$$|e^i\rangle = \frac{iZ}{k_0} b|j^i\rangle$$

$$\text{obj}: 0.5\cdot \mathrm{Im}\{\langle e^i|T\rangle\} + \underbrace{e_{vac}}_{\langle j^i|b|j^i\rangle}$$

only take the real part

$$\text{constraints}: \mathrm{Re}\{\langle e^i|P|T\rangle\} - \langle T|[P(X^{-1\dagger} - b_0^{\dagger})]^S|T\rangle = 0$$

$$\mathrm{Im}\{\langle e^i|P|T\rangle\} - \langle T|[P(X^{-1\dagger} - b_0^{\dagger})]^A|T\rangle = 0$$

$$L = \mathrm{Im}\{\langle e^i|T\rangle\} + e_{vac} + \lambda_1\left[\mathrm{Re}\{\langle e^i|P|T\rangle\} - \langle T|[P(X^{-1\dagger} - b_0^{\dagger})]^S|T\rangle\right]$$
$$+\lambda_2\left[\mathrm{Im}\{\langle e^i|P|T\rangle\} - \langle T|[P(X^{-1\dagger} - b_0^{\dagger})]^A|T\rangle\right]$$

$$= \frac{\langle e^i|T\rangle - \langle T|e^i\rangle}{2j} + e_{vac} + \lambda_1\left[\frac{\langle e^i|P|T\rangle + \langle T|P|e^i\rangle}{2} - 2\langle T|[P(X^{-1\dagger} - b_0^{\dagger})]^S|T\rangle\right]$$

$$+\lambda_2\left[\frac{\langle e^i|P|T\rangle - \langle T|P|e^i\rangle}{2j} - 2\langle T|[P(X^{-1\dagger} - b_0^{\dagger})]^A|T\rangle\right]$$

$$\frac{\partial L}{\langle T|} = -\frac{|e^i\rangle}{2j} + \lambda_1\left[\frac{P|e^i\rangle}{2} - 2\left(P(X^{-1\dagger} - b_0^{\dagger})\right)^S|T\rangle\right] + \lambda_2\left[\frac{P|e^i\rangle}{2j} - 2\left(P(X^{-1\dagger} - b_0^{\dagger})\right)^A|T\rangle\right] = 0$$

$$\underbrace{\left[\left(P(X^{-1\dagger} - b_0^{\dagger})\right)^S + \left(P(X^{-1\dagger} - b_0^{\dagger})\right)^A\right]}_{A}|T\rangle = \underbrace{-\frac{|e^i\rangle}{4j} + \lambda_1\frac{P|e^i\rangle}{4} + \lambda_2\frac{P|e^i\rangle}{4j}}_{b}$$