## Exploring the system

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## Discussion

$$H_i^0 \left( \frac{g_i}{m_i} - 1 \right) - S\mu \,\bar{H} = (1 - \mu)H_i$$

$$H_i^0 \left( \frac{g_i}{m_i} - 1 \right) = (1 + S\mu - \mu) \,\bar{H}$$

$$(1 - \mu)H_i = H_i^0 \left( \frac{g_i}{m_i} - 1 \right) - \frac{\mu}{1 + S\mu - 1} \sum_j H_j^0 \left( \frac{g_j}{m_j} - 1 \right)$$

Plugging in  $g_i$  on both sides and knowing that NPP =  $\sum_i g_i H_i$ . We get:

$$(1 - \mu) \text{ NPP} = \sum_{i} \left[ m_i H_i^0 \left( \frac{g_i^2}{m_i^2} - \frac{g_i}{m_i} \right) \right] - \frac{\mu}{1 + S\mu - 1} \sum_{i} \sum_{j} \left[ m_i H_i^0 \left( \frac{g_i g_j}{m_i m_j} - \frac{g_i}{m_i} \right) \frac{H_i^0}{H_j^0} \right]$$

Define:

$$(*) = m_i H_i^0 \left( \frac{g_i^2}{m_i^2} - \frac{g_i}{m_i} \right)$$

$$(1 - \mu) \text{ NPP} = \left( 1 - \frac{\mu}{1 + S\mu - 1} \right) \sum_i (*) - \frac{\mu}{1 + S\mu - 1} \sum_{i,j} m_i H_i^0 \left( \frac{g_i g_j}{m_i m_j} - \frac{g_i}{m_i} \right) \frac{H_i^0}{H_j^0}$$

Or what is the same:

$$(1 - \mu) \text{ NPP} = \left(1 - \frac{\mu}{1 + S\mu - 1}\right) \sum_{i} (*) - \frac{\mu}{1 + S\mu - 1} \frac{g_j}{m_j} H_i^0 m_i \sum_{i,j} \left(\frac{g_j}{m_j} - 1\right) \frac{H_i^0}{H_j^0}$$

Assuming  $\sum_{i} Sp_i = 1$ , then  $\sum p_i$  NPP is equal to NPP, where p is a vector.

Finally, with no competition  $(\mu = 0)$ , we have:

$$\frac{g_i}{m_i} = 1 + \sqrt{\frac{4p_i \, \text{NPP}}{F_i}}$$

So you need:

$$p_i \text{ NPP} > \frac{F_i}{4} \text{ where } F_i = m_i H_i^0$$

## Continuation