Predator Implementation

Extended Model Equations

We now extend the model to include predators. The dynamics of herbivores H_i and predators P_k are given by:

Herbivores:

$$\frac{1}{H_i}\frac{dH_i}{dt} = m_i\left(\left[\frac{g_i}{m_i} - 1\right] - \frac{H_i + \sum_{j\neq i}\beta_{ij}H_j}{H_i^0}\right) - \sum_k \frac{f_{ki}P_k}{H_i}$$

I believe $\sum_k \frac{f_{ki}P_k}{H_i}$ is the way to go since the effect of H_i is embedded in the functional response $f_{ki} = \frac{a_k H_i}{1+a_k h_k H_i}$. Right?

Predators:

$$\frac{1}{P_k}\frac{dP_k}{dt} = e_k \sum_i f_{ki} - m_k$$

where:

- f_{ki} is the functional response of predator k on prey i.
- e_k is the conversion efficiency of predator k.
- m_k is the mortality rate of predator k.

Functional Response

For now, we assume a Type II functional response:

$$f_{ki} = \frac{a_k H_i}{1 + a_k h_k H_i}$$

where:

- a_k is the attack rate of predator k.
- h_k is the handling time of predator k.

Adjustments to the Herbivore Dynamics

Including predation, the herbivore dynamics become:

$$\frac{dH_i}{dt} = H_i m_i\left(\frac{g_i}{m_i} - 1\right) - \frac{H_i m_i}{H_i^0}\left(H_i + \mu\sum_{j\neq i} H_j\right) - \sum_k f_{ki}P_k$$

At equilibrium, we have:

$$H_i m_i\left(\frac{g_i}{m_i} - 1\right) - \frac{H_i m_i}{H_i^0}\left(H_i + \mu\sum_{j\neq i} H_j\right) = \sum_k f_{ki}P_k$$

Net Primary Productivity with Predators

I assume, predation redistributes energy from herbivores to predators but should not change the total NPP input into the system. **Hence, I assume the NPP consumed by herbivores should remain NPP $= \sum_{i \in S} g_i H_i$?**

Parametrization Adjustments

To account for predation in the parametrization, we can adjust the effective mortality rate of herbivores:

$$m_i^{\text{eff}} = m_i + \frac{1}{H_i} \sum_k f_{ki} P_k$$

Our parametrization of g_i can be modified to include m_i^{eff}:

$$\frac{g_i}{m_i^{\text{eff}}} = \sqrt{\frac{1 + \mu(S^* - 1)}{S^*} \cdot \frac{\text{NPP}}{H_i^0 m_i^{\text{eff}}}}$$

Thus, this adjustment should ensure that the growth rates g_i are sufficient to balance both natural mortality and predation. **Am I wrong?**