

# Introduction to Quantum Mechanics

Spring 2013

Final Exam: Monday, August 12, 2013

THIS IS A CLOSED BOOK EXAM\*

EXAM RULES: All notes, as well as all books, notebooks, note cards, *etc.* must be inside closed and latched briefcases or inside closed and zippered backpacks. Briefcases and backpacks must be under your desk. Students should be well separated.

You may write your answers in English or German.

\*You are allowed three A4 pages with notes on both sides. You must hand these sheets in with your exam. They are worth points!

Write your name in the upper right hand corner of each page.

SHOW YOUR WORK!!

You have 120 minutes to complete the exam.

Name: \_\_\_\_\_

Student Number: \_\_\_\_\_

“Cheat Sheets”: \_\_\_\_\_/10

Problem 1: \_\_\_\_\_/50

Problem 2: \_\_\_\_\_/40

Problem 3: \_\_\_\_\_/35

Problem 4: \_\_\_\_\_/25

Problem 5: \_\_\_\_\_/40

Extra Credit: \_\_\_\_\_/4

Total: \_\_\_\_\_/200

Mark: \_\_\_\_\_

Name: \_\_\_\_\_

**1. Short Answer:**

(a) Circle True or False (30 points possible)

(Do not guess: +2 for correct answer, 0 for no answer, -2 for incorrect answer)

- T            My name and student number are on the front page of this exam.
- T    F        Two Hermitian operators will always commute.
- T    F        The Dirac delta function is not in Hilbert space.
- T    F        If a particle is described by a wave packet, its energy is always well defined.
- T    F        If an electron approaches a potential barrier of any finite height, it can always be reflected.
- T    F        If a particle is in a non-stationary state, the measurement of its energy must yield one of several values.
- T    F         $\frac{d}{dt} \int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 0$  arises only for unphysical solutions to the 1D Schrödinger equation.
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- T    F        Given a system of two identical particles, the exchange operator  $\hat{P}$  always commutes with the Hamiltonian operator  $\hat{H}$ .
- T    F        Given a specific shell in a hydrogenic atom, all of the subshells have the same energy.
- T    F        Every classical observable can be represented in quantum mechanics by a Hermitian operator.
- T    F        In metals at  $T=0K$ , the electronic states are filled up to the Fermi level.
- T    F        If a quantum mechanical observable is measured, one possible result is always the expectation value of the observable.
- T    F        An uncertainty relation will exist for any two observables that have operators that do not commute.
- T    F        Electronic bands exist in semiconductor and insulators, but not metals.

Name: \_\_\_\_\_

- (b) If a particle is placed in a sphere of radius  $\rho$  where the potential  $V(r)$  is described by

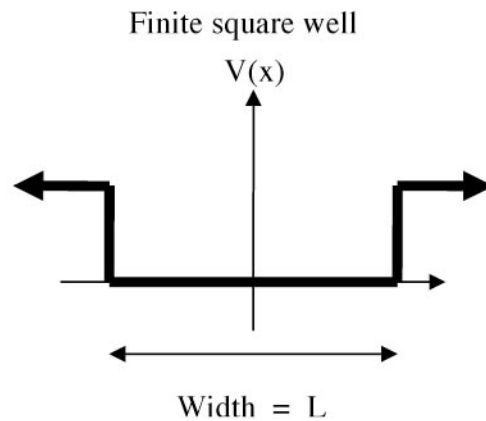
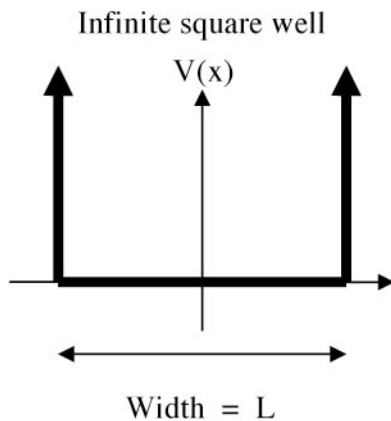
$$V(r) = \begin{cases} 0 & \text{for } r \leq \rho \\ \infty & \text{for } r > \rho \end{cases}$$

where  $r$  is the radial coordinate. Write down the angular components to the particle's wave function? (5 points)

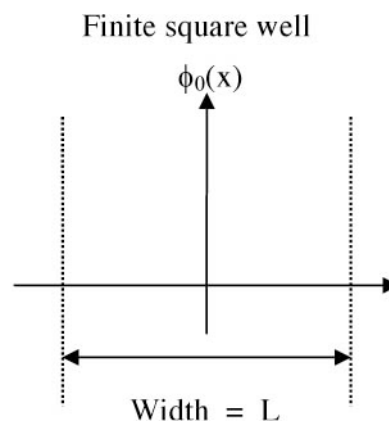
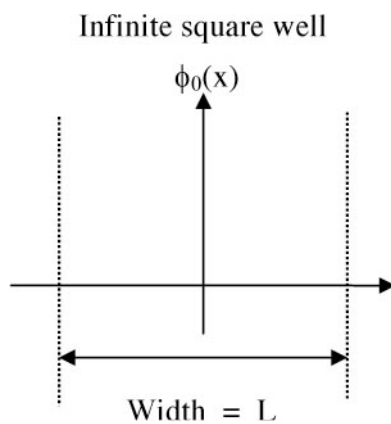
- (c) Give an example of a quantum mechanical system that has both a discrete and continuous part to its spectrum. (5 points)

Name: \_\_\_\_\_

(d) Given the two square wells as shown below with the same width,



sketch the wavefunction for a particle in the ground state on each of the corresponding plots below (5 points).



(e) In part (d), do you expect the energy of the ground state of the finite square well to be less than, equal to, or greater than the energy of the ground state of the infinite square well? Explain your answer. (5 points)

Name: \_\_\_\_\_

**2. Short Answer:**

(a) Write the hydrogen eigenstate  $\psi_{32\pm 1}$ . Explain any constants in your answer. (10 points)

(b) What is the electronic configuration and term symbol for the ground state of Al? (10 points)

Name: \_\_\_\_\_

- (c) Find the ground-state energy for a system of  $N$  noninteracting identical particles that are confined to a one-dimensional infinite square well when the particles are (i) bosons and (ii) spin  $1/2$  fermions. (10 points)

- (d) For the  $N$  bosons in part (c), write down the ground-state time-independent wave function. (10 points)

Name: \_\_\_\_\_

3. An operator  $\hat{A}$ , representing observable  $A$ , has two normalized eigenstates  $\psi_1$  and  $\psi_2$  with eigenvalues  $a_1$  and  $a_2$ , respectively. Operator  $\hat{B}$ , representing observable  $B$ , has two normalized eigenstates  $\phi_1$  and  $\phi_2$  with eigenvalues  $b_1$  and  $b_2$ , respectively. The eigenstates are related by

$$\psi_1 = (3\phi_1 + 4\phi_2)/5$$

$$\psi_2 = (4\phi_1 - 3\phi_2)/5$$

- (a) Observable  $A$  is measured, and the eigenvalue  $a_2$  is obtained. What is the state of the system immediately after this measurement? (5 points)

- (b) If observable  $B$  is now measured [after the first measurement in part (a)], what are the possible results, and what are their probabilities? (10 points)

Name: \_\_\_\_\_

(c) Right after the measurement of  $B$  in part (b),  $A$  is measured again. You were not told the outcome of the  $B$  measurement before the  $A$  measurement. What is the probability of getting  $a_2$  after the three measurements ( $A$ ,  $B$ , and then  $A$ )? (15 points)

(d) Would your answer change in part (c) if you were told the outcome of the  $B$  measurement? How? (5 points)



Name: \_\_\_\_\_

4. We prepare a simple harmonic oscillator with the following normalized wavefunction:

$$\Psi(x, 0) = \left( \frac{9\beta^2}{\pi} \right)^{\frac{1}{4}} e^{-9(\beta x)^2/2}$$

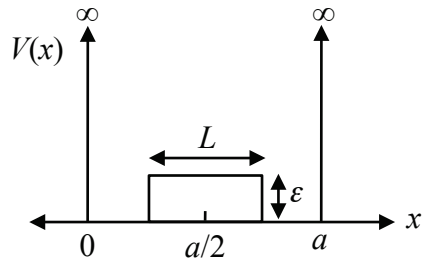
where  $\beta \equiv \sqrt{\frac{m\omega}{\hbar}}$ .

We then immediately measure the energy of the oscillator in this state. What is the probability of getting the ground state energy? (25 points)

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5. A one-dimensional infinite square well has a potential step centered in the middle as shown below.



- (a) Calculate the energy of the ground state to first order. Then evaluate it for  $L=a/10$ . (25 points)

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- (b) Using first-order perturbation theory, determine how much of the  $n=2$  eigenstate from the standard infinite square well (*i.e.* without the potential step in the middle) is contained in the lowest energy eigenstate of the perturbed infinite square well from (a). (15 points)

Name: \_\_\_\_\_

Extra Credit: (1 point each)

The following questions are about computer operating systems:

1. What year was the Linux kernel first released?
2. What year was Microsoft Windows 1.0 first released?
3. What year was the Apple Macintosh, which utilized the earliest version of Mac OS, released?
4. What year was Android 1.0 released?

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