpha_{kt,u} + \beta \times \iota^{\$}_{ikt,u} + \gamma \times \log \text{Variable costs}_{it} + v^{\ell}_{ikt,u}, \tag{16}$$

where i is a firm, k is a product-market defined as an export destination and eight digit product pair, t is a month, u is a unit of measurement, and ℓ is a currency. We regress the log unit value $p^{\ell}_{ikt,u}$ on an indicator $\iota^{\$}_{ikt,u}$ that takes the value one if the transaction is invoiced in dollars and zero otherwise. We also include a product-market and time fixed effect $\alpha_{kt,u}$ to account for market specific variation, including local demand and supply shocks. Given that high prices could reflect high markups or low marginal costs, we also control for the firm's total variable costs. The coefficient of interest, β, is identified by comparing transactions invoiced in dollars to transactions invoiced in euros, holding fixed the eight digit product, the destination market, the month, and the unit of measurement.

We report the results in Table 2 for different sets of fixed effects. In all cases, the coefficient on the dollar invoicing indicator is positive and significant. In our preferred specification, shown in Column (4), dollar invoicing predicts 16% higher unit values in a market. This suggests that firms invoicing in dollars systematically charge higher markups than those that invoice in euros.

Table 2: Invoicing and markups at the market level

Dependent Variable:	log Unit Values			
Model:	(1)	(2)	(3)	(4)
Variables				
Dollar invoicing	0.249	0.236	0.132	0.150
	(0.071)	(0.074)	(0.057)	(0.069)
log Costs	0.027	0.015	-0.012	-0.013
	(0.012)	(0.018)	(0.010)	(0.012)
Fixed-effects				
Destination-CN8-Unit	Yes	–	Yes	–
Year-Month	Yes	–	Yes	–
Destination-CN8-Unit-Year-Month	–	Yes	–	Yes
Fit statistics				
Firms	173,345	173,345	173,345	173,345
Observations	25,983,322	25,983,322	25,983,322	25,983,322
R^2	0.969	0.985	0.641	0.765
Within R^2	0.006	0.005	0.001	0.001
Weights	Exports	Exports	Equal	Equal

Note. This table reports the result from a linear regression of log unit values on an indicator for dollar invoicing and controls. Standard errors are clustered by firm.

Invoicing and markups at the firm level. Our second test provides direct evidence supporting the fact that high markups and foreign currency invoicing are positively correlated at the firm level. To measure firm-level markups, we follow De Loecker and Warzynski (2012). This approach requires additional assumptions on firm production, but provides a direct empirical counterpart to our theory. We start from the observation that firms' cost minimization pins down markups (Hall, 1988). Specifically, assuming that firm can flexiblity adjust a competitively priced input, say materials V, the cost minimization problem implies that markups are given by

$$\mu_{ijt} = \frac{\theta^V_{ijt}}{\Omega^V_{ijt}},$$

where i is a firm, j is a sector, Ω^V_{ijt} is the costs-to-sales ratio, and θ^V_{ijt} is the output elasticity with respect to materials. The cost-to-sales ratio can be directly measured from balance sheet data, but the output elasticity cannot. We assume that firms within a two-digit sector share the same translog production function with labor, capital, and materials as inputs. The translog family is flexible and nests the Cobb–Douglas specification as a special case. Under this assumption, output elasticities are a function of observable inputs and production function coefficients, with

$$\theta^V_{ijt} = \beta^V_j + \beta^{KV}_j K_{ijt} + \beta^{LV}_j L_{ijt} + 2\beta^{VV}_j V_{ijt},$$

where V denotes material inputs, K capital, and L labor, and β_j^m is a production function coefficient. All inputs are in logarithms. Although production coefficients are the same for firms within the same sector, input usage differs. Therefore, variation in markups within a sector reflects both variation in the material costs-to-sales ratio and variation in output elasticities.

We structurally estimate production function coefficients following De Ridder et al. (2022), who build on the seminal work of Olley and Pakes (1996), Levinsohn and Petrin (2003), and Ackerberg et al. (2015). We bootstrap our production function estimation procedure to find standard errors for production function coefficients. Given the large error bands, we use a simple empirical Bayes shrinkage estimator to limit the impact of measurement error. We describe our estimation procedure and results in Appendix C.

Equipped with these estimates of markups at the level of the firm, we measure to the relationship between them and invoicing currency use. Figure 3 illustrates our results graphically when firms are indexed by size, and shows a clear positive association between foreign currency invoicing and markups.

We also regress firm-level markups on the share of exports invoiced in foreign currencies to measure the relationship without indexing on size

$$\text{Markups}_{ijt} = \alpha_{jt} + \beta \times \text{Non-euro share}_{ijt} + v_{ijt}.$$

Here α_{jt} is an industry-time fixed effect, where industry is measured at the 4-digit level. The coefficient of interest, β, is identified by comparing the markups of firms within the same industry and year but have different invoicing shares. We use two different outcomes, markups measured in levels or in logs, and show the results in Table 3. Consistent with our market-level evidence, we systematically find a positive correlation between markups and foreign currency invoicing. The effects are economically significant: a one standard deviation increase in foreign currency invoicing (20 percentage points) predicts an increase in markups of about 11 percentage points.

3.4 Persistence of invoicing

We now show that invoicing is extremely persistent in our data. This provides support for one our main theoretical assumptions, namely that firms do not update their invoicing decisions following small exchange rate shocks. We estimate a simple univariate regression

$$\text{Euro share}_{ikt+h} = \alpha_h + \beta_h \cdot \text{Euro share}_{ikt} + v_{ikt+h},$$

at the market level and at the firm level for $h = 1$ and $h = 4$, where t is a quarter. We report the results in Table 4. We are interest in the autocorrelation

Table 3: Invoicing and at the firm level

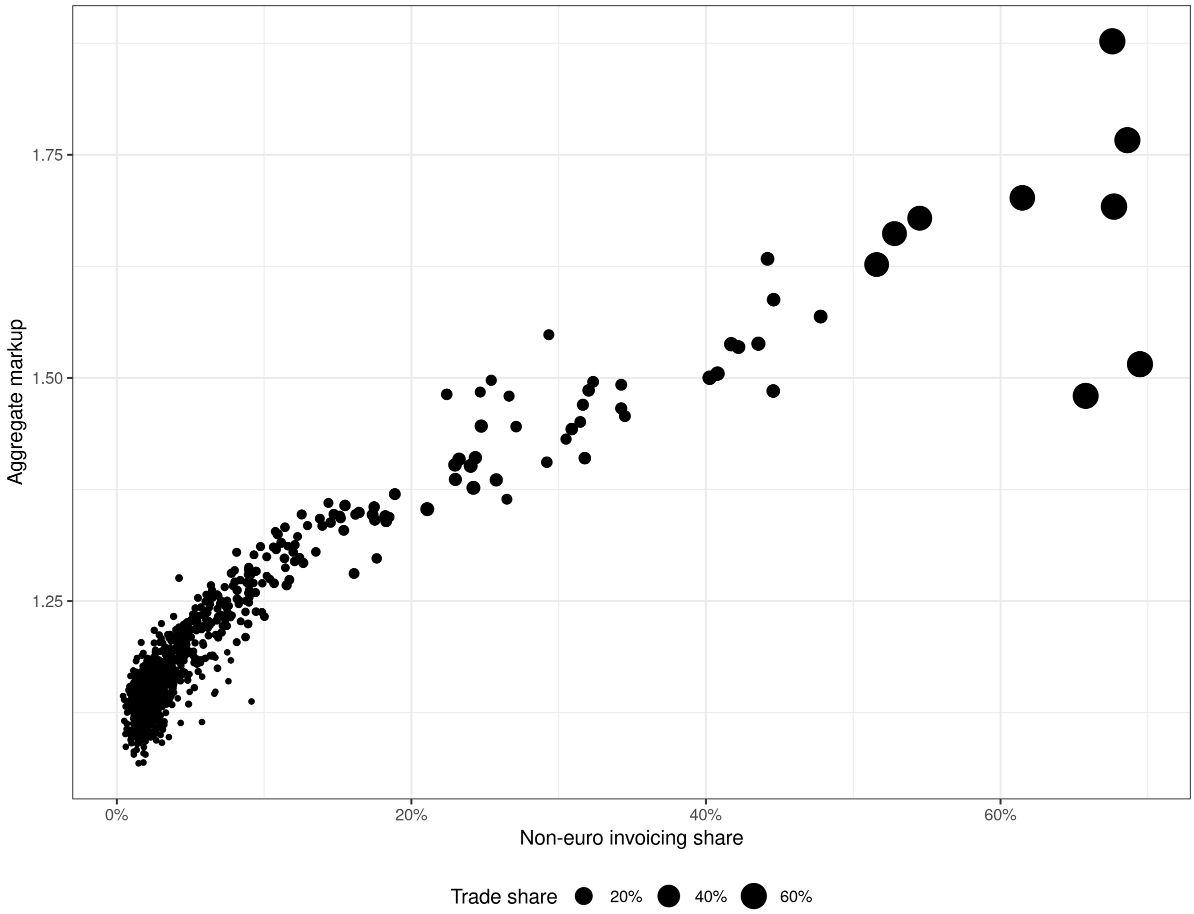
Dependent Variables:	$\log \mu$		μ	
Model:	(1)	(2)	(3)	(4)
Variables				
Non-euro share	0.059	0.060	0.102	0.102
	(0.005)	(0.005)	(0.010)	(0.010)
Fixed-effects				
Sector	Yes	–	Yes	–
Year	Yes	–	Yes	–
Sector-Year	–	Yes	–	Yes
Fit statistics				
Firms	119,637	119,637	119,637	119,637
Observations	484,479	484,479	484,479	484,479
R^2	0.371	0.375	0.483	0.487
Within R^2	0.001	0.001	0.001	0.001

Note. This table reports the result from a linear regression of the share of non-euro exports on markups. The non-euro share is computed as the fraction of exports to destinations outside the eurozone denominated in currencies other than the euro. Markups are estimated through production function estimation, as described in the text. Standard errors are clustered by firm.

coefficient β_h and in the quality of the fit. At the market level, the autocorrelation is about 0.94 and the R^2 greater than 0.85 even for a one year lag. At the firm level, the autocorrelation is close to 0.75 and the R^2 greater than 0.50.

We interpret these results as showing that there is little adjustment in invoicing, especially at the market level. Although there is considerable evidence that currency invoicing is an endogenous decision, at any point in time, the current invoicing distribution is an excellent predictor of the future invoicing distribution. This is in line with the considerable invoicing persistence previously documented by Amiti et al. (2022) and Corsetti et al. (2022).

Figure 3: Binned scatterplot of markups and foreign currency invoicing



Note. This figure shows the aggregate markup and average non-euro share of exports from 2011 to 2020. We sort firms into size bins according to export values. For each bin, we compute the total share of exports to destinations outside the eurozone invoiced in currencies other than the euro, and the harmonic average of firm-level markups weighted by export shares. Firm-level markups are computed using production function estimation as described in the text. The size of the points is proportional to the export share to destinations outside the eurozone.

Table 4: Invoicing stickiness

Dependent Variables:	Euro share$_{t+1}$	Euro share$_{t+4}$	Euro share$_{t+1}$	Euro share$_{t+4}$
Model:	(1)	(2)	(3)	(4)
Variables				
Euro share$_t$	0.940	0.937	0.743	0.745
	(0.004)	(0.004)	(0.003)	(0.004)
Fit statistics				
Observations	9,188,518	4,274,863	1,478,191	995,120
R^2	0.882	0.874	0.547	0.547
Level	Market	Market	Firm	Firm

Note. This table reports the results from a regression of the future euro share on the current euro share at the market level and firm level. The euro share is computed as the fraction of exports to destinations outside the eurozone denominated in euros. Standard errors are clustered by firms.

4 Dynamic general equilibrium model

In this section, we embed the partial equilibrium model of Section 2 into a multi-country, open economy, dynamic general equilibrium setting. We make two important additions: first, we explicitly model the production network with roundabount linkages; second, we add international asset trade which endogenously pins down exchange rates. The dynamics of aggregate variables depend in a tractable way on microeconomic primitives, which we calibrate to realistic firm-level distributions.

Our model builds on the classic open economy New Keynesian framework of Galí and Monacelli (2005) and its extension by Gopinath et al. (2020). As in those models, competition is monopolistic, prices adjust infrequently, firms use intermediate inputs, and goods are invoiced in several currencies. However, our model differs from this classic literature in two important ways. First, building on recent work by Baqaee et al. (2023a), we introduce firm heterogeneity by allowing for arbitrary variation in markups and pass-through within a sector. Second, we allow for an arbitrary distribution of invoicing at the firm level. This lets our model match the rich invoicing patterns observed in the data (Amiti et al., 2022; Corsetti et al., 2022), while also nesting the three main paradigms of the literature, local currency pricing, producer currency pricing, and dominant or vehicle currency pricing.

4.1 Model setting

The model we present here contains three countries, Cobb–Douglas technology, and roundabout production. All of our theoretical results extend to settings with a finite number of countries, constant returns to scale technology, and input-output linkages. Our framework is therefore flexible enough to nest most quantitative models commonly used in international macroeconomics.

Countries. There are three countries indexed by $c \in \{H, D, F\}$. Country H represents the eurozone and uses the euro as its currency; country D represents the United States and uses the dollar; country F represents the rest of the world and uses a local, unspecified, currency. The role of the dollar as the dominant currency will be reflected in our calibration, but there is nothing ex-ante different about country D in our model. We let E_c^d be the price of country c currency in the currency of country d. For example, an increase in E_H^D represents an appreciation of the euro relative to the dollar.

Households. There is a continuum of households in country c indexed by $h \in [0, 1]$. These households consume a final good $C_{ct}(h)$, provide a variety of labor $L_{ct}(h)$, and set the wage for that variety of labor $W_{ct}(h)$. Per-period utility is identical for all consumer and all countries, and takes the standard form

$$U(C_{ct}, L_{ct}) = \frac{C_{ct}^{1-\gamma}}{1-\gamma} - \frac{L_{ct}^{1+1/\zeta}}{1+1/\zeta},$$

where household dependence is dropped for convenience. Consumption is in a country-specific final good, described below. Households in c maximize the discounted sum of utility

$$\mathbf{E}_t \left[\sum_{k \geq 0} \beta^k U(C_{ct+k}, L_{ct+k}) \right],$$

subject to the budget countraint expressed in their domestic currency

$$\overline{P}_{ct} C_{ct} + \sum_d E^c_{dt} q_{cdt} B_{cdt+1} + \mathbf{E}_t[\mathcal{M}_{t,t+1} D_{ct+1}] = W_{ct} L_{ct} + \Pi_{ct} + D_{ct} + \sum_d E^c_{dt}(1 + i_{dt-1}) B_{cdt}.$$

Here, $\overline{P}_{ct} C_{ct}$ is consumption expenditure, $W_{ct} L_{ct}$ is labor income, Π_{ct} is remitted profits from domestic firms, and $\mathcal{M}_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal payoffs. Households have access to a full set of state-contingent securities denominated in their domestic currency, and D_{ct} denotes the time t value of this domestic currency security portfolio. Households can also trade risk-free bonds denominated in the currency of any country. B_{cdt} denotes holdings of the currency d denominated bond, which pays interest rate i_{dt}. There are wedges $q_{cd,t}$ associated with buying foreign currency denominated bonds, with $q_{cd,t} = q^{-1}_{dc,t}$ and $q_{cc,t} = 1$.

Wage rigidities. Households set wages in their country's currency, and are subject to wage rigidities as in Galí (2015). Each period the household may adjust their wage with probability δ_w. Given firms' cost minimizing behavior, described below, households of type h face the downward sloping labor demand curve

$$L_{ct}(h) = \left(\frac{W_{ct}(h)}{W_{ct}} \right)^{-\nu} L_{ct},$$

where $\nu > 1$ is the elasticity of subsitution across labor varieties and W_{ct} is the aggregate wage.

Production. Each country produces and exports destination-specific sectoral goods. These sectoral goods are then combined into one country-specific final good with constant elasticity of substitution σ. The final good is used both for consumption by local households and for production by local firms. Specifically, the final good $\overline{Y}_d$ in country d and its price $\overline{P}_d$ are given by

$$\overline{Y}_d = \left[\sum_{j\in\mathcal{J}(d)} \omega_j Y_j^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \quad ; \quad \overline{P}_d = \left[\sum_{j\in\mathcal{J}(d)} \omega_j^{\sigma} P_j^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

where ω_j is a taste shifter. We let $j = (c,d)$ be the sector located in country c selling to destination d and $\mathcal{J}(d) = \{(H,d),(D,d),(F,d)\}$ be the set of sectors selling to destination d.

Differentiated goods. Sectoral production is as in Section 2. In each sector j, there is a continuum Θ_j of differentiated goods producers with measure one. As in Basu (1995), firms combine labor and intermediate goods with Cobb–Douglas technology, so that

$$y_i = A_i L_i^{1-\alpha} X_i^{\alpha},$$

where y_i is firm output, X_i is intermediate input quantity, A_i is firm productivity, and $L_i = \left(\int_0^1 L_i(h)^{\frac{\nu-1}{\nu}} \mathrm{d}h\right)^{\frac{\nu}{\nu-1}}$ is labor input. This implies that the sectoral marginal cost for sector $j = (c,d)$ located in country c and selling to country d is

$$M_j = \frac{W_c^{1-\alpha} P_c^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \cdot E_c^d,$$

where $W_c = \left(\int_0^1 W_c(h)^{1-\nu} \mathrm{d}h\right)^{\frac{1}{1-\nu}}$ is the aggregate wage. Firm i's marginal cost can thus be written as $M_i = M_j / A_i$.

Price setting. Each differentiated firm is assigned a currency ℓ in which it invoices. The firm sets prices in currency ℓ to maximize discounted profit flows in its home currency. In each period, a firm may adjust its price with probability δ_p, which is common across firms. The price setting problem for a firm in sector j located in c and selling to d in currency ℓ is thus

$$\max_{\bar{p}_{it}^{\ell}} \mathbf{E}_t \left[\sum_{k\geq 0} \mathcal{M}_{t,t+k}(1-\delta_p)^k y_{it+k} E_{dt+k}^c \left(E_{\ell t+k}^d \bar{p}_{it+k}^{\ell} - M_{it+k}\right)\right],$$

where M_i is the firm's marginal cost resulting from cost minimization, and the residual demand curve is identical to Section 2.

Financial markets. Complete markets within each country leads to the typical Euler equation pricing domestic bonds

$$\beta(1+i_{ct})\mathbf{E}_t \left[\left(\frac{C_{ct+1}}{C_{ct}}\right)^{-\gamma} \frac{\overline{P}_{ct}}{\overline{P}_{ct+1}}\right] = 1,$$

where $(1+i_{ct}) = 1/\mathbf{E}_t[\mathcal{M}_{t,t+1}]$ is the return on a one-period riskless bond paying off one unit of domestic currency at time $t+1$.

From the internationally traded risk-free bonds, the Backus-Smith condition between each pair of countries (c, c') is

$$\mathbf{E}_t\left[\left(\frac{C_{c,t+1}}{C_{c,t}}\right)^{-\gamma}\frac{\overline{P}_{c,t}}{\overline{P}_{c,t+1}}\frac{E^c_{c',t+1}}{E^c_{c',t}}\right] = \mathbf{E}_t\left[\left(\frac{C_{c',t+1}}{C_{c',t}}\right)^{-\gamma}\frac{\overline{P}_{c',t}}{\overline{P}_{c',t+1}}\right]q_{cc',t}.$$

Where $q_{cc',t}$ is the wedge associated with holding foreign currency denominated bonds, or a risk-sharing wedge.

Note that the microfoundation of this risk sharing wedge is not important for our purposes. For example, they could be microfounded by segmented international financial markets with limits to arbitrage, as in Jeanne and Rose (2002) and Gabaix and Maggiori (2015). We only use it to generate variation in the exchange rate which is exogenous of the other fundamentals in our model.

Monetary policy. The monetary authority in each country c follows a standard Taylor rule which determines the nominal interest rate

$$1+i_{ct} = \mathbf{E}_t\left[\left(\frac{\overline{P}_{ct+1}}{\overline{P}_{ct}}\right)^{\phi_\pi}\right]\left(\frac{\overline{Y}_{ct}}{\overline{Y}_c}\right)^{\phi_y}V^P_{ct}.$$

where V^P_t is a monetary policy shock.

Equilibrium. Equilibrium conditions are as follows:

1. Taking $\{\overline{P}_{ct}, \Pi_{ct}(h), W_{ct}, L_{ct}, i_{dt}, E^c_{dt}\}$ as given, households of type h in each country choose $\{C_{ct}(h), L_{ct}(h), D_{ct}(h), B_{cdt}(h), \overline{W}_{ct}(h)\}$ to maximize expected utility, subject to the budget constraints, labor demand curves, and wage setting rigidities.
2. Taking upstream prices as given, final good aggregators and sectoral composite good aggregators choose inputs to minimize per unit costs.
3. Taking $\{W_{ct}, \overline{P}_{ct}, P_{odt}, Y_{odt}, E^c_{dt}\}$ as given, intermediate firms in every sector choose $\{\overline{p}_{it}\}$ to maximize discounted future profits, subject to the firm's production function, residual demand curve, and price setting rigidities.
4. Final goods markets clear for every country c

$$\overline{Y}_{ct} = \int_0^1 C_{ct}(h)\mathrm{d}h + \sum_d \int_{\Theta_{cd}} X_i \mathrm{d}i,$$

labor markets clear for every country c and households type h

$$\sum_d \int_{\Theta_{cd}} L_{it}(h)\mathrm{d}i = L_{ct}(h),$$

and all currency d denominated bonds are in zero net supply

$$\sum_c B_{cdt} = 0.$$

4.2 Three-block log-linear representation

We now analyze the dynamics of the open economy New Keynesian model around a zero inflation steady state. The log-linearized model has a tractable three-block representation. It can be calibrated from a small number of simple microeconomic moments that capture the average levels of markups, pass-through, and currency use in invoicing, and the heterogeneity in pass-through and invoicing currency use. Detailed derivations are given in Appendix A.

Production block. The first block describes the production side of the model. From firm profit maximization, we find a Phillips' curve for each sector $j = (c, d)$

$$\begin{aligned} \mathrm{d}\log \pi_{j,t} &= \beta \mathbf{E}_t[\mathrm{d}\log \pi_{j,t+1}] + \frac{\theta_p}{\overline{\mu}_j}\mathbf{E}_{\lambda_j}[1-\rho_i]\mathrm{d}\log A_{j,t} + \theta_p \mathbf{E}_{\lambda_j}[\rho_i]\mathrm{d}\log \frac{M_{j,t}}{P_{j,t}} \\ &\quad + \sum_\ell \left[\beta \mathbf{E}_{\lambda_j}[\iota_i^\ell]\left(\mathbf{E}_t[\mathrm{d}\log E^\ell_{d,t+1} - \mathrm{d}\log E^\ell_{d,t}]\right) - \mathbf{E}_{\lambda_j}[\iota_i^\ell]\left(\mathrm{d}\log E^\ell_{d,t} - \mathrm{d}\log E^\ell_{d,t-1}\right)\right], \end{aligned}$$

where $\theta_p = \frac{\delta_p}{1-\delta_p}(1-\beta(1-\delta_p))$ and E^ℓ_{jt} is the exchange rate from currency ℓ to the destination country of sector j. Relative to the traditional open economy model, the Phillips curve is augmented with an invoicing term capturing international nominal ridigities and a productivity term capturing real rigidities. The dynamics of productivity are endogenous and tightly linked to exchange rate movements

$$\begin{aligned} \mathrm{d}\log A_{jt} &= \frac{1}{\kappa_A}\mathrm{d}\log A_{jt-1} + \frac{\beta}{\kappa_A}\mathbf{E}_t[\mathrm{d}\log A_{jt+1}] + \overline{\mu}_j\theta_p \operatorname{Cov}_{\lambda_j}(\varsigma_i, \rho_i)\,\mathrm{d}\log\frac{M_{jt}}{P_{jt}} \\ &\quad + \overline{\mu}_j \sum_\ell \left[\operatorname{Cov}_{\lambda_j}\left(\varsigma_i, \iota_i^\ell\right)\left(\beta \mathbf{E}_t[\mathrm{d}\log E^\ell_{d,t+1} - \mathrm{d}\log E^\ell_{d,t}] - \left(\mathrm{d}\log E^\ell_{d,t} - \mathrm{d}\log E^\ell_{d,t-1}\right)\right)\right], \end{aligned}$$

where $\kappa_A = 1 + \beta + \theta_p\left(1 + \operatorname{Cov}_{\lambda_j}(\varsigma_i, \rho_i)\right)$. As in the static model, changes in productivity reflect heterogeneity in pass-through and heterogeneity in invoicing.

Household block. The second block is standard and describes the household side of the model. It consists of an Euler equation for each country

$$\mathrm{d}\log C_{c,t} = \mathbf{E}_t\left[\mathrm{d}\log C_{c,t+1}\right] - \frac{1}{\gamma}\left(\mathrm{d}\log i_{c,t} + \mathbf{E}_t[\mathrm{d}\log \pi_{c,t+1}]\right),$$

as well as a wage-setting equation

$$\mathrm{d}\log \pi^w_{c,t} = \beta \mathbf{E}_t\left[\mathrm{d}\log \pi^w_{c,t+1}\right] - \theta_w\left(\mathrm{d}\log\frac{W_{c,t}}{P_{c,t}} - \gamma\mathrm{d}\log C_{c,t} - \frac{1}{\zeta}\mathrm{d}\log L_{c,t}\right),$$

where $\theta_w = \frac{\delta_w}{1-\delta_w}\frac{1-\beta(1-\delta_w)}{1+\nu/\zeta}$.

Closing block. The third block closes the model. Goods and labor markets clear. Central banks in each country follow a Taylor rule

$$\mathrm{d}\log i_{c,t} = \phi_\pi \mathbf{E}_t[\pi_{c,t+1}] + \phi_y \mathrm{d}\log Y_{c,t} + \mathrm{d}\log V^P_{c,t}.$$

Finally, exchange rate dynamics are pinned down by a risk-sharing condition between each pair of countries

$$\gamma \mathrm{d}\log C_{c,t} + \mathrm{d}\log P_{c,t} + \mathrm{d}\log V^R_{cc',t} = \gamma \mathrm{d}\log C_{c',t} + \mathrm{d}\log P_{c',t} + \mathrm{d}\log E^c_{c',t}.$$

Here, V^P and V^R are monetary policy and risk-sharing shocks, respectively.

5 Calibration

Sufficient statistics. As described in Section 2, we provide a list of portable statistics (Nakamura and Steinsson, 2018) that are sufficient to describe firm heterogeneity in a wide class of open economy macro models. Table 5 summarizes these statistics, and the values used in our calibration. In practice, our ability to allow for distributions of firm-level variables that vary across countries, sectors, and export markets is limited by available data. Although we think that this variation is interesting and quantitatively relevant, we abstract away from it in our calibration. Researchers studying different countries or sectors can easily update our calibration and use the model to conduct new quantitative exercises.

Table 5: Sufficient statistics

	Moment	**Value**
Demand system		
Aggregate markup	$\overline{\mu}$	1.13
Pass-through	$\mathbf{E}_\lambda[\rho_i]$	0.45
Pass-through heterogeneity	$\mathrm{Cov}_\lambda(\varsigma_i, \rho_i)$	0.26
Invoicing of exports to the US		
Euro share	$\mathbf{E}_\lambda[\iota_i^€]$	0.40
Dollar share	$\mathbf{E}_\lambda[\iota_i^\$]$	0.60
Euro heterogeneity	$\mathrm{Cov}_\lambda(\varsigma_i, \iota_i^€)$	0.19
Dollar heterogeneity	$\mathrm{Cov}_\lambda(\varsigma_i, \iota_i^\$)$	−0.19
Invoicing of exports to the RoW		
Euro share	$\mathbf{E}_\lambda[\iota_i^€]$	0.73
Dollar share	$\mathbf{E}_\lambda[\iota_i^\$]$	0.19
Local share	$\mathbf{E}_\lambda[\iota_i^L]$	0.08
Euro heterogeneity	$\mathrm{Cov}_\lambda(\varsigma_i, \iota_i^€)$	0.14
Dollar heterogeneity	$\mathrm{Cov}_\lambda(\varsigma_i, \iota_i^\$)$	−0.10
Local heterogeneity	$\mathrm{Cov}_\lambda(\varsigma_i, \iota_i^L)$	−0.04

Demand system. Ideally, we would compute these sufficient statistics directly from firm-level data. However, while firm sales shares and invoicing are directly observed, markups and pass-through are not. Instead, we rely on the demand system calibration of Baqaee et al. (2023b), who use identified moments in pass-through heterogeneity across firm size from Amiti et al. (2019). Assuming that firm characteristics vary only as a function of size $s \in [0,1]$, they show that markups solve the ordinary differential equation

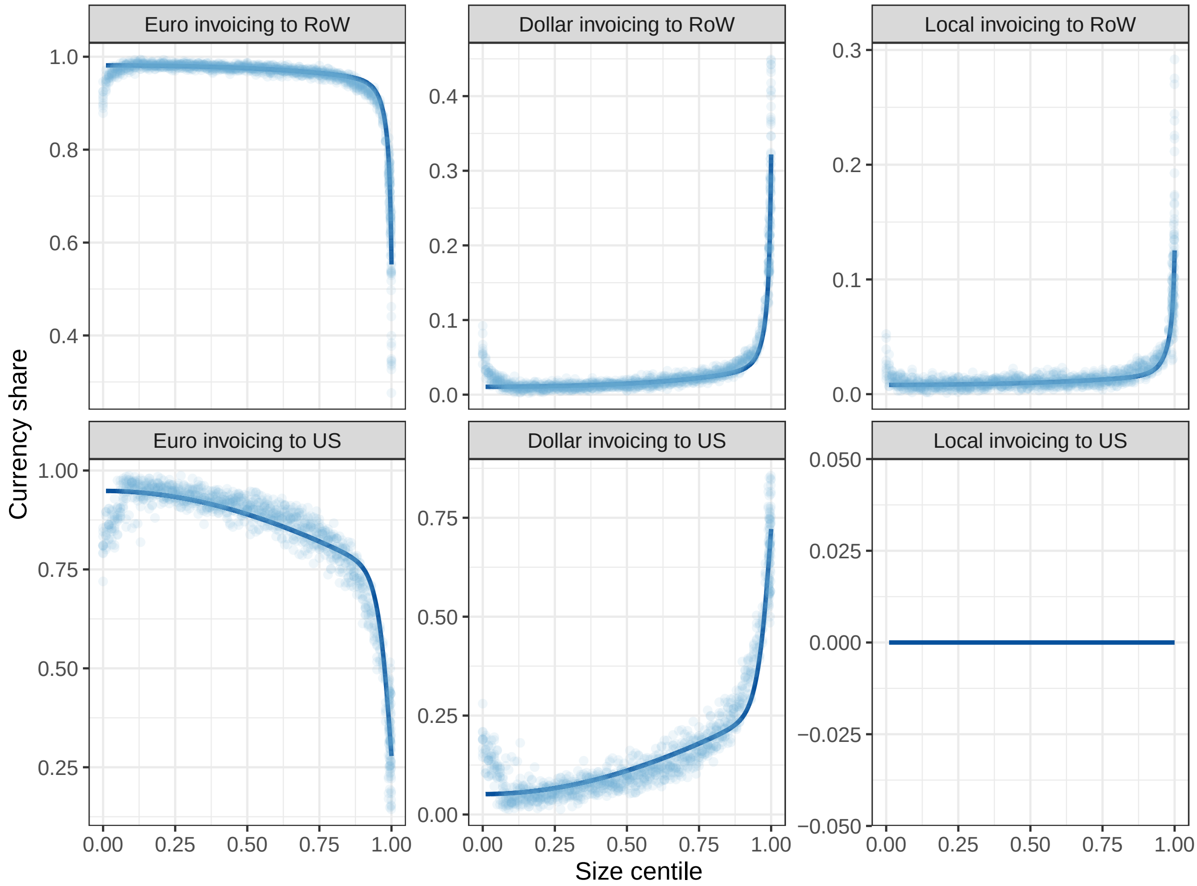
$$\frac{\mu'(s)}{\mu(s)} = (\mu(s)-1)\frac{1-\rho(s)}{\rho(s)}\frac{\lambda'(s)}{\lambda(s)} \tag{17}$$

up to a boundary condition which can be chosen to match the value of the aggregate markup. We directly use their estimated pass-through distribution ρ, which is designed to match the empirical estimates of Amiti et al. (2019), and the implied markup distribution μ. We refer the reader to Appendix D for additional details.

The assumption that pass-through, markups, and invoicing depend only on firm size is restrictive. However, it has both theoretical and empirical support. Theoretically, market shares are sufficient statistics for markups and pass-through in models of oligpolistic competition such as Atkeson and Burstein (2008). Empirically, we find that firm size captures meaningful variation in invoicing and Burstein et al. (2020) show that market shares covary with markups using similar data.

Sales shares and invoicing. We estimate the distribution of sales shares and invoicing by sorting firms into size bins. For each bin, we compute the sales share of that bin, as well as the currency shares for the euro, the dollar, and local currencies. We then fit exponential functions to these data, and obtain smooth functions $\lambda(s)$ for sales and $\iota^{\ell}(s)$ for invoicing in currency ℓ, where $s \in [0,1]$ is the size bin. Figure 4 shows the calibrated invoicing distributions.

Figure 4: Calibrated currency invoiced distributions



Note. This figure shows the empirical currency invoicing distribution (dots) and our calibrate distribution (solid line). Each dot corresponds to a firm size bin and a year.

Computing the statistics. Having calculated the above distributions, we can now compute the sufficient statistics shown in Table 5. Invoicing moments are from the perspective of the eurozone (country H), and are applied to the rest of the world (country F) so the two are symmetric. We calibrate the aggregate shares of invoicing currency use in the United States (country D) to match the shares of US exports invoiced in euros, dollars, and other currencies, but do not allow for heterogeneity in country D invoicing. This is because the dollar is the dominant currency, so the invoicing pattern of US exporters likely differs from that of other countries.

External parameters Table 6 lists the other parameter values of our calibration. The time period is one quarter. We use standard parameters for the discount rate, intertemporal elasticity of substitution, elasticity of substitution across labor varieties, and monetary policy (Galí, 2015). The Frisch elasticity of labor supply is 0.5, in line with microeconomic estimates (Chetty et al., 2013). The Calvo price flexibility is 0.4, implying an average price duration of about 10 months.

This is between the median duration for export prices of 6 months as measured by Fitzgerald and Haller (2013) and 11 months as measured by Gopinath et al. (2010). The elasticity of subsitution between domestic and foreign products is 2, in line with estimates from Feenstra et al. (2018) which finds values ranging from 1 to 4 depending on the industry. We set the Calvo wage flexibility to 0.15, which implies an average wage duration of about one and a half year. Finally, we calibrate the home-bias to produce consumption share of 0.7 on domestic goods, as in Gopinath et al. (2020). Finally, the intermediates intensity in production is 2/3.

Table 6: External parameters

	Parameter	**Value**
Household preferences		
Discount rate	β	0.99
Intertemporal rate of substitution	γ	2
Frisch elasticity of labor supply	ζ	0.50
Elasticity of substitution across industry varieties	η	2
Elasticity of substitution across labor varieties	ν	4
Technology		
Intermediates intensity	α	2/3
Nominal rigidities		
Price flexibility	δ_p	0.40
Wage flexibility	δ_w	0.15
Monetary policy		
Output gap targeting	ϕ_y	0.5/4
Inflation targeting	ϕ_π	1.50
Expenditure shares		
Expenditure share on domestic goods	λ_D	0.70

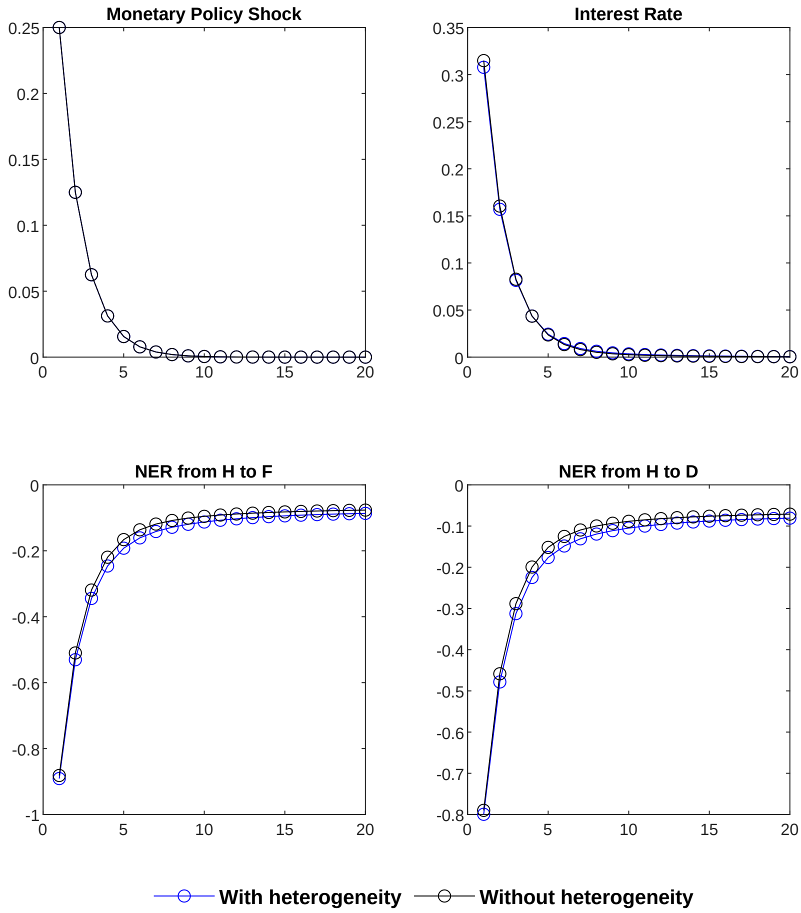
6 Quantitative results

We now use our calibrated dynamic model to quantify the importance of firm heterogeneity for exchange rate transmission and for dollar monetary policy spillovers in general equilibrium. To do so, we will compare shock responses of macroeconomic variables under two scenarios: a scenario with firm heterogeneity, as described in Section 5, and a counterfactual scenario without firm heterogeneity, in which the covariances in Table 5 are set to zero for all countries and sectors.

6.1 How does firm heterogeneity impact the international transmission of monetary policy shocks?

The first shock we consider is a 25 basis points monetary tightening in the eurozone (country H), used to simulate a euro appreciation. We compare impulse responses with and without firm heterogeneity. Figure 5 shows the monetary policy shock and the responses of interest rate and bilateral exchange rates in country H. The presence of firm heterogeneity slightly changes the central bank's interest rate path following the shock, and thus results in slightly different exchange rate dynamics. However, the differences are small enough that we feel comfortable comparing the transmission of this exchange rate appreciation under the two scenarios.

Figure 5: Eurozone (country H) exchange rate shock



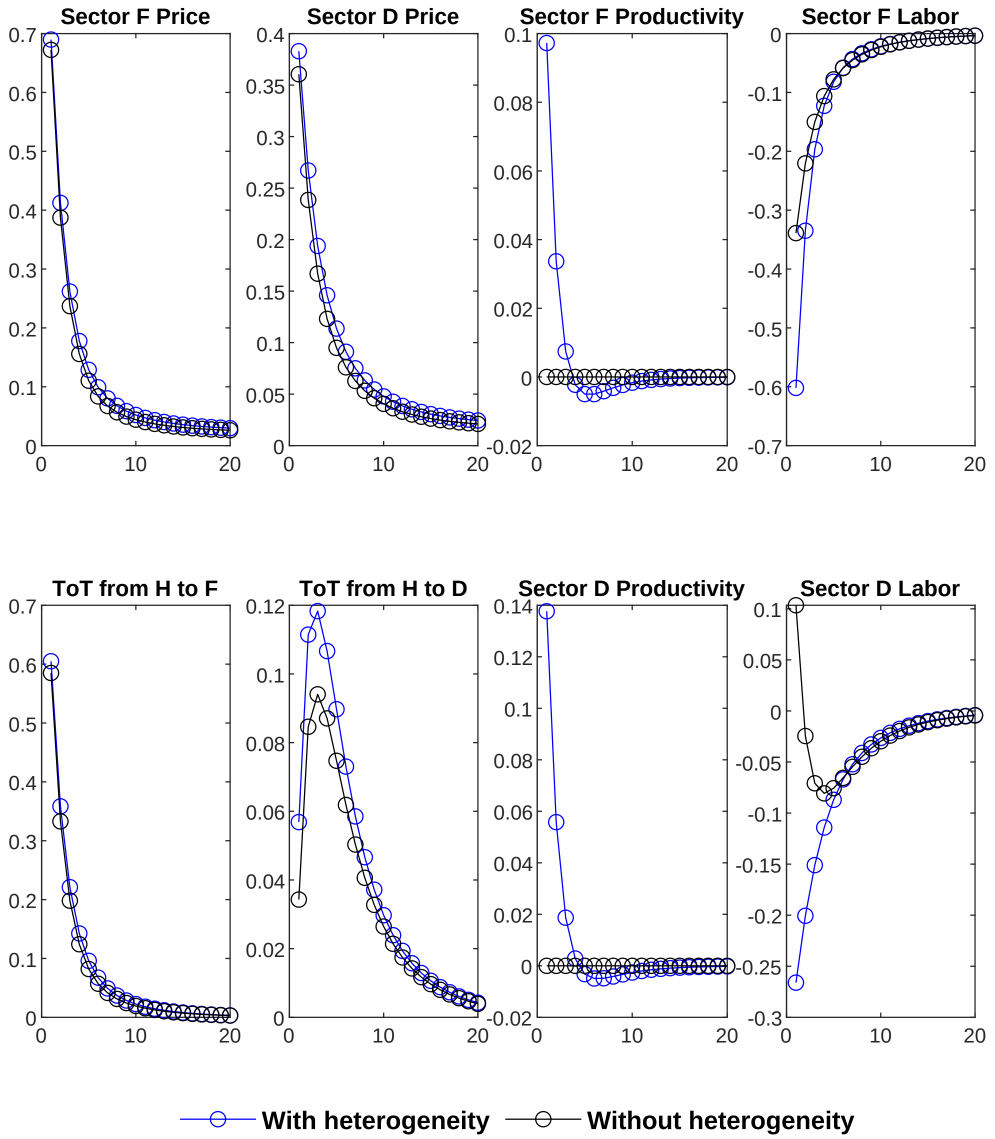
Note. This figure shows impulse responses of country H financial variables following a 25bp monetary policy shock in country H. The heterogeneous firms calibration is described in the text. The homogeneous firms calibration uses identical numbers with all covariances set to zero.

Figure 6 shows the impulse responses of eurozone (country H) variables to the euro appreciating. The most striking impact of firm heterogeneity is the large productivity improvements in both export sectors. This is because the key covariances $\text{Cov}_\lambda(\varsigma, \iota^\$)$ and $\text{Cov}_\lambda(\varsigma, \iota^L)$ are both negative. The euro appreciation increases the export prices of small firms, which use the euro, relative to large firms, which use the dollar and the destination currency. This induces an expenditure switching effect, reallocating resources toward large firms. Since large firms have low demand elasticities (high markups) and are inefficiently small, productivity improves in the exporting sectors. When firms are homogeneous, there is no reallocation and thus no productivity response.

The appreciation of the euro makes country H euro exports more expensive compared to other countries' goods, decreasing total country H exports and thus factor demand in exporting sectors. The heterogeneous firms case sees a greater fall in labor supplied to each export sector because the foreign expenditure switching effect is coupled with productivity increases. One could interpret this as the unemployment effect of currency appreciation being greater with firm heterogeneity. Alternatively, in a world with frictional labor markets or adjustment costs in factors of production, some of this decrease in factor use would instead translate into greater decreases in overall production in the presence of heterogeneity.

Prices are higher with heterogeneity in both sectors because of the strategic complementarity in price setting. In exports to the rest of the world (country F), firms setting prices in euros face more elastic demand than firms setting prices in dollars or in the local currency, so the euro appreciation increases the relative price of exporters facing more elastic demand. In response, competitors strategically adjust their prices by more than if the demand elasticities were the same across firms. This effect directly feeds into the term of trade response. Recall that there is no invoicing heterogeneity for firms in country D – thus there is no offsetting force in prices for imports in H from D, which is why terms of trade responses are different in shape and magnitude.

Figure 6: Impulse responses of Eurozone (country H) bilateral variables



Note. This figure shows impulse responses of country H variables following a 25bp monetary policy shock in country H. The heterogeneous firms calibration is described in the text. The homogeneous firms calibration uses identical numbers with all covariances set to zero. Prices are expressed in the destination market currency.

6.2 Are dollar shocks special because large firms use the dollar?

We now analyze the implications of firm heterogeneity for dollar shocks. The dollar is special in part because it is used in bilateral trade, even when neither trading partner is the United States (Gopinath and Itskhoki, 2022). This implies that the dollar exchange rate, rather than the bilateral exchange rate, is passed through to prices (Gopinath et al., 2020; Boz et al., 2022). In addition to this channel, our model is designed to capture an additional feature of the dollar, which is that dollar invoicing is overwhelmingly done by large firms (Figure 1).

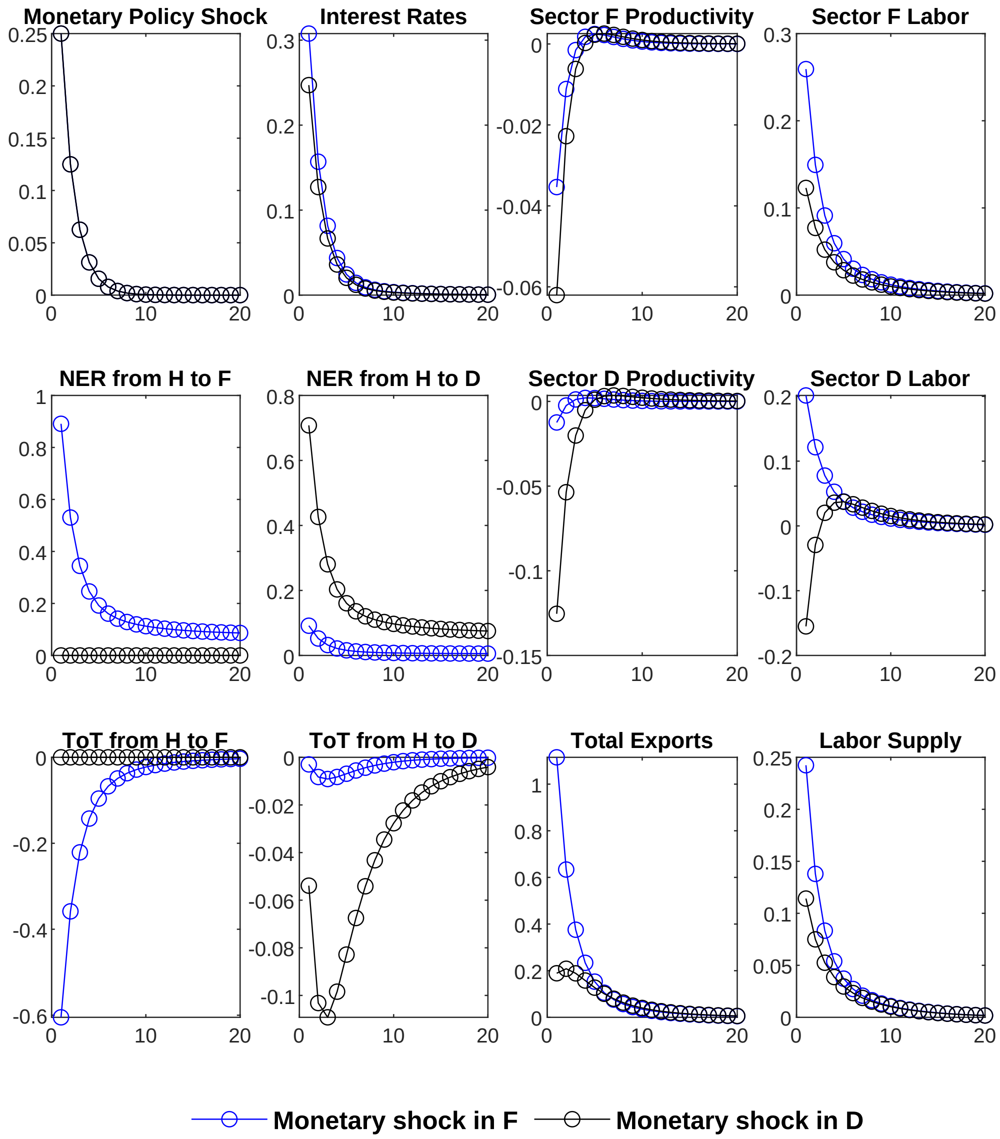
Dollar monetary policy. To illustrate the specialness of the dollar, we compare responses in the eurozone (country H) to identical monetary policy tightening shocks in the US (country D) and in the rest of the world (country F). The US monetary tightening we think of as a dollar shock, and the RoW tightening as a local currency shock. Recall that in the calibration, the only difference between countries is the currency use in trade and the the distributions of invoicing – the three countries are all the same size. This means that differential responses in the eurozone can entirely be attributed to differences in invoicing.

Figure 7 shows the impulse responses. In both cases, the euro depreciates and the term of trade falls relative to the country tightening its monetary policy. The country F shock induces a slight depreciation of the euro with respect to dollar, through general equilibrium forces. The country D shock does not induce a change in the H to F term of trade because the two countries are perfectly symmetric. Therefore, their responses to the dollar shock exactly offset in their bilateral exchange rate and terms of trade.

The key difference between the shocks comes in the productivity responses. When country F tightens its monetary policy, misallocation increaes in the sector exporting to F. However, when country D tightens its monetary policy, misallocation increases in both in the sector exporting to D and the sector exporting to F. This is because while country F currency is only used by firms exporting to country F, the dollar is used when exporting to both country D and country F. The response of exports and labor in the eurozone is smaller when the dollar appreciates than when country F's currency appreciates. This is because reallocation effects are stronger for dollar shocks, due to the fact that dollar invoicing is more prevalent. Thus, the impact of expenditure switching is not as large.

These results illustrate how both the aggregate shares of currency use and invoicing heterogeneity matter. The aggregate shares determine the average incidence of expenditure switching and, through this, movements in term of trade, total exports, and labor. Currency use across firms determines the productivity responses, which feeds into factor use and prices. The pervasiveness of dollar invoicing by large firms in all sectors means that reallocation takes place in all export sectors in response to a dollar shock, thereby increasing the spillover of US monetary policy into productivity.

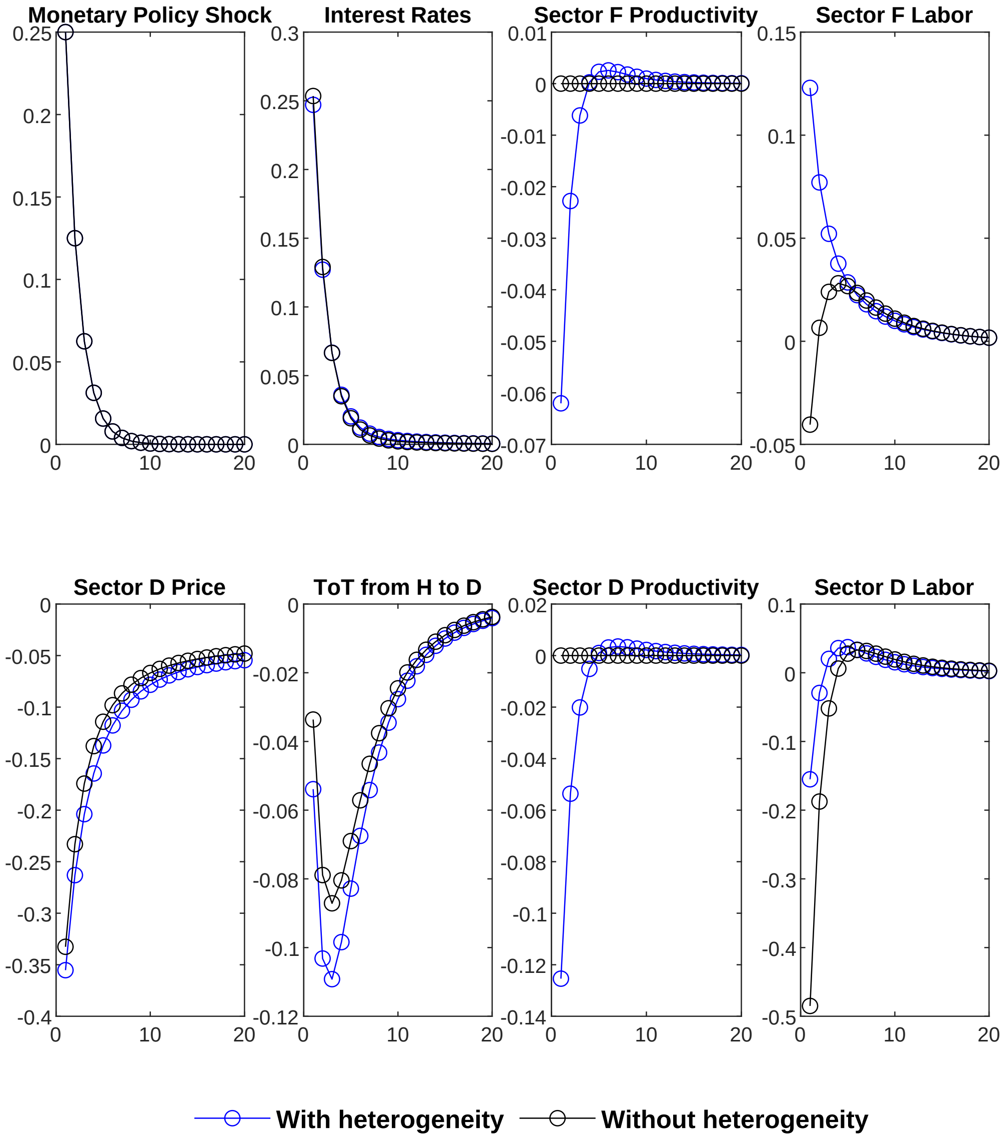
Figure 7: Comparing monetary policy in the US (country D) to the RoW (country F)



Note. This figure shows impulse responses for country H variables following a 25bp monetary policy shock in either country F or country D. The first two panels show the monetary policy shock and interest rate movement in country F and country D.

Isolating the effect of firm heterogeneity. In Figure 8 we zoom in on the implications of firm heterogeneity for dollar shocks. As before, we compare impulse responses in the eurozone (country H) to a US monetary tightening under our baseline calibration and under an alternative calibration that shuts down firm heterogeneity. As we can see, the impact of this shock is similar to that of the monetary tightening in country H. Heterogeneity primarily impacts export sector productivity and factor use. The interesting fact is that, as mentioned above, the productivity effect is present in sector exporting to country F in addition to the sector exporting to country D, due to the dominance of the dollar.

Figure 8: The importance of firm heterogeneity for US monetary policy (country D)



*Note.*This figure shows impulse responses to country H variables following a 25bp monetary policy shock in country D. The heterogeneous firms calibration is described in the text. The homogeneous firms calibration uses identical numbers with all covariances set to zero. Price variables are expressed in the destination currency.

7 Conclusion

This paper proposes a new framework to analyze the importance of firm heterogeneity, particularly currency invoicing heterogeneity, for the transmission of exchange rate shocks. We provide nonparametric formulas for local changes in sectoral prices, markups, and productivity, which depend on a small number of sufficient statistics. Using an administrative dataset from France, we document the importance of invoicing heterogeneity across and within sector. We then calibrate our sufficient statistics within three-country New Keynesian model, and show that the presence of firm heterogeneity change the effect of exchange rate shocks on productivity and labor.

Our work could be extended in a number of ways. First, incorporating realistic frictions in labor supply, for example by introducing several types of labor that are imperfectly substitutable, could help characterize the implications of currency movements for workers in the trade sector. Second, our framework can be embedded in a quantitative production network model allowing for additional input-ouput linkages and cross-sector heterogeneity. Provided that rich enough data are available, our sufficient statistic calibration approach can easily be adapted to this application.

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Appendices

A Proofs

A.1 Static model

Proof of Proposition 1. Log-linearizing the definition of the aggregator $\mathcal{P}_j$ yields

$$\mathrm{d}\log\mathcal{P}_j = \mathbf{E}_{\lambda_j}\left[\varsigma_i \mathrm{d}\log p_i\right].$$

Plugging microeconomic exchange rate pass-through, we can solve for the changes in the aggregator

$$\mathrm{d}\log\mathcal{P}_j = \frac{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]}\mathrm{d}\log M_j + \frac{\mathbf{E}_{\lambda_j}[\sigma_i(1-\delta_i)\iota_i]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]}\mathrm{d}\log E_j.$$

We now show how to rewrite this formula in terms of sufficient statistics. Note that

$$\begin{aligned}\mathbf{E}_{\lambda_j}[\sigma_i(1-\delta_i)\iota_i] = {} & \mathbf{E}_{\lambda_j}[1-\delta_i]\mathbf{E}_{\lambda_j}[\sigma_i]\,\mathrm{Cov}_{\lambda_j(1-\delta_j)}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\iota_i\right)\\ & + \frac{\mathbf{E}_{\lambda_j}[\sigma_i(1-\delta_i)]}{\mathbf{E}_{\lambda_j}[1-\delta_j]}\Big(\mathbf{E}_{\lambda_j}[1-\delta_i]\mathbf{E}_{\lambda_j}[\iota_i] + \mathrm{Cov}_{\lambda_j}(\iota_i, 1-\delta_i)\Big).\end{aligned}$$

Therefore

$$\begin{aligned}\mathrm{d}\log\mathcal{P}_j = {} & \frac{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]}\mathrm{d}\log M_j + \frac{\frac{\mathbf{E}_{\lambda_j}[\sigma_i(1-\delta_i)]}{\mathbf{E}_{\lambda_j}[1-\delta_j]}}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]} \times \mathbf{E}_{\lambda_j}[1-\delta_i]\mathbf{E}_{\lambda_j}[\iota_i]\mathrm{d}\log E_j\\ & + \frac{\frac{\mathbf{E}_{\lambda_j}[\sigma_i(1-\delta_i)]}{\mathbf{E}_{\lambda_j}[1-\delta_j]}}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]} \times \mathrm{Cov}_{\lambda_j}(\iota_i, 1-\delta_i)\mathrm{d}\log E_j\\ & + \frac{\mathbf{E}_{\lambda_j}[1-\delta_i]\mathbf{E}_{\lambda_j}[\sigma_i]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]} \times \mathrm{Cov}_{\lambda_j(1-\delta_i)}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\iota_i\right)\mathrm{d}\log E_j.\end{aligned}$$

Now recall that $\mathrm{d}\log P_j = \mathbf{E}_{\lambda_j}[\mathrm{d}\log p_i]$. Therefore

$$\mathrm{d}\log P_j = \mathbf{E}_{\lambda_j}[\delta_i\rho_i]\mathrm{d}\log M_j + \mathbf{E}_{\lambda_j}[\delta_i(1-\rho_i)]\mathrm{d}\log\mathcal{P}_j + \mathbf{E}_{\lambda_j}[(1-\delta_i)\iota_i]\mathrm{d}\log E_j.$$

It follows that

$$
\begin{aligned}
\mathrm{d}\log P_j = &\left(\mathbf{E}_{\lambda_j}[\delta_i\rho_i] + \frac{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i]\mathbf{E}_{\lambda_j}[\delta_i(1-\rho_i)]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]}\right)\mathrm{d}\log M_j \\
&+\left(1+\frac{\mathbf{E}_{\lambda_j}[\delta_i(1-\rho_i)]\frac{\mathbf{E}_{\lambda_j}[\sigma_i(1-\delta_i)]}{\mathbf{E}_{\lambda_j}[1-\delta_j]}}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]}\right)\mathbf{E}_{\lambda_j}[1-\delta_i]\mathbf{E}_{\lambda_j}[\iota_i]\mathrm{d}\log E_j \\
&+\left(1+\frac{\mathbf{E}_{\lambda_j}[\delta_i(1-\rho_i)]\frac{\mathbf{E}_{\lambda_j}[\sigma_i(1-\delta_i)]}{\mathbf{E}_{\lambda_j}[1-\delta_j]}}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]}\right)\mathrm{Cov}_{\lambda_j}(\iota_i, 1-\delta_i)\mathrm{d}\log E_j \\
&+\frac{\mathbf{E}_{\lambda_j}[\delta_i(1-\rho_i)]\mathbf{E}_{\lambda_j}[1-\delta_i]\mathbf{E}_{\lambda_j}[\sigma_i]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]}\mathrm{Cov}_{\lambda_j(1-\delta_i)}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}, \iota_i\right)\mathrm{d}\log E_j.
\end{aligned}
$$

We can simply define

$$
\begin{aligned}
\kappa_M &= \mathbf{E}_{\lambda_j}[\delta_i\rho_i] + \frac{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i]\mathbf{E}_{\lambda_j}[\delta_i(1-\rho_i)]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]}, \\
\kappa_\iota &= 1+\frac{\mathbf{E}_{\lambda_j}[\delta_i(1-\rho_i)]\frac{\mathbf{E}_{\lambda_j}[\sigma_i(1-\delta_i)]}{\mathbf{E}_{\lambda_j}[1-\delta_j]}}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]}, \\
\kappa_\sigma &= \frac{\mathbf{E}_{\lambda_j}[\delta_i(1-\rho_i)]\mathbf{E}_{\lambda_j}[1-\delta_i]\mathbf{E}_{\lambda_j}[\sigma_i]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]}.
\end{aligned}
$$

Note that $\kappa_M, \kappa_\sigma \in [0,1]$ while $\kappa_\iota \geq 1$.

Proof of Proposition 2. We have

$$
\mathrm{d}\log A_j = \mathrm{d}\log\overline{\mu}_j + \mathrm{d}\log M_j - \mathrm{d}\log P_j = -\mathbf{E}_{\lambda_j}\left[\frac{\overline{\mu}_j}{\mu_i}\mathrm{d}\log\frac{\lambda_i}{\mu_i}\right] - \mathbf{E}_{\lambda_j}[\mathrm{d}\log\mu_i] = \mathrm{Cov}_{\lambda_j}\left(-\frac{\overline{\mu}_j}{\mu_i}, \mathrm{d}\log\frac{\lambda_i}{\mu_i}\right).
$$

Now note that $\lambda_i/\mu_i = M_j y_i/P_j Y_j$. Therefore

$$
\mathrm{d}\log A_j = \mathrm{Cov}_{\lambda_j}\left(-\frac{\overline{\mu}_j}{\mu_i}, \mathrm{d}\log\frac{y_i}{Y_j}\right) = \mathrm{Cov}_{\lambda_j}\left(-\frac{\overline{\mu}_j}{\mu_i}, \mathrm{d}\log y_i\right).
$$

The log-linearized residual demand curve writes $\mathrm{d}\log y_i/Y_j = -\sigma_i\mathrm{d}\log p_i/\mathcal{P}_j$. It follows that

$$
\begin{aligned}
\mathrm{d}\log A_j &= \mathbf{E}_{\lambda_j}\left[\frac{\overline{\mu}_j}{\mu_i}\times\sigma_i\mathrm{d}\log\frac{p_i}{\mathcal{P}_j}\right] + \mathbf{E}_{\lambda_j}\left[\sigma_i\mathrm{d}\log\frac{p_i}{\mathcal{P}_j}\right] = \mathbf{E}_{\lambda_j}\left[\overline{\mu}_j(\sigma_i-1)\mathrm{d}\log\frac{p_i}{\mathcal{P}_j}\right] \\
&= \overline{\mu}_j\left(\mathcal{P}_j - P_j\right) = \overline{\mu}_j\mathrm{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}, \mathrm{d}\log p_i\right).
\end{aligned}
$$

This completes the proof of the first part of the proposition. For the second part, we use our results on ERPT, so that

$$\begin{aligned}\operatorname{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}, \mathrm{d}\log p_i\right) = &\left[\operatorname{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}, \delta_i\rho_i\right) + \frac{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i]\operatorname{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}, \delta_i(1-\rho_i)\right)}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]}\right]\mathrm{d}\log M_j \\ &+ \left[\frac{\mathbf{E}_{\lambda_j}[\sigma_i(1-\delta_i)\iota_i]\operatorname{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}, \delta_i(1-\rho_i)\right)}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]} + \operatorname{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}, (1-\delta_i)\iota_i\right)\right]\mathrm{d}\log E_j.\end{aligned}$$

Focusing on the marginal cost component, note that

$$\begin{aligned}\operatorname{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}, \delta_i\rho_i\right) &= \mathbf{E}_{\lambda_j}[\delta_i]\frac{\mathbf{E}_{\lambda_j}\left[\delta_i\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}\rho_i\right]}{\mathbf{E}_{\lambda_j}[\delta_i]} - \mathbf{E}_{\lambda_j}[\delta_i]\frac{\mathbf{E}_{\lambda_j}\left[\delta_i\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}\right]}{\mathbf{E}_{\lambda_j}[\delta_i]}\frac{\mathbf{E}_{\lambda_j}[\delta_i\rho_i]}{\mathbf{E}_{\lambda_j}[\delta_i]} + \mathbf{E}_{\lambda_j}[\delta_i\rho_i]\left(\frac{\mathbf{E}_{\lambda_j}\left[\delta_i\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}\right]}{\mathbf{E}_{\lambda_j}[\delta_i]} - 1\right) \\ &= \mathbf{E}_{\lambda_j}[\delta_i]\operatorname{Cov}_{\lambda_j\delta_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}, \rho_i\right) + \mathbf{E}_{\lambda_j\delta_j}[\rho_i]\operatorname{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}, \delta_i\right).\end{aligned}$$

Therefore, the first terms in brackets is equal to

$$\begin{aligned}&\frac{\mathbf{E}_{\lambda_j}[\delta_i]\mathbf{E}_{\lambda_j}[\sigma_i(1-\delta_i)]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]}\operatorname{Cov}_{\lambda_j\delta_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}, \rho_i\right) + \frac{\mathbf{E}_{\lambda_j\delta_j}[\rho_i]\mathbf{E}_{\lambda_j}[\sigma_i(1-\delta_i)] + \mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]}\operatorname{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_i}[\sigma_i]}, \delta_i\right) \\ &= \frac{\mathbf{E}_{\lambda_j}[\delta_i]\mathbf{E}_{\lambda_j}[\sigma_i]\mathbf{E}_{\lambda_j}[(1-\delta_i)]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]}\operatorname{Cov}_{\lambda_j\delta_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}, \rho_i\right) \\ &+ \frac{\mathbf{E}_{\lambda_j\delta_j}[\rho_i]\mathbf{E}_{\lambda_j}[\sigma_i(1-\delta_i)] + \mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i] - \mathbf{E}_{\lambda_j}[\delta_i]\operatorname{Cov}_{\lambda_j\delta_j}(\sigma_i, \rho_i)}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]}\operatorname{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_i}[\sigma_i]}, \delta_i\right) \\ &= \underbrace{\frac{\mathbf{E}_{\lambda_j}[\delta_i]\mathbf{E}_{\lambda_j}[\sigma_i]\mathbf{E}_{\lambda_j}[1-\delta_i]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]}}_{\theta_\rho/\overline{\mu}_j}\operatorname{Cov}_{\lambda_j\delta_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}, \rho_i\right) + \underbrace{\frac{\mathbf{E}_{\lambda_j\delta_j}[\rho_i]\mathbf{E}_{\lambda_j}[\sigma_i]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i + \sigma_i(1-\delta_i)]}}_{\theta_\delta/\overline{\mu}_j}\operatorname{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_i}[\sigma_i]}, \delta_i\right).\end{aligned}$$

We now turn to the exchange rate component. Proceeding similarly

$$\operatorname{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}, (1-\delta_i)\iota_i\right) = \mathbf{E}_{\lambda_j}[1-\delta_i]\operatorname{Cov}_{\lambda_j(1-\delta_j)}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}, \iota_i\right) - \operatorname{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]}, \delta_i\right)\mathbf{E}_{\lambda_j(1-\delta_j)}[\iota_i].$$

Therefore, the second brackets write

$$
\begin{aligned}
&\left(\frac{\mathbf{E}_{\lambda_j}[\sigma_i(1-\delta_i)\iota_i]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i+\sigma_i(1-\delta_i)]}\left(1-\mathbf{E}_{\lambda_j\delta_j}[\rho_i]\right)-\mathbf{E}_{\lambda_j(1-\delta_j)}[\iota_i]\right)\mathrm{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\delta_i\right)\\
&-\frac{\mathbf{E}_{\lambda_j}[\sigma_i(1-\delta_i)\iota_i]\mathbf{E}_{\lambda_j}[\delta_i]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i+\sigma_i(1-\delta_i)]}\mathrm{Cov}_{\lambda_j\delta_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\rho_i\right)+\mathbf{E}_{\lambda_j}[1-\delta_i]\,\mathrm{Cov}_{\lambda_j(1-\delta_j)}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\iota_i\right)\\
&=\left(\frac{\mathbf{E}_{\lambda_j}[\sigma_i(1-\delta_i)]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i+\sigma_i(1-\delta_i)]}\mathbf{E}_{\lambda_j(1-\delta_j)}[\iota_i]\mathbf{E}_{\lambda_j\delta_j}[1-\rho_i]-\mathbf{E}_{\lambda_j(1-\delta_j)}[\iota_i]\right)\mathrm{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\delta_i\right)\\
&-\frac{\mathbf{E}_{\lambda_j}[\sigma_i(1-\delta_i)]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i+\sigma_i(1-\delta_i)]}\mathbf{E}_{\lambda_j(1-\delta_j)}[\iota_i]\mathbf{E}_{\lambda_j}[\delta_i]\,\mathrm{Cov}_{\lambda_j\delta_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\rho_i\right)\\
&+\left(\frac{\mathbf{E}_{\lambda_j}[\sigma_i]\mathbf{E}_{\lambda_j}[(1-\delta_i)]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i+\sigma_i(1-\delta_i)]}\left[-\mathbf{E}_{\lambda_j}[\delta_i]\,\mathrm{Cov}_{\lambda_j\delta_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\rho_i\right)+\mathbf{E}_{\lambda_j\delta_j}[1-\rho_i]\,\mathrm{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\delta_i\right)\right]+\mathbf{E}_{\lambda_j}[1-\delta_i]\right)\\
&\times\mathrm{Cov}_{\lambda_j(1-\delta_j)}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\iota_i\right)\\
&=\mathbf{E}_{\lambda_j(1-\delta_j)}[\iota_i]\left(\frac{\mathbf{E}_{\lambda_j}[\sigma_i(1-\delta_i)]\mathbf{E}_{\lambda_j\delta_j}[1-\rho_i]+\mathbf{E}_{\lambda_j}[\delta_i]\,\mathrm{Cov}_{\lambda_j\delta_j}(\sigma_i,\rho_i)}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i+\sigma_i(1-\delta_i)]}-1\right)\mathrm{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\delta_i\right)\\
&-\frac{\mathbf{E}_{\lambda_j}[\sigma_i]\mathbf{E}_{\lambda_j}[(1-\delta_i)\iota_i]\mathbf{E}_{\lambda_j}[\delta_i]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i+\sigma_i(1-\delta_i)]}\mathrm{Cov}_{\lambda_j\delta_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\rho_i\right)\\
&+\left(\frac{\mathbf{E}_{\lambda_j}[\sigma_i]\mathbf{E}_{\lambda_j}[(1-\delta_i)]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i+\sigma_i(1-\delta_i)]}\left[-\mathbf{E}_{\lambda_j}[\delta_i]\,\mathrm{Cov}_{\lambda_j\delta_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\rho_i\right)+\mathbf{E}_{\lambda_j\delta_j}[1-\rho_i]\,\mathrm{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\delta_i\right)\right]+\mathbf{E}_{\lambda_j}[1-\delta_i]\right)\\
&\times\mathrm{Cov}_{\lambda_j(1-\delta_j)}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\iota_i\right)\\
&=-\mathbf{E}_{\lambda_j(1-\delta_j)}[\iota_i]\overline{\mu}_j^{-1}\theta_\delta\,\mathrm{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\delta_i\right)-\mathbf{E}_{\lambda_j(1-\delta_j)}[\iota_i]\overline{\mu}_j^{-1}\theta_\rho\,\mathrm{Cov}_{\lambda_j\delta_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\rho_i\right)\\
&+\frac{\mathbf{E}_{\lambda_j}[(1-\delta_i)]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i+\sigma_i(1-\delta_i)]}\left[-\mathbf{E}_{\lambda_j}[\delta_i]\,\mathrm{Cov}_{\lambda_j\delta_j}(\sigma_i,\rho_i)+\mathbf{E}_{\lambda_j\delta_j}[1-\rho_i]\,\mathrm{Cov}_{\lambda_j}(\sigma_i,\delta_i)+\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i]+\mathbf{E}_{\lambda_j}[\sigma_i(1-\delta_i)]\right]\\
&\times\mathrm{Cov}_{\lambda_j(1-\delta_j)}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\iota_i\right)\\
&=-\frac{\mathbf{E}_{\lambda_j(1-\delta_j)}[\iota_i]}{\overline{\mu}_j}\left[\theta_\delta\,\mathrm{Cov}_{\lambda_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\delta_i\right)+\theta_\rho\,\mathrm{Cov}_{\lambda_j\delta_j}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\rho_i\right)\right]+\frac{\theta_\iota}{\overline{\mu}_j}\mathrm{Cov}_{\lambda_j(1-\delta_j)}\left(\frac{\sigma_i}{\mathbf{E}_{\lambda_j}[\sigma_i]},\iota_i\right).
\end{aligned}
$$

Here, we define

$$
\begin{aligned}
\theta_\iota&=\overline{\mu}_j\times\frac{\mathbf{E}_{\lambda_j}[\sigma_i]\mathbf{E}_{\lambda_j}[\delta_i\rho_i+(1-\delta_i)]}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i+\sigma_i(1-\delta_i)]}\times\mathbf{E}_{\lambda_j}[1-\delta_i]\\
&=\overline{\mu}_j\times\frac{\mathbf{E}_{\lambda_j}[\sigma_i]\left(1-\mathbf{E}_{\delta_j\lambda_j}[1-\rho_i]\mathbf{E}_{\lambda_j}[\delta_i]\right)}{\mathbf{E}_{\lambda_j}[\sigma_i\delta_i\rho_i+\sigma_i(1-\delta_i)]}\times\mathbf{E}_{\lambda_j}[1-\delta_i]
\end{aligned}
$$

Sufficient statistics. We now show that all of the parameters defined above can be written in terms of the following moments: $\overline{\mu}$, $\mathbf{E}_\lambda[\delta_i]$, $\mathbf{E}_{\delta\lambda}[\rho_i]$, $\mathbf{E}_\lambda[\iota_i]$ $\mathrm{Cov}_\lambda(\varsigma_i,\delta_i)$, $\mathrm{Cov}_{\lambda\delta}(\varsigma_i,\rho_i)$, and $\mathrm{Cov}_{\lambda(1-\delta)}(\varsigma_i,\iota_i)$. We drop the j indices for simplicity. Note the following facts:

$$
\begin{aligned}
\mathbf{E}_\lambda[\delta_i\rho_i] &= \mathbf{E}_{\delta\lambda}[\rho_i]\mathbf{E}_\lambda[\delta_i]\\
\mathbf{E}_\lambda[\sigma_i\delta_i\rho_i] &= \mathbf{E}_{\lambda\delta}[\sigma_i\rho_i]\mathbf{E}_\lambda[\delta_i] = \mathrm{Cov}_{\lambda\delta}(\sigma_i,\rho_i)\mathbf{E}_\lambda[\delta_i] + \mathbf{E}_{\lambda\delta}[\rho_i]\mathbf{E}_\lambda[\sigma_i]\mathbf{E}_\lambda[\delta_i] + \mathrm{Cov}_\lambda(\sigma_i,\delta_i)\mathbf{E}_{\lambda\delta}[\rho_i],\\
\mathbf{E}_\lambda[\delta_i(1-\rho_i)] &= \mathbf{E}_{\delta\lambda}[1-\rho_i]\mathbf{E}_\lambda[\delta_i],\\
\mathbf{E}_\lambda[\sigma_i(1-\delta_i)] &= \mathrm{Cov}_\lambda(\sigma_i,1-\delta_i) + \mathbf{E}_\lambda[\sigma_i]\mathbf{E}_\lambda[\delta_i].
\end{aligned}
$$

It follows immediately that

$$
\begin{aligned}
\kappa_M &= \mathbf{E}_{\delta\lambda}[\rho_i]\mathbf{E}_\lambda[\delta_i] + \frac{\left(\mathrm{Cov}_{\lambda\delta}(\varsigma_i,\rho_i)\mathbf{E}_\lambda[\delta_i] + \mathbf{E}_{\lambda\delta}[\rho_i]\mathbf{E}_\lambda[\delta_i] + \mathrm{Cov}_\lambda(\varsigma_i,\delta_i)\mathbf{E}_{\lambda\delta}[\rho_i]\right)\mathbf{E}_{\delta\lambda}[1-\rho_i]\mathbf{E}_\lambda[\delta_i]}{1-\mathrm{Cov}_\lambda(\varsigma_i,\delta_i)(1-\mathbf{E}_{\lambda\delta}[\rho_i]) - \mathbf{E}_\lambda[\delta_i]\left(1-\mathbf{E}_{\lambda\delta}[\rho_i] - \mathrm{Cov}_{\lambda\delta}(\varsigma_i,\rho_i)\right)},\\
\kappa_\iota &= 1 + \frac{\mathbf{E}_{\delta\lambda}[1-\rho_i]\mathbf{E}_\lambda[\delta_i]\times\left(1-\frac{\mathrm{Cov}_\lambda(\varsigma_i,\delta_i)}{\mathbf{E}_\lambda[1-\delta_i]}\right)}{1-\mathrm{Cov}_\lambda(\varsigma_i,\delta_i)(1-\mathbf{E}_{\lambda\delta}[\rho_i]) - \mathbf{E}_\lambda[\delta_i]\left(1-\mathbf{E}_{\lambda\delta}[\rho_i] - \mathrm{Cov}_{\lambda\delta}(\varsigma_i,\rho_i)\right)}\\
\kappa_\sigma &= \frac{\mathbf{E}_{\delta\lambda}[1-\rho_i]\mathbf{E}_\lambda[\delta_i]\mathbf{E}_\lambda[1-\delta_i]}{1-\mathrm{Cov}_\lambda(\varsigma_i,\delta_i)(1-\mathbf{E}_{\lambda\delta}[\rho_i]) - \mathbf{E}_\lambda[\delta_i]\left(1-\mathbf{E}_{\lambda\delta}[\rho_i] - \mathrm{Cov}_{\lambda\delta}(\varsigma_i,\rho_i)\right)}.
\end{aligned}
$$

Given our above result, it is obvious that θ_ρ, θ_δ, and θ_ι can be written as a function of our sufficient statistics. The only required change is in the denominator, which is the same as for the κ coefficients.

A.2 Dynamic model

Proposition 3. *The production block describes the evolution of allocative efficiency and prices for each sector.*

$$
\begin{aligned}
\mathrm{d}\log A_{jt} &= \frac{1}{\kappa_A}\mathrm{d}\log\mathrm{d}\log A_{jt-1} + \frac{\beta}{\kappa_A}\mathbf{E}_t[\mathrm{d}\log A_{jt+1}] + \theta_p\,\mathrm{Cov}_{\lambda_j}(\varsigma_i,\rho_i)\,\mathrm{d}\log\frac{M_{jt}}{P_{jt}}\\
&-\overline{\mu}_j\sum_\ell\left[\mathrm{Cov}_{\lambda_i}\left(\varsigma_i,\iota_i^\ell\right)\left(\beta\mathbf{E}_t[-\mathrm{d}\log E^d_{\ell t+1} + \mathrm{d}\log E^\ell_{jt}] - \mathrm{d}\log E^d_{\ell t-1} + \mathrm{d}\log E^d_{\ell t}\right)\right] \qquad \text{(TFP)}\\
\mathrm{d}\log\pi_{jt} &= \beta\mathbf{E}_t[\mathrm{d}\log\pi_{jt+1}] + \frac{\theta_p}{\overline{\mu}_j}\mathbf{E}_{\lambda_j}[1-\rho_i]\mathrm{d}\log A_{jt} + \theta_p\mathbf{E}_{\lambda_j}[\rho_i]\mathrm{d}\log\frac{M_{jt}}{P_{jt}}\\
&+\beta\sum_\ell\left[\mathbf{E}_{\lambda_j}[\iota_i^\ell]\left(\mathbf{E}_t[-\mathrm{d}\log E^d_{\ell t+1} + \mathrm{d}\log E^d_{\ell t}]\right) + \mathbf{E}_{\lambda_j}[\iota_i^\ell]\left(\mathrm{d}\log E^d_{\ell t} - \mathrm{d}\log E^d_{\ell t-1}\right)\right], \qquad \text{(PC)}
\end{aligned}
$$

where $\theta_p = \frac{\delta_p}{1-\delta_p}(1-\beta(1-\delta_p))$ *and* $\kappa_A = 1+\beta+\theta_p\left(1+\mathrm{Cov}_{\lambda_j}(\varsigma_i,\rho_i)\right)$. *The household block describes the evolution of wages and consumption.*

$$
\begin{aligned}
\mathrm{d}\log\pi_{wt} &= \beta\mathbf{E}_t\left[\mathrm{d}\log\pi_{wt+1}\right] - \theta_w\left(\mathrm{d}\log\frac{W_t}{P_t} - \gamma\mathrm{d}\log C_t - \frac{1}{\zeta}\mathrm{d}\log L_t\right) \qquad \text{(Wage PC)}\\
\mathrm{d}\log C_t &= \mathbf{E}_t\left[\mathrm{d}\log C_{t+1}\right] - \frac{1}{\gamma}\left(\mathrm{d}\log i_t + \mathbf{E}_t[\mathrm{d}\log\pi_{t+1}]\right) \qquad \text{(EE)}\\
& \qquad \text{(Risk-sharing)}
\end{aligned}
$$

The final block contains the monetary policy rule and market clearing.

$$\mathrm{d}\log i_t = \phi_\pi \mathbf{E}_t[\pi_{t+1}] + \phi_y \mathrm{d}\log Y_t + \mathrm{d}\log V_t \tag{TR}$$

$$\mathrm{d}\log \pi_t = \sum_j \lambda_j \mathrm{d}\log \pi_{jt} \tag{Aggregate PC}$$

$$\mathrm{d}\log C_t = \sum_j \lambda_j \mathrm{d}\log C_{jt} \tag{Aggregate consumption}$$

$$\mathrm{d}\log L_t = \sum_j \alpha_j \mathrm{d}\log L_{jt} \tag{Labor market clearing}$$

where the λ_j are the equilibrium expenditure shares of each country on sector j, and the α_j are the equilibrium labor shares of each country on sector j.

Proof of Proposition 3. We only give details for the sectoral Phillips curve and the endogenous TFP dynamics, as the other equations are standard.

Sectoral Phillips curve. The firm's first order condition for a firm that sets its price in currency ℓ is

$$\mathbf{E}_t\left[\sum_{k\geq 0}(1-\delta_i)^k E^c_{dt+k}\mathcal{M}_{jt}y_i\left(\frac{\widetilde{p}^\ell_{it}E^d_{\ell t}}{\mathcal{P}_{jt+k}}\right)\left(\widetilde{p}^\ell_{jt}E^d_{\ell t+k}-\mu_i\left(\frac{\widetilde{p}^\ell_{it}E^d_{\ell t+k}}{\mathcal{P}_{jt+k}}\right)M_{it+k}\right)\right]=0.$$

Here, $E^d_{\ell t}$ is the exchange rate from currency ℓ to the destination currency and E^c_{dt} is the exchange rate from the destination currency to the home currency. Log-linearizing this condition around the zero-inflation deterministic steady state, where exchange rates are normalized to one, we get

$$\mathbf{E}_t\left[\sum_{k\geq 0}(1-\delta_i)^k\beta^k\left(\mathrm{d}\log\widetilde{p}^\ell_{it}+\mathrm{d}\log E^d_{\ell t+k}-\mathrm{d}\log M_{jt+k}+\frac{1-\rho_i}{\rho_i}\left(\mathrm{d}\log\widetilde{p}^\ell_{it}+\mathrm{d}\log E^d_{\ell t+k}-\mathrm{d}\log\mathcal{P}_{jt+k}\right)\right)\right]=0.$$

Rearranging

$$\begin{aligned}\mathrm{d}\log\widetilde{p}^\ell_{it}&=(1-(1-\delta_i)\beta)\mathbf{E}_t\left[\sum_{k\geq 0}(1-\delta_i)^k\beta^k\left(\rho_i\mathrm{d}\log M_{jt+k}+(1-\rho_i)\mathrm{d}\log\mathcal{P}_{jt+k}-\mathrm{d}\log E^d_{\ell t+k}\right)\right]\\&=(1-(1-\delta_i)\beta)\left(\rho_i\mathrm{d}\log M_{jt}+(1-\rho_i)\mathrm{d}\log\mathcal{P}_{jt}-\mathrm{d}\log E^d_{\ell t}\right)+(1-\delta_i)\beta\mathbf{E}_t\mathrm{d}\log\widetilde{p}^\ell_{it+1}.\end{aligned}$$

Now, if the firm is hit by a Calvo shock it targets its new ideal price which is converted to the destination currency. If it is not, its price stays at its old level in its invoicing currency and is converted to the destination currency at the new exchange rate. Therefore,

$$\begin{aligned}\mathrm{d}\log p_{it+1}&=\delta_i\left(\mathrm{d}\log\widetilde{p}^\ell_{it+1}+\iota^\ell_i\mathrm{d}\log E^d_{\ell t+1}\right)+(1-\delta_i)\left(\mathrm{d}\log p_{it}+\iota^\ell_i\mathrm{d}\log E^d_{\ell t+1}-\iota^\ell_i\mathrm{d}\log E^d_{\ell t}\right)\\&=\delta_i\mathrm{d}\log\widetilde{p}^\ell_{it+1}+(1-\delta_i)\mathrm{d}\log p_{it}+\iota^\ell_i\mathrm{d}\log E^d_{\ell t+1}-(1-\delta_i)\iota^\ell_i\mathrm{d}\log E^d_{\ell t},\end{aligned}$$

where ι^ℓ_i is an indicator for whether the firm invoices in currency ℓ, and $E^d_{\ell t+1}$ is the exchange rate from currency ℓ to the destination currency. Note that the price p_{it+1} is expressed in the destination currency. Rearranging

$$\mathrm{d}\log \widetilde{p}^{\ell}_{it+1} = \frac{1-\delta_i}{\delta_i}\mathrm{d}\log \pi_{it+1} + \mathrm{d}\log p_{it+1} - \frac{1}{\delta_i}\iota^{\ell}_i \mathrm{d}\log E^d_{\ell t+1} + \frac{1-\delta_i}{\delta_i}\iota^{\ell}_i \mathrm{d}\log E^d_{\ell t}.$$

where $\mathrm{d}\log \pi_{it+1} = \mathrm{d}\log p_{it+1} - \mathrm{d}\log p_{it}$. This implies

$$\begin{aligned}
&\mathrm{d}\log \widetilde{p}^{\ell}_{it} - (1-\delta_i)\beta \mathbf{E}_t\left[\mathrm{d}\log \widetilde{p}^{\ell}_{it+1}\right] + (1-(1-\delta_i)\beta)\mathrm{d}\log E^d_{\ell t} \\
&= \frac{1-\delta_i}{\delta_i}\pi_{it} - \frac{1}{\delta_i}\iota_i \mathrm{d}\log E^d_{\ell t} + \mathrm{d}\log p_t + \frac{1-\delta_i}{\delta_i}\iota^{\ell}_i E^d_{\ell t-1} + (1-(1-\delta_i)\beta)\iota^{\ell}_i \mathrm{d}\log E^d_{\ell t} \\
&- (1-\delta_i)\beta \mathbf{E}_t\left[\mathrm{d}\log \pi_{it+1}\right] + \beta(1-\delta_i)\frac{1}{\delta_i}\iota^{\ell}_i \mathbf{E}_t\left[\mathrm{d}\log E^d_{\ell t+1}\right] - \beta(1-\delta_i)\mathbf{E}_t\left[p_{it+1}\right] - \beta(1-\delta_i)\frac{1-\delta_i}{\delta_i}\iota^{\ell}_i \mathrm{d}\log E^d_{\ell t} \\
&= \frac{1-\delta_i}{\delta_i}\left[\pi_{it} - \beta \mathbf{E}_t\left[\mathrm{d}\log \pi_{it+1}\right] + \theta_p \mathrm{d}\log p_t + \iota^{\ell}_i \mathrm{d}\log E_{jt-1} - (1+\beta)\iota^{\ell}_i \mathrm{d}\log E^d_{\ell t} + \iota^{\ell}_i \mathbf{E}_t\left[\mathrm{d}\log E^d_{\ell t+1}\right]\right]
\end{aligned}$$

where $\theta_p = \delta_i(1-(1-\delta_i)\beta)/(1-\delta_i)$. We can plug this into the recursive formula above, yielding

$$\begin{aligned}
\mathrm{d}\log \pi_{it} - \beta \mathrm{d}\log \pi_{it+1} = \theta_p &\left[\rho_i(\mathrm{d}\log M_{jt} - \mathrm{d}\log P_{jt}) + (1-\rho_i)(\mathrm{d}\log \mathcal{P}_{jt} - \mathrm{d}\log P_{jt})\right] \\
&+ \beta\iota^{\ell}_i\left(\mathbf{E}_t[-\mathrm{d}\log E^d_{\ell t+1} + \mathrm{d}\log E^d_{\ell t}]\right) + \iota^{\ell}_i\left(\mathrm{d}\log E^d_{\ell t} - \mathrm{d}\log E^d_{\ell t-1}\right).
\end{aligned}$$

From now on, we assume that $\delta_i = \delta$ is the same for all firms within a sector. Integrating over firms and summing over invoicing choices, and using the fact that

$$\mathrm{d}\log A_{jt} = \overline{\mu}_j \times \left(\mathrm{d}\log \mathcal{P}_{jt} - \mathrm{d}\log P_{jt}\right),$$

we get

$$\begin{aligned}
\mathrm{d}\log \pi_{jt} = \beta \mathbf{E}_t[\mathrm{d}\log \pi_{jt+1}] &+ \frac{\theta_p}{\overline{\mu}_j}\mathrm{d}\log A_{jt} + \theta_p(\mathbf{E}_{\lambda_j}[\rho_i]\mathrm{d}\log M_{jt} + \mathbf{E}_{\lambda_j}[1-\rho_i]\mathcal{P}_{jt}) \\
&+ \sum_{\ell}\left[\beta \mathbf{E}_{\lambda_j}[\iota^{\ell}_i]\left(-\mathrm{d}\log E^d_{\ell t+1} + \mathrm{d}\log E^d_{\ell t}\right) + \mathbf{E}_{\lambda_j}[\iota^{\ell}_i]\left(\mathrm{d}\log E^d_{\ell t} - \mathrm{d}\log E^d_{\ell t-1}\right)\right].
\end{aligned}$$

Productivity. Recall that $\mathrm{d}\log \mathcal{P}_{jt+1} - \mathrm{d}\log \mathcal{P}_{jt} = \mathbf{E}_{\lambda_j}\left[\varsigma_i \mathrm{d}\log \pi_{it}\right]$. Hence

$$\begin{aligned}
&\mathrm{d}\log \mathcal{P}_{jt} - \mathrm{d}\log \mathcal{P}_{jt-1} - \beta \mathbf{E}_t\left[\mathrm{d}\log \mathcal{P}_{jt+1} - \mathrm{d}\log \mathcal{P}_{jt}\right] \\
&= \theta_p\left(\mathbf{E}_{\lambda_j}[\varsigma_i\rho_i](\mathrm{d}\log M_{jt} - \mathrm{d}\log \mathcal{P}_{jt})\right) \\
&+ \beta \mathbf{E}_{\lambda_j}[\varsigma_i\iota^{\ell}_i]\left(\mathbf{E}_t[-\mathrm{d}\log E^d_{\ell t+1} + \mathrm{d}\log E^d_{\ell t}]\right) + \mathbf{E}_{\lambda_j}[\varsigma_i\iota^{\ell}_i]\left(\mathrm{d}\log E^d_{\ell t} - \mathrm{d}\log E^d_{\ell t-1}\right)
\end{aligned}$$

Now,

$$\begin{aligned}
\mathrm{d}\log \mathcal{P}_{jt} - \mathrm{d}\log \mathcal{P}_{jt-1} - \beta \mathbf{E}_t\left[\mathrm{d}\log \mathcal{P}_{jt+1} - \mathrm{d}\log \mathcal{P}_{jt}\right] = &\frac{1}{\overline{\mu}_j}\left[(1+\beta)\mathrm{d}\log A_{jt} - \mathrm{d}\log A_{jt-1} - \beta \mathbf{E}_t[\mathrm{d}\log A_{jt+1}]\right] \\
&+ \mathrm{d}\log \pi_{jt} - \beta \mathbf{E}_t[\mathrm{d}\log \pi_{jt+1}].
\end{aligned}$$

Combining the two and the New Keynesian Phillips' curve, we find

$$\frac{1}{\overline{\mu}_j}\left[(1+\beta)\mathrm{d}\log A_{jt} - \mathrm{d}\log A_{jt-1} - \beta\mathbf{E}_t[\mathrm{d}\log A_{jt+1}]\right]$$

$$= -\frac{\theta_p\left(1+\left(\mathbf{E}_{\lambda_j}[\varsigma_i\rho_i] - \mathbf{E}_{\lambda_j}[\rho_i]\right)\right)}{\overline{\mu}_j}\mathrm{d}\log A_{jt} + \theta_p\left(\mathbf{E}_{\lambda_j}[\varsigma_i\rho_i] - \mathbf{E}_{\lambda_j}[\rho_i]\right)(\mathrm{d}\log M_{jt} - \mathrm{d}\log P_{jt})$$

$$+\left(\mathbf{E}_{\lambda_j}[\varsigma_i\iota_i^{\ell}] - \mathbf{E}_{\lambda_j}[\iota_i^{\ell}]\right)\left(\beta\left(\mathbf{E}_t[-\mathrm{d}\log E^d_{\ell t+1} + \mathrm{d}\log E^d_{\ell t}]\right) + \left(\mathrm{d}\log E^d_{\ell t} - \mathrm{d}\log E^d_{\ell t-1}\right)\right).$$

Solving for $\mathrm{d}\log A_{jt}$ gives the desired result.

B Data appendix

B.1 Dataset construction

We combine two data sources:

1. detailed transaction-level data provided by the customs administration (DGDDI);
2. balance sheet data provided by the fiscal administration (DGFIP) and processed by the statistics office (INSEE) and provided to researchers in a form called FARE. For each year t, there are two available versions of FARE, t and $t+1$. We use the former.

Procedure description. French customs measure trade at the transaction level for each legal units, identified with a unique administrative number called "SIREN." Some legal unit that are not registered in France but in neighboring countries such as Switzerland or Belgium also enter these data. The customs data are nearly but not entirely exhaustive, as discussed in Bergounhon et al. (2018). FARE is exhaustive and contains balance sheet data for all French firms. For most firms, balance sheet information is reported at the legal unit level. For some very large firms, it is reported at a more aggregated level called "contour" which captures the economic perimeter of a holding company and its subsidiaries. The INSEE provides researcher with contour files that allows researchers to recover the list of legal entities included in a contour and merge FARE with the customs data.

Cleaning procedure before merging. We do not apply any cleaning step to the FARE dataset before merging. We will clean some of the variables post-merging, but the dataset is pre-processed by the INSEE. For customs data, we follow the cleaning procedures documented on Isabelle Méjean's webiste.[2]

Merging procedure. We use the following merging procedure. Before merging, we compute the wage bill and input costs in the FARE data.

1. Customs data are provided in four different files by the administration, separated by sign (import or export) and destination type (eurozone or not). For each year,

[2] See http://www.isabellemejean.com/FrenchCustomsData.html. Specifically, the code files `BasicCleaning.do`, `ExpDEBCleaning.do`, and `ImpDEBCleaning.do`.

we combine these files to build a cross-section of trade transactions. We then merge this cross-section with FARE using the SIREN identifier.

2. We collect SIREN from the trade cross-section that do not have a match in the FARE data. We then check whether these SIREN belong to a contour profile. When that is the case, we aggregate all monthly transactions (values and quantities) at the contour level, keeping heterogeneity in month, year, destination, product, unit of measurement, and currency level.[3]

3. Our final dataset consists of firms that were matched in step 1 or 2 in a dataset, with the understanding that a firm is either a SIREN or a contour.

Because FARE is exhaustive, observations that are not matched in step 1 or step 2 should correspond to firms located outside of France. We manually checked that this was the case for the largest missing observations for a few years in our panel. An informal computation for one specific year suggests that, after manually removing large offenders, our final dataset contains over 99 percent of French trade.

B.2 Variables definition

Unit values. We define unit values as trade volume in euros divided by quantity. Trade volume is systematically reported in our dataset. For quantities, we preferably use the number of units reported by the customs administration; in other cases, we use the weight in kilograms. More specifically

1. When the number of units variable `units` is strictly positive, and the measurement unit `typeunit` is nonmissing, we define unit values as

$$\texttt{unitval} = \frac{\texttt{valstat}}{\texttt{units}}.$$

2. In other cases, if the weight variable `kgs` is strictly positive, and the flag `indicmasse` is equal to one, we define unit values as

$$\texttt{unitval} = \frac{\texttt{valstat}}{\texttt{kgs}}.$$

Our unit value measure is defined at the level of a firm, 8-digit product category, destination, currency of invoicing, and month. When measuring prices at the quarterly level, we sum `valstat` and `units/kgs` separately and use the results to compute unit values.[4] We check that the measurement unit `typeunit` is unique for a given product and year. It is important to keep track of the unit in which prices are measured for comparisons across firms or time periods. In practice, we find that units values are almost always measured in the same unit across firms within an 8-digit product category and a destination, with little to no variation in units across time.

Proxying for prices with unit value is standard in the literature on exchange rate pass-through but it has drawbacks. First, even at the disaggregated 8-digit level, one

[3] In the data, these variables are respectively called `mois`, `an`, `pyod`, `nc8`, `typeunit`, and `devfac`. We also keep heterogeneity in the `indicmasse` dimension.

[4] We found that using other methods of aggregation, such as an average of monthly unit values weighted by `valstat`, lead to almost identical results.

observation may aggregate several transactions of different products. Second, it does not account for possible variations in product quality across transactions and time.

Price changes and price indices. As explained above, we proxy for prices using unit values which are export value divided by quantity at the firm, eight-digit product, export destination, and quarter level. Next, we calculate quarterly log differences in unit values and aggregate them using trade volume weights at the sector (two-digit) and destination level. Specifically, the change $\Delta \log P_{sdt}$ in the price index for sector s and destination d in quarter t is

$$\Delta \log P_{sdt} = \sum_{ik \in N_{sdt-1}} \lambda_{ikt-1} \Delta \log p_{ikt},$$

where $\Delta \log p_{ikt}$ is the change in unit values for firm i and product-destination k, λ_{ikt-1} is the share of firm i in the total exports of sector s to destination d, and N_{sdt-1} is the set of firm and product pairs in sector s that export to destination d in quarter $t-1$.

We apply several filters in constructing sectoral price indices. We drop changes in log unit values that are above 3 or below -3, which roughly corresponds to prices multiplied or divided by more than twenty over a quarter. We also impose that there be at least 5 distinct firms exporting to a given destination in a given sector in order to compute the price index.

Balance sheet variables. The main variables we construct from the balance sheet data are as follows:

- Wage bill is the sum of wages (`redi_r216`) and social security payments (`redi_r217`).
- Material costs is purchases of goods (`redi_r210`) minus changes in inventories of goods (`redi_r211`) plus purchases of commodities (`redi_r212`) minus changes in inventories of commodities (`redi_r213`). Therefore

$$\begin{aligned}\text{Input costs} = {} & \text{Purchases of goods} - \Delta\text{Inventories of goods} \\ & + \text{Purchases of commodities} - \Delta\text{Inventories of commodities.}\end{aligned}$$

- Other costs (which includes services, some R&D spending, etc.) is outside purchases and external costs (`redi_214`).
- Total variable costs is the sum of the wage bill, material costs, and other costs.
- Sales is gross turnover (`redi_r310`).
- Industry is the four-digit code in the APE nomenclature (`ape_diff`).
- Sector is the two-digit code in the APE nomenclature.

C Production function estimation

Estimation procedure. As explained in the main text, markups are given by

$$\mu_{it} = \frac{\theta^V_{it}}{\Omega_{it}},$$

where Ω_{it} is the cost-to-sales ratio, which is observed, and θ_{it}^V is the output elasticity with respect to materials, which is not.

We estimate this output elasticity following the procedure outlined in De Ridder et al. (2022). We estimate a translog production function at the two-digit sector level. Their procedure has two steps, to which we add a third step that tries to deal with measurement error in production function coefficients. The three steps are

1. purge revenues from measurement error,
2. estimate productivity autocorrelation and production function parameters jointly using a method of moments estimator,
3. shrink estimates to limit the influence of outliers using empirical Bayesian methods.

Purging step. We have an unbalanced panel where each observation is a firm i and a year t. All firms belong to the same sector j. For simplicity, we omit sector subscripts unless necessary. All variables are in logs. Let V_{it} be material input, K_{it} capital input, L_{it} labor input, ω_{it} firm productivity, and Y_{it} output. We estimate the production function

$$Y_{it} = \omega_{it} + F(K_{it}, L_{it}, V_{it}).$$

We assume that material inputs can be flexibly adjusted, and that the demand for material can be written as $V_{it} = v(\omega_{it}, \Xi_{it})$, where Ξ_{it} is a vector of variables other than productivity that determine input choices. Under a monotonicity assumption, productivity can be inferred from input choices. Specifically, assuming that for any Ξ, the function $\omega \mapsto v(\omega, \Xi)$ is increasing, we have $\omega_{it} = v^{-1}(V_{it}, \Xi_{it})$. Therefore

$$\begin{aligned} Y_{it} &= v^{-1}(V_{it}, \Xi_{it}) + F(K_{it}, L_{it}, V_{it}) \\ &= \Phi(K_{it}, L_{it}, V_{it}, \Xi_{it}), \end{aligned}$$

where $\Phi(K, L, V, \Xi) = v^{-1}(V, \Xi) + F(K, L, V)$. We estimate this relationship by regressing output on a time fixed effect and a third degree polynomial in capital, materials, labor, as well as sales shares computed at the 4-digit level. We then collect the fitted values from this procedure, which we denote $\widehat{\Phi}_{it}$.

Production function estimation. We assume that productivity follows an AR(1) process

$$\omega_{it} = \rho \times \omega_{it-1} + \xi_{it}.$$

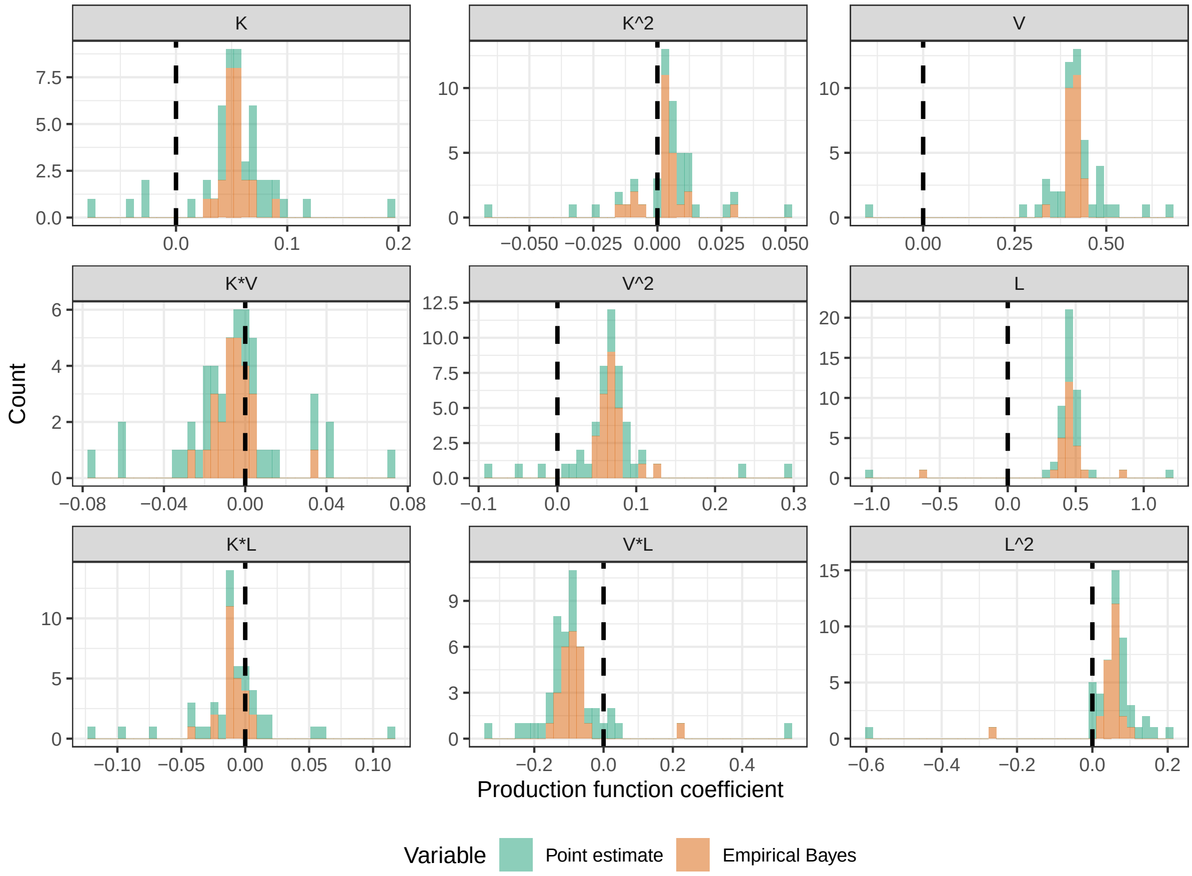
Given production function parameters and autocorrelation estimates, we construct productivity as

$$\widehat{\omega}_{it} = \widehat{\Phi}_{it} - \widehat{F}(K_{it}, L_{it}, V_{it}),$$

and the productivity innovation as

$$\widehat{\xi}_{it} = \widehat{\omega}_{it} - \widehat{\rho} \times \widehat{\omega}_{it-1}.$$

Figure A.1: Empirical Bayes shrinkage of production function coefficients



Note. This figure shows the estimated production function coefficients. In green, the point estimates before shrinkage. In orange, the empirical Bayes estimates.

Then, given an information set $\mathcal{I}_{it}$, we have

$$\mathbf{E}\left[\xi_{it} \mid \mathcal{I}_{it-1}\right] = 0.$$

We then use the moments $\mathbf{E}[\widehat{\omega}_{ijt-1}\widehat{\xi}_{ijt}] = 0$, along with moments for each type of input depending on timing as the set of identifying moments for GMM estimation of the production function parameters.

Empirical Bayes. At the end of the two steps described above, we have vector of production function coefficients for each sector. We estimate standard errors for each parameter using a block bootstrap at the firm level. Let $\widehat{\theta}_j = (\theta_{j1}, \ldots, \theta_{jk})$ be the production function coefficient estimates for sector j and let $\widehat{s}_j = \left(\widehat{s}_{j1}, \ldots, \widehat{s}_{jk}\right)$ be the associated standard errors estimates. Given the relatively large standard errors (Figure A.1), we are concerned with the possibility that measurement error will contaminate our markups estimates. To adress this issue, we use a simple shrinkage estimator. The main intuition behind our estimator is that it shrinks noisy estimates toward the cross-sectional mean. Such methods are well established in statistics and economics, and yield substantial improvements in a wide variety of contexts.

Formally, the setup is as follows. We first assume that for any sector j and parameter k

$$\widehat{\theta}_{jk} \mid \theta_{jk}, s_{jk} \sim \mathcal{N}(\theta_{jk}, s_{jk}^2).$$

This assumption is based on the central limit theorem. In principle, it holds at least asymptotically. We further assume that the production function coefficient θ_{jk} in a given sector j is drawn according to a normal distribution

$$\theta_{jk} \mid s_{jk} \sim \mathcal{N}(\mu_k, \sigma_k^2).$$

Here, the grand mean μ_k and the variance σ_k^2 are hyperparameters. While this type of hierarchical model is commong in the literature, it relies on the strong assumption that variation in production technologies across sectors is normally distributed. Furthermore, it does not capture the fact that output elasticities may be correlated across different input types. However, a theoretical analysis of production technologies across sectors is beyond the scope of this paper. Under our assumptions, the optimal estimator is

$$\theta_{jk} = \mathbf{E}\left[\theta_{jk} \mid \widehat{\theta}_{jk}, s_{jk}\right] = \frac{s_{jk}^2}{s_{jk}^2 + \sigma_k^2}\mu_k + \frac{\sigma_k^2}{s_{jk}^2 + \sigma_k^2}\theta_{jk}.$$

Of course, we do not know the grand mean μ_k and variance σ_k^2. The empirical Bayes approach simply replaces these values by their sample counterparts, so that

$$\widehat{\theta}_{jk}^{\text{EB}} = \frac{\widehat{s}_{jk}^2}{\widehat{s}_{jk}^2 + \widehat{\sigma}_k^2}\widehat{\mu}_k + \frac{\widehat{\sigma}_k^2}{\widehat{s}_{jk}^2 + \widehat{\sigma}_k^2}\widehat{\theta}_{jk}.$$

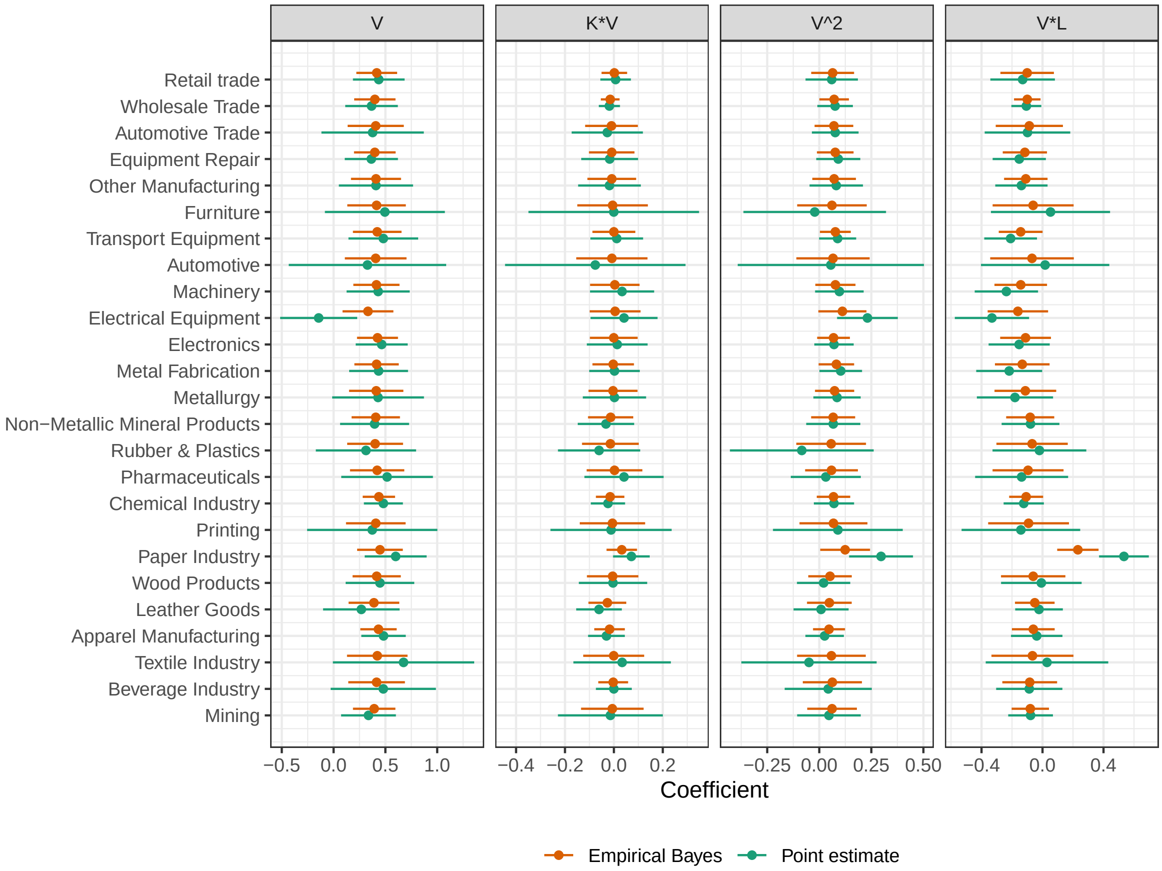
We construct confidence intervals for the Bayes estimator $\widehat{\theta}_{jk}^{\text{EB}}$ following Armstrong et al. (2022). One implementation issue is that the naive plug-in estimator for σ_k^2 might not be positive in finite sample. We address this by using the finite sample estimator proposed by Armstrong et al. (2022) and implemented in their R package `ebci`. We refer the reader to Appendix A of their paper for details.

Results. We show our production function coefficient estimates in Figure A.1. The initial point estimates are shown in green, and the empirical Bayes estimates in orange. By construction, the empirical Bayes estimates are less dispersed and closer to a normal. We note that a number of seemingly anomalous estimates (such as negative coefficients in material inputs and squared matrial inputs) disappear after shrinking estimates. Figure A.2 further breaks downs the estimates by sector for the coefficients used in estimating markups.

Dataset construction. For each 2-digit industry we estimate a translog production function in material costs, wage bill, and physical capital stock. The variables are constructed as described above. Firm revenues are deflated by EU-KELMS price indices for gross output in the firm's 2-digit industry, materials are deflated by price indices for intermediate inputs, and the wage bill and capital are deflated by the GDP deflator.

As noted by Burstein et al. (2020), there are two drawbacks to using deflated firm revenues to proxy for quantities. First, an aggregate is used rather than firm level prices,

Figure A.2: Empirical Bayes shrinkage of production function coefficients



Note. This figure shows the estimated production function coefficients. In green, the point estimates before shrinkage. In orange, the empirical Bayes estimates. Confidence intervals are computed following Armstrong et al. (2022).

so the measure is inexact. Second, it does not admit imperfect competition on the demand side. However, Burstein et al. (2020) they take steps to account for these drawbacks and find that markups estimated while including these controls have high pairwise correlations with what we estimate, and that first differences of the markups measures are highly correlated as well. The level of markups is also important for the results of the quantitative model, but we use an alternative calibration for that which admits variation in the average markup.

Table A.1: Production function estimates and summary statistics

	Elasticity		Markups				N
Description	Avg.	(S.d.)	Q1	Med.	Q3	Avg.	Obs.
Mining	0.41	(0.18)	1.64	2.49	3.68	2.82	1,402
Beverages	0.65	(0.16)	1.02	1.23	1.59	1.52	6,057
Textile	0.55	(0.15)	1.22	1.45	1.83	1.50	6,861
Wearing apparel	0.51	(0.12)	1.08	1.38	1.83	1.44	7,688
Leather	0.45	(0.12)	0.99	1.22	1.53	1.12	2,857
Lumber and wood	0.54	(0.12)	1.06	1.25	1.52	1.28	6,499
Paper	2.07	(0.84)	3.67	4.88	6.29	6.25	4,942
Printing	0.44	(0.15)	1.43	1.74	2.12	1.76	7,521
Coke and petroleum	0.55	(0.16)	1.09	1.27	1.58	1.39	11,266
Chemicals	0.53	(0.14)	1.30	1.69	2.32	1.92	2,141
Pharmaceuticals	0.52	(0.11)	1.12	1.31	1.59	1.31	14,439
Rubber and platic	0.47	(0.18)	1.24	1.52	1.98	1.92	6,573
Other non-metal	0.59	(0.19)	1.16	1.40	1.72	1.37	3,365
Basic metals	0.43	(0.18)	1.33	1.60	1.98	1.59	31,513
Fabricated metal products	0.49	(0.16)	1.11	1.34	1.71	1.62	9,993
Computer, optic, and electrical	0.58	(0.25)	1.15	1.40	1.69	1.78	7,570
Electrical equipment	0.51	(0.14)	1.08	1.25	1.49	1.28	19,192
Machinery and equipment	0.68	(0.16)	1.19	1.38	1.62	1.54	5,137
Motor vehicles	0.49	(0.19)	1.05	1.29	1.71	2.01	2,697
Other transport equipment	0.52	(0.15)	1.27	1.54	1.86	1.69	4,947
Other manufacturing	0.44	(0.17)	1.17	1.43	1.83	1.49	10,617
Repair	0.46	(0.17)	1.12	1.36	1.77	1.70	12,295
Retail and wholesale mot.v.	0.73	(0.18)	0.93	1.03	1.15	1.11	30,005
Other wholesale trade	0.67	(0.17)	0.95	1.06	1.22	1.18	199,596
Other retail trade	0.59	(0.14)	0.91	1.05	1.28	1.27	69,394

Note. This table presents the results for the production function estimation procedure described in the main text from 2011 to 2019 for each two-digit sector. Elasticities are the output elasticities with respect to materials. The mean, median, standard deviation, and quartiles are computed over the entire panel. For the average markup, we compute the harmonic sales-weighted average of firm-level markups for each year and present the time-series average.

D Details on the model calibration

Distributions of pass-through and markups We use the exact same data as Baqaee et al. (2023b) for pass-through and markups. Their full description of their calibration can be found in Appendix A of their paper.

Distributions of sales shares and invoicing Let $s \in [0, 1]$ the a firm size centile in the data, and $\Lambda(s)$ be the share of sales for firms up to that centile s

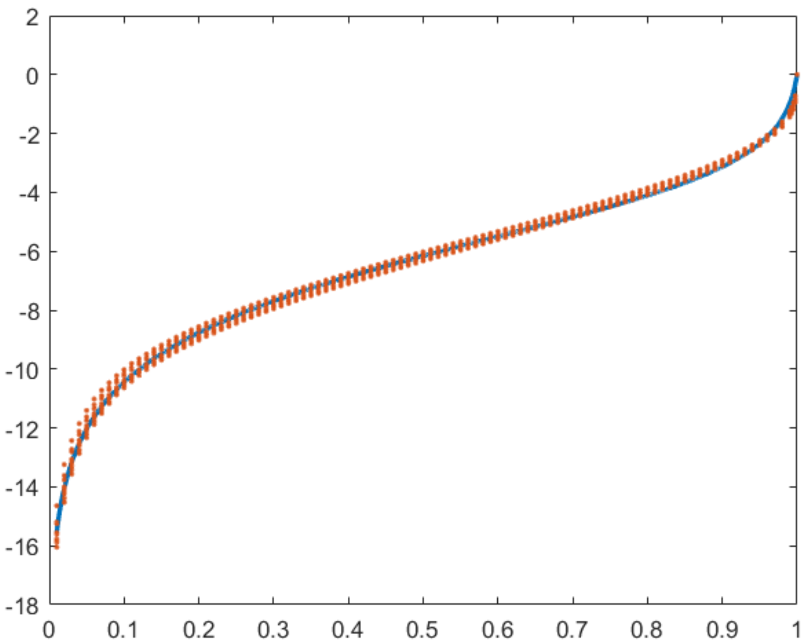
$$\Lambda(s) = \int_0^s \lambda(s) ds$$

where $\lambda(s)$ is the sales share density. We fit a smooth curve to $\log(\Lambda(s))$ of the form

$$\log(\Lambda(s)) = x_1 * \left(\log \left(\frac{s^{x_2}}{(1+h) - s^{x_3}} \right) + \log(h) \right)$$

where h is the step size of the quantiles, and then numerically differentiate to find $\lambda(s)$. The calibrated functions and data can be seen in the Figure A.3.

Figure A.3: Fitted distribution of $\log(\Lambda(s))$



For invoicing, we observe the exports from France to country d invoiced in currency ℓ, where d is either the US or the Rest of the World, and ℓ is either the Euro, the Dollar, or the local destination currency (when non-dollar). For each pair (ℓ, d), we compute from the data the share of exports in currency ℓ to country d within each centile for firms. When then fit this data to functions of the form

$$\log(\iota_{d\ell}(s)) = x_1 + x_2 s^2 + x_3 s^{x_4}$$

The data and fitted distributions are reported in Figure 4 in the main text.

E Endogenous currency choice with switching costs

To provide theoretical justification for our assumption that firms do not change their currency of invoicing in response to small exchange rate shocks, we consider a dynamic model of endogenous currency choice close to Gopinath et al. (2010), with three important differences. First, we allow for an arbitrary number of markets and currencies, in line with the evidence that firms using foreign currencies also export to many markets and may exploit within-firm strategic complementarities (Corsetti et al., 2022). Second, we introduce switching costs to rationalize the observed persistence of invoicing. Third, we model invoicing as a continuous decision, which helps keep the model tractable and simplifies the intuition.

Setup. Time is discrete and infinite. We consider a firm choosing an invoicing bundle ℓ within the K-dimensional simplex Δ_K[5]. In the case of a firm exporting one good and choosing between producer, local, and dominant currency pricing, the set of acceptable bundles is the two-dimension simplex, each vertex representing one of the extreme invoicing paradigms. In each period, the firm can reset its currency bundle with probability $\delta \in (0,1)$. The profits of the firm depend on its invoicing decisions and on an exogenous and stationary Markov chain $(\theta_t)_{t\geq 0}$ which takes values in $\mathbf{R}^N$.

The value function of the firm that chooses currency bundle ℓ_t is

$$v(\ell_t,\theta_t) = \pi(\ell_t,\theta_t) + \beta\delta\mathbf{E}_t\left[v(\ell_t,\theta_{t+1})\right] + \beta(1-\delta)\mathbf{E}_t\left[w(\ell_t,\theta_{t+1})\right].$$

Here $\beta \in (0,1)$ is the discount rate and w is the value of the firm when it is allowed to reset its currency, which is given by

$$w(\ell,\theta) = \max_{\ell'\in\Delta_N}\left\{v(\ell',\theta) - S\mathbf{1}\left\{\ell' \neq \ell\right\}\right\}.$$

The parameter $S > 0$ captures a fixed switching cost.

Infrequent switching. Consider a firm at time $t+1$ with currency bundle ℓ chosen at an initial state θ. Conditional on being able to update its currency bundle, it will do so if

$$v(\ell',\theta_{t+1}) - v(\ell,\theta) \geq S,$$

where ℓ' is the optimal choice for state θ_{t+1} in the absence of switching cost. Assuming that the initial optimal bundle was interior and that the value function is differentiable around (ℓ,θ), a first order Taylor expansion reveals that the firm will switch currencies if

$$\nabla_\theta v(\ell,\theta)\cdot(\theta_{t+1}-\theta) \geq S, \tag{18}$$

[5]We can accomodate an arbitrary number of invoicing decisions by considering Cartesian products of the simplex. We do not do so to alleviate notations.

up to error terms that we neglect. The ex-ante probability of switching is therefore approximately

$$\sum_{s\geq 0} \delta(1-\delta)^s \times \mathbf{P}\left[\nabla_\theta v(\ell,\theta)\cdot(\theta_{t+s+1}-\theta) \geq S \mid \theta_t\right]. \tag{19}$$

This equation shows that switching cost can rationalize persistence in invoicing choice in a wide class of endogenous currency choice models. In this model, currency switching is less frequent when the probability of updating is low, when switching costs are high, or when the probability of large movements in the underlying state is low.

F Additional empirical results

Table A.2: Exchange rate pass-through to product-level prices

Dependent Variables:	$\Delta_1 \log p$	$\Delta_2 \log p$	$\Delta_3 \log p$	$\Delta_4 \log p$
Model:	(1)	(2)	(3)	(4)
Variables				
$\Delta_1 \log E^\ell$	0.741			
	(0.026)			
$\Delta_2 \log E^\ell$		0.729		
		(0.028)		
$\Delta_3 \log E^\ell$			0.688	
			(0.029)	
$\Delta_4 \log E^\ell$				0.638
				(0.031)
Fixed-effects				
Year-Quarter-Destination	Yes	Yes	Yes	Yes
CN8-Destination	Yes	Yes	Yes	Yes
Fit statistics				
Firms	39,142	39,142	39,142	39,142
Observations	3,769,250	3,769,250	3,769,250	3,769,250
R^2	0.029	0.042	0.054	0.068
Within R^2	0.001	0.001	0.002	0.002

Note. This table reports the results from a linear regression of log change in unit values on the log change in the exchange rate for the currency of invoicing. The specification is

$$\Delta_h \log p^\ell_{ijdt} = \alpha_{dt} + \alpha_{jd} + \beta\Delta_h \log E^\ell_{dt} + u^\ell_{ijdt},$$

where i is a firm, j is a product measured at the 8-digit level, d is a destination country, t is a year-quarter, ℓ is a currency, and $\Delta_h x_t = x_{t+h} - x_t$. The exchange rate from currency ℓ to the destination currency d is E^ℓ_{dt}. The regression includes destination-time fixed effects α_{dt} and product-destination fixed effects α_{jd}. Standard errors are clustered by firm.

Table A.3: Exchange rate pass-through to product-level quantities

Dependent Variables:	$\Delta_1 \log q$	$\Delta_2 \log q$	$\Delta_3 \log q$	$\Delta_4 \log q$
Model:	(1)	(2)	(3)	(4)
Variables				
$\Delta_1 \log E^{\ell}$	-0.209			
	(0.069)			
$\Delta_2 \log E^{\ell}$		-0.111		
		(0.047)		
$\Delta_3 \log E^{\ell}$			-0.102	
			(0.047)	
$\Delta_4 \log E^{\ell}$				-0.101
				(0.045)
Fixed-effects				
Year-Quarter-Destination	Yes	Yes	Yes	Yes
CN8-Destination	Yes	Yes	Yes	Yes
Fit statistics				
Firms	39,142	39,142	39,142	39,142
Observations	3,769,250	3,769,250	3,769,250	3,769,250
R^2	0.031	0.038	0.044	0.051
Within R^2	0.000	0.000	0.000	0.000

Note. This table reports the results from a linear regression of log change in quantities on the log change in the exchange rate for the currency of invoicing. Quantities are measured in the product-destination specific unit of measurement provided by the customs data. When it is not available, we use the transaction weight in kilograms. The specification is

$$\Delta_h \log q^{\ell}_{ijdt} = \alpha_{dt} + \alpha_{jd} + \beta \Delta_h \log E^{\ell}_{td} + u^{\ell}_{ijdt},$$

where i is a firm, j is a product measured at the 8-digit level, d is a destination country, t is a year-quarter, ℓ is a currency, and $\Delta_h x_t = x_{t+h} - x_t$. The exchange rate from currency ℓ to the destination currency d is E^{ℓ}_{dt}. The regression includes destination-time fixed effects α_{dt} and product-destination fixed effects α_{jd}. Standard errors are clustered by firm.

Table A.4: Predictive power of size for invoicing

Dependent Variable:	Foreign invoicing share							
Model:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Variables								
Constant	-116.69		30.52		-6.36		25.30	
	(12.11)		(20.40)		(0.21)		(0.49)	
log Exports	8.65	6.72	-7.53	0.06	1.09	1.08	-5.43	-5.04
	(0.72)	(0.45)	(2.28)	(2.98)	(0.02)	(0.02)	(0.11)	(0.11)
$(\log \text{Exports})^2$			0.43	0.19			0.31	0.30
			(0.06)	(0.09)			(0.01)	(0.01)
Fixed-effects								
Sector-Year	–	Yes	–	Yes	–	Yes	–	Yes
Fit statistics								
Firms	241,460	241,460	241,460	241,460	241,460	241,460	241,460	241,460
Observations	921,973	921,973	921,973	921,973	921,973	921,973	921,973	921,973
R^2	0.41	0.61	0.42	0.61	0.03	0.06	0.06	0.09
Within R^2		0.18		0.19		0.02		0.05
BIC	9,554,308.37	9,132,603.89	7,936,595.17	8,621,498.28	7,939,192.61	7,993,021.76	7,904,421.63	7,962,800.54
Weights	Exports	Exports	Exports	Exports	Equal	Equal	Equal	Equal

Note. This table shows the results for a regression of the invoicing share on exports. The invoicing share is the share of exports to destinations outside the eurozone invoiced in another currency than the euro, computed at the firm and calendar year level and expressed in percents. The exports is the total value in euros of exports to destinations outside the eurozone computed at the firm and calendar year level. Standard errors are clustered by firm.