Finite impulse response filtering
FIR windowed-sinc filters

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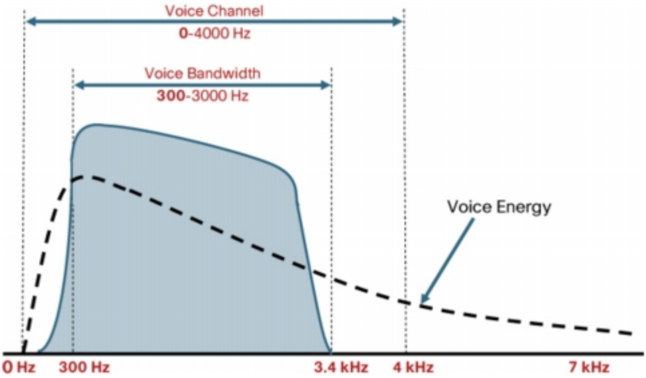
Classification of discrete filters

Table: Classification of discrete filters

	Finite impulse response (FIR)	Infinite impulse response (IIR)
Filtering in time domain	Moving average	Leaky Integrator
Filtering in frequency domain	Windowed-sinc Filters Equiripple Minimax	Bilinear z-transform

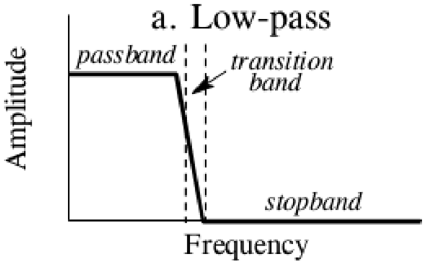
Information in frequency domain

- Information of a signal is contained in frequency response, phase and amplitude.
- **Many samples** in the signal are needed for frequency analysis.
- The **frequency response** shows how information in frequency domain is being changed.
- Examples: telephone voice channel, equalizer...

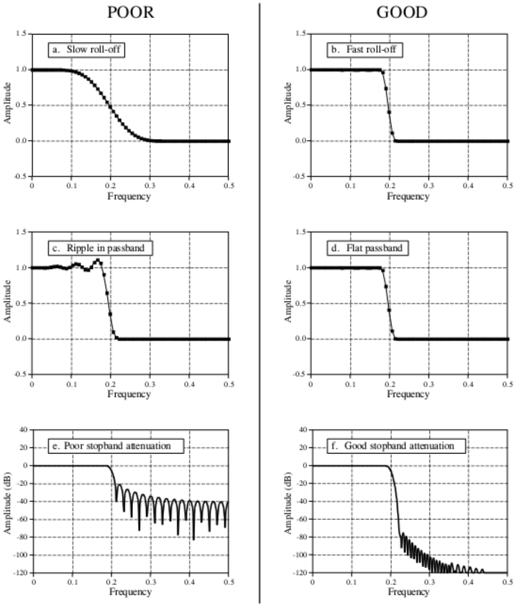


Frequency domain parameters

- Passband.
- Stopband.
- Cut-off frequency.
- Transition band (fast roll-off).
- Passband ripple.
- Stopband ripple.



Frequency response



Strategy of filtering by windowed-sinc

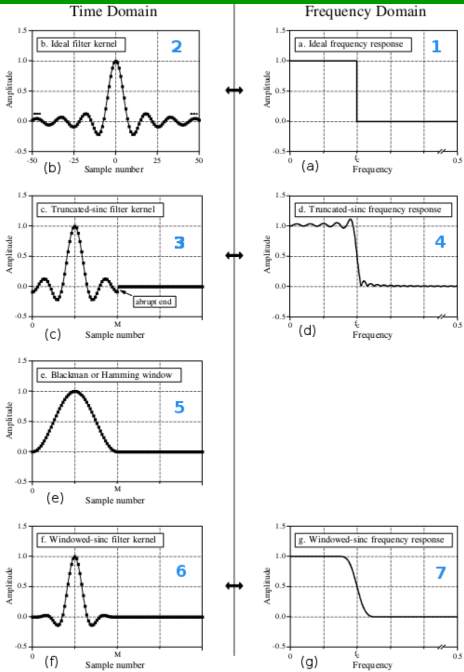
- Taking the Inverse Fourier Transform of an ideal frequency response (1) produces an ideal sinc filter kernel (2, impulse response) with **infinite** length.

$$h_s[n] = \frac{\sin(\pi f[n]/f_s)}{(\pi f[n]/f_s)}$$

- To get around this problem, ideal sinc filter is **truncated** to M+1 points, symmetrically chosen around the main lobe, where M is an even number.
- Truncated-sinc (3) produces the **Gibbs phenomenon** in frequency response (4), no matter how long M is made.
- Multiplying the truncated-sinc ($h_{st}[n]$, 3) by the Blackman window ($w[n]$, 5) results in a **windowed-sinc filter kernel** ($h_w[n]$, 6) with frequency response (7).

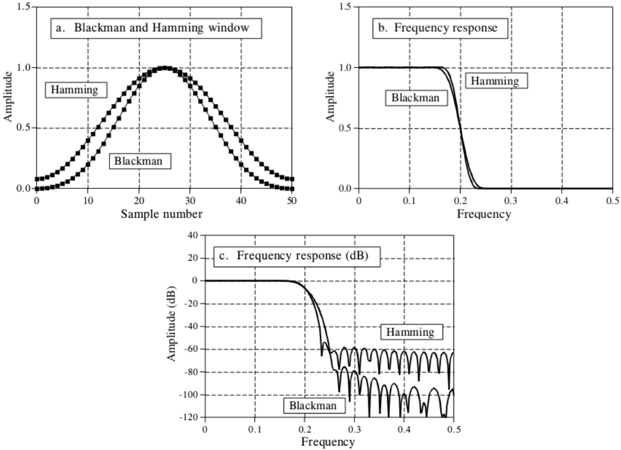
$$h_w[n] = h_{st}[n] \cdot w[n]$$

$$y[n] = h_w[n] * x[n]$$



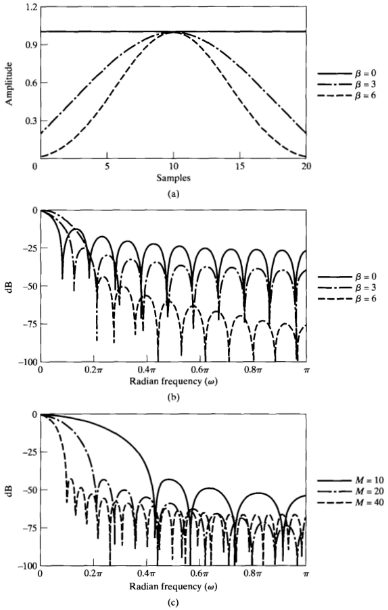
Differences between Blackman and Hamming

- The two windows have M = 50 (51 points)
- Which of these two windows should you use? It's a trade-off between parameters.
- The Hamming window has about a 20% faster roll-off than the Blackman.
- However, the Blackman has a better stopband attenuation, -74dB (-0.02%) vs. -53dB (-0.2%).
- The Blackman has a passband ripple of only about 0.02%, while the Hamming is typically 0.2%.
- In general, the Blackman should be your first choice; a slow roll-off is easier to handle than poor stopband attenuation.



Kaiser window filter

- The Kaiser window has two parameters:
    - Lenght, M+1.
    - Shape parameter, β.
- Trade-off between side-lobe amplitude and main-lobe width.



Normalized performances of windowed-sinc filters

Name of window function $w[n]$	Transition width ΔF in (Hz), (normalised)	Pass-band ripple A_p in (dB)	Ripple δ_p, δ_s	Side-lobe level in (dB)	Stop-band attenuation A_s in (dB)
Rectangular	0.9/N	0.741	0.089	-13	21
Hanning	3.1/N	0.0546	0.063	-31	44
Hamming	3.3/N	0.0194	0.0022	-41	53
Blackman	5.5/N	0.0017	0.000196	-57	74
Kaiser β=4.54	2.93/N	0.0274			50
β=5.65	3.63/N	0.00867			60
β=6.76	4.32/N	0.00275			70
β=8.96	5.71/N	0.000275			90

FIR structures

$$H(z) = b_0 + b_1 z^{-1} + \ldots + b_{M-1} z^{M-1}$$

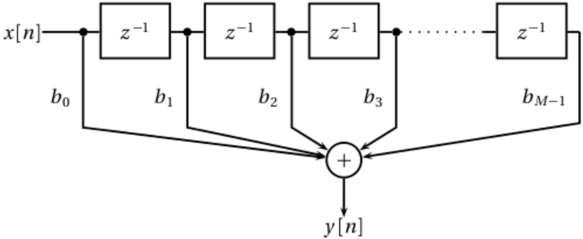


Figure 7.22 Direct FIR implementation.

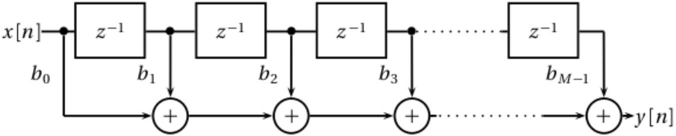


Figure 7.23 Transversal FIR implementation.

Bibliography

1 Steven W. Smith, The Scientist and Engineer's Guide to Digital Signal Processing. Chapter 16. www.dspguide.com.

2 Paolo Prandoni and Martin Vetterli. Signal processing for communications. Taylor and Francis Group, LLC. 2008. Sections 5.4 and 5.5 https://www.sp4comm.org/.