A General Measure of Bargaining Power for Non-Cooperative Games

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Abstract

Despite recent advances, no general methods for computing bargaining power in non-cooperative games exist. We propose a number of axioms such a measure should satisfy and show that they characterise a unique function. The principle underlying this measure is that the influence of a player can be assessed according to how much changes in this player's preferences affect equilibrium outcomes. Considering specific classes of games, our approach nests existing measures of power. We present applications to cartel formation, the non-cooperative model of the household, legislative bargaining as well as to Nash-in-Nash bargaining.

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1 Introduction

Bargaining power and its sources have long interested economists and social scientists more generally. Examples include bargaining between buyers and sellers (Dunlop & Higgins 1942, Taylor 1995, Loertscher & Marx 2022), cartel members (Napel & Welter 2021), employers and labour unions (Hamermesh 1973, Svejnar 1986, Manning 1987), husband and wife (Basu 2006, Browning et al. 2013, Anderberg et al. 2016), the members of a political alliance (Diermeier et al. 2003, Francois et al. 2015), or legislators (Snyder et al. 2005, Kalandrakis 2006, Napel & Widgrén 2006, Ali et al. 2019, Nunnari 2021). In cooperative game theory, a vast literature deriving power indices exists with the Shapley-Shubik index (Shapley & Shubik 1954) and the Penrose-Banzhaf index (Penrose 1946, Banzhaf 1965) being the most famous examples. Cooperative game theory, however, does not model the process through which players interact and thus is not able to answer questions such as how the bargaining power of a player depends on their ability to make a counter offer, delay agreement, or veto certain outcomes. In non-cooperative game theory, on the other hand, the structure of the interaction between players forms an explicit part of a game, but in this context much less effort has been invested in developing measures of power. A common approach is to assume complete information, transferable utility, and self-interested players, in which case bargaining reduces to the division of a fixed surplus. In such settings, which we will refer to as surplus-division games (or SD games for short), power can be measured by the expected share of the surplus that each participant receives. But if utility is non-transferable or at least one player feels some degree of altruism, the utility a player achieves in equilibrium need not be informative about this player's bargaining power. To see this, consider the following example: Three countries form a military alliance and need to decide how to respond to foreign aggression. Country A is hawkish, Country B is dovish, and Country C prefers a measured response. If the agreed policy coincides with that favoured by Country C, it is not clear whether this outcome is due to the dominance of Country C or represents a compromise between countries A and B. How can we quantify the bargaining power of each country?

In this paper, we provide a measure of bargaining power that can be applied to any non-cooperative game of bargaining, including games of incomplete information, but also to mechanisms or even social choice functions. As the above example shows, the outcome of the game alone may not fully reveal each players' bargaining power. The fundamental idea underlying our approach is that we can instead calculate a player's power based on the effect of hypothetical changes in this player's preferences, holding all other aspects of the game fixed. In the case of the military alliance, for example, we can consider what would happen to the agreement if country C was dovish or hawkish instead of moderate. If country C

has little influence, a change in this country's preferences would leave the outcome largely unchanged. If country C is very powerful, on the other hand, the outcome would always remain close to the one favoured by country C.

Before providing further specifics, it is useful to clarify what we mean by power. We follow Max Weber in defining power as "the probability that one actor within a social relationship will be in a position to carry out his own will despite resistance, regardless of the basis on which this probability rests" (Weber 1947, p. 152). Several aspects of this definition are worth highlighting: First, as in the example of the military alliance, getting what you want per se does not constitute power, unless this occurs against the will of others. Second, the definition focuses on an actor's de facto ability to achieve an outcome, without regard to the distribution of formal decision making rights. This focus on outcomes aligns well with how economists generally treat power, for instance when equating power with a player's surplus share. Finally, this definition of power is broad and subsumes different ways of achieving a desired outcome such as authority, coercion or persuasion.

Our approach of shifting a player's preferences is well-suited to assess power as defined above. If the preferred outcome of country B in the initial example changes but the actual outcome remains the same, this reveals that in the initial equilibrium country B got what it wanted through the will of others rather than in spite of it. More generally, the size of the effect of the shift in preferences provides exactly the information we need to quantify country B's de facto power. The simplicity of this approach notwithstanding, any number of measures of bargaining power can be constructed on its basis. To guide our choice between these measures, we specify four Axioms that such a function should satisfy. These axioms reflect the basic principle outlined above: The Axiom of Null players, for instance, states that a player should be assigned a bargaining power of zero if changes in their utility function never have any effect on the outcome of a game. The Axiom of Local Dictators, on the other hand, posits that some player n should be assigned a power of one if, starting from the vector of players' actual utility function, any shift in the utility function of player n produces the same outcome as if all other players' preferences were aligned with those of player n. To specify the third axiom, we introduce the concept of a compound game, which is a lottery that determines the game to be played. The Axiom of Compound games states that the bargaining power assigned to a player in a compound game should be a weighted average of the bargaining power in each constituent game. The final axiom requires for a broad class of games that no player can be assigned a bargaining power greater than that of a local dictator.

Our main result establishes that these four axioms characterise a unique function with a clear interpretation: the bargaining power of player n is calculated based on how much the outcome of the game is affected if the utility function of player n is replaced with that of the player most opposed to them, with the actual utility function of player n serving as a metric that quantifies the size of the impact. The effect of the shift in player n's preferences is then expressed relative to the one that would occur if this player was a local dictator. Bargaining power calculated in this way thus answers the question of how much a player is able to influence the outcome of the game compared to a local dictator.

Though most useful in other settings, it is instructive to examine the properties of our measure when applied to SD games. We establish conditions under which our measure is equal to the expected share of the total surplus a player receives in equilibrium and thus equivalent to the conventional approach to calculating bargaining power in this context. Whereas the two approaches often coincide, they can also produce notably different results as illustrated by the following example: Suppose there are two players who need to divide a cake and each player's utility is given by their share. With probability .9 the whole cake is given to player 1 and the game ends. With the remaining probability, player 2 is given the opportunity to propose a split. If player 1 accepts such an offer, the split proposed by player 2 is implemented. If player 1 rejects, both players receive nothing. In the unique subgame perfect equilibrium of this game, player 2 proposes to keep the whole cake and player 1 accepts. The share of the cake (and of the available surplus) that player 1 receives in expectation is therefore equal to .9. However, the preferences of player 1 do not matter for the outcome. For example, the outcome of the game would not change even if player 1 preferred to give all of the cake to player 2. Given that our measure is based on the degree to which changes in a player's preferences lead to changes in the outcome, it assigns player 1 a bargaining power of zero rather than 0.9. Player 1 is likely to get their will, but they do not impose it.

The bargaining power our measure assigns to a player is conditional on players' preferences, which is in line with the well-known fact that aspects of preferences, such as impatience or risk aversion, can matter for a player's ability to achieve favourable outcomes. It can also be of interest to abstract from preferences and evaluate power as determined by the rules of the game only, for example when designing institutions before players' preferences are known. Such an ex ante measure of power can be constructed based on our ex post measure by specifying a distribution that players' preferences are drawn from and then calculating expected ex post power under said distribution. When applied to weighted voting games, we show that under suitable choices of the distribution of players' preferences the ex ante version of our measure reproduces the Shapley-Shubik index and the Penrose-Banzhaf index.

We provide four additional applications of our theory, the first of which is cartel formation. If firms are unable to make transfers between cartel members due to the risk of being caught out, firms may negotiate over individual production quantities. Knowing the influence that each firm had on the agreement can provide a basis for apportioning compensation in case of conviction, for instance. We show that under mild assumptions our measure of bargaining power takes a particularly simple form in this setting and becomes equal to a firm's profit in equilibrium divided by the profit this firm would achieve if it was a monopolist. With asymmetric costs or demand elasticities the latter number may differ widely between cartel members. Even a firm with a small market share may thus turn out to wield considerable influence.

The second application we consider is intra-household decision-making. The literature of the economics of the household has an intrinsic interest in the distribution of power between husband and wife and its underlying determinants. While the collective model of the household features explicit bargaining weights, in non-cooperative models power is an implicit product of the entire environment. Our measure can be used to quantify bargaining power in this setting and reveal the driving factors through comparative statics. We illustrate this in the context of a model analysed by Bertrand et al. (2020) and show that even a small gender wage gap can lead to a significant difference in the bargaining power of husband and wife.

As a third example, we examine bargaining power in the context of the legislative process of the European Union. This complex process involves three institutions and various stages, making it difficult to judge each participant's relative influence. Our results indicate that, contrary to public perception, the European Commission is less powerful than the European Council and the European Parliament. We use the example of the Commission to illustrate how applying our measure to slightly modified extensive forms can reveal which aspects of the rules of the game give a player more or less influence. We show that the Commission's veto is a key source of influence, while the ability to formulate the first proposal is not. Such insights are valuable for institutional design where achieving a balanced distribution of power often is important.

Further applications of interest abound. For instance, our measure can be applied to Nash-in-Nash bargaining (Horn & Wolinsky 1988, Collard-Wexler et al. 2019), which has recently been used extensively in applied work (Crawford & Yurukoglu 2012, Gowrisankaran et al. 2015, Ho & Lee 2017, Crawford et al. 2018). Bagwell et al. (2021) estimate a Nash-in-Nash model of WTO negotiations in order to study how different bargaining protocols affect outcomes. The parameters they estimate include bargaining weights for bilateral negotiations. Our measure could be applied to the estimated model to assign each country an overall bargaining power and to study how bargaining power is affected by changes to institutional rules. In our final application, we show that the relationship between a player's bargaining weights in Nash-in-Nash bargaining and their bargaining power is not straightforward: due to externalities between bilateral negotiations, increasing a player's bargaining weight in one negotiation can even lower the player's overall bargaining power.

The remainder of this paper is organised as follows: In Section 2, we place our study in the context of the literature. Section 3 derives our measure of bargaining power and explores its properties. Some extensions of the basic theory are introduced in Section 4. Section 5 presents applications, while Section 6 concludes.

2 Related Literature

Our main contribution to the literature is to provide a method for calculating the bargaining power of a player that can be applied to any non-cooperative model of bargaining. In cooperative game theory, a vast literature exists that develops power indices for so-called simple games with a particular interest in voting games (see, for example, Penrose 1946, Shapley & Shubik 1954, Banzhaf 1965, Deegan & Packel 1978, Johnston 1978, Holler 1982, Owen & Shapley 1989). Since a non-cooperative game can generally not be expressed as an in some sense equivalent cooperative game,[1] there is no general way to apply power indices intended for cooperative games to non-cooperative games. In non-cooperative game theory, by contrast, the only approach to measuring power that is widely applied is to assume complete information, transferable utility and selfish players, in which case power can be measured by the share of the total surplus a player receives (Taylor 1995, Haller & Holden 1997, Kambe 1999, Fréchette et al. 2005, Snyder et al. 2005, Kalandrakis 2006, Ali et al. 2019). Yet, transferable utility is a strong assumption since it requires that players have access to a common currency with constant marginal utility (Myerson 1991, p. 384). When utility is non-transferable or information is incomplete, it is in some cases possible to express the equilibrium of the bargaining game as a weighted average of each player's most preferred outcome, either in terms of physical outcomes or in terms of utilities. In games with more than two players such weights are often not unique, however, as in the example of the military alliance we provide in the introduction. Larsen & Zhang (2025) follow this approach to derive a measure of bargaining power for two-player games.[2] Their measure assigns a player a high bargaining power if their utility is close to their best-possible outcome. The same is not

[1] Papers that connect cooperative and non-cooperative game theory typically seek to provide a non-cooperative justification for a cooperative solution concept by finding a specific non-cooperative game that generates the same distribution of payoffs. See, for example, Hart & Mas-Colell (1996), Krishna & Serrano (1996) and Laruelle & Valenciano (2008).

[2] The focus of Larsen & Zhang (2025) is on mechanisms. Relatedly, Basteck & Lojkine (2025) develop measures of freedom and power specifically for mechanisms.

necessarily true for our measure, as illustrated by the example in the introduction where player 1 is given a high share of the surplus regardless of their choices and thus assigned a bargaining power of zero.

Steunenberg et al. (1999) develop a power measure for games where players' utilities are a function of the distance between the outcome and their ideal point. They assume a distribution that players' preferences and the status quo are drawn from and that the power of a player is inversely proportional to the average distance between their ideal point and the outcome across all possible draws. This procedure cannot calculate power conditional on a specific constellation of preferences.

Napel & Widgrén (2004) introduce the idea of measuring power based on shifts in players' preferences. They propose a measure for games with a one-dimensional outcome space and suggests different ways in which their approach can potentially be generalised. While our measure can be applied to a wider set of games, another key difference between our approach and theirs is that Napel & Widgrén focus on marginal shifts in preferences, while we consider large shifts. A drawback of marginal shifts is that they may not reveal the full extent of a player's influence. To see this, consider the following example: Two players need to agree on a point on the real line. Each players' utility is equal to minus the distance between the chosen point and their ideal point. The ideal point of player 1 is equal to 1, that of player 2 equal to 2, and there is a status quo given by 2.5. The game simply consists in player 1 making a take-it-or-leave-it offer to player 2. Player 2 only accepts if the offer is weakly above 1.5 and player 1 thus offers 1.5. A marginal shift in the ideal point of player 1 leaves the outcome unchanged and the measure of Napel & Widgrén thus assigns player 1 a bargaining power of zero. However, player 1 clearly has an influence on the outcome of the game. Our measure assigns both players a bargaining power of .5.

We thus go beyond the existing literature by providing a new measure of bargaining power, which is the first measure that can be applied to any non-cooperative game of bargaining. Furthermore, we provide the first axiomatization of a measure of bargaining power in the field of non-cooperative game theory. While the measure is novel, the underlying conception of power aligns well with the existing literature in that other measures of power equally focus on a player's de facto ability to impose a desired outcome.

3 A Measure of Bargaining Power

In this section we present our approach to measuring bargaining power. We start by formally defining the setting in which we develop our theory.

3.1 Theoretical Framework

We start by introducing notation that is convenient for our purpose to describe what is otherwise a standard game. Let $\Gamma = (\mathcal{N}, \mathcal{T}, O, \mathbf{u})$ be an extensive form game. $\mathcal{N}$ denotes the set of players with $N = |\mathcal{N}|$ and $2 \leq N < \infty$. $\mathcal{T}$ is the "game tree", which we use here in a broader sense than is typically the case to refer to a full description of the order of moves, including those by nature, and the information structure of the game. The set of all possible outcomes of the game is given by O and contains at least two elements, that is, $|O| \geq 2$. The preferences of player n over the set O are represented by a utility function u_n and $\mathbf{u}$ is the vector of all players' utility functions. The set $\mathcal{U}$ collects all distinct utility functions contained in $\mathbf{u}$.

From an ex ante perspective, an equilibrium of Γ generates a probability distribution over outcomes due to possible moves of nature or mixed strategies. We assume there exists a function μ^* that maps vectors of utility functions $\mathbf{u} \in \mathcal{U}^N$ into probability measures over the set of outcomes O, holding all other elements of Γ fixed. This assumption is satisfied if the equilibrium of Γ is always unique, possibly subject to some method of equilibrium selection. We provide an extension to games with multiple equilibria in Section 4.2.[3]

The indirect utility function of player n is defined as the expected utility of the player under the equilibrium distribution $\mu^*(\mathbf{u})$ over outcomes, that is,

$$v_n(u_n, \mathbf{u}) = \int_O u_n(o) \; d\mu^*(\mathbf{u}) \; .$$

Given that the measure μ^* corresponds to the probability distribution over outcomes at the beginning of the game prior to any moves of nature, the indirect utility of a player represents their ex ante utility.

Note that the utility function of player n appears twice in the definition of the indirect utility function: once explicitly and once as part of the vector $\mathbf{u}$. Importantly, we do not require these utility functions to coincide. The indirect utility function can thus be used to evaluate how some player n would feel about "hypothetical" outcomes that would occur if their utility function contained in $\mathbf{u}$ was different from their actual utility function. We henceforth refer to the vector $\mathbf{u}$ contained in the definition of the game Γ as players' "endowed" utility functions. To avoid confusion, we follow the convention that u_n always refers to the endowed utility function of player n and $\mathbf{u}$ to the vector of endowed utility functions, while symbols such as u' or $\mathbf{u}'$ denote arbitrary (vectors of) utility functions drawn from the set $\mathcal{U}$. Since we never consider indirect utilities where the first argument is different from

[3] We abstract from issues such as equilibrium existence or measurability, which may require additional restrictions on utility functions in practice.

player n's endowed utility function, we simplify notation by suppressing dependence on the first argument and simply write $v_n(\mathbf{u})$.

We refer to the indirect utilities that arise if all players were to share the same preferences as agreement payoffs. To define these formally, let $\mathbf{1}_{u'}$ be an N-vector such that each element is equal to the same utility function $u' \in \mathcal{U}$, that is, $\mathbf{1}_{u'} = (u', u', ..., u') \in \mathcal{U}^N$.

Definition 1 (Agreement Payoffs)**.** *An agreement payoff of player n is an indirect utility of the form $v_n(\mathbf{1}_{u'})$ for some $u' \in \mathcal{U}$.*

In many games, the payoff $v_n(\mathbf{1}_{u_n})$ under agreement on player n's endowed utility function represents the best feasible payoff from player n's perspective.[4] In a public goods game, for example, agreement on player n's utility function would imply an equilibrium where all players apart from player n contribute.

Up to this point, Γ could be any extensive form game. In order to apply our measure of bargaining power, however, Γ needs to satisfy some conditions. First, we require players' indirect utility functions to be finite-valued.

Assumption 1 (Finite Indirect Utilities)**.** *For any player n and any vector $u' \in \mathcal{U}^N$, $-\infty < v_n(\mathbf{u}') < \infty$.*

Furthermore, we require some disagreement among players.

Assumption 2 (Conflict of Interest)**.** *For any player n there exists a player m such that $v_n(\mathbf{1}_{u_n}) > v_n(\mathbf{1}_{u_m})$.*

Assumption 2 states that every player strictly prefers agreement on their endowed utility function over agreement on the endowed utility function of at least one other player. This assumption requires not only that there are two players with distinct preferences, but also that players collectively have at least some influence on the outcome. Assumption 2 thus rules out any "game" where the outcome is independent of any player's choices. On the other hand, a game where all players have the same most-preferred alternative can satisfy Assumption 2 as long as players do not have the ability to implement the mutually preferred outcome with certainty and some players disagree in their ranking of other outcomes. Assumption 2 could thus be summarised as requiring that there is a conflict of interest between players regarding the outcomes that are actually achievable. Since bargaining is a way to resolve a conflict of interest, Assumption 2 represents an essential feature of a bargaining game.

[4]If the equilibrium concept is Nash equilibrium and all players have the same preferences as n, a strategy profile that maximises the payoff of player n constitutes a Nash equilibrium since no player has a profitable deviation. If there is a unique equilibrium it follows that $v_n(\mathbf{1}_{u_n})$ is equal to the maximal payoff of player n across all strategy profiles, assuming that a maximal payoff exists.

The measure of bargaining power that we derive based on a list of axioms below can be applied to any game that satisfies Assumptions 1 and 2. However, the axioms determine a unique function only for games that satisfy an additional assumption. This assumption states that if the endowed utility functions of two players are not identical, then neither are the corresponding agreement payoffs.

Assumption 3 (Regularity). *If $u_n \neq u_m$ for $n, m \in \mathcal{N}$, then $v_n(\mathbf{1}_{u_n}) \neq v_n(\mathbf{1}_{u_m})$.*

Denote the set of indirect utility functions of player n generated by all games satisfying Assumptions 1 to 3 by $\mathcal{V}_n$.

An important class of games in our context are SD games, which are defined as follows:

Definition 2 (SD Games). *A game of surplus division satisfies*

$$O = \{o \in [0,1]^N | \sum_{n=1}^{N} o_n \leq 1\}$$

and each player's utility function is given by $u_n(o) = o_n$.

The outcome of an SD game is a vector that assigns each player a share of the available surplus and each player's utility is equal to the share they receive. A possible misconception is that players' endowed utility functions are identical in this setting. However, maximising the share of the surplus of player n is not the same as maximising the share of the surplus of some other player m.

3.2 Example and Intuition

We use the following example to illustrate the concepts:

Example 1. *Consider a game with outcome space $O = [0,1]$ and three players. The utility function of player $n \in \{1,2,3\}$ is given by $u_n(o) = -|o - i_n|$, where i_n is the ideal point of player n. Let $i_1 = 0$, $i_2 = 1/2$, and $i_3 = 1$. Since players' endowed utility functions only differ in ideal points, it is possible to write the indirect utilities as $v_n(i_1, i_2, i_3)$.*

The game starts with a move of nature that determines which, if any, of the players can subsequently choose the outcome of the game. Player n is chosen with probability λ_n. With probability λ_4, however, nature determines that $o = 0$.

In Example 1, the influence of each player is increasing in the probability that this player is selected to choose the outcome. The approach we follow here in order to quantify the power of a player is to introduce changes in a player's preferences and observe to what extent doing so changes the outcome of the game. In Example 1, the expected outcome of the game is equal to $\lambda_2/2 + \lambda_3$ since each player implements their own ideal point if given the opportunity. If we assign player 1 the ideal point of player 3 instead, the expected outcome would equal $\lambda_1 + \lambda_2/2 + \lambda_3$. The question then arises how to quantify differences in the outcomes of games and an approach that is always possible is to compare outcomes in terms of the utilities they imply for a player. The indirect utility as we define it here is designed for this purpose. Given that player 1 in Example 1 is risk neutral, their utility in the actual equilibrium of the game is $v_1(i_1, i_2, i_3) = -|\lambda_2/2 + \lambda_3 - i_1|$, which is simply equal to $-(\lambda_2/2 + \lambda_3)$. In the counterfactual game where player 1 is assigned the ideal point of player 3, we already calculated the expected outcome to be equal to $\lambda_1 + \lambda_2/2 + \lambda_3$. The indirect utility function of player 1 evaluates this hypothetical outcome under player 1's endowed utility function, that is,

$$v_1(i_3, i_2, i_3) = -|\lambda_1 + \lambda_2/2 + \lambda_3 - i_1| = -(\lambda_1 + \lambda_2/2 + \lambda_3) \ .$$

The difference $v_1(i_1, i_2, i_3) - v_1(i_3, i_2, i_3)$ is therefore equal to λ_1, which illustrates that the indirect utility function of a player provides information about this player's bargaining power. In general, we may want to normalize this quantity in some way since a simple difference in utilities depends on the scale of players' utility functions. In addition, there is also the question which shifts in players' preferences should be taken into account. We therefore pursue an axiomatic approach in the following section.

We can also use Example 1 to illustrate the concept of an agreement payoff. If all players shared the ideal point of player 3, for instance, the outcome of the game would be equal to 1 unless nature determines the outcome to be equal to 0 with probability λ_4. The agreement payoff $v_3(i_3, i_3, i_3)$ of player 3 is therefore given by $-|(\lambda_1 + \lambda_2 + \lambda_3) - 1| = -\lambda_4$. The best feasible payoff from player 3's perspective given the rules of the game thus only coincides with their highest possible payoff of zero if $\lambda_4 = 0$. The example shows that the difference between the two payoffs indicates the degree of control that players collectively have over the outcome of the game.

Any of the games given in Example 1 satisfy Finite Indirect Utilities, while Conflict of Interest and Regularity hold if and only if $\lambda_4 < 1$: as long as at least one player has some control over the outcome, players strictly prefer agreement on their own over agreement on any other ideal point.

3.3 Axioms

Our aim is to derive a function $\rho_n : \mathcal{V}_n \to \mathbb{R}$ that uses the information contained in the indirect utility function of a player to assign this player a number that indicates their bargaining power.[5] Below we introduce axioms that this function should satisfy, which require the following definitions. Throughout, we refer to Example 1 for illustrative purposes.

First, a player n is a local dictator if—given the endowed utility functions of the remaining players—the outcome of the game always equals the one that would arise if all other players shared the utility function of player n, no matter what this function actually is. Let $(u'', \mathbf{u}'_{-n})$ represent the vector of utility functions created by taking some vector $\mathbf{u}'$ and replacing the utility function of player n with some function $u'' \in \mathcal{U}$.

Definition 3 (Local Dictator)**.** *Player n in some game Γ is said to be a local dictator if $\mu^*(u', \mathbf{u}_{-n}) = \mu^*(\mathbf{1}_{u'})$ for any $u' \in \mathcal{U}$.*

We refer to a player satisfying Definition 3 as a local dictator rather than simply as a dictator since the property pertains only to a specific vector of other players' preferences rather than to any such vector. In Example 1, a player n satisfies the definition of a local dictator if and only if $\lambda_n = 1 - \lambda_4$. The definition therefore does not imply that a local dictator has the ability to implement their most preferred outcome with certainty. Instead, the defining property of a local dictator is that their influence over the outcome is equal to the collective influence of all players.

A null player, on the other hand, is a player who never affects the outcome.

Definition 4 (Null Player)**.** *Player n in some game Γ is said to be a null player if $\mu^*(\mathbf{u}') = \mu^*(u'', \mathbf{u}'_{-n})$ for any $\mathbf{u}' \in \mathcal{U}^N$ and $u'' \in \mathcal{U}$.*

Assumption 2 rules out that a player could simultaneously be a local dictator and a null player.[6] In Example 1, player n is a null player if and only if $\lambda_n = 0$, which implies that player n is both null and a local dictator if and only if $\lambda_4 = 1$. As explained above, however, the latter case violates the Assumption of Conflict of Interest.

[5] Note that it would in principle be possible to let the bargaining power of a player depend on all players' indirect utility functions rather than just their own. Doing so would have the potential advantage that the sum of bargaining powers can be normalized to equal one, for example. However, as we argue in Section 3.5, such a normalisation would not be compatible with our axioms in any case. Without a clear reason to include other players' payoffs, we instead opt for a simpler measure.

[6] A player can be both a local dictator and a null player only if $\mu^*(\mathbf{1}_{u'}) = \mu^*(\mathbf{1}_{u''})$ for any $u', u'' \in \mathcal{U}$. To see this, suppose there exist $u', u'' \in \mathcal{U}$ such that $\mu^*(\mathbf{1}_{u'}) \neq \mu^*(\mathbf{1}_{u''})$. Then n being a local dictator implies $\mu^*(u', \mathbf{u}_{-n}) = \mu^*(\mathbf{1}_{u'}) \neq \mu^*(\mathbf{1}_{u''}) = \mu^*(u'', \mathbf{u}_{-n})$. It follows that n is not null, which would require $\mu^*(u', \mathbf{u}_{-n}) = \mu^*(u'', \mathbf{u}_{-n})$. Assumption 2 is thus sufficient to ensure that a player cannot be a local dictator and a null player at once since it implies that not all agreement outcomes are equal.

Finally, a compound game is a game that starts with a random draw that determines which of a number of other games is played. Importantly, all players are aware of which game is selected and—given that equilibrium is assumed to be unique—the behaviour of players is thus identical to the case where each game is played in isolation. The constituent games of a compound game need to be compatible in the sense that they share the same sets of outcomes, players, and utility functions.

Definition 5 (Compound Game)**.** Γ *is said to be a compound game if*

i. there exists a finite set of games $\mathbf{\Gamma} = \{\Gamma_1, \Gamma_2, ..., \Gamma_G\}$ *that differ only in terms of their respective game trees, and*

ii. Γ *begins with a commonly-observed move of nature selecting one game from* $\mathbf{\Gamma}$ *to be played subsequently, and each game* $\Gamma_g \in \mathbf{\Gamma}$ *is chosen with probability* λ_g*.*

We write $\Gamma = \sum_{g=1}^{G} \lambda_g \Gamma_g$.

Any of the games in Example 1 can be seen as a compound game.

We now state and discuss the axioms that we impose on the measure of bargaining power ρ_n.

Axiom A1 (Null Players)**.** *If player* n *is a null player in a game* Γ *with their associated indirect utility function given by* v_n*, then* $\rho_n(v_n) = 0$*.*

Axiom A2 (Local Dictators)**.** *If player* n *is a local dictator in a game* Γ *with their associated indirect utility function given by* v_n*, then* $\rho_n(v_n) = 1$*.*

Axioms A1 and A2 impose that a local dictator is assigned a higher bargaining power than a null player and further normalise the power of such players to one and zero, respectively.

Axiom A3 (Compound Games)**.** *Let* $\Gamma = \sum_{g=1}^{G} \lambda_g \Gamma_g$ *and denote by* v_n*,* $v_{1,n}$*, ...,*$v_{G,n}$ *the corresponding indirect utility functions of some player* n*. If all constituent games* Γ_1 *to* Γ_G *share the same agreement payoffs, then for any player* n

$$\rho_n(v_n) = \sum_{g=1}^{G} \lambda_g \rho_n(v_{g,n}) \ .$$

The Axiom of Compound Games states that the bargaining power of a player in a compound game Γ should be equal to a weighted average of the bargaining power of this player in each of the constituent games of Γ. This property is desirable since equilibrium uniqueness and the assumption that players are aware of which game is selected ensure that behaviour in each constituent game is the same as if this game were played on its own. The outcome of the game as a whole is thus a weighted average of the outcomes in each constituent game, as are the indirect utility functions. Furthermore, the assumption of equal agreement payoffs included in the axiom implies that players collectively have the same degree of control over the outcome of each game. The meaning of being a local dictator is thus the same across games. In Example 1, consider the case that $\lambda_4 = 0$, which implies that the players have full control over the outcome of the game. Then the probability that player n is able to choose the outcome, λ_n, is an obvious measure of this player's bargaining power. The example indicates that it is natural to think of the bargaining power of a player in a compound game as their expected power across constituent games as required by the axiom.

Axioms 1 to 3 are sufficient to produce a unique measure of bargaining power in two-player games. For games with more than two players we require an additional axiom.

Axiom A4 (Intermediate Payoffs)**.** *Let $\underline{u}(n)$ denote a utility function that yields the lowest agreement payoff for player n, that is, $v_n(\mathbf{1}_{\underline{u}(n)}) \leq v_n(\mathbf{1}_{u'}) \ \forall \ u' \in \mathcal{U}$. If an indirect utility function v_n satisfies*

$$v_n(\mathbf{1}_{\underline{u}(n)}) \leq v_n(u', \mathbf{u}_{-n}) \leq v_n(\mathbf{1}_{u_n}) \qquad \forall \ u' \in \mathcal{U}, \tag{1}$$

then $\rho_n(v_n) \leq 1$.

The Axiom of Intermediate payoffs addresses a potential drawback of a measure of bargaining power satisfying Axioms 1 to 3. Ideally, no player should ever be assigned a higher bargaining power than a local dictator. Axiom A4 imposes $\rho_n \leq 1$ for a broad class of games, namely games for which all the indirect utilities of a player lie in between their payoff under agreement on their endowed utility function and their worst agreement payoff. While we are not aware of a game discussed in the literature that violates the second inequality in Equation (1), violations of the first inequality usually involve Pareto-inefficient equilibria as in the example presented in Section 5.3.

3.4 The Main Result

For the purpose of stating the main result, denote by $\underline{\mathcal{U}}_n$ the set of utility functions such that agreement on any of these functions generates the lowest possible agreement payoff for player n, that is, $\underline{\mathcal{U}}_n = \arg\min_{u' \in \mathcal{U}} u_n(\mathbf{1}_{u'})$. To understand the relevance of this set, first suppose there was a player with utility function u_m such that the indirect utility of player n under agreement on this utility function, $v_n(\mathbf{1}_{u_m})$, is close to the indirect utility of player n under agreement on their own utility function. This indicates that the utility functions u_n

and u_m rank outcomes in a comparable fashion. If the indirect utility function of some other player k generates a much lower agreement payoff for player n, on the other hand, then there is an accordingly larger difference between the utility functions u_n and u_k. The set $\underline{\mathcal{U}}_n$ thus contains the utility functions that are most different from that of player n. In the context of games satisfying the Assumption of Regularity the set $\underline{\mathcal{U}}_n$ contains a unique element.

We can now state our main result:

Theorem 1. *A function $\rho_n : \mathcal{V}_n \to \mathbb{R}$ satisfies Axioms A1, A2, A3, and A4 if and only if*

$$\rho_n(v_n) = \frac{1}{|\underline{\mathcal{U}}_n|} \sum_{u' \in \underline{\mathcal{U}}_n} \frac{v_n(\mathbf{u}) - v_n(u', \mathbf{u}_{-n})}{v_n(\mathbf{1}_{u_n}) - v_n(\mathbf{1}_{u'})} \ . \tag{2}$$

Proof. See Appendix A. □

The measure of bargaining power introduced by Theorem 1 has a straightforward interpretation. Each of the terms of the sum calculates the effect that a change in the preferences of player n has on the outcome, with the endowed utility function of player n serving as a metric. The effect is then expressed as a share of the one that would occur if player n was a local dictator, which is given by $v_n(\mathbf{1}_{u_n}) - v_n(\mathbf{1}_{u'})$. This quantity is averaged over the largest-possible shifts in the preferences of player n as given by the elements of the set $\underline{\mathcal{U}}_n$. The question answered by the function ρ_n is simply how much influence player n has on the outcome of the game relative to that of a local dictator.

As pointed out above, it holds for games satisfying Assumption 3 that $\underline{\mathcal{U}}_n$ contains a unique element and Equation (2) could be simplified. However, expressing the function as a sum over the set $\underline{\mathcal{U}}_n$ ensures that ρ_n can also be applied to games violating Assumption 3. The focus on the largest-possible shifts in preferences is a consequence of Axiom A4 and thus helps avoid values of ρ_n above one. In addition, large shifts in preferences are more likely to fully reveal a player's power than small shifts as illustrated by the example discussed in Section 2.

In Appendix A we present the proof of Theorem 1 as a series of lemmas that clearly show the additional restrictions that each axiom imposes on the shape of the function ρ_n. First, the Axiom of Compound Games has the consequence that ρ_n must be an affine function on a class of games sharing the same outcome sets, sets of players, and agreement payoffs. To see this, note that the indirect utilities of a player in a compound game $\Gamma = \sum_{g=1}^{G} \lambda_g \Gamma_g$ are a weighted average of the indirect utilities of each constituent game: $v_n = \sum_{g=1}^{G} \lambda_g v_{g,n}$. If all constituent games share the same agreement payoffs, the Axiom of Compound Games requires

$$\rho_n\left(\sum_{g=1}^{G}\lambda_g v_{g,n}\right) = \sum_{g=1}^{G}\lambda_g \rho_n(v_{g,n}) \ .$$

Given that ρ_n is a function of a finite number of utilities, which are real numbers, affinity implies the functional form

$$\rho_n(v_n) = \beta + \sum_{\mathbf{u}' \in \mathcal{U}^N} \alpha(\mathbf{u}')\ v_n(\mathbf{u}')\ ,$$

where β and each $\alpha(\mathbf{u}')$ are real numbers. The value of these coefficients must be constant across games with equal agreement payoffs, but may differ between such classes of games. In other words, the coefficients may be functions of agreement payoffs.

The Axiom of Null Players imposes $\rho_n(v_n) = 0$ if player n is a null player. The definition of a null player implies that any indirect utilities $v_n(\mathbf{u}')$ and $v_n(u'', \mathbf{u}'_{-n})$, which differ only in the included utility function of player n, take the same value. However, the definition does not pin down the level of these payoffs. For ρ_n to take the value zero in any game in which player n is a null player, it is thus necessary that ρ_n can be expressed as a function of differences of indirect utilities $v_n(\mathbf{u}') - v_n(u'', \mathbf{u}'_{-n})$. Since any such difference is equal to zero when n is null, the constant β must also be equal to zero.

Note that $(u', \mathbf{u}_{-n})$ is a vector of utility functions that differs from the vector of endowed utility functions only in the utility function of player n. The definition of a local dictator restricts any utility $v_n(u', \mathbf{u}_{-n})$ to equal $v_n(\mathbf{1}_{u'})$. n being a local dictator does not, however, restrict the values of other indirect utilities where the utility functions of players other than n differ from their endowed utility functions. To ensure that $\rho_n(v_n) = 1$ if n is a local dictator as required by the Axiom of Local Dictators, ρ_n thus cannot depend on indirect utilities other than those of the form $v_n(u', \mathbf{u}_{-n})$.[7]

The above arguments establish that ρ_n takes the shape

$$\rho_n(v_n) = \sum_{(u', u'') \in \mathcal{U}^2} \alpha(u', u'')\ [v_n(u', \mathbf{u}_{-n}) - v_n(u'', \mathbf{u}_{-n})]\ . \tag{3}$$

It is then possible to factor out an arbitrary non-zero number C in the form

$$\rho_n(v_n) = C \sum_{(u', u'') \in \mathcal{U}^2} \frac{\alpha(u', u'')}{C}\ [v_n(u', \mathbf{u}_{-n}) - v_n(u'', \mathbf{u}_{-n})]\ .$$

[7] As was pointed out above, the coefficients used to calculate ρ_n may depend on the values of agreement payoffs. The final expression for ρ_n given in Theorem 1 therefore contains agreement payoffs in additions to indirect utilities of the form $v_n(u', \mathbf{u}_{-n})$.

Since the values of coefficients are at this point undetermined, we can redefine their values to include the division by C. In order to satisfy the Axiom of Local Dictators, the constant C multiplying the sum must be equal to one divided by the value that the remaining part of the expression takes in case player n is a local dictator, that is,

$$\rho_n(v_n) = \frac{\sum_{(u',u'')\in\,\mathcal{U}^2} \alpha(u',u'')\ [v_n(u',\mathbf{u}_{-n}) - v_n(u'',\mathbf{u}_{-n})]}{\sum_{(u',u'')\in\,\mathcal{U}^2} \alpha(u',u'')\ [v_n(\mathbf{1}_{u'}) - v_n(\mathbf{1}_{u''})]}\ .$$

If the set of utility functions $\mathcal{U}$ contains only two elements, the preceding expression simplifies to the form given by Theorem 1. The role of the Axiom of Intermediate Payoffs is thus to pin down the values of the α-coefficients in the case of more than two endowed utility functions. The function ρ_n as given by Theorem 1 clearly satisfies Axiom A4: If an indirect utility satisfies Equation (1), then it follows from the definition of the set $\underline{\mathcal{U}}_n$ that any denominator in Equation (2) represents the biggest difference between any indirect utilities of player n. The fraction as a whole can therefore not be greater than one and the same holds for ρ_n. Suppose, instead, that ρ_n put non-zero weight on some additional indirect utility $v_n(u'',\mathbf{u}_{-n})$ where u'' is not an element of $\underline{\mathcal{U}}_n$ nor equal to u_n. Then it would be possible to construct an indirect utility v_n' such that $\rho_n(v_n') > 1$, violating Axiom A4. As a starting point, let v_n be an indirect utility satisfying Equation (1) such that player n is a local dictator and thus $\rho_n(v_n) = 1$. Now construct a second indirect utility v_n' based on v_n by shifting the value of the payoff $v_n(u'',\mathbf{u}_{-n})$ slightly up or down. Since ρ_n is monotonic in the value of the indirect utility $v_n(u'',\mathbf{u}_{-n})$, one such shift must produce a value of ρ_n above one.

3.5 Additional Properties

In this section, we discuss properties of the function ρ_n introduced by Theorem 1 that are not directly stated in the axioms. For example, the Axiom of Compound Games implies that ρ_n is a continuous function when restricted to a class of games that share equal agreement payoffs. In fact, ρ_n turns out to be a continuous function in general, which follows since Assumption 2 guarantees that the denominator in Equation (2) is not equal to zero for any $v_n \in \mathcal{V}_n$. This is an attractive property since it implies that players are assigned a similar bargaining power in games that generate similar indirect utility functions. Furthermore, the function ρ_n is invariant under affine transformations of players' utility functions, which is reassuring since such transformations do not affect behaviour.

Negative bargaining power. A perhaps unexpected property of the function ρ_n is that it can generate values below zero. Intuitively, assigning a player negative bargaining power would seem justified if they are worse off than if they were a null player. However, the preceding definition is generally not meaningful in games with more than two players, since a null player can experience a range of payoffs depending on what the remaining players agree upon. The following statement, which is equivalent in the context of two-player games, applies to any number of players: a player has negative bargaining power if they benefit from making a commitment to act in the interest of the players most opposed to them.[8] Only under this condition does the function ρ_n generate a negative value, since it can be formally expressed as $u_n(\mathbf{u}) < u_n(u', \mathbf{u}_{-n})$ for $u' \in \underline{\mathcal{U}}_n$, implying a negative numerator in Equation (2). The ability to generate negative bargaining power is thus an informative feature of the measure. In Section 5.3 we discuss an example in which a second-mover would benefit from the ability to commit to supporting a first-mover's position, implying negative bargaining power.

Relation to Shapley value. It is also instructive to compare the properties of our measure of bargaining power to those of the Shapley value. The Shapley value is a solution concept for cooperative games and thus assigns each player a payoff, while our measure is intended for non-cooperative games. Nevertheless, both are functions that take a description of a game and assign a real number to each player and two of the four axioms that define the Shapley value are in fact related to axioms imposed by us. In particular, both approaches rely on an Axiom of Null Players and the definition of a null player is similar in both contexts. In addition, our Axiom of Compound Games is a weaker version of the Axiom of Linearity imposed on the Shapley value. As a consequence, ρ_n is not a linear function and only affine on subsets of games sharing the same agreement payoffs. Shapley's Axiom of Anonymity is not required for our result, even though the function ρ_n is also invariant to the re-labelling of players. On the contrary, the Axioms of Local Dictators and Intermediate Payoffs are unique to our setting. The clearest point of departure, however, is that the Axiom of Efficiency requires the payoffs assigned to players by the Shapley value to add up to one. Such a normalisation is not compatible with our axioms. The reason is that in the equilibrium of some games all players may be indistinguishable from null players in the sense that no individual player could change the outcome even if they tried. All players are then assigned a bargaining power of zero. A situation of this type can arise, for example, in an equilibrium of a voting game where no player's ballot can swing the outcome. An advantage

[8] In a two-player game, n being a null player implies $v_n(\mathbf{u}) = v_n(u_{-n}, u_{-n})$ and being a null player thus entails the same payoff as acting in the interest of the most opposed player.

of not normalizing the sum of power coefficients is that this sum reveals information about the nature of the game, namely the degree to which players mutually block each other from affecting the outcome.

Comparison to surplus-division games. A final characteristic we want to highlight is the relationship between our measure and the share of the surplus that a player receives in an SD game, which is commonly used to assess a player's bargaining power in that setting. As the following result demonstrates, the two approaches coincide under certain conditions.

Proposition 1. *In an SD game, $\rho_n(v_n) = v_n(\mathbf{u})$ if the outcomes $\mu^*(u', \mathbf{u}_{-n})$, $\mu^*(\mathbf{1}_{u_n})$, and $\mu^*(\mathbf{1}_{u'})$ are Pareto efficient for any $u' \in \underline{\mathcal{U}}_n$.*

Proof. See Appendix B. $\square$

The proof of Proposition 1 proceeds by using the definition of an SD-game and the assumption of Pareto efficiency to determine the values of the indirect utilities entering ρ_n. First, Pareto efficiency implies that one player receives the whole surplus if all players agree that this would be the ideal outcome. Accordingly, $v_n(\mathbf{1}_{u_n}) = 1$ and $v_n(\mathbf{1}_{u'}) = 0$ for any $u' \neq u_n$. In addition, under the vector of utility functions $(u', \mathbf{u}_{-n})$ all players prefer to redistribute surplus from player n to some other player, and Pareto efficiency therefore implies $v_n(u', \mathbf{u}_{-n}) = 0$. Substituting accordingly in Equation (2) yields the desired result. Intuitively, efficiency of the agreement payoffs implies that the players collectively have full control over the allocation of the surplus and so would a local dictator. A player's bargaining power thus depends on what share of the total surplus they have under their individual control. Efficiency of the outcomes $\mu^*(u', \mathbf{u}_{-n})$ further implies that any part of the surplus that player n receives in equilibrium is actually due to their influence, rather than simply assigned to them due to some feature of the rules of the game (recall the example in the introduction). The player's surplus share then fully reflects their bargaining power. Inefficiency of any of the outcomes listed in Proposition 1 implies that $\rho_n(v_n) = v_n(\mathbf{u})$ does not hold in general, even though the equality can arise coincidentally.

3.6 Examples

Before presenting more substantive applications of our measure of bargaining power in Section 5, we provide a detailed illustration of its use in the context of the ultimatum game and the choice of an optimal auction.

Example 2 (Ultimatum Game)**.** *Consider a game of surplus division with two players, p and r, and with an outcome space and utility functions as given in Definition 2. Player*

p, the proposer, offers a split of a dollar (o_p, o_r) and player r, the respondent, may accept or reject. If the respondent accepts, the offer of player p is implement, while both of them receive zero otherwise.

In the unique subgame perfect equilibrium of the ultimatum game the proposer offers the split $(1,0)$ and the respondent accepts. Since this is an efficient SD game, Proposition 1 tells us that the bargaining power of the proposer is equal to one and that of the respondent equal to zero. We nevertheless derive the bargaining power of player p as a simple illustration of how to calculate bargaining power in practice. Beyond the equilibrium payoff of player p, we also need to determine the value of their agreement payoffs for the cases that the proposer is assigned the endowed utility function of the respondent and vice versa. In the former case, the proposer wants to maximise the share of the dollar that player r receives and thus proposes the split $(0,1)$, which is accepted. The indirect utility function of player p evaluates this hypothetical outcome using player p's endowed utility function. We therefore have $v_p(u_r, u_r) = 0$. If the respondent is assigned the endowed utility function of the proposer, on the other hand, the proposer continues to offer $(1,0)$ and thus $v_p(u_p, u_p) = 1$. The game accordingly satisfies Assumptions 1 to 3 and it holds that $\underline{\mathcal{U}}_p = \{u_r\}$. It follows that

$$\begin{aligned}\rho_p(v_p) &= \frac{v_p(u_p, u_r) - v_p(u_r, u_r)}{v_p(u_p, u_p) - v_p(u_r, u_r)} \\ &= \frac{1-0}{1-0} \\ &= 1\ .\end{aligned}$$

Example 3 (Selling an Object to Multiple Buyers)**.** *Consider a game with a seller who wants to sell an object to one of $N-1$ buyers. The value of the object to the seller is equal to zero, while y_n denotes the value of buyer n, which is private information. Buyers' values are drawn independently from a uniform distribution on the interval $[0,1]$ at the beginning of the game. Once nature has drawn values, the seller selects a mechanism in which the buyers subsequently participate. Let the identity of the player who receives the object be given by $B \in \{1, ..., N\}$ where $B = 1$ indicates that the seller keeps the object. $P \in \mathbb{R}_+^{N-1}$ is a vector of monetary transfers from the buyers to the seller. Accordingly $O = \{1, ..., N\} \times \mathbb{R}_+^{N-1}$. The endowed utility function of the seller is $u_1(B, P) = \sum_{n=2}^{N} P_n$ while that of some buyer n is given by $u_n(B, P) = \mathbb{1}_{B=n} \cdot y_n - P_n$. The strategy set of the seller is restricted to mechanisms that are individually rational and budget-balanced so that the payoffs of all players are non-negative ex ante.*

As is well known, in the equilibrium of the above game the seller chooses a second-price auction with a reserve price and all buyers bid their value. In order to calculate the bargaining power of the seller, we also need to determine the payoff of the seller when the seller is assigned the utility function of a specific buyer as well as the seller's agreement payoffs. First, consider the counterfactual game where the seller wants to maximise the utility of some buyer n. Note that the game tree and the information structure remain the same as in the original game. Players thus maintain their private information and the seller has access to the same mechanisms. The best the seller can do for a buyer is thus to choose a mechanism that assigns them the good for free. Under the seller's endowed utility function this implies a payoff of zero, that is, $v_1(u_n, \mathbf{u}_{-1}) = 0$. The same is true if all players are assigned the utility function of some buyer n and we have $v_1(\mathbf{1}_{u_n}) = 0$ for any $n \neq 1$. It remains to determine the agreement payoff $v_1(\mathbf{1}_{u_1})$. Under an individually-rational mechanism, the best outcome the seller could hope for even under complete information is to assign the object to the buyer with the highest value and receive a payment equal to this value. If buyers want to maximise the utility of the seller, the seller can actually achieve this outcome by running a first-price auction.[9] Given that we have defined the indirect utilities as ex ante expected payoffs, we have $v_1(\mathbf{1}_{u_1}) = E[y_{(1)}]$, where $y_{(1)}$ is the highest value. Accordingly,

$$\begin{aligned}
\rho_1(v_1) &= \frac{1}{|\underline{\mathcal{U}}_1|} \sum_{u' \in \underline{\mathcal{U}}_1} \frac{v_1(\mathbf{u}) - v_1(u', \mathbf{u}_{-1})}{v_1(\mathbf{1}_{u_1}) - v_1(\mathbf{1}_{u'})} \\
&= \frac{1}{N-1} \sum_{u' \in \mathcal{U} \setminus u_1} \frac{v_1(\mathbf{u}) - 0}{E[y_{(1)}] - 0} \\
&= \frac{v_1(\mathbf{u})}{E[y_{(1)}]} \,.
\end{aligned}$$

The bargaining power of the seller is calculated by comparing their equilibrium payoff to the best-possible outcome under complete information.[10] The seller is thus assigned a bargaining power below one due to the information rent that buyers receive in equilibrium. Larsen & Zhang (2025) effectively introduce several measures by considering different benchmark payoffs. When first-best payoffs are chosen as the relevant benchmark, our measure coincides with theirs in this particular setting.

A general point that we can illustrate in this context is that, according to our measure,

[9] In a game where a buyer is assigned the utility function of the seller, this buyer would in principle be willing to transfer their entire wealth to the seller. Each buyer must therefore be given a budget constraint that determines the maximum payment they could make to the seller. The logical assumption is that this maximum payment is equal to the buyer's value.

[10] Analogous calculations show that the bargaining power of each buyer is equal to their equilibrium payoff divided by their expected valuation, which is their expected payoff if they are given the good for free.

positive bargaining power requires that a player makes a choice. Restraining the seller's strategy set to only choosing a reserve price but not the auction format itself would lead to a lower bargaining power for the seller. If the reserve price was also exogenously given, the seller would be assigned a power of zero.

4 Extensions

4.1 Ex Ante Power and Relation to Voting Power Indices

Our measure of bargaining power calculates power based on the endowed utility functions and power may depend on preferences. In some sense this is natural: for example, it is generally held that more impatient negotiators are at a disadvantage. In some cases, and in particular for the purpose of institutional design, it can nevertheless be of interest what degree of influence the rules of the game assign to each player independently of preferences. Napel & Widgrén (2004) distinguish in this context between an ex ante and an ex post perspective, that is, assessments of power before or after players' preferences have been revealed. Following their approach, we can use our ex post measure to calculate power from an ex ante perspective. Doing so requires specifying a distribution F that players' preferences are drawn from and ex ante power is simply equal to expected ex post power under F. Depending on the chosen distribution, it may be possible to calculate this expectation exactly, such as when F has finite support. Otherwise, expected power can be calculated numerically by drawing preferences, calculating ex post power, and repeating this process until the mean across draws converges. Denote by $\bar{\rho}_n(F)$ the ex ante bargaining power of player n under the distribution F calculated based on the ex post measure ρ_n.

In practice, care needs to be taken with respect to preference profiles that violate Assumption 2, since the value of ρ_n is not defined in such cases. One option is to specify F such that such cases do not occur. Alternatively, it may be possible to resolve the problem by assigning a default value when ρ_n is not defined. For example, if players' utility functions are identical, it may be reasonable to assign each player a power of zero or of $1/N$. In other games, such as the example that follows, a natural extension of ρ_n exists.

We now use the ex post and ex ante measures ρ_n and $\bar{\rho}_n$ to investigate the relationship between our theory and the literature on voting power indices, which calculate the power of players in weighted voting games. In such games, a committee decides whether to accept or reject a proposal. The outcome space is equal to $\{0, 1\}$, where 1 corresponds to acceptance of the proposal, while 0 indicates rejection. It is typically assumed that players have strict preferences over the two outcomes and it is then without loss of generality to let all players' utility functions be given either by u^0 or by u^1, where $u^i(o) = 1$ if $o = i$ and $u^i(o) = 0$ otherwise. Beyond the set of players, a weighted voting game is characterised by a voting rule, which consists of a quota $q > 0$ and a vector of weights $w \in \mathbb{R}_+^N$, one for each member of the committee. Players simply vote in favour of or against the proposal and the proposal is accepted if and only if the sum of all players' weights who vote in favour is at least equal to q. All players voting in favour is sufficient for acceptance, that is, $\sum_{n=1}^N w_n \geq q$. Assume players vote sincerely. Denote by $S \subseteq \mathcal{N}$ the set of players who prefer acceptance under the endowed utility functions $\mathbf{u}$. In the language of cooperative game theory, the players in S form a coalition and the value V of the game indicates whether a coalition wins: $V(S) = 1$ if $\sum_{n \in S} w_n \geq q$ and $V(S) = 0$ otherwise.

Under any given constellation of preferences $\mathbf{u}$ and the corresponding profile of votes, player n is said to be pivotal if them changing their vote would change the outcome of the game. Since such a player satisfies the definition of a local dictator, the measure ρ_n assigns them a power of 1. If a player is not pivotal, their preferences do not matter for the outcome and $\rho_n = 0$. Note, however, that agreement among the players implies that Assumption 2 is violated and the value of ρ_n is not defined. It seems natural to introduce the convention that in such unanimous games (that is, $S = \emptyset$ or $S = \mathcal{N}$), $\rho_n = 1$ if player n is pivotal and $\rho_n = 0$ otherwise. We then have the following result:

Proposition 2. *Let v_n^S denote the indirect utility of player n corresponding to a weighted voting game where the set of players S prefers acceptance. Assume $\rho_n(v_n^{S=\emptyset}) = 1$ if $w_n \geq q$ and $\rho_n(v_n^{S=\emptyset}) = 0$ otherwise. Also assume $\rho_n(v_n^{S=\mathcal{N}}) = 1$ if $\sum_{m \in S \setminus n} w_m < q$ and $\rho_n(v_n^{S=\mathcal{N}}) = 0$ otherwise. Then there exist distributions F_{PB} and F_{SS} such that $\bar{\rho}_n(F_{PB})$ is equal to the Penrose-Banzhaf index and $\bar{\rho}_n(F_{SS})$ is equal to the Shapley-Shubik index.*

Proof. See Appendix B. ☐

Under suitable choices of the distribution of preferences F, $\bar{\rho}_n(F)$ is thus equal to the Shapley-Shubik index or the Penrose-Banzhaf index. These indices are based on cooperative game theory, and showing that they are equivalent to $\bar{\rho}_n(F)$ is possible since a weighted voting game is a rare case of a game that can naturally be expressed in a cooperative or a non-cooperative form. In general, however, voting power indices cannot be applied to non-cooperative games, for which our measure is intended.

4.2 Games with Multiple Equilibria

Above we considered games with a unique equilibrium under any of the possible constellations of players' utility functions, or at least games where equilibrium uniqueness applies under some suitable refinement. It is clear that multiplicity of equilibria can make meaningful statements about bargaining power impossible. For example, in the Baron Ferejohn model (Baron & Ferejohn 1989) any distribution of the surplus can be supported by some subgame perfect equilibrium if there are sufficiently many legislators and these are sufficiently patient. The approach that we propose here makes it possible to apply our measure to games with multiple equilibria, but the informativeness of the measure depends on the severity of the multiplicity problem.

Let $\Sigma(\mathbf{u}')$ denote the set of probability measures μ^* that correspond to the equilibria that exist under some vector of utility functions $\mathbf{u}'$. Instead of assuming that we can assign a probability of one to a particular equilibrium as we did above, we can choose a more general approach and specify a probability distribution over possible equilibria. It is typically not obvious how to choose this distribution and we provide a solution below that avoids the need to make any commitment in this regard. For now, however, assume that for each $\mathbf{u}'$ we can specify a probability measure $\sigma_{\mathbf{u}'}$ on $\Sigma(\mathbf{u}')$. We can then define the indirect utility of player n as

$$v_n(\mathbf{u}') = \int_{\Sigma(\mathbf{u}')} \int_O u_n(o) \; d\mu^* \; d\sigma_{\mathbf{u}'} \; .$$

The measure of bargaining power of Theorem 1 can then be computed based on this indirect utility function without any further adjustments. What is more, the definitions and axioms presented above can be adapted to this more general setting with only minor changes and the proof of Theorem 1 applies verbatim. For example, the definition of a Null Player in a game with multiple equilibria would require that changes in this player's utility function have no effect on the probability distribution over equilibria.

In order to ensure that ρ_n yields sensible results, the probability distribution over equilibria under agreement on the utility function of some player n should always be chosen such that the equilibrium that maximises the utility of player n is assigned probability one. Beyond affecting the exact meaning of being a local dictator, this choice guarantees that the denominator of Equation (2) is always greater than zero.

In the likely case that it is not obvious how to assign a probability to each equilibrium, we propose to calculate the range of bargaining powers implied by all possible probability distributions over equilibria. Due to the affinity of the measure ρ_n, the bargaining power assigned to a player under any given distribution over equilibria is equal to a weighted average over the bargaining powers assigned under each individual equilibrium. To determine the range of bargaining powers implied by all possible distributions over equilibria it is therefore sufficient to calculate the highest and the lowest bargaining power implied by the individual equilibria. In doing so, the same default equilibrium selection under agreement as outlined in the previous paragraph should be applied.

5 Applications

5.1 Cartel Formation

The formation of a cartel arguably constitutes a setting of non-transferable utility since monetary transfers could be used as evidence of collusion in court. Since the production levels that maximise joint profits may imply wide disparities between the profits of individual cartel members, quantities may be subject to negotiation. Suppose, for example, that N firms produce a homogeneous good, where each firm has a constant marginal cost c_n that differs between firms. In this case the sum of profits would be maximised if only the firm with the lowest cost produces, but in the absence of a means to redistribute these profits the remaining firms clearly have no incentive to agree to such terms. If the firms are later found by the authorities to have engaged in collusive behaviour, the relative influence of each firm in bringing about the agreement could be used for the purpose of apportioning compensation.[11] In order to determine this relative influence, it may not be sufficient to know the market share or cost structure of each firm, for instance because a relatively small or inefficient firm could be pulling above its weight due to political clout or connections to organised crime. Non-cooperative cartel formation is a subject of ongoing research (Abe 2021, Korsten & Samuel 2023) and providing a fully-specified model is beyond the scope of this paper. Yet, our measure of bargaining power takes a particularly simple form in this setting under weak assumptions about the underlying process. These assumptions are *i*) that a firm's profit is fully reflective of its payoff in the game, which is reasonable if other forms of compensation are not possible, and *ii*) that if a firm's utility function is replaced with that of another firm, it ceases production, implying an indirect utility of zero.[12] Under these conditions our measure of bargaining power becomes equal to a firm's equilibrium profit divided by this firm's individual monopoly profit. Simply relying on market shares or shares of total profits may thus not accurately reflect a firm's role in the formation of the cartel. The reason is that total production or total profits do not provide a relevant benchmark at the individual level. The highest-possible profit an inefficient firm could hope for may be substantially lower than that of a competitor with lower costs. To illustrate, consider a case with three

[11] Napel & Welter (2021, 2022) respectively propose using the Shapley-Shubik index and the Shapley value to assign relative responsibility for damages to the members of a cartel. The drawback of these approaches is that one has to assume that a cartel among any subgroup of firms is associated with a unique vector of production quantities, precluding bargaining among cartel members.

[12] To simplify the discussion, we assume that firms do not have fixed costs.

firms and unit costs that are given by $c_1 = 0.1$, $c_2 = 0.2$, and $c_3 = 0.3$ and an inverse market demand equal to $P = 1 - Q$ where Q is total production. Then individual monopoly profits are given by 0.20, 0.16 and 0.12 in ascending order of costs. The best possible payoff thus differs substantially across firms and dividing individual by total equilibrium profits would overstate the bargaining power of efficient firms and understate that of inefficient firms.

5.2 Household Bargaining

The literature on intra-household decision making has an inherent interest in the determinants of the balance of power between spouses. One approach, namely the collective model of the household (Chiappori 1988, 1992), assumes efficient outcomes while the distribution of resources is determined by explicit parameters for male and female bargaining power. The main competitor is the non-cooperative model of the household (Lundberg & Pollak 1994, Konrad & Lommerud 1995, Browning et al. 2010, Lechene & Preston 2011), which instead assumes that husband and wife play a Nash equilibrium. In this case, bargaining power is an implicit product of the decision-making environment. We use an application of this framework presented in Bertrand et al. (2020) to demonstrate how our approach can be used to evaluate the bargaining power of household members. We focus on the second period of the model, after a man and a woman have decided to form a household. At this point of the game, husband and wife simultaneously decide how to allocate one unit of time between remunerated work and the production of a public good within the household. For simplicity, we assume that there are no spillovers from private consumption. The utility of household member $g \in \{m, f\}$ is then given by

$$u_g(t_g, t_{-g}) = (1 - t_g)w_g + \beta \log(t_m + t_f) \ ,$$

where $t_g \in [0, 1]$ is the share of time spent on producing the public good, w_g is the gender-specific wage, and β determines the weight of public good consumption relative to private consumption. We follow Bertrand et al. (2020) and assume a gender wage gap, $w_f < w_m$, and $\beta < w_m$. Under these assumptions the man works full-time while the woman stays home if $w_f < \beta$ and works part-time otherwise.

In order to calculate players' bargaining powers, we also need to determine the equilibrium if the husband maximises the utility of the wife and vice versa. Without spillovers from earnings, maximising the utility of the partner implies dedicating all available time to producing public goods. If the wife shares the utility function of the husband, the latter always works full time. In the reverse situation, the wife also stays home if her wage is sufficiently low and works part-time or full-time for higher wages. Given that the husband's behaviour differs across these two scenarios for all parameter constellations under consideration, the two agreement payoffs of each player are not equal and the game satisfies the Assumption of Conflict of Interest. For β sufficiently large, on the other hand, both partners would always prefer to stay home and there is no disagreement.

Figure 1 plots the bargaining powers of husband and wife as a function of the female wage w_f for the cases $\beta = 0.2$ and $\beta = 0.6$, assuming $w_m = 1$. For $w_f < \beta$, the wife devotes all her time to the production of public goods, which is also the behaviour that maximises the utility of the husband. Accordingly, the husband is assigned a bargaining power of one and the wife a bargaining power of zero. Once her wage becomes sufficiently high, the wife finds it attractive to work part time. Doing so increases her utility and lowers that of her husband, leading to a more equal distribution of power. However, the power of the wife is substantially lower than that of the husband even if her wage is almost equal to his. The reason is that even a slightly lower opportunity cost of domestic labour on part of the wife allows the husband to free-ride on her effort. For $w_f = w_m$, the equilibrium remains unique under agreement on one player's utility function. However, multiple equilibria exist under the endowed utility functions and bargaining power depends on the probability assigned to each equilibrium (see Section 4.2). The figure assigns probability one to the equilibrium where the husband works full-time, which may be due to a social convention. Assigning the same probability to all equilibria, in contrast, would lead to equal bargaining power and a discontinuity at $w_f = w_m$.

As Figure 1 shows, a higher value of the public good β polarises the distribution of bargaining power, since the wife reduces her labour supply while the husband continues to free-ride. A possible interpretation is that modern appliances that generate a more quickly declining marginal productivity of housework lead to greater equality within the household.

5.3 Legislative Bargaining in the European Union

Real-life bargaining protocols can be complicated, making it difficult to understand how much power they confer on each participant. We illustrate this by applying our theory to a sightly simplified version of the procedure most commonly used by the European Union when passing legislation, namely the Ordinary Legislative Procedure. The game has three players: the Commission, the Council and the Parliament. These players bargain over a law $p \in [-1, 1]$, which would replace the status quo $q \in [-1, 1]$. Each player evaluates the final policy, which may be equal to the status quo, according to the utility function

$$u_n(p) = -(p - i_n)^2 \ ,$$

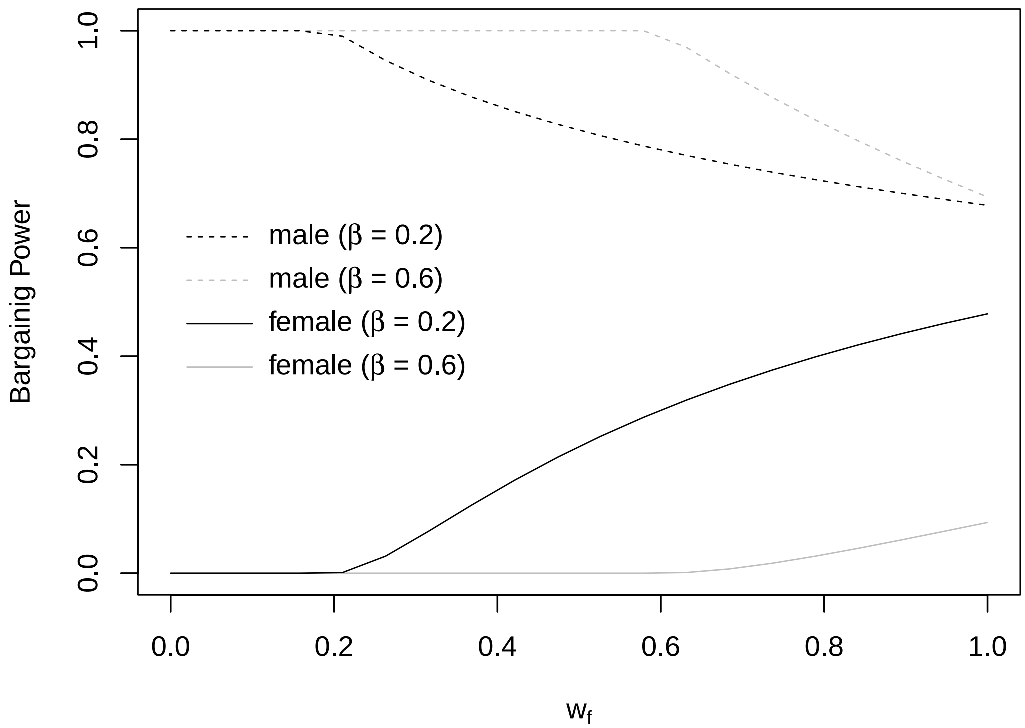

Figure 1: Bargaining Power of Husband and Wife

Notes: The figure plots the bargaining power of husband (dashed lines) and wife (solid lines) against the wife's wage w_f, assuming the husband's wage w_m is equal to 1. Black lines correspond to a value of β of 0.2, while grey lines correspond to $\beta = 0.6$.

where i_n is the ideal point of player n.

The game starts with the decision of the Commission to initiate the legislative process. If it declines to do so, the status quo remains in place, while the Commission makes an initial proposal otherwise. This ability of the Commission to determine whether new legislation can even be considered is known as gate-keeping in the political science literature (Denzau & Mackay 1983), but referred to as the Commission's right of initiative in the context of the EU. If the Commission decides to make an initial proposal, this proposals is then forwarded to the Parliament, which first decides to accept or not accept the proposal. In the latter case, the Parliament can either reject the proposal, leaving the status quo in place, or produce an amended proposal, which concludes the first reading in the Parliament. At this point, the Commission may withdraw the proposal, once more leaving the status quo in place. The first reading in the Council proceeds as the first reading in the Parliament, with the only difference that acceptance leads to the proposal in its current shape becoming the final policy. The same is true of the second reading in the Parliament, which is reached if the Council decides to introduce amendments of its own. The second reading in the Council, on the other hand, differs from the first reading in that the Council does not produce an amended proposal. Instead, if the Council neither accepts nor rejects, the third reading commences, in which the Council and the Parliament negotiate directly. We assume that the third reading produces a policy equal to a weighted average of the most preferred elements of the winset of the Council and the Parliament, where the winset contains all elements of the interval $[-1, 1]$ that each player prefers over the status quo. The weights on the most preferred elements of the Council and the Parliament are given by w and $1-w$, respectively. Any decision to accept or reject a proposal is subject to utility shocks drawn from an extreme value distribution with standard deviation 1, generating logistic choice probabilities. Final utilities are given by the utility $u_n(p)$ from the final policy plus any utility shocks realized during negotiations. In addition, the Commission pays a utility cost k if it decides to introduce an initial proposal.

We illustrate the usefulness of our measure by calculating power in an ex ante sense as in Section 4.1: We set $w = k = 0.5$ and draw ideal points and the status quo uniformly from $[-1, 1]$. For each draw, we calculate the indirect utility functions and each player's bargaining power and then repeat this procedure until the average across draws converges.[13]

		Commission	Council	Parliament
(1)	Baseline	0.09	0.14	0.20
(2)	No Right of Initiative	0.08	0.25	0.34
(3)	No Veto	0.08	0.04	-0.02
(4)	Only First Proposal	0.01	0.38	0.43

Table 1: Bargaining Power in the Legislative Process of the European Union

Notes: The table presents the bargaining powers of the three legislative institutions of the EU in the baseline model and in three alternative scenarios: "No Right of Initiative" modifies the baseline model by forcing the Commission to always make an initial proposal; "No Veto" refers to a version where the Commission has no veto after making the first proposal; and "Only First Proposal" removes both the veto of the Commission and its right to leave the status quo in place by not making an initial proposal.

The results in the first row of Table 1 show that the Parliament has the strongest bargaining power while the Commission is least influential. As discussed in Crombez et al. (2006), some observers claim that the right of initiative puts the Commission in a dominant

[13]In practice, to speed up convergence, we draw at random from an evenly-spaced grid between -1 and 1 with 100 elements and omit draws for which the ideal points of all players coincide, which would violate the Assumption of Conflict of Interest.

position, but our results do not confirm this view. The additional rows in Table 1 show what exactly the sources of the Commission's power are by considering modified versions of the legislative bargaining game. The second row of Table 1 presents a scenario where the Commission is forced to make an initial proposal. The third row of Table 1, on the other hand, removes the veto of the Commission after the first reading in Parliament. The Commission's power coefficient remains largely unchanged in both scenarios, indicating that the right of initiative and the ability to veto proposals are substitutes from the Commission's perspective: being forced to initiate legislation is not too costly since the Commission can veto proposals, while losing the veto has a limited impact as long as the Commission can prevent unwanted policy changes by not initiating the legislative process. The consequences for the other players are different though. Forcing the Commission to initiate legislation gives them more opportunities to shape policy and their bargaining power increases. Without the ability to subsequently veto proposals, in contrast, the Commission only initiates legislation when the preferences of the Council and the Parliament work in the Commission's favour, thus diminishing the influence of the Council and the Parliament. The bargaining power of the Parliament even turns negative in this scenario. In fact, the power of both the Council and the Parliament can be negative for specific constellations of ideal points and status quos. Negative bargaining power reveals that a player would benefit from committing to playing in the interest of the player most opposed to them (see Section 3.5). Suppose, for instance, that the Parliament and the Council share the same ideal point, which is closer to that of the Commissions than to the status quo. There thus exist policies that all actors prefer to the status quo. However, if the Commission is close to the status quo, it declines to initiate legislation because it foresees that the Council and the Parliament will subsequently implement a policy that is worse for the Commission than the status quo. In this situation the Parliament and the Council would be better off if they could credibly commit to accepting an initial proposal from the Commission equal to the Commission's ideal point.

In the final row of Table 1, the Commission is forced to initiate legislation for any status quo and does not have the ability to veto proposals. The only form in which the Commission intervenes in the legislative process is therefore by formulating the initial proposal without the ability to pre-emptively keep the status quo in place.[14] Yet, this right to formulate the first proposal is a very minor source of influence and the Commission's bargaining power is close to zero. We thus conclude that the limited power of the Commission derives from its right of initiative and its veto, but to a negligible extent from the ability to produce the

[14]The reason why the Commission may derive bargaining power from the ability to formulate the first proposal is due to the presence of idiosyncratic preference shocks. Without these shocks, the Parliament would only accept the initial proposal if it likes it at least as much as the proposal the Parliament would subsequently make itself. This constraint would reduce the bargaining power of the Commission to zero.

initial proposal.

5.4 Nash Bargaining and Nash-in-Nash Bargaining

Nash bargaining takes as given a set of outcomes $O = X \cup \{d\}$ where X is the set of possible agreements and d is the outcome in case of disagreement. Two players bargain over O and the resulting outcome is the one that maximises the Nash product $u_1(o)^{\omega_1} \cdot u_2(o)^{\omega_2}$ with $\omega_n \in [0,1]$ and $\omega_1 + \omega_2 = 1$.[15] Based on this so-called Nash solution, we can construct an outcome function μ^* and indirect utility functions (v_1, v_2) as defined above. Note that when both players are assigned the utility function of player n, the Nash solution yields the outcome that maximises the utility function of this player. Denote this outcome by o_n^*. Applying our measure of bargaining power to this setting yields

$$\rho_n(v_n) = \frac{v_n(\mathbf{u}) - u_n(o_{-n}^*)}{u_n(o_n^*) - u_n(o_{-n}^*)},$$

which is the share of the maximal utility gain over the preferred outcome of the other player that player n achieves in equilibrium. When utility is transferable and the utility possibility frontier is therefore linear, ρ_n coincides with player n's surplus share, which in turn is equal to ω_n (see, for example, Muthoo 1999, pp. 35-36). In general, however, ρ_n does not equal ω_n, which reflects that bargaining power is not necessarily fully captured by the bargaining weights, but can also be due to the shape of the utility possibility frontier (Binmore et al. 1986, p. 186). Figure 2 provides an illustration: The solid concave curve is an example of a utility possibility frontier, while the solid convex curve represents the level set of the Nash product that is tangent to the utility possibility frontier when $\omega_1 = 0.45$ and $\omega_2 = 0.55$. Player 1 is assigned a bargaining power of 0.8 in this case as their utility is equal to 0.4 compared to a maximal utility of 0.5 while the best-possible outcome for Player 2 implies a utility of zero for player 1. The dashed lines correspond to a second utility possibility frontier and bargaining weights $\omega_1 = 0.6$ and $\omega_2 = 0.4$. While Player 1 has a higher bargaining weight and achieves a higher equilibrium utility in this example, the increase in their maximal utility implies a lower bargaining power of 0.65.

The preceding discussion has implications for Nash-in-Nash bargaining (Horn & Wolinsky 1988, Collard-Wexler et al. 2019), which is a solution concept for a situation where multiple actors bargain bilaterally and each bilateral agreement can affect other agreements through externalities. Bilateral negotiations are modelled as Nash bargaining and the overall equilibrium represents a Nash equilibrium in Nash bargains. In this setting, it is possible that

[15] The Nash product is usually expressed as $(u_1(o) - u_1(d))^{\omega_1} \cdot (u_2(o) - u_2(d))^{\omega_2}$, but it is always possible to normalise utility functions such that $u_n(d) = 0$ for both players.

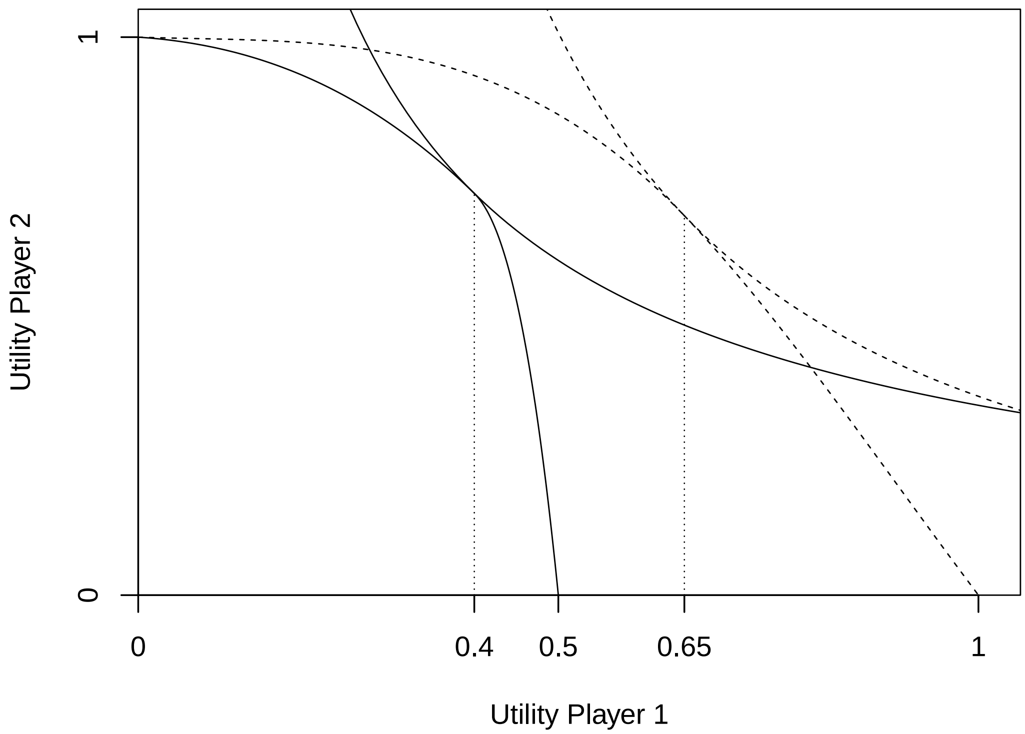

Figure 2: Examples of Nash bargaining

Notes: The figure provides two examples of utility possibility frontiers, given by the solid and dashed concave lines, respectively. The convex lines show level sets of the Nash product, with tangency points determining the outcome under Nash bargaining. In the example indicated by solid lines the bargaining weights are $\omega_1 = 0.45$ and $\omega_2 = 0.55$ while in the example indicated by dashed lines we have $\omega_1 = 0.6$ and $\omega_2 = 0.4$.

an increase in a player's bargaining weight in a specific bilateral negotiation generates an externality affecting the agreement between some other parties, which in turn generates a feedback effect on the utility possibility frontier in the original negotiation. As Figure 2 illustrates, a potential consequence is that an increase in the bargaining weight of a player leads to a higher equilibrium utility for this player, but at the same time lowers their bargaining power. Alternatively, a player may be involved in multiple bilaterals and a higher bargaining weight in one of these could lower their bargaining power in a separate negotiation. At least in settings where utility is non-transferable, the overall bargaining power of a player can therefore not be assumed to be equal to a simple average of bargaining weights in bilateral negotiations nor to be a monotone function of their equilibrium utility.

6 Conclusion

Bargaining power is a key element of economic, political and social relations. Many central questions in these fields are analysed through the lenses of non-cooperative games, for which measures of bargaining power, however, have been proposed only for specific settings. This paper introduces a novel method for measuring power in any non-cooperative game of bargaining. The power of a player is calculated as the extent to which shifts in this player's preferences change the outcome of the game relative to the change that would occur if the player in question was a dictator. Since our measure is calculated based on a mapping from players' utility functions to utilities, it can equally be applied to calculate how much power a player has under a specific mechanism or social choice function. We show that no other measure satisfies a number of axioms. As the application to legislative bargaining in the EU shows, our measure is particularly valuable when assessing the role of procedural rules in determining the influence of a player. Such insights are crucial, for example, when designing institutions that aim to achieve a specific distribution of power among agents. How do changes to judicial proceedings affect bargaining power in out-of-court settlements? Does the most-favoured-nation principle give large countries an outsized influence in WTO negotiation rounds? What are the implications of different protocols for climate negotiations (Harstad 2023) for each participants' influence on the final agreement? Our measure can shed light on these and many related questions.

Appendix

A Proof of Theorem 1

This appendix contains the proof of Theorem 1. The proof is presented in the context of a fixed class of indirect utility functions $\bar{\mathcal{V}}_n \subset \mathcal{V}_n$ that correspond to games sharing a common outcome space, set of players, and agreement payoffs.

We start by presenting three lemmas that successively introduce sharper restrictions on the function $\rho_n(v_n)$.

Lemma 1. *A function $\rho_n : \bar{\mathcal{V}}_n \to \mathbb{R}$ satisfies Axiom A3 if and only if*

$$\rho_n(v_n) = \beta + \sum_{\mathbf{u}' \in \mathcal{U}^N} \alpha(\mathbf{u}')\ v_n(\mathbf{u}')\ ,$$

where β and all $\alpha(\mathbf{u}')$ are real numbers.

Proof. Note that the domain of a player's indirect utility function v_n is the set $\mathcal{U}^N$, which has a finite number of elements. ρ_n is therefore a function of a finite vector of utilities, which are real numbers.

Let $\Gamma = \sum_{g=1}^{G} \lambda_g \Gamma_g$ with corresponding indirect utility functions v_n, $v_{1,n}$, ..., $v_{G,n} \in \bar{\mathcal{V}}_n$. Given that the class $\bar{\mathcal{V}}_n$ was defined to contain indirect utilities sharing the same agreement payoffs, Axiom A3 requires

$$\sum_{g=1}^{G} \lambda_g\ \rho_n(v_{g,n}) = \rho_n(v_n) = \rho_n\left(\sum_{g=1}^{G} \lambda_g v_{g,n}\right)\ ,$$

where the second equality follows since the indirect utilities of a compound game are a convex combination of the indirect utilities of the constituent games. ρ_n is therefore an affine function on $\bar{\mathcal{V}}_n$. Given that it was established above that ρ_n is a function of a finite vector of real numbers, affinity of ρ_n is satisfied if and only if ρ_n takes the form given in the statement of the lemma. $\square$

Lemma 2. *A function $\rho_n : \bar{\mathcal{V}}_n \to \mathbb{R}$ satisfies Axioms A1 and A3 if and only if*

$$\rho_n(v_n) = \sum_{\substack{(\mathbf{u}', u'') \\ \in\ \mathcal{U}^{N+1}}} \alpha(\mathbf{u}', u'')\ [v_n(\mathbf{u}') - v_n(u'', \mathbf{u}'_{-n})]\ ,$$

where all $\alpha(\mathbf{u}', u'')$ are real numbers.

Proof. Given the functional form of ρ_n established in Lemma 1, it needs to be shown what additional restrictions Axiom A1 imposes. It will be shown that it must be possible to formulate ρ_n as a function of differences in payoffs of the form $v_n(\mathbf{u}') - v_n(u'', \mathbf{u}'_{-n})$. To see this, suppose that after rearranging the terms of ρ_n to form pairs of utilities of the preceding kind, there remain K payoffs $v_n(\tilde{\mathbf{u}}^1)$,...,$v_n(\tilde{\mathbf{u}}^K)$ with non-zero coefficients for which no pair can be formed. For any pair $\tilde{\mathbf{u}}^k$ and $\tilde{\mathbf{u}}^j$ of the underlying vectors of utility functions it must be the case that the two vectors differ in the utility function of some player other than n, since it would otherwise be possible to form an additional pair of indirect utilities of the above form. Let v_n correspond to a game where n is a null player and thus $\rho_n(v_n) = 0$. Since all differences in payoffs of the form $v_n(\mathbf{u}') - v_n(u'', \mathbf{u}'_{-n})$ are equal to zero if player n is null, we have

$$\rho_n(v_n) = \beta + \sum_{k=1}^{K} \alpha(\tilde{\mathbf{u}}^k)\ v_n(\tilde{\mathbf{u}}^k) = 0\ . \tag{4}$$

If there exist multiple games in $\bar{\mathcal{V}}_n$ such that n is null and the sum $\sum_{k=1}^{K} \alpha(\tilde{\mathbf{u}}^k)\ v_n(\tilde{\mathbf{u}}^k)$ differs across some of these games, then the preceding equality cannot hold for all such games and Axiom A1 would be violated. Suppose therefore that n being null implies a fixed value of this sum across all elements of $\bar{\mathcal{V}}_n$. It will be shown that this assumption can only be satisfied if it holds for any individual vector $\tilde{\mathbf{u}}^k$ that the utility functions of all players other then n contained in $\tilde{\mathbf{u}}^k$ are equal. To the contrary, suppose that there exists a vector $\tilde{\mathbf{u}}^k$ such that for two players m and j it holds that $\tilde{u}^k_m \neq \tilde{u}^k_j$. At least one of these functions must be different from u_n. Without loss of generality, suppose $\tilde{u}^k_m \neq u_n$. Then we can construct two games, Γ_m and Γ_j, such that n is null in both games and it holds that $\mu^*_m(\tilde{\mathbf{u}}^k) = \mu^*_m(\mathbf{1}_{\tilde{u}^k_m})$ and $\mu^*_j(\tilde{\mathbf{u}}^k) = \mu^*_j(\mathbf{1}_{u_n})$ while $\mu^*_m(\tilde{\mathbf{u}}^t) = \mu^*_j(\tilde{\mathbf{u}}^t)$ for any $1 \leq t \leq K$ such that $t \neq k$. To see that this construction is possible, recall that any two of the K vectors of utility functions under consideration must differ in the utility function of some player other than n and n being null therefore does not restrict the values of the corresponding payoffs. Assumption 3 implies $v_n(\tilde{\mathbf{u}}^k) \neq v_n(\tilde{\mathbf{u}}^j)$. But since all other relevant payoffs of player n are identical across the two games it follows that Equation (4) cannot be satisfied for both of them, which is the desired contradiction. For any vector $\tilde{\mathbf{u}}^k$ there thus exists a utility function u^k such that the utility functions of all players other than n contained in $\tilde{\mathbf{u}}^k$ are equal to u^k. If n is a null player, it follows that $v_n(\tilde{\mathbf{u}}^k) = v_n(\mathbf{1}_{u^k})$ and in order to satisfy Equation (4) it must hold that $\beta = -\sum_{k=1}^{K} \alpha(\tilde{\mathbf{u}}^k)\ v_n(\mathbf{1}_{u^k})$. However, this contradicts that it is impossible to pair any of the payoffs $v_n(\tilde{\mathbf{u}}^k)$ with another of the form $v_n(u', \tilde{\mathbf{u}}^k_{-n})$. Given that all such pairs are zero if player n is null, it further follows that ρ_n cannot contain any additional constant. $\square$

Lemma 3. *A function $\rho_n : \bar{\mathcal{V}}_n \to \mathbb{R}$ satisfies Axioms A1, A2, and A3 if and only if*

$$\rho_n(v_n) = \frac{\sum_{(u',u'')\in\,\mathcal{U}^2}\ \alpha(u',u'')\ [v_n(u',\mathbf{u}_{-n}) - v_n(u'',\mathbf{u}_{-n})]}{\sum_{(u',u'')\in\,\mathcal{U}^2}\ \alpha(u',u'')\ [v_n(\mathbf{1}_{u'}) - v_n(\mathbf{1}_{u''})]}\ ,$$

where all $\alpha(u',u'')$ are real numbers such that the denominator in the preceding expression is not equal to zero.

Proof. As a first step, it will be shown that an additional restriction implied by Axiom A2 is that ρ_n can only depend on indirect utilities of the form $v_n(u',\mathbf{u}_{-n})$ for some $u' \in \mathcal{U}$, that is, indirect utilities under vectors of utility functions that differ from the vector of endowed utility functions only in the utility function of player n. Given the functional form established by Lemma 2, suppose that ρ_n depends on a pair of indirect utilities $v_n(\mathbf{u}') - v_n(u'',\mathbf{u}'_{-n})$ with a non-zero coefficient, where the utility function of some player other than n included in the vector $\mathbf{u}'$ differs from their endowed utility function. At least one of these payoffs is not an agreement payoff and, without loss of generality, let this be the payoff $v_n(\mathbf{u}')$. Suppose v_n corresponds to a game where player n is a local dictator and $\rho_n(v_n) = 1$. Since n being a local dictator does not restrict the payoff $v_n(\mathbf{u}')$, we can construct a second indirect utility v_n' where n continues to be a local dictator by changing this payoff while holding v_n otherwise constant. Given the already established functional form of ρ_n, the perturbation in $v_n(\mathbf{u}')$ increases or decreases the value of $\rho_n(v_n')$ relative to $\rho_n(v_n)$, violating Axiom A2.

We have thus established that

$$\rho_n(v_n) = \sum_{(u',u'')\in\,\mathcal{U}^2} \alpha(u',u'')\ [v_n(u',\mathbf{u}_{-n}) - v_n(u'',\mathbf{u}_{-n})]\ . \tag{5}$$

Let $C \neq 0$ be some real number. We can rewrite

$$\rho_n(v_n) = C \sum_{(u',u'')\in\,\mathcal{U}^2} \frac{\alpha(u',u'')}{C}\ [v_n(u',\mathbf{u}_{-n}) - v_n(u'',\mathbf{u}_{-n})]\ .$$

Since the exact values of the coefficients are as of yet undetermined, we can redefine them to include the division by C and simply write

$$\rho_n(v_n) = C \sum_{(u',u'')\in\,\mathcal{U}^2} \alpha(u',u'')\ [v_n(u',\mathbf{u}_{-n}) - v_n(u'',\mathbf{u}_{-n})]\ . \tag{6}$$

Under Axiom A2, n being a local dictator implies

$$C \sum_{(u',u'')\in\,\mathcal{U}^2} \alpha(u',u'')\;[v_n(\mathbf{1}_{u'}) - v_n(\mathbf{1}_{u''})] \;=\; 1\;.$$

Solving for C and substituting back into Equation (6) yields the desired result. Any such function satisfies Axiom A2 as long as the coefficients are chosen such that the value of C is not equal to zero. $\square$

It needs to be shown that the function given in the statement of Theorem 1 is the only function among those given by Lemma 3 that satisfies Axiom A4. If $|\mathcal{U}| = 2$, Lemma 3 pins down a unique function corresponding to the one given in the statement of Theorem 1. It remains to consider the case $|\mathcal{U}| > 2$. Note that Assumption 3 implies that the set $\underline{\mathcal{U}}_n$ contains a unique element. Denote this utility function by $\underline{u}(n)$. It then holds by definition that $v_n(\mathbf{1}_{\underline{u}(n)}) = \min_{u'\in\mathcal{U}} v_n(\mathbf{1}_{u'})$. Furthermore, Equation (2) simplifies to

$$\rho_n(v_n) = \frac{v_n(\mathbf{u}) - v_n(\underline{u}(n), \mathbf{u}_{-n})}{v_n(\mathbf{1}_{u_n}) - v_n(\mathbf{1}_{\underline{u}(n)})}\;.$$

If v_n satisfies Equation (1), the denominator in the preceding equation constitutes the largest possible difference between any two indirect utilities of player n. The fraction as a whole can therefore not exceed one and the measure given by Theorem 1 satisfies Axiom A4.

To conclude the proof, it will be shown that any other measure ρ'_n of the form given by Lemma 3 violates Axiom A4. For any such measure, it is the case that the value of this function depends on an indirect utility $v_n(u'', \mathbf{u}_{-n})$ such that $u'' \notin \{u_n, \underline{u}(n)\}$. Let v_n be an indirect utility function that satisfies Equation (1) such that player n is a local dictator. Since any measure given by Lemma 3 satisfies Axiom A2, we have $\rho'_n(v_n) = 1$. Assumption 3 and Equation (1) together imply $\min_{u'\in\mathcal{U}} v_n(\mathbf{1}_{u'}) < v_n(u'', \mathbf{u}_{-n}) < v_n(\mathbf{1}_{u_n})$. We can therefore construct a second indirect utility v'_n that also satisfies Equation (1) by perturbing the payoff $v_n(u'', \mathbf{u}_{-n})$ upwards or downwards by a sufficiently small number while keeping all other indirect utilities fixed. Due to the affinity of ρ'_n it holds for one of these perturbations that $\rho'_n(v'_n) > 1$.

B Additional Proofs

Proof of Proposition 1. Pareto efficiency implies that if all players agree that a unique outcome would be optimal, then the equilibrium of the game must produce this outcome with certainty. In an SD game, under the vector of utility functions $\mathbf{1}_{u_m}$ all players agree that player m should receive the whole surplus. Pareto efficiency of the outcomes $\mu^*(\mathbf{1}_{u_n})$ and $\mu^*(\mathbf{1}_{u_m})$ for $m \neq n$ thus implies $v_n(\mathbf{1}_{u_n}) = 1$ and $v_n(\mathbf{1}_{u_m}) = 0$. Furthermore, Pareto efficiency implies $v_n(u_m, \mathbf{u}_{-n}) = 0$ since under the vector of utility functions $(u_m, \mathbf{u}_{-n})$ all players other than n prefer more for themselves while player n prefers more for player m. Using all of the above to substitute in Equation 2, it follows that

$$\rho_n(v_n) \;=\; \frac{1}{|\underline{\mathcal{U}}_n|} \sum_{u' \in \underline{\mathcal{U}}_n} \frac{v_n(\mathbf{u}) - 0}{1 - 0} \;=\; v_n(\mathbf{u}) \;. \qquad \square$$

Proof of Proposition 2. We start by calculating the value of ρ_n for a given vector of endowed utility functions $\mathbf{u} \notin \{\mathbf{1}_{u^0}, \mathbf{1}_{u^1}\}$. It is clear that the agreement outcome under the vector of preferences $\mathbf{1}_{u^0}$ ($\mathbf{1}_{u^1}$) is equal to 0 (1) with certainty. If player n is pivotal, the outcome coincides with that preferred by player n, which implies $v_n(\mathbf{u}) = 1$ and $v_n(u', \mathbf{u}_{-n}) = 0$ for $u' \in \{u^0, u^1\} \setminus u_n$. It follows that $\rho_n = 1$ if player n is pivotal. If player n is not pivotal, switching the preference of player n has no consequence for the outcome and $\rho_n = 0$. It follows that $\rho_n(v_n^S) = V(S \cup \{n\}) - V(S)$ if $n \notin S$ and $\rho_n(v_n^S) = V(S) - V(S \setminus \{n\})$ if $n \in S$.

Define $F_{PB}(\mathbf{u}) = 1/2^N$ and $F_{SS}(\mathbf{u}) = [|S|! \cdot (N - |S|)!]/(N+1)!$. We can now establish that

$$\begin{aligned}
\bar{\rho}_n(F_{PB}) &= \sum_{S \subseteq \mathcal{N}} \frac{1}{2^N} \rho_n(v_n^S) \\
&= \sum_{\substack{S \subseteq \mathcal{N} \\ n \notin S}} \frac{1}{2^N} \rho_n(v_n^S) + \sum_{\substack{S \subseteq \mathcal{N} \\ n \in S}} \frac{1}{2^N} \rho_n(v_n^S) \\
&= 2 \sum_{\substack{S \subseteq \mathcal{N} \\ n \notin S}} \frac{1}{2^N} \rho_n(v_n^S) \\
&= \sum_{\substack{S \subseteq \mathcal{N} \\ n \notin S}} \frac{1}{2^{N-1}} \left[V(S \cup \{n\}) - V(S) \right] \;,
\end{aligned}$$

where the third equality follows from the fact that for every $S \subseteq \mathcal{N}$ such that $n \notin S$ there exists exactly one $S' \subseteq \mathcal{N}$ such that $n \in S'$ and $S = S' \setminus \{n\}$. Since pivotality of player n only depends on the other players' preferences, it thus holds that $\rho_n(v_n^S) = \rho_n(v_n^{S'})$. Furthermore,

$$\begin{aligned}
\bar{\rho}_n(F_{SS}) &= \sum_{S \in \mathcal{N}} \frac{|S|! \cdot (N - |S|)!}{(N+1)!} \rho_n(v_n^S) \\
&= \sum_{\substack{S \subseteq \mathcal{N} \\ n \in S}} \Bigg[\frac{|S|! \cdot (N - |S|)!}{(N+1)!} \rho_n(v_n^S) \\
&\qquad + \frac{(|S|-1)! \cdot (N - |S| + 1)!}{(N+1)!} \rho_n(v_n^{S \setminus n}) \Bigg] \\
&= \sum_{\substack{S \subseteq \mathcal{N} \\ n \in S}} \Bigg[\frac{|S|! \cdot (N - |S|)!}{(N+1)!} \\
&\qquad + \frac{(|S|-1)! \cdot (N - |S| + 1)!}{(N+1)!} \Bigg] \rho_n(v_n^S) \\
&= \sum_{\substack{S \subseteq \mathcal{N} \\ n \in S}} \frac{(|S|-1)! \cdot (N - |S|)!}{N!} \left[V(S) - V(S \setminus \{n\}) \right] \ ,
\end{aligned}$$

where the third equality holds since $\rho_n(v_n^S) = \rho_n(v_n^{S \setminus n})$, which follows as the value of ρ_n only depends on whether player n is pivotal, which in turn only depends on the preferences of other players. □

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