Exploring the system

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Discussion

$$H_i^0\left(\frac{g_i}{m_i}-1\right)-S\mu\,\bar{H}=(1-\mu)H_i$$

$$H_i^0\left(\frac{g_i}{m_i}-1\right)=(1+S\mu-\mu)\,\bar{H}$$

$$(1-\mu)H_i=H_i^0\left(\frac{g_i}{m_i}-1\right)-\frac{\mu}{1+S\mu-1}\sum_j H_j^0\left(\frac{g_j}{m_j}-1\right)$$

Plugging in g_i on both sides and knowing that $\mathrm{NPP}=\sum_i g_iH_i$. We get:

$$(1-\mu)\,\mathrm{NPP}=\sum_i\left[m_iH_i^0\left(\frac{g_i^2}{m_i^2}-\frac{g_i}{m_i}\right)\right]-\frac{\mu}{1+S\mu-1}\sum_{i,j}\left[m_iH_i^0\left(\frac{g_ig_j}{m_im_j}-\frac{g_i}{m_i}\right)\frac{H_i^0}{H_j^0}\right]$$

Define:

$$(*)=m_iH_i^0\left(\frac{g_i^2}{m_i^2}-\frac{g_i}{m_i}\right)$$

$$(1-\mu)\,\mathrm{NPP}=(1-\frac{\mu}{1+S\mu-1})\sum_i(*)-\frac{\mu}{1+S\mu-1}\sum_{i,j}m_iH_i^0\left(\frac{g_ig_j}{m_im_j}-\frac{g_i}{m_i}\right)\frac{H_j^0}{H_i^0}$$

Or what is the same:

$$(1-\mu)\,\mathrm{NPP}=(1-\frac{\mu}{1+S\mu-1})\sum_i(*)-\frac{\mu}{1+S\mu-1}g_iH_i^0\sum_{i,j}\left(\frac{g_j}{m_j}-1\right)\frac{H_j^0}{H_i^0}$$

Scenarios

No competition

$$\mathrm{NPP} = \sum_i m_i H_i^0 \left(\frac{g_i^2}{m_i^2} - \frac{g_i}{m_i} \right)$$

$$\mathrm{NPP} = \sum_i \left(\frac{H_i^0 g_i^2}{m_i} - H_i^0 g_i \right)$$

Since $\sum_i p_i = 1$ and $\mathrm{NPP}_i = p_i \mathrm{NPP}$:

$$\mathrm{NPP}_i = \frac{H_i^0 g_i^2}{m_i} - H_i^0 g_i = H_i^0 \left(\frac{g_i^2}{m_i} - g_i \right)$$

Rearranging into a quadratic equation:

$$H_i^0 \left(\frac{g_i^2}{m_i} - g_i - \frac{\mathrm{NPP}_i}{H_i^0} \right) = 0.$$

$$H_i^0 \left(g_i^2 - m_i g_i - m_i \frac{\mathrm{NPP}_i}{H_i^0} \right) = 0.$$

Since $H_i^0 \neq 0$:

$$g_i^2 - m_i g_i - m_i \frac{\mathrm{NPP}_i}{H_i^0} = 0.$$

Since $g_i > 0$, we select the positive root:

$$g_i = \frac{m_i + \sqrt{m_i^2 + \frac{4 m_i \, \mathrm{NPP}_i}{H_i^0}}}{2}$$

To express as $\frac{g_i}{m_i}$. IMPORTANT! Factor out m_i^2 from the square root

$$g_i = \frac{m_i + m_i \sqrt{1 + \frac{4 \, \mathrm{NPP}_i}{m_i H_i^0}}}{2}$$

Hence, since $\mathrm{NPP}_i = p_i \mathrm{NPP}$ & $m_i H_i^0 = F_i$:

$$\frac{g_i}{m_i} = \frac{1 + \sqrt{1 + \frac{4 p_i \mathrm{NPP}}{F_i}}}{2}$$

If we assume that $4 p_i \mathrm{NPP} \gg 1$ and then, the condition for $\frac{g_i}{m_i} > 1$ is:

$$p_i \, \mathrm{NPP} > \frac{F_i}{4}$$

No approximation But if we don't make the assumption $4p_i\text{NPP} \gg 1$, then for $\frac{g_i}{m_i} \leq 1$:

$$\frac{1+\sqrt{1+\frac{4p_i\text{NPP}}{F_i}}}{2} \leq 1$$

Which leads to:

$$\frac{4p_i\,\text{NPP}}{F_i} \leq 0$$

But given that $p_i \geq 0$ and $\text{NPP} \geq 0$ and $F_i \geq 0$, it's mathematically impossible that $\frac{g_i}{m_i} \leq 1$.

High g_i/m_i & No competition

For simplification, we focus on the case where $g_i \gg m_i$.

$$(1-\mu)\,\text{NPP} = (1-\frac{\mu}{1+S\mu-1})\sum_i m_i H_i^0\left(\frac{g_i^2}{m_i^2}\right) - \frac{\mu}{1+S\mu-1}\sum_{i,j} m_i H_i^0\left(\frac{g_i g_j}{m_i m_j}\right)\frac{H_j^0}{H_i^0}$$

If no competition ($\mu = 0$), we have:

$$\text{NPP} = \sum_i m_i H_i^0\left(\frac{g_i^2}{m_i^2}\right)$$

Since $\sum_i p_i = 1$ and $\text{NPP}_i = p_i\text{NPP}$:

$$\text{NPP}_i = m_i H_i^0\left(\frac{g_i^2}{m_i^2}\right)$$

$$\frac{g_i}{m_i} = \sqrt{\frac{\text{NPP}_i}{m_i H_i^0}} = \sqrt{\frac{\text{NPP}_i}{F_i}}$$

$$\frac{g_i}{m_i} = \sqrt{\frac{p_i\text{NPP}}{F_i}}$$

Thus, the condition is $p_i\text{NPP} > F_i$ where $F_i = m_i H_i^0$