Typical stages in digital signal processing

D/A Conversion

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Summary

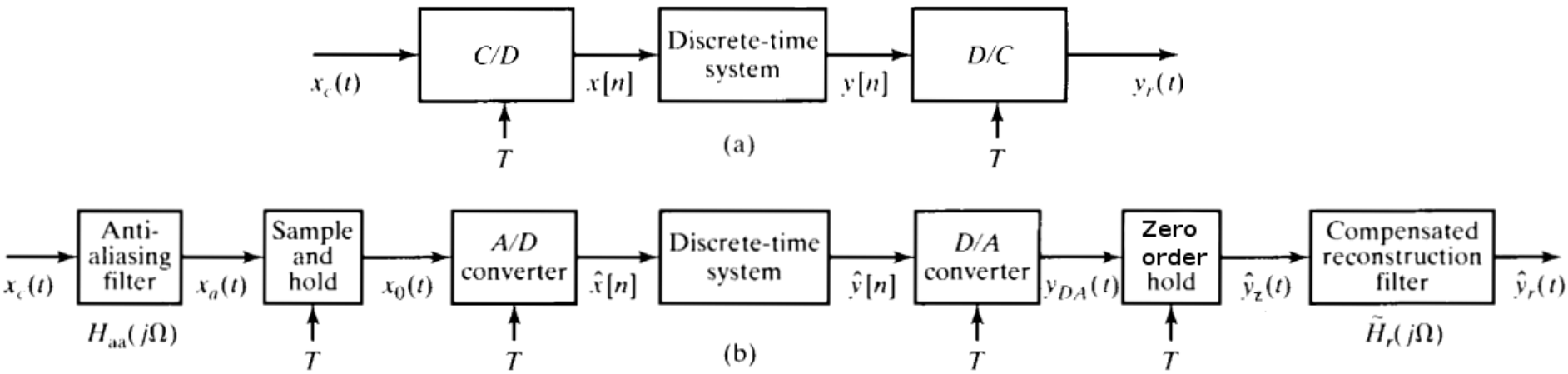
1. D/A Conversion
2. Ideal D/A Converter
3. Zero-order holder
4. Reconstruction Filter
5. Reconstruction Filtering Strategies

D/A Conversion

- The DAC (Digital-Analog Converter) reverses the ADC process.
- It converts an abstract finite-precision number (usually a fixed-point binary number) into a physical quantity (voltage).

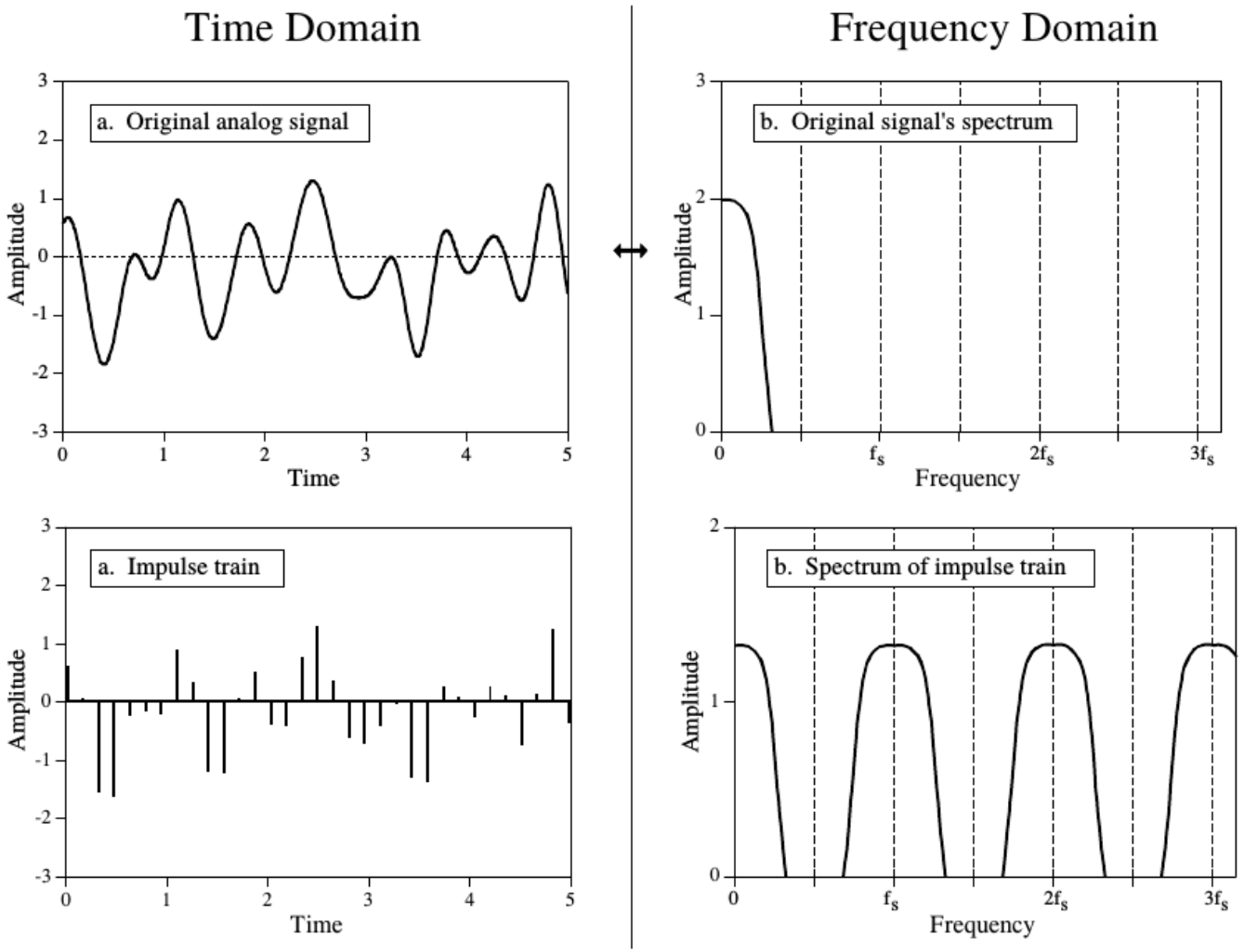
Some important DAC features:

- **Resolution**: the number of possible output levels the DAC is designed to reproduce. This is usually stated as the number of bits of the DAC.
- **Maximum sampling rate**: the maximum speed at which the DACs circuitry can operate and still produce correct output.



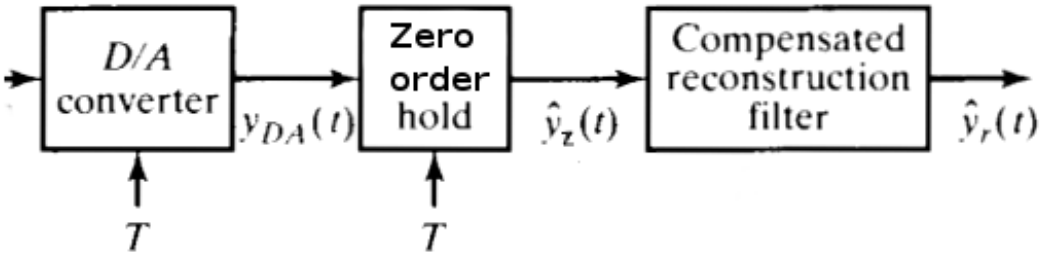
Ideal D/A Converter

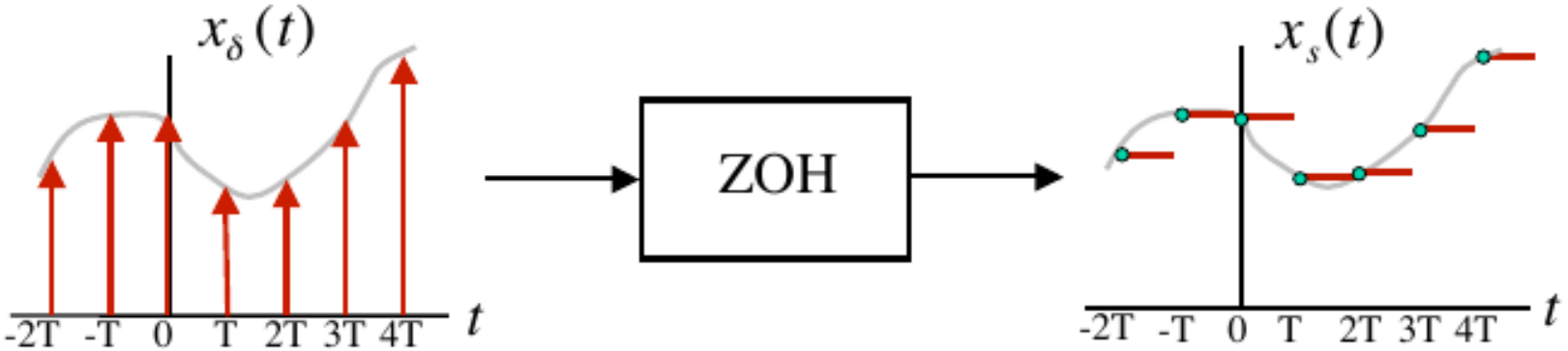
- DAC decodes the signal making a conversion from a bit sequence to an impulse train.
- The impulse train contains a duplication of analog signal spectrum.
- The original analog signal is reconstructed by passing this impulse train through a low-pass filter, with cutoff frequency equal to $f_s/2$.



Zero-order holder

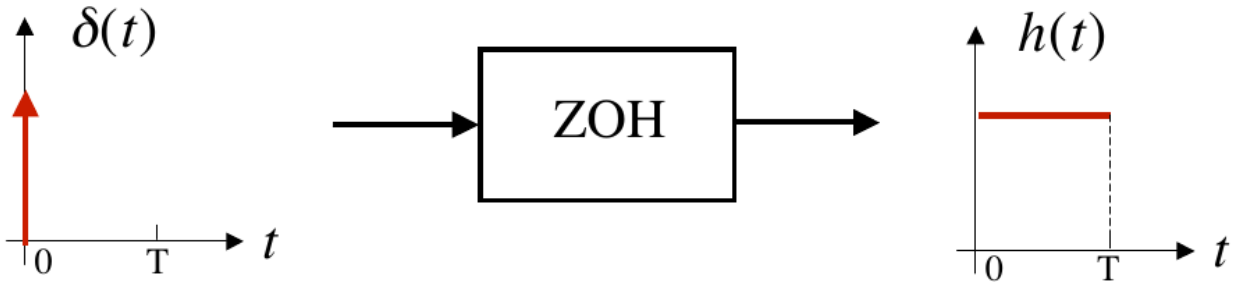
- Impulse train is a mathematical method pure.
- DACs operate by **holding** the last value until another sample is received.
- **Zero-order holder** interpolates analog values between times T, $2T$, $3T$, ..., and produces the staircase appearance.





Zero-order holder, impulse response

The impulse response of a zero-order holder.



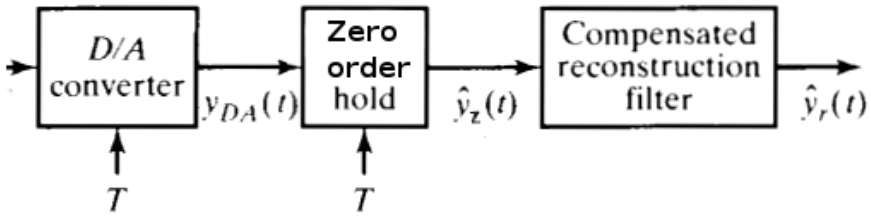
In the frequency domain:

$$H_{ZOH}(f) = \mathcal{F}\left\{ rect\left(\frac{t - T_S/2}{T_S} \right) \right\} = \frac{sin\left(\pi f / f_s\right)}{\left(\pi f / f_s\right)} = sinc\left(\pi f / f_s\right) \tag{1}$$

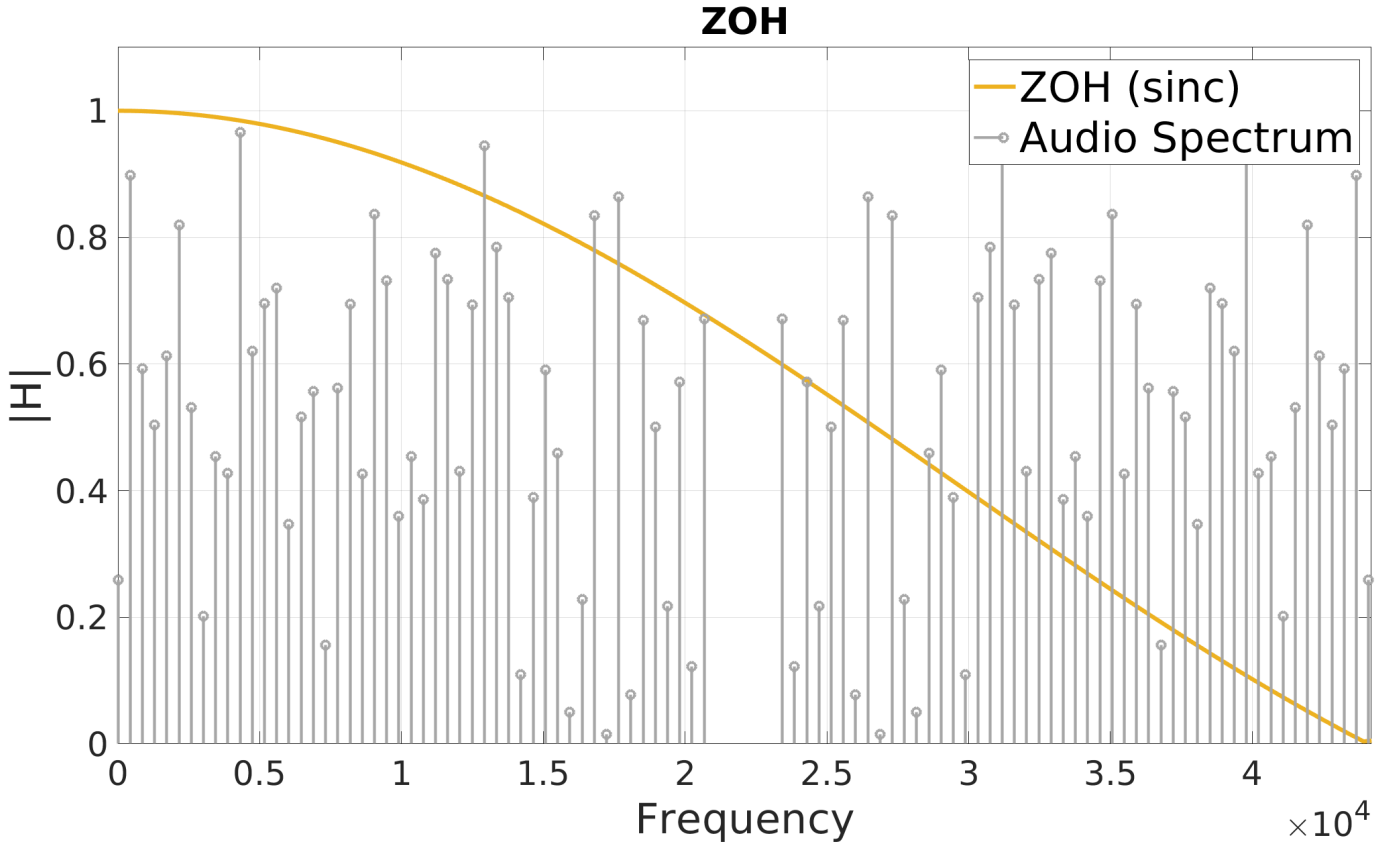
sinc: sine cardinal function.

Zero-order holder, frequency analysis

- ZOH is the convolution of the impulse train with a rectangular pulse.
- In the frequency domain, ZOH Fourier transform (sinc) is being multiplied by the impulse train spectrum.

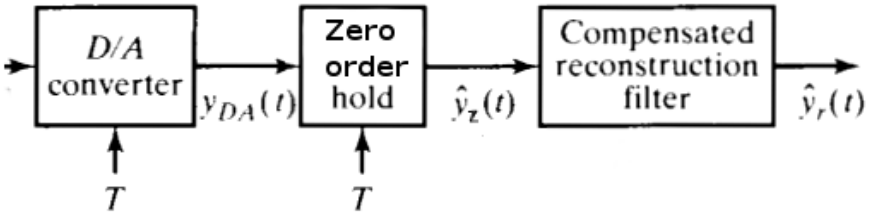


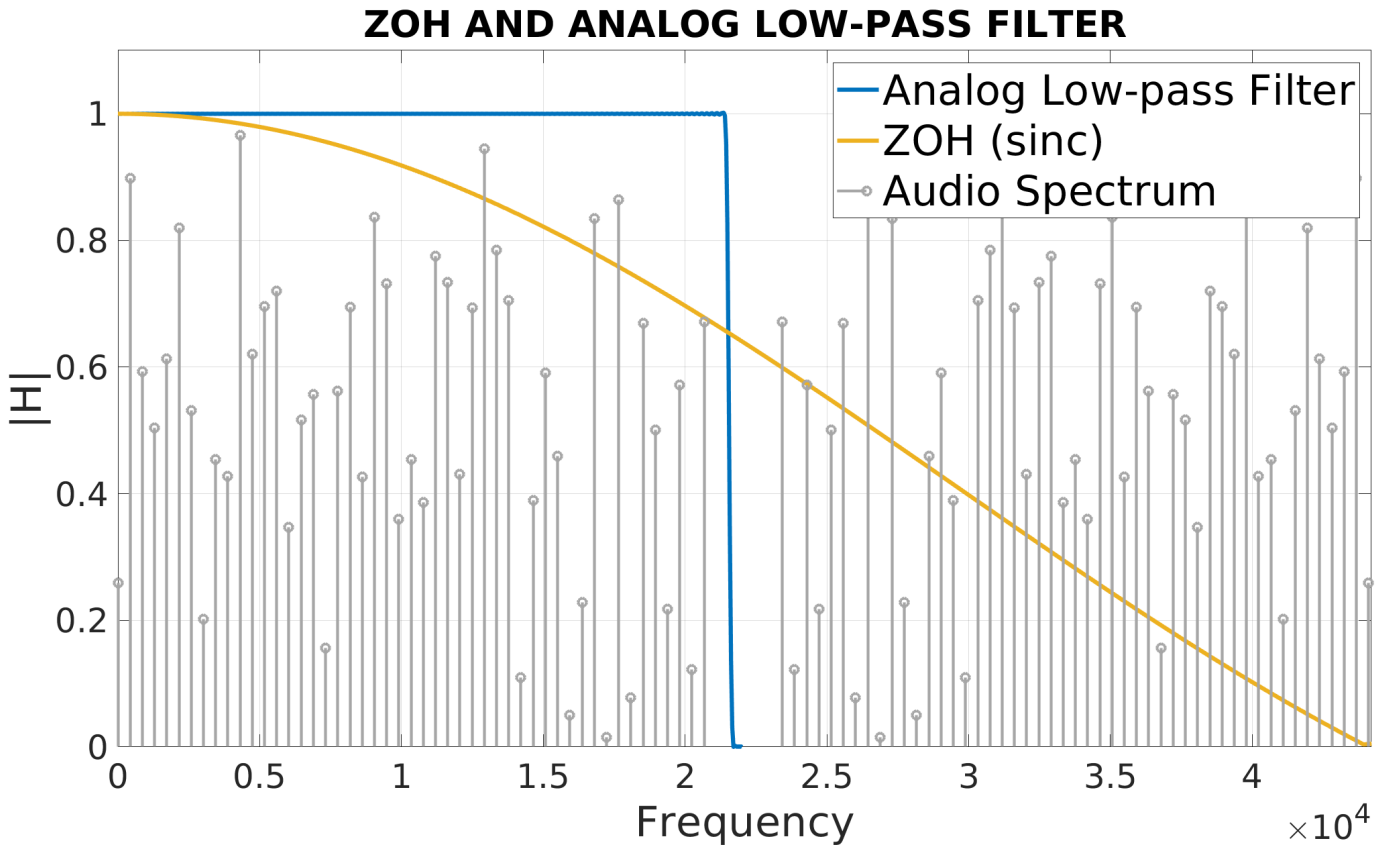
Example for audio: $f_N = 22050$ Hz, $f_s = 44100$ Hz.



Zero-order holder, frequency analysis, 2

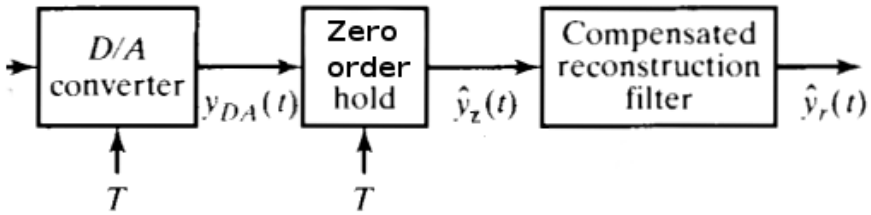
An analog low-pass filter is required to cut frequencies > 22010 Hz.

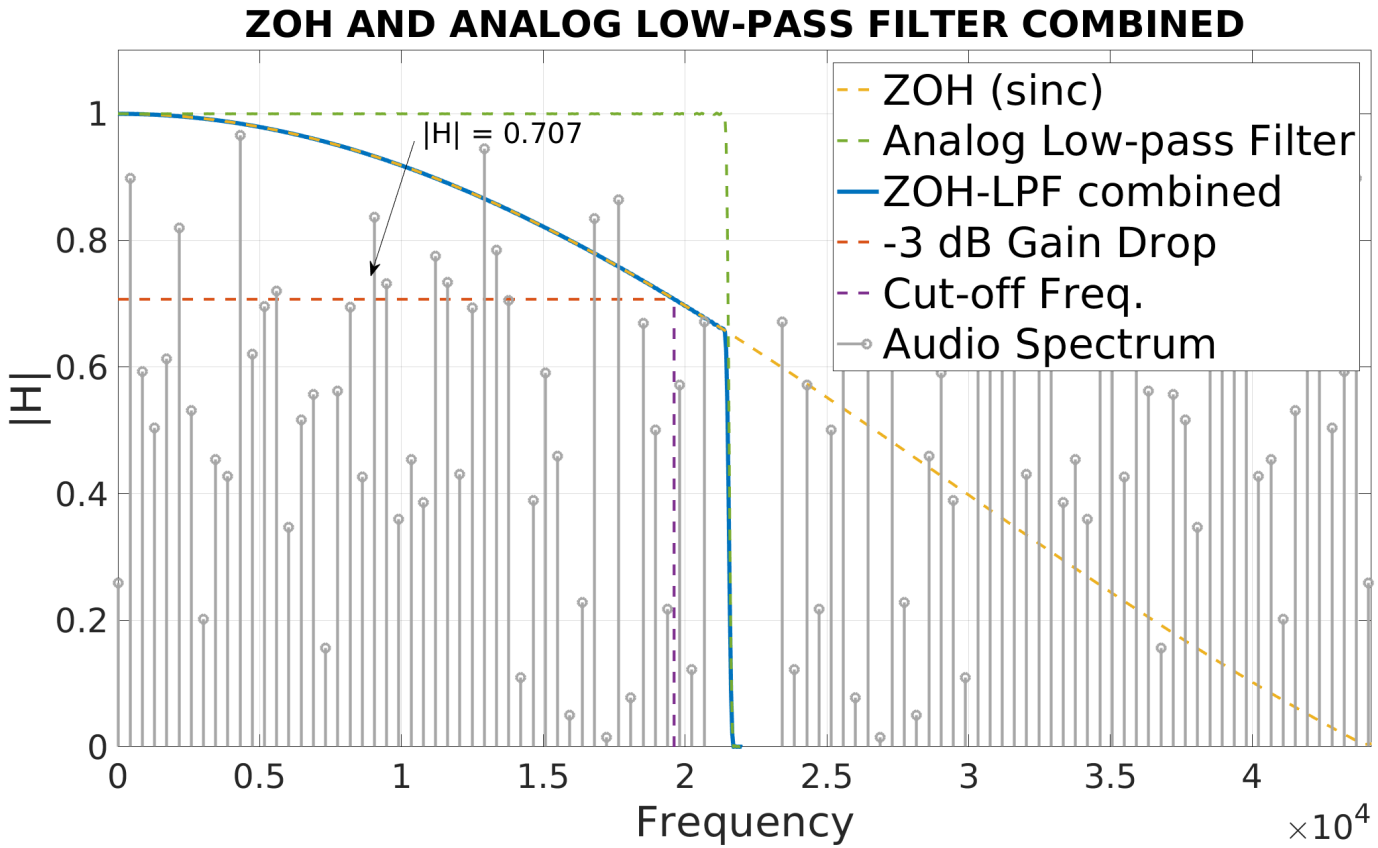




Zero-order holder, frequency analysis, 3

- The ZOH spectrum (sinc) produces a gain drop of - 3 dB.
- The cut-off frequency is decreased to $\approx$ 20 kHz.

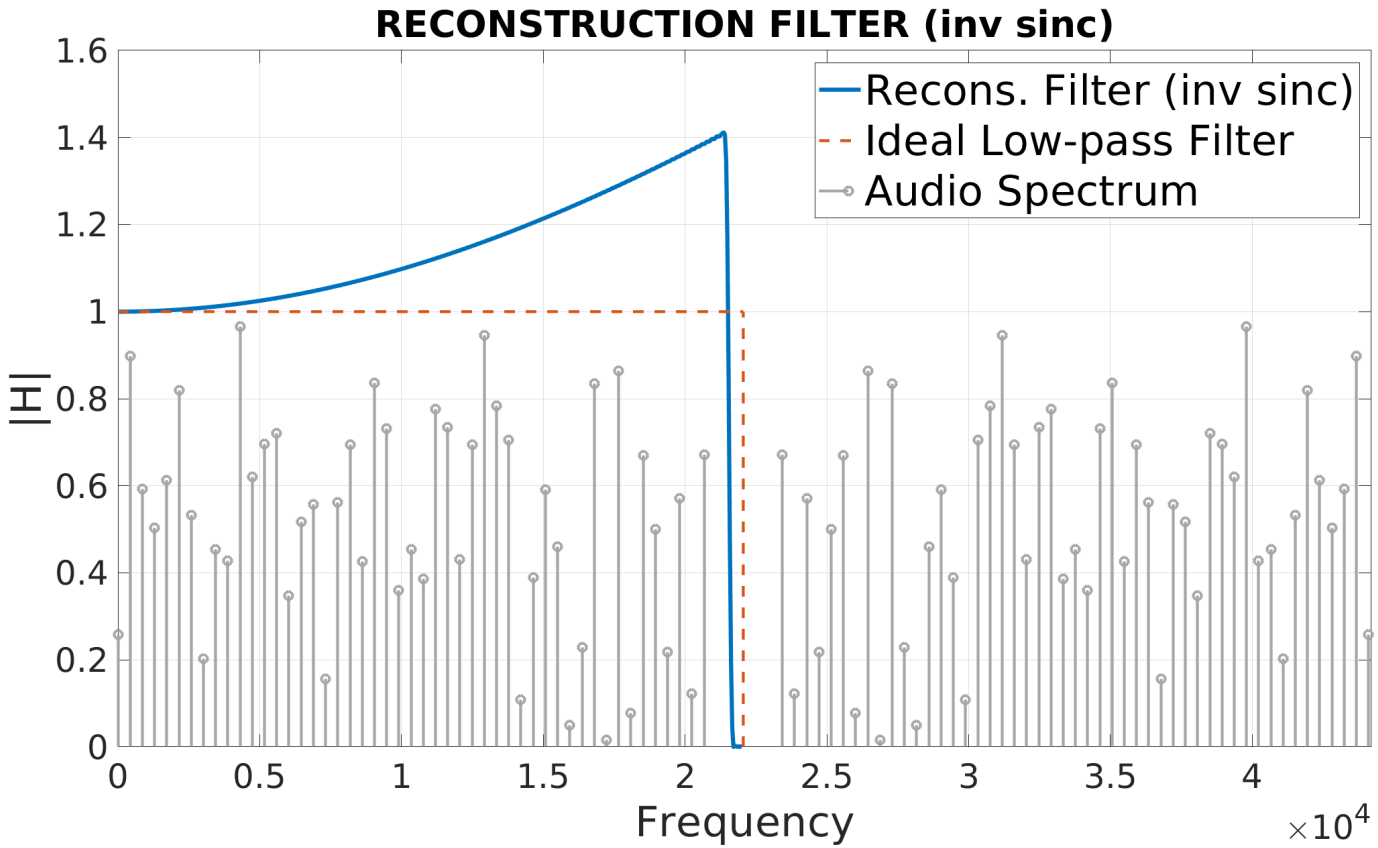




Reconstruction Filter

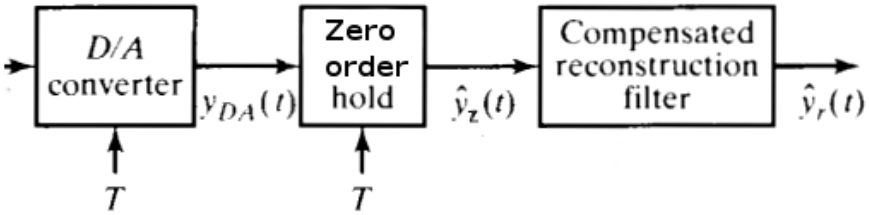
- RF removes all frequencies above $f_s/2$.
- It boosts the frequencies by the reciprocal of the zeroth-order holder's effect.
- It is also known as invert sinc filter (inv sinc).

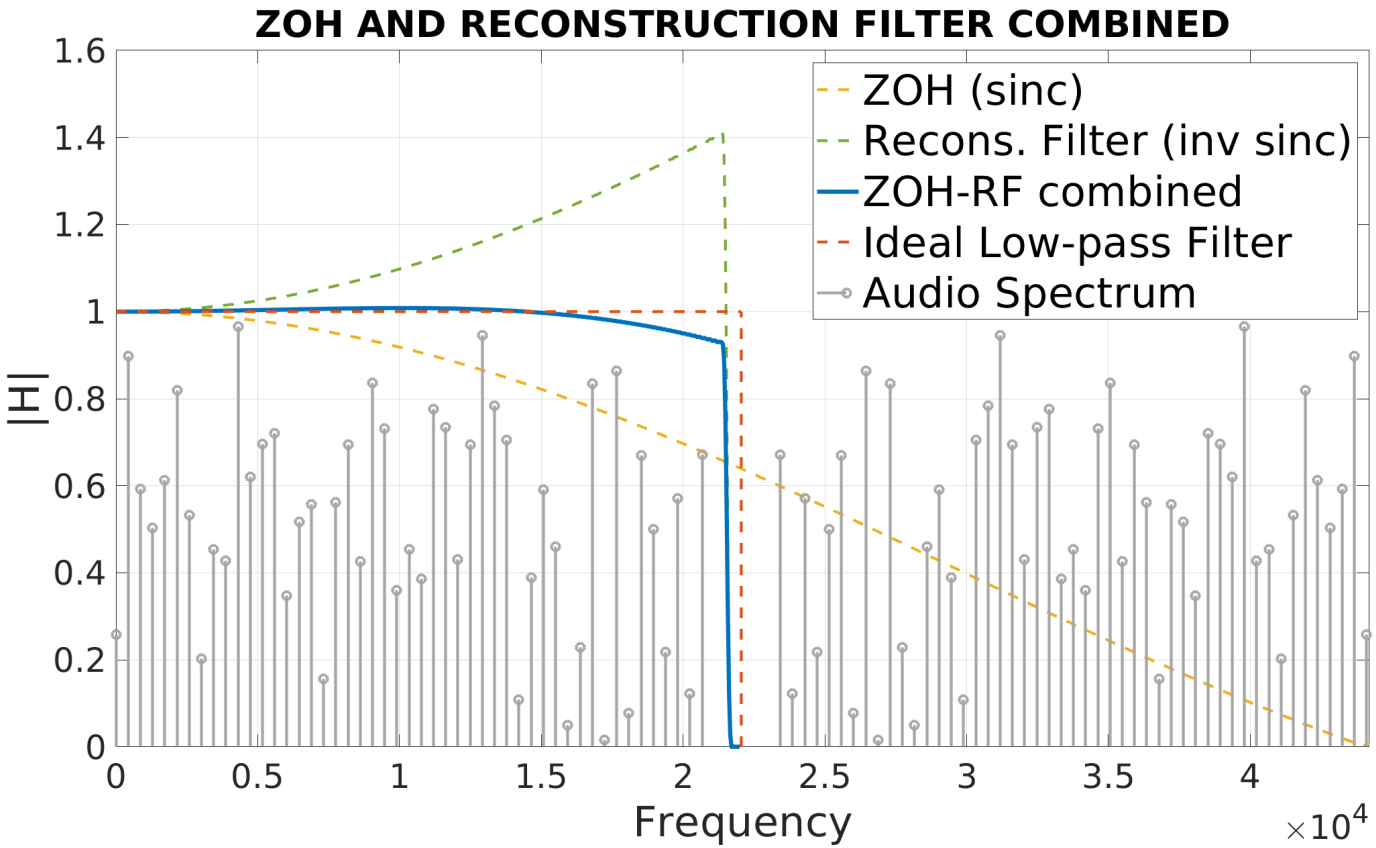
$$H_{RF}(f) = \frac{(\pi f/f_s)}{sin\,(\pi f/f_s)} = 1/sinc\,(\pi f/f_s) \tag{2}$$



Reconstruction Filter, 2

The reconstruction filter compensates the gain drop.





Reconstruction Filtering Strategies

1) **Ignore** the effect of the zero-order holder and **accept** the consequences.

2) **Pre-equalizing**: digital filter to remove the sinc effect [3].

3) **Post-equalizing**: analog filter to remove the sinc effect [3].

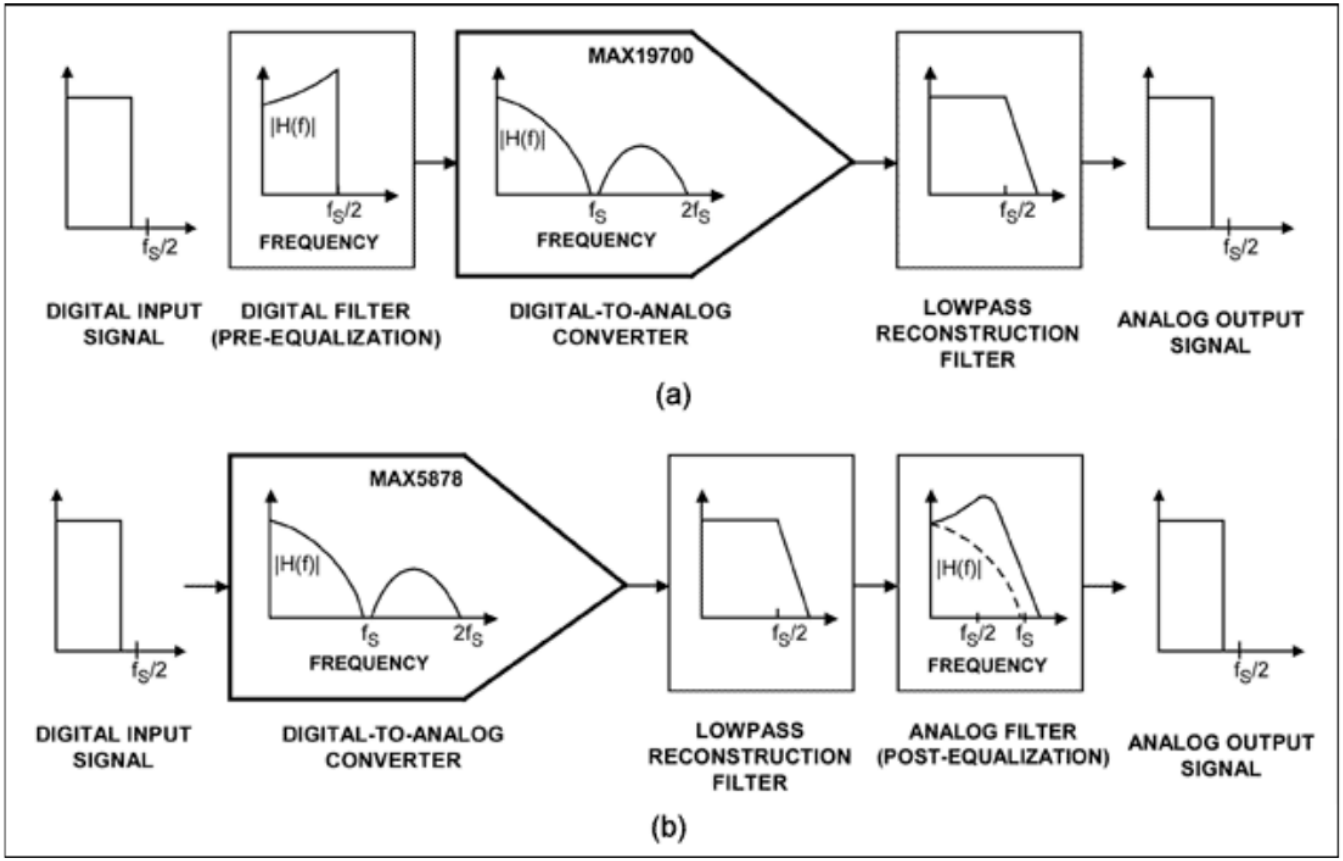
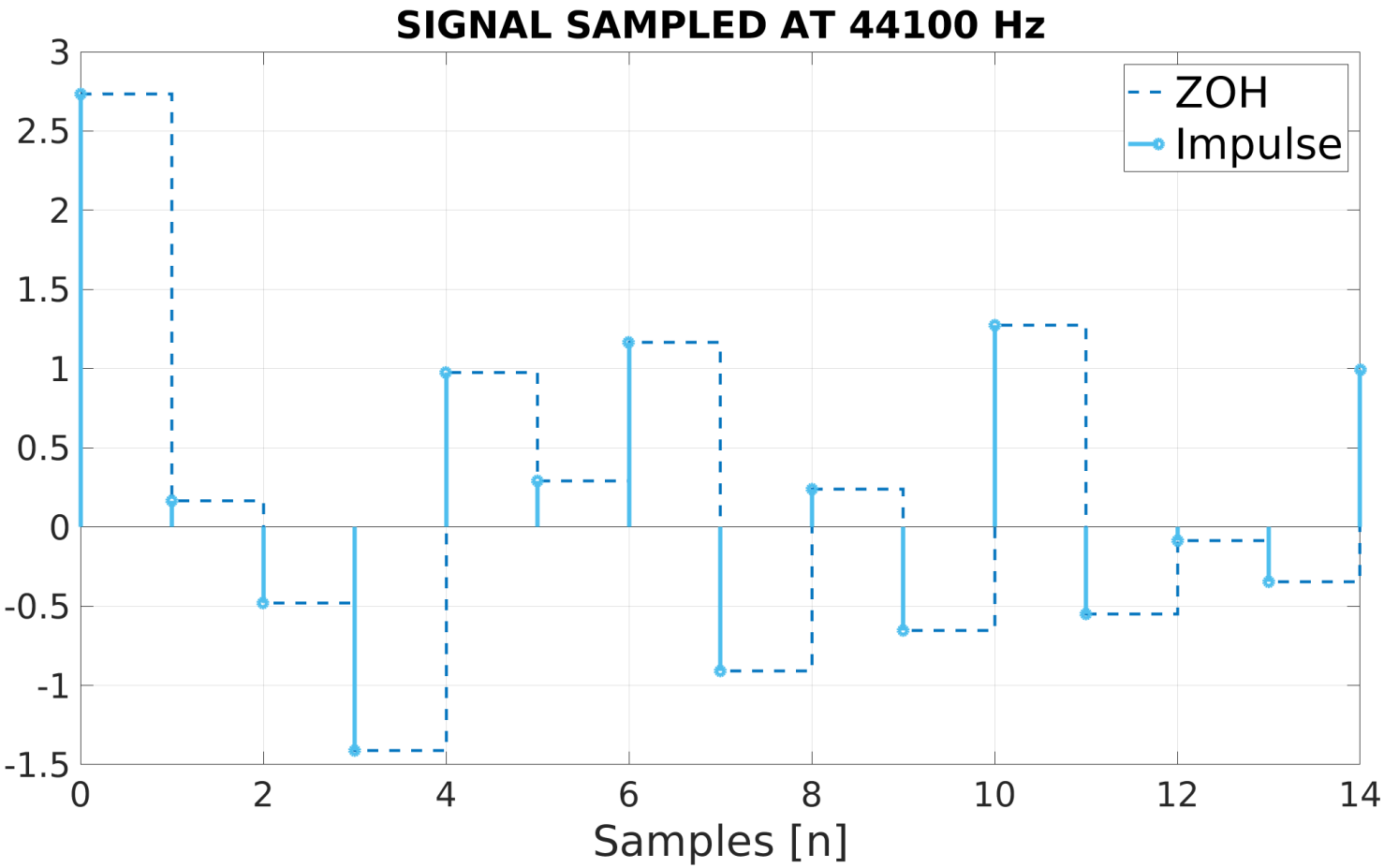


*Figure 4. A pre-equalization digital filter is used to cancel the effect of sinc rolloff in a DAC (**a**). As an alternative, you can use a post-equalization analog filter for the same purpose (**b**).*

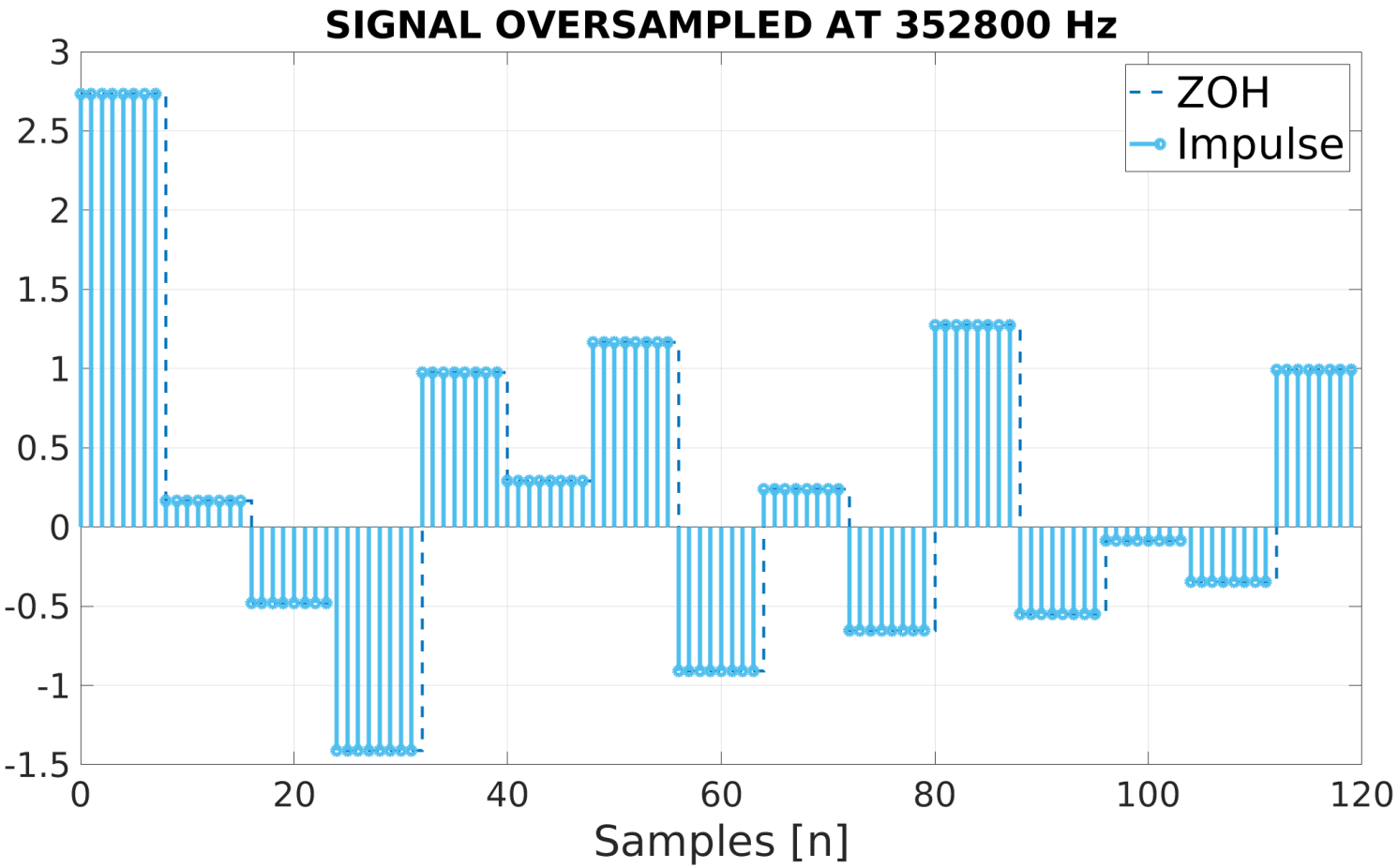
Reconstruction Filtering Strategies, Oversampling

In CD players, data sampling rate is 44.1 kHz.



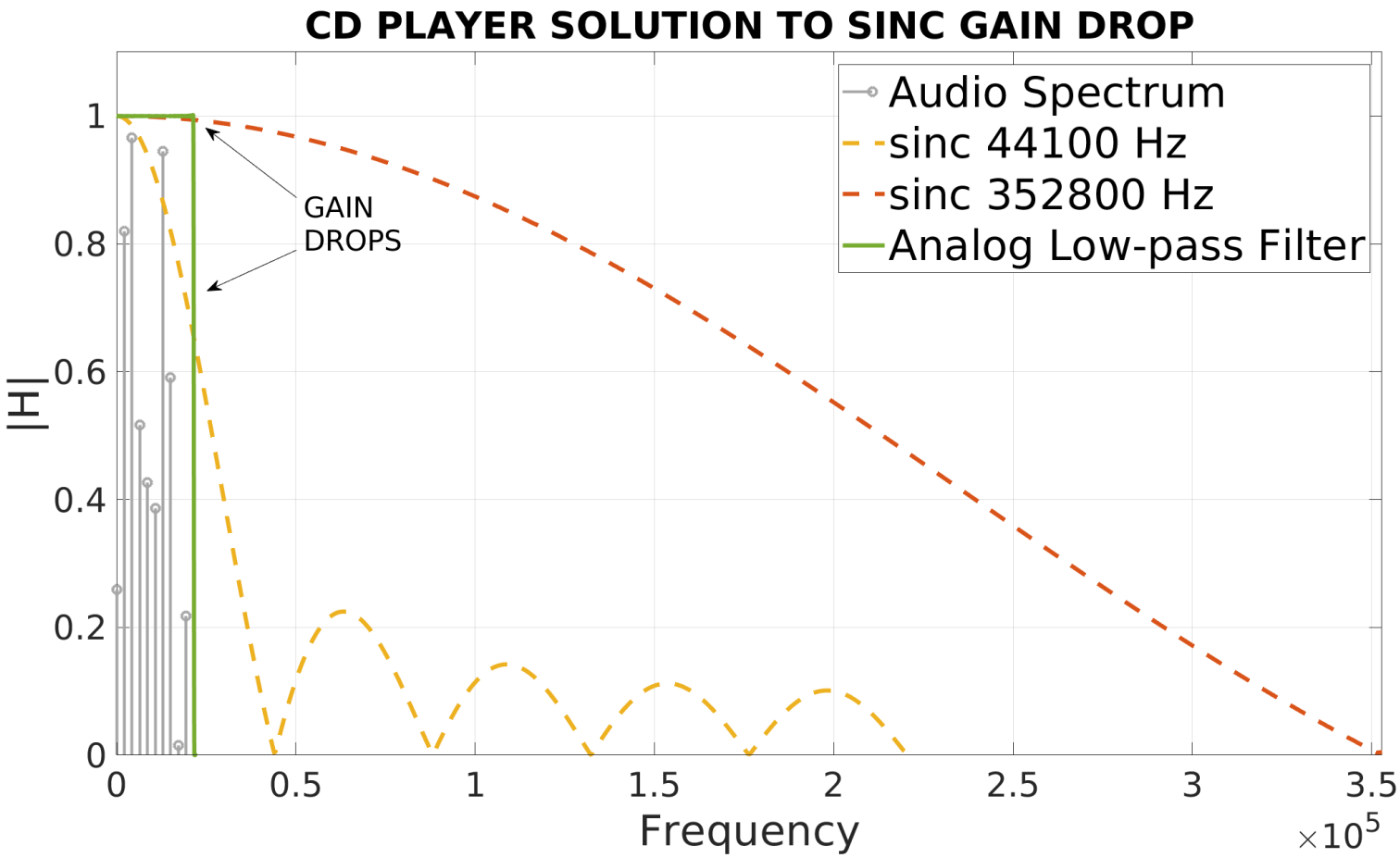
Reconstruction Filtering Strategies, Oversampling, 2

Data is oversampled by a factor of 8 to 352.8 kHz.



Reconstruction Filtering Strategies, Oversampling, 3

- At 80 % of Nyquist frequency, the output amplitude is attenuated by 2.42dB.
- Distortion for ZOH is effectively eliminated by using oversampling.



Bibliografía

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3. Maxim Integrated. Equalizing Techniques Flatten DAC Frequency Response. Application Note 3853. August 2012.