NPP parametrization of a food web

Consider the trophic level above primary producers (herbivores):

$$\frac{\dot{H}_i}{H_i} = m_i \left([\frac{g_i}{m_i} - 1] - \frac{H_i + \sum \beta_{ij} H_j}{H_i^0} \right)$$

NPP is

$$\text{NPP} = \sum_{i \in \mathcal{S}} g_i H_i$$

Where $\mathcal{S}$ is the set of S^* surviving herbivores. If μ is the mean competition strength between herbivores than we propose to parametrize the model via a knowledge of NPP. Given the natural mortality rate m_i of herbivores and H_i^0 a characteristic density then

$$\frac{g_i}{m_i} = \sqrt{\frac{1 + \mu(S^* - 1)}{S^*} \frac{\text{NPP}}{H_i^0 m_i}}$$

Or, equivalently, if we denote F_i the minimal NPP needed for the maintenance of a single population of species i then

$$\frac{g_i}{m_i} = \sqrt{\frac{1 + \mu(S^* - 1)}{S^*} \frac{\text{NPP}}{F_i}}; \; H_i^0 = F_i/m_i$$

To see why we make this choice, suppose for simplicity that $\beta_{ij} \equiv \mu < 1$ (mean field competition). Then at equilibrium

$$(1 - \mu) H_i + \mu S^* \bar{H} = \left(\frac{g_i}{m_i} - 1 \right) H_i^0$$

so

$$(1 + \mu(S^* - 1)) \bar{H} = \frac{1}{S^*} \sum_{j \in \mathcal{S}} \left(\frac{g_j}{m_j} - 1 \right) H_j^0$$

So

$$(1 - \mu) H_i = \left(\frac{g_i}{m_i} - 1 \right) H_i^0 - \frac{\mu S^*}{1 + \mu(S^* - 1)} \frac{1}{S^*} \sum_{j \in \mathcal{S}} \left(\frac{g_j}{m_j} - 1 \right) H_j^0$$

then, given our parametrization,

$$(1-\mu) \sum_{i \in \mathcal{S}} g_i H_i \approx (1+\mu(S^*-1)) \text{NPP} \left\{ 1 - \frac{\mu S^*}{1 + \mu(S^* - 1)} \times \frac{1}{S^{*2}} \sum_{i,j \in \mathcal{S}} \sqrt{\frac{m_i H_j^0}{m_j H_i^0}} \right\}$$

if we can assume that $\frac{1}{S^{*2}} \sum_{i,j \in \mathcal{S}} \sqrt{\frac{m_i H_j^0}{m_j H_i^0}} \approx 1$ t we get to $\sum_{i \in \mathcal{S}} g_i H_i \approx \text{NPP}$.