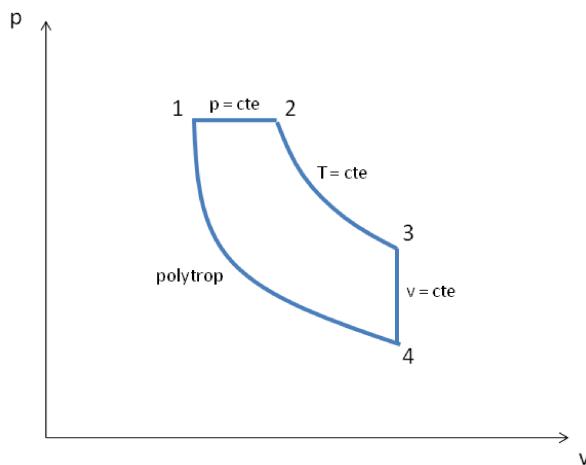


Thermodynamik I – Lösung Rechenübung 3

Aufgabe 1

a) p-v Diagramm:



b) Volumina und Druck bei jedem Zustand:

Zustand 1:

$$T_1 = 400 \text{ K}, p_1 = 10 \text{ bar}$$

$$V_1 = \frac{mRT_1}{p_1} = \frac{m \frac{\bar{R}}{M_{Luft}} T_1}{p_1} = \frac{1.5 \text{ kg} \frac{8.314 \text{ J/(mol} \cdot \text{K)}}{28.97 \text{ g/mol}} 400 \text{ K}}{10^6 \text{ Pa}} = 0.172 \text{ m}^3$$

Zustand 2:

$$T_2 = 550 \text{ K}, p_2 = 10 \text{ bar}$$

$$V_2 = \frac{mRT_2}{p_2} = 0.237 \text{ m}^3$$

Zustand 3:

$$T_3 = T_2 = 550 \text{ K}, p_3 = 7.5 \text{ bar}$$

$$V_3 = \frac{mRT_3}{p_3} = 0.316 \text{ m}^3$$

Zustand 4:

$$V_4 = V_3 = 0.316 \text{ m}^3$$

$$p_4 = p_1 \left(\frac{V_1}{V_4} \right)^n = 10 \text{ bar} \left(\frac{0.172 \text{ m}^3}{0.316 \text{ m}^3} \right)^{1.5} = 4.03 \text{ bar}$$

$$T_4 = \frac{p_4 V_4}{m R} = 295.4 \text{ K}$$

c) Arbeit und Wärmeübergang vom Zustand 1 nach 2:

Arbeit:

$$W_{12} = \int_{V_1}^{V_2} p dV = p \Delta V = 10^6 \text{ Pa} (0.237 \text{ m}^3 - 0.172 \text{ m}^3) = 65 \text{ kJ}$$

Wärmeübergang:

$$\Delta U = Q - W \Rightarrow Q_{12} = \Delta U_{12} + W_{12}$$

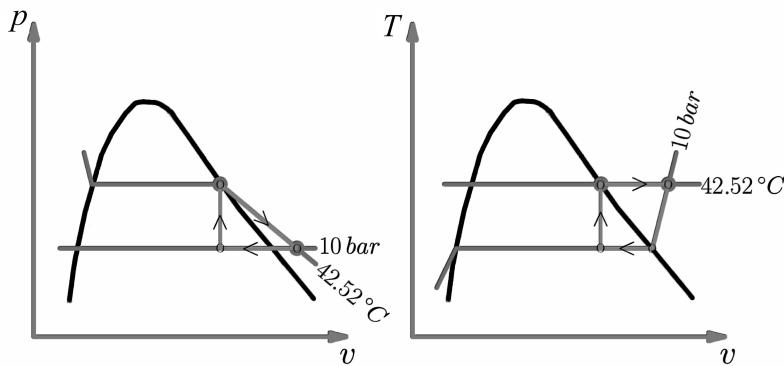
$$\begin{aligned} Q_{12} &= \Delta U_{12} + W_{12} = m(u_2 - u_1) + W_{12} \\ &= 1.5 \text{ kg} \cdot (396.86 \frac{\text{kJ}}{\text{kgK}} - 286.16 \frac{\text{kJ}}{\text{kgK}}) + 65 \text{ kJ} = 231 \text{ kJ} \end{aligned}$$

Alternativ:

$$\begin{aligned} Q_{12} &= \Delta U_{12} + W_{12} \cong m [c_{v_{\bar{T}=475K}} (T_2 - T_1)] + W_{12} \\ &= 1.5 \text{ kg} \cdot 0.7375 \frac{\text{kJ}}{\text{kgK}} (550 \text{ K} - 400 \text{ K}) + 65 \text{ kJ} = 231 \text{ kJ} \end{aligned}$$

Aufgabe 2

Mit Hilfe von Tabelle A-13 ff.:



Zustand 1: $p_1 = 10 \text{ bar}$ $T_1 = T_{\text{sat}} = 24.89^\circ C$

$$v_1 = v_f + x(v_g - v_f) = 1.6584 \cdot 10^{-3} \text{ m}^3/\text{kg} + 0.6 \cdot (0.1285 - 1.6584 \cdot 10^{-3}) \text{ m}^3/\text{kg}$$

$$= \underline{\underline{0.07776 \text{ m}^3/\text{kg}}}$$

$$u_1 = u_f + x(u_g - u_f) = 296.10 \text{ kJ/kg} + 0.6 \cdot (1334.66 - 296.10) \text{ kJ/kg}$$

$$= \underline{\underline{919.24 \text{ kJ/kg}}}$$

Zustand 2: $v_2 = v_1$ gesättigter Dampf

$$u_2 = \text{durch Interpolation } u_2 = \underline{\underline{1341.26 \text{ kJ/kg}}}$$

$$T_2 = \text{durch Interpolation } T_2 = \underline{\underline{42.52^\circ C}}$$

Zustand 3: $p_3 = p_1 = 10 \text{ bar}$ $T_3 = T_2 = \underline{\underline{42.52^\circ C}}$

aus Tabelle A-15: $v_3 = 0.14027 \text{ m}^3/\text{kg}$ $u_3 = 1374.95 \text{ kJ/kg}$

$$\underbrace{KE}_{=0} + \underbrace{PE}_{=0} + U = Q - W$$

1 → 2: $W_{12} = 0$ da isochor!

$$Q_{12} = m(u_2 - u_1) = 2 \text{ kg} (1341.258 - 919.236) \text{ kJ/kg} = \underline{\underline{844.0 \text{ kJ}}}$$

2 → 3: $Q_{23} = 228 \text{ kJ}$

$$W_{23} = Q_{23} - m(u_3 - u_2) = 228 \text{ kJ} - 2 \text{ kg} (1374.95 - 1341.28) \text{ kJ/kg} = \underline{\underline{160.6 \text{ kJ}}}$$

3 → 1 : $W_{31} = \int_{V_3}^{V_1} p dV = mp(v_1 - v_3) = \underline{\underline{-125.0 \text{ kJ}}}$

$$Q_{31} = m(u_1 - u_3) + W_{31} = 2 \text{ kg} (919.24 - 1374.95) \text{ kJ/kg} + (-125.0 \text{ kJ}) = \underline{\underline{-1036.4 \text{ kJ}}}$$

$$W_{\text{Prozess}} = W_{12} + W_{23} + W_{31} = 0 + 160.66 \text{ kJ} + (-125 \text{ kJ}) = \underline{\underline{35.6 \text{ kJ}}}$$

$W_{\text{Prozess}} > 0$ also leistet das System Arbeit!

Aufgabe 3

Folgendes kann den Wasserdampftafeln entnommen werden:

Zustand 1:	$p_1 = 5 \text{ bar}$	$T_1 = 160^\circ C$
	$v_1 = 0.3835 \text{ m}^3/\text{kg}$	$u_1 = 2575.2 \text{ kJ/kg}$

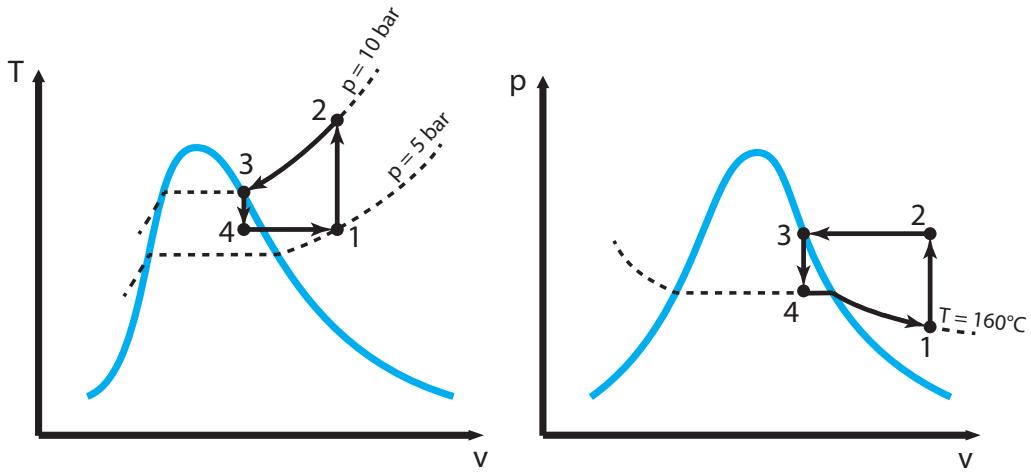
Zustand 2:	$p_2 = 10 \text{ bar}$
	$v_2 = v_1$

Zustand 3:	$p_3 = 10 \text{ bar}$	$u_2 = 3231.8 \text{ kJ/kg}$
	$v_3 = 0.1944 \text{ m}^3/\text{kg}$	$u_3 = 2583.6 \text{ kJ/kg}$

Zustand 4:	$v_4 = v_3$	$T_4 = T_1 = 160^\circ C$
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$$x = \frac{v - v_f}{v_g - v_f} \text{ damit folgt für } x_4 = \frac{0.1944 - 1.102 \cdot 10^{-3}}{0.3071 - 1.102 \cdot 10^{-3}} = \underline{\underline{0.6318}}$$

$$u_4 = x_4 u_{Dampf} + (1 - x_4) u_{Flüssig} = \underline{\underline{1871.2 \text{ kJ/kg}}}$$



$$\Delta U = Q - W$$

$$\Delta U = m \cdot \Delta u$$

$$\underline{1 \rightarrow 2:} W_{12} = 0 \quad mu_{12} = Q_{12} = m(u_2 - u_1) = \underline{\underline{656.6 \text{ kJ}}}$$

$$\underline{2 \rightarrow 3:} W_{23} = m \int_{v_2}^{v_3} p \cdot dv = mp(v_3 - v_2) = \underline{\underline{-189.1 \text{ kJ}}}$$

$$Q_{23} = m(u_3 - u_2) + W_{23} = \underline{\underline{-837.3 \text{ kJ}}}$$

$$\underline{3 \rightarrow 4:} W_{34} = 0 \quad Q_{34} = m(u_4 - u_3) = \underline{\underline{-712.4 \text{ kJ}}}$$

$$\underline{4 \rightarrow 1:} W_{41} = Q_{41} - m(u_1 - u_4) = \underline{\underline{111.8 \text{ kJ}}}$$

$$W_{Prozess} = \sum_{i=1}^4 W_i = \underline{\underline{-77.3 \text{ kJ}}} \quad Q_{Prozess} = \sum_{i=1}^4 Q_i = \underline{\underline{-77.3 \text{ kJ}}}$$

$$\sum (Q_i - W_i)_{Kreisprozess} = 0$$

$W_{Prozess} < 0$: Arbeit wird dem System zugeführt

$Q_{Prozess} < 0$: Wärme wird vom System abgeführt