Typical stages in digital signal processing

Aliasing prefiltering

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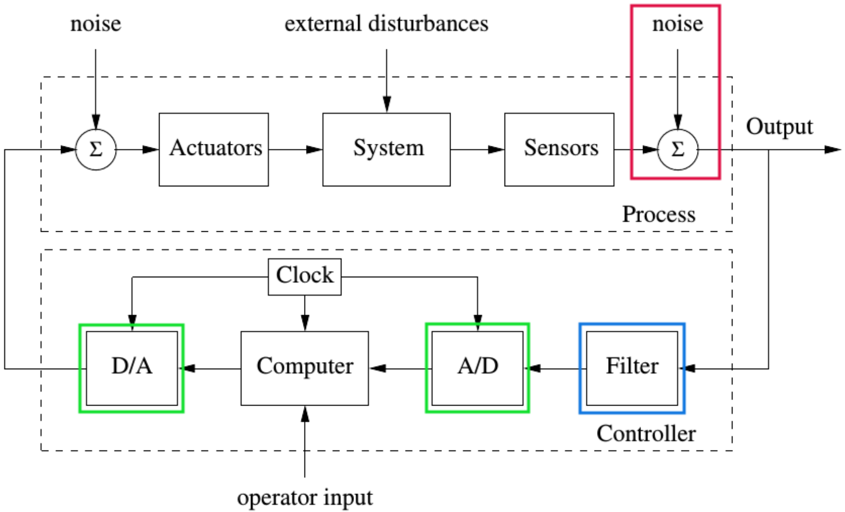
Summary

1. DSP in a the context of control systems
2. Digital processing of analog signals
3. Sampling signals in the frequency domain
   - Periodic sampling
   - Frequency-domain representation of sampling
4. Aliasing prefiltering

DSP in a the context of control systems



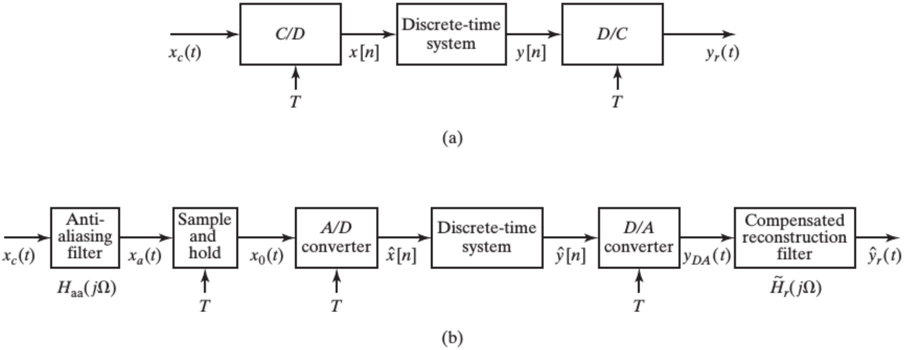


Figure 4.47 (a) Discrete-time filtering of continuous-time signals. (b) Digital processing of analog signals.

Periodic sampling

The discrete-time representation of a continuous-time signal is obtained through periodic sampling from a continuous-time signal $x_c(t)$ according to,

$$x[n] = x_c(nT), \qquad -\infty < n < \infty\,, \tag{1}$$

where T is the sampling period, and $f_s = 1/T$ is the sampling frequency, or $\Omega_s = 2\pi/T$ in radians/s

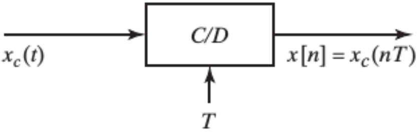


Figure 4.1 Block diagram representation of an ideal continuous-to-discrete-time (C/D) converter.

It is convenient to represent the sampling process mathematically in the two stages.

1. An impulse train $s(t)$ is multiplied by a continuous-time signal $x_c(t)$.
2. The continuous-time signal $x_s(t)$ is transformed to a discrete-time sequence $x[n]$.

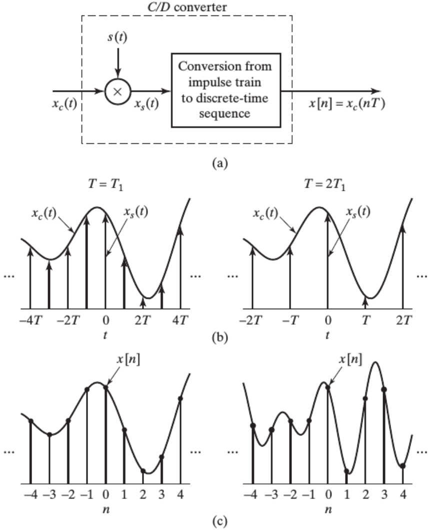


Figure 4.2 Sampling with a periodic impulse train, followed by conversion to a discrete-time sequence. (a) Overall system. (b) $x_s(t)$ for two sampling rates. (c) The output sequence for the two different sampling rates.

Nyquist-Shannon Sampling Theorem

Let $x_c(t)$ be a bandlimited signal with,

$$X_c(j\Omega) = 0 \;\text{ para }\; |\Omega| \geq \Omega_N \,. \tag{2}$$

Then $x_c(t)$ is uniquely determined by its samples $x[n] = x_c(nT)$, $n = 0, \pm 1, \pm 2, \ldots$ if,

$$\Omega_s = \frac{2\pi}{T} \geq 2\Omega_N \,. \tag{3}$$

The frequency Ω_N is commonly referred to as the **Nyquist frequency**, and the frequency $2\Omega_N$ as the **Nyquist rate**.

The Nyquist rate is the minimum sampling frequency in order to be able of reconstructing $x_c(t)$.

Frequency-domain representation of sampling

$x_s(t)$ is obtained multiplying $x_c(t)$ by a periodic impulse train $s(t)$,

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)\,, \tag{4}$$

$$x_s(t) = x_c(t)\, s(t)\,, \tag{5}$$

$$= x_c(t) \sum_{n=-\infty}^{\infty} \delta(t - nT)\,, \tag{6}$$

$$= \sum_{n=-\infty}^{\infty} x_c(nT)\, \delta(t - nT) \qquad \text{by sifting property.} \tag{7}$$

The Fourier transform of the periodic impulse train $s(t)$ is the periodic impulse train,

$$S(j\Omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\Omega - k\Omega_s), \qquad \text{where } \Omega_s = \frac{2\pi}{T}\,. \tag{8}$$

The Fourier transform of $x_s(t)$ is the continuous-variable convolution of $X_c(j\Omega)$ and $S(j\Omega)$,

$$X_s(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega)\,, \tag{9}$$

$$X_s(j\Omega) = \frac{1}{T} X_c[j(\Omega - k\Omega_s)]\,. \tag{10}$$

Frequency-domain representation of sampling, 2

- Fourier transform of $x_s(t)$ consists of periodically repeated copies of $X_c(j\Omega)$
- These copies are shifted by integer multiples of the sampling frequency.
- It is evident that

$\Omega_s - \Omega_N \geq \Omega_N$, or,

$\Omega_s \geq 2\Omega_N$

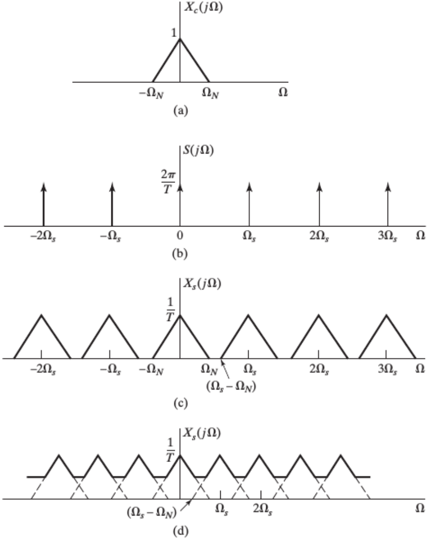


Figure 4.3 Frequency-domain representation of sampling in the time domain. (a) Spectrum of the original signal. (b) Fourier transform of the sampling function. (c) Fourier transform of the sampled signal with $\Omega_S > 2\Omega_N$. (d) Fourier transform of the sampled signal with $\Omega_S < 2\Omega_N$.

The aliasing filter is an **analog** low-pass or band-pass filter.

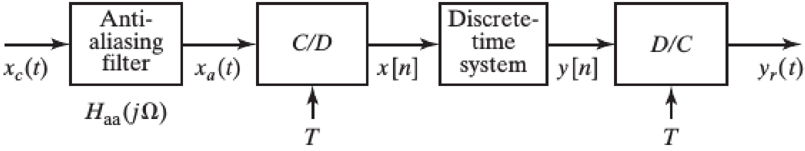
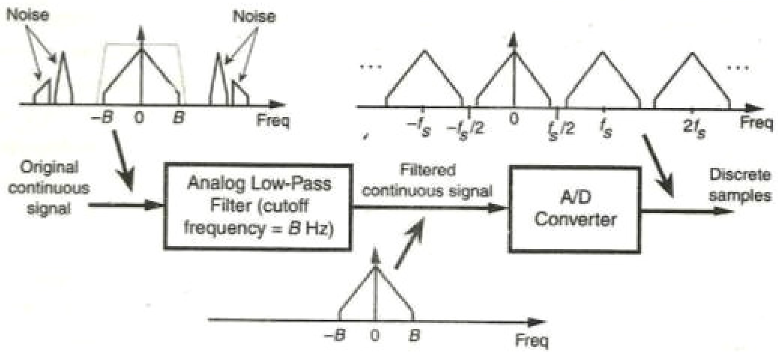


Figure 4.48 Use of prefiltering to avoid aliasing.

Even if the signal is naturally bandlimited (as music), wideband additive noise may fill in the higher frequency range, and as a result of sampling, these noise components would be aliased into the low-frequency band.



Aliasing prefiltering, example

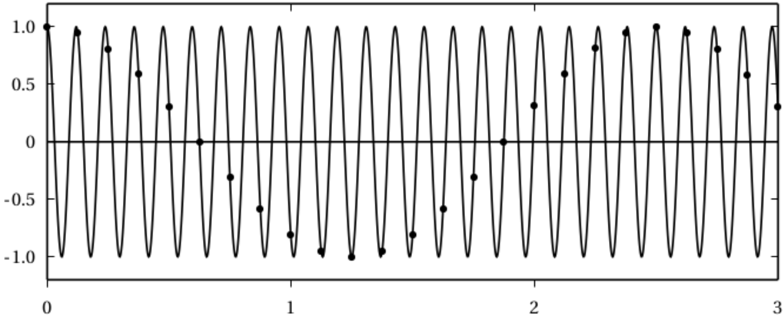


Figure 9.8 Example of aliasing: a sinusoid at 8400 Hz, $x(t) = \cos(2\pi \cdot 8400t)$ (solid line) is sampled at $F_s = 8000$ Hz. The sampled values (dots) are indistinguishable from those of at 400 Hz sinusoid sampled at F_s.

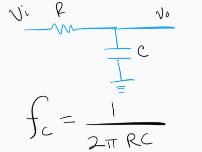


Figure: RC low-pass filter.

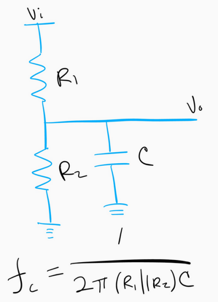


Figure: RC low-pass filter with voltage divider.

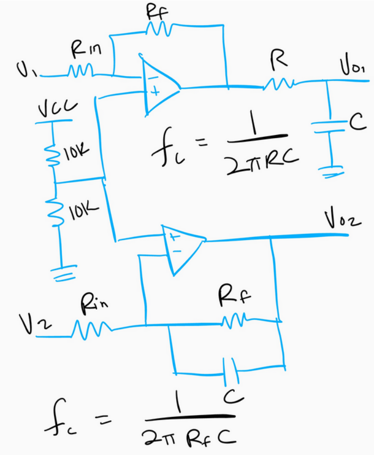


Figure: The top circuit is filtered using a passive filter while the bottom part uses an active filter.

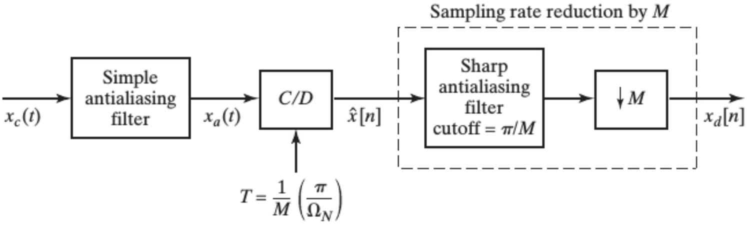


Figure 4.49 Using oversampled A/D conversion to simplify a continuous-time antialiasing filter.

Oversampling frequency response

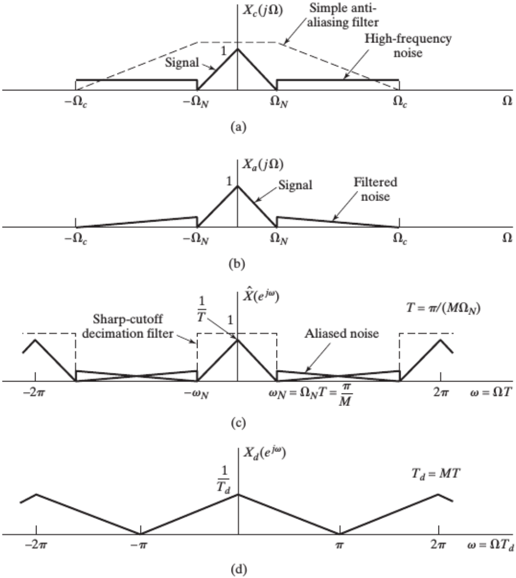


Figure 4.50 Use of oversampling followed by decimation in C/D conversion.

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