

# How Political Parties Shape Electoral Competition

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This version: 22 Sep 2014

Latest draft: [www.nmotz.com/NMpartyf.pdf](http://www.nmotz.com/NMpartyf.pdf)

## Abstract

Across established democracies a particular pattern can be observed: Political competition is intense in national elections, but much less so at regional levels. In fact, it is not uncommon that a single party monopolizes a regional office over long periods of time. In this paper I present a model of party formation that is able to explain this pattern. A key feature is that voters are poorly informed about the policies that individual candidates stand for at regional levels. As in Snyder & Ting (2002), parties provide some of this information by allowing only certain types of politicians to join. In order to compete, a party therefore needs to attract and retain the right kind of politicians. I demonstrate the existence of an equilibrium with two parties, one centre-left and one centre-right. These parties are able to monopolize regional elections through offering career prospects at the national level to local politicians. This prevents them from joining more extreme parties targeting particular states. Preventing entry of this kind is important to existing parties, as they would otherwise risk losing their core support in the national election. Competition at the national level, on the other hand, results from both parties being able to recruit moderate politicians. I demonstrate that this equilibrium is unique whenever it exists, under the condition that party leaders are allowed to collude and merge their parties. Overall, the model then predicts the number of parties to vary between two and three.

**Keywords:** Political parties, electoral competition.

**JEL Classification:** D72.

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\*I am grateful to my supervisors Ian Preston and Guy Laroque. Omer Ali was very helpful in obtaining some of the data used in this paper. I would also like to thank Antonio Cabrales, Aureo de Paula, and Lukas Wenner for helpful comments and discussions.

# 1 Introduction

Across developed democracies a common pattern can be observed. At the national level elections are competitive: Multiple parties participate and more than one of them stands a chance of emerging as the winner. Accordingly, a single party rarely manages to hold on to power for more than two or three electoral cycles. In contrast, regional monopolies are commonly observed. In the United States, Maryland and Massachusetts are strongholds of the Democratic Party, while Texas has been dominated by the Republican Party since the early nineties. In the German state of Bavaria the Christian Social Union has been in control of the state legislature for almost 60 years. Many departments in the south of France have long been held by the Socialist Party.

To demonstrate this pattern more systematically I collect election results for a number of federal states. Federal states feature regional elections that form a natural counterpart to national level elections, namely elections of state governments. State boundaries are also less subject to manipulation by politicians compared to other types of administrative units. My sample consists of Australia, Austria, Canada, Germany, and the United States. Using this data, I construct a measure of how competitive elections in a particular region are, as described in detail in section 2. The results are presented in figure 1. A lower number implies a higher degree of competition.

Figure 1 clearly shows that federal elections are typically about as competitive as the most competitive states in the respective country, while in each country there are states where competition is substantially lower than at the federal level.<sup>1</sup> How can this pattern be explained? A simple explanation would be to argue that parties choose their platforms in order to maximize their success at the national level (and that local candidates are for some reason tied to running on this platform as well). Due to heterogeneity in voter preferences across different states, parties then do well in some states and less well in others as a by-product. This answer may seem convincing when considering a country such as Australia where state elections are less competitive than general elections, but not by a wide margin. However, this line of reasoning does not explain how parties maintain the regional monopolies we see, especially in Austria and the US. For these countries the data seems to indicate that voter preferences in some states are far more homogeneous and potentially more extreme than in the respective country as a whole. Why is there no entry of regional parties

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<sup>1</sup>Are more in-depth discussion of the figure is provided in section 3.

Figure 1: Political Competitiveness



Notes: Each circle represents elections in a given state, while crosses stand for federal-level elections. A higher number implies a lower degree of competition.

that are better able to cater to the particular views of voters in such states than national parties are? And if something prevents entry at the regional level, why is this force not at play at the national level as well? Providing a satisfactory answer to these questions requires a concept of what exactly the role of political parties is.

In the model that I construct in this paper—and in line with a growing body of empirical research to be discussed below—the policy choices of a politician are largely determined by this politician’s preferences. Voters therefore care about the preferences of politicians, but are initially poorly informed about these. Following Snyder & Ting (2002), parties provide some of this information by not allowing politicians of all political shades to join. Seeing that a politician is a member of a particular party thus tells voters that this politician must fall into a specific region of the political spectrum. In contrast, there would be no role for parties in the model if voters were fully informed.

In order to compete, parties thus need to attract the right kind of politicians.

Importantly, politicians also care about their chances of getting elected. This concern is a main driver of the choice of party affiliation. A key insight that emerges from the model is that political parties that are successful in national elections can maintain regional monopolies because they offer career prospects at the federal level. As an example, consider a state like Nebraska where the Republican Party currently controls all major elected offices. Suppose strongly conservative members of the Republican Party in Nebraska could form a separate party and do equally well in state elections. This would have the benefit of eliminating internal competition for nominations from politicians belonging to the more moderate wing of the party. It would, however, also deprive members of the newly formed party of any chance of advancing to the federal level. If these career prospects are valuable enough, conservative politicians in Nebraska prefer to remain a part of the Republican Party, which can then maintain its hold on the state.

While the moderate wing of the Republican Party is a burden to conservatives in a right-leaning state, the conservative wing is a detriment to the electoral chances of the party at the national level. It makes the party more extreme and thus less attractive to voters in the political centre. The national party itself might therefore have an incentive to try to exclude the most conservative members. But this would result in the establishment of a more extremist party and the risk of a split in the conservative vote. This threat of entry is the force that prevents the national party from moderating itself in the model and may explain why the establishment of the Republican Party has been relatively accommodating towards the radical Tea Party movement.

The main result of the paper is that there exists an equilibrium where two parties are formed, one centre-left and one centre-right. Both parties win with equal probability at the federal level while dominating some state elections. This equilibrium is maintained by the forces described above: Neither party can shift further towards the centre without inducing entry of a third party, while in equilibrium such entry is precluded as politicians have no incentive to deviate towards joining a new party.

The main reason that the federal election is competitive in this equilibrium is that both parties are able to recruit moderate politicians that are attractive to the federal median voter. Moderates have the option of joining either party and tend to affiliate themselves to the one that does well in their state. Consequently, a state with a leftist (rightist) median voter becomes a source of moderate candidates for the centre-left (centre-right) party. That all moderate politicians

sort into the same party is impossible.

I also demonstrate that under certain conditions the two-party equilibrium referred to so far is the unique equilibrium whenever it exists. Generally, the model allows for the formation of a huge number of parties, many of which play no role in elections at the federal level. These equilibria disappear if the possibility of mergers between parties is allowed for. The model then predicts that the number of parties will vary between two and three, with three parties existing only if politicians value success at the federal level relatively little or are unlikely to achieve it. Collusion between party leaders can, however, not result in the formation of a single party dominating all elections. Such a party would have to accommodate all types of politicians in order to prevent entry. Even if this was possible, intense internal competition for nominations would ensue, making some politicians willing to defect to a new party. Deterrence of entry is impossible.

To the best of my knowledge, the contrast between political competition at the state and at the federal level has previously not been demonstrated as clearly as in figure 1. Accordingly, no explicit explanations have been proposed either. A model that could be applied to this question is provided by Callander (2005), who studies competition between two parties in multiple single-member districts with threat of entry at the district level. Parties, which are not explicitly modelled, are free to choose any platform. Callander finds that the threat of entry leads to the divergence of party platforms, similar to this paper. The mechanism through which entry is deterred is different though. In addition, the equilibrium presented by Callander requires specific assumptions on the distribution of voters across districts, while the restrictions imposed on voter distributions in this paper are mild. This is because entry in the model of Callander implies the loss of one district, while entry has much wider consequences in the current model as explained above. Previous contributions to the literature on political competition with entry consider only a single district (Palfrey 1984, Osborne 1993, 2000).

I also add to the literature on party formation (Jackson & Moselle 2002, Levy 2004, Morelli 2004, Osborne & Tourky 2008, most notably) by integrating the concept of parties as “informative labels” proposed by Snyder & Ting (2002) into a model where the number of parties is entirely endogenous. Furthermore, I introduce the tradeoff between internal and external competition as a driver of politicians’ affiliation decisions, which arises very naturally in the model.

The rest of the paper is organized as follows: Section 2 details the con-

struction of the measure of political competitiveness I use and addresses some alternative explanations of the pattern displayed in figure 1. In section 3 I discuss a number of empirical results that lend support to some of the assumptions made in the model, which is laid out in section 4. Section 5 gives the theoretical results. Robustness of the results to relaxing some of the assumptions made in the basic version of the model is discussed in section 6. Section 7 concludes.

## 2 Measuring Competitiveness

I want to compare the competitiveness of national level elections to the competitiveness of regional elections. In selecting countries to include in my sample I focus on federal states for two main reasons: First of all, federal states have stable regional boundaries that are less subject to manipulation by politicians than is the case for other kinds of administrative units. This rules out gerrymandering as an explanation of regional monopolies. In addition, state elections carry some weight, making it harder to argue that the formation of parties is entirely driven by considerations regarding the national level. My sample consists of state and federal elections for the countries Austria, Australia, Canada, Germany and the US.<sup>2</sup> For all these countries, with exception of the US, the elections I consider are for state and federal parliaments. In the US I compare popular voter shares for the presidential election with results of gubernatorial elections. My data generally includes all such elections between 1945 and June 2014. For the US I restrict the sample to elections held after the passage of the civil rights act of 1964. Prior to this event the Democratic Party dominated the US South, partially through limiting the ability of African-Americans to vote. In Germany I include only the 11 states belonging to the Federal Republic of Germany prior to 1990.

The most commonly used measure of the competitiveness of an election is the vote margin between the highest and the second-highest vote getter. Denote this vote margin for an election at time  $t$  in administrative unit  $r$  by  $d_r^t$ , where  $r$  stands either for a particular state or the federal level of a country. I then

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<sup>2</sup>Election results were retrieved from the following sources:

Austria and Germany: [www.parties-and-elections.eu/](http://www.parties-and-elections.eu/)

Australia: [elections.uwa.edu.au/](http://elections.uwa.edu.au/)

Canada: [www.electionalmanac.com](http://www.electionalmanac.com)

US presidential elections: [www.ropercenter.uconn.edu/elections/common/pop\\_vote.html](http://www.ropercenter.uconn.edu/elections/common/pop_vote.html)

US gubernatorial elections up to 1990: ICPSR (1995)

US gubernatorial elections after 1990: [library.cqpress.com/elections/](http://library.cqpress.com/elections/) .

measure the competitiveness of elections in region  $r$  by computing average vote margins over time:

$$\frac{1}{T} \sum_t d_r^t,$$

where  $T$  is the total number of elections in region  $r$  included in the sample. These are the values displayed in figure 1.

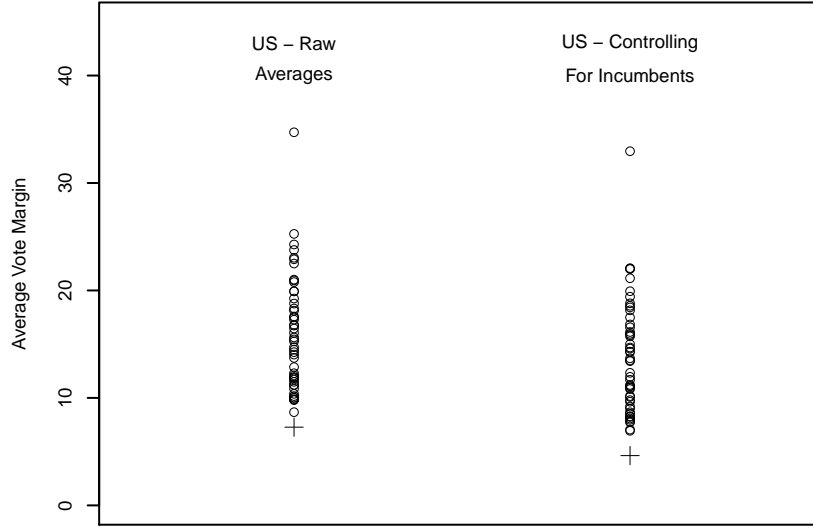
Austria, Germany, and the US show the same pattern of highly contested federal elections and wide vote margins in at least some states. The picture for Australia is similar, but less extreme. In fact, no Australian state is dominated by one of the two main parties of the country.<sup>3</sup> It would seem that this is a consequence of relatively homogeneous distributions of voters across states. In Canada, on the other hand, competition at the federal level is relatively low. This reflects the success of the Liberal Party in the nineteenth century, but also the landslide victories of the Progressive Conservative Party in 1958 and 1984. The more important difference between Canada and the other countries, however, is not visible in the picture: Canadian federal parties are only loosely connected to state parties and successful regional parties that play no role at the federal level exist. Such regional parties can be observed mostly in countries with strong regional identities such as Canada, Belgium, or Spain. In some sense their presence highlights the question of why such parties fail to exist in other countries.

To conclude this section, I will briefly discuss two other factors beyond heterogeneity in voter preferences that may play a role in shaping the outcomes shown in figure 1. The first one is the role of incumbents running for re-election. It is well known that at least in the US incumbents tend to enjoy an electoral advantage. Some states in the US also have less strict term limits for governors. This raises the question to what extent the difference between federal and state elections is a consequence of the presence of incumbents at either level. I demonstrate for the US that this factor is of minor importance. To do so, I run a regression of presidential and gubernatorial vote margins in my sample on a set of state dummies as well as an indicator for the presence of an incumbent. The coefficient on the incumbent dummy indeed turns out to be highly significant with a magnitude of slightly more than five percent. The effect on the remaining coefficients is small, as can be seen in figure 2. The first column in this graph reproduces the raw average voter margins for the US as shown

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<sup>3</sup>I treat the coalition of the Liberal Party and the National Party as a single party. Keeping them separate makes Australian elections look somewhat less competitive.

Figure 2: Political Competitiveness in the US



Notes: Each circle represents governors elections in a given state, while crosses stand for presidential elections.

before. The second column plots predicted vote margins with the incumbent dummy set to zero. The presence of incumbents does not appear to be a main driver of the low competitiveness of many gubernatorial elections.

A second concern I want to address is that differential rates of turnout at state and federal elections might be of importance. Indeed, turnout tends to be lower in state elections than in federal elections. It would be hard to argue though that this lower turnout causes higher vote margins. On the contrary, there is evidence that low turnout is a consequence of higher expected vote margins (Levine & Palfrey 2007). As such, turnout is itself a measure of competitiveness.

### 3 Related Empirical Evidence

This section will discuss empirical evidence supportive of some of the assumptions featured in the model or the general ideas behind it. First of all, a growing



literature investigates the determinants of the policy choices of elected officials. Chattopadhyay & Duflo (2004) and Bhalotra & Clots-Figueras (2014) find that policy preferences of politicians matter. Both papers establish that an exogenous increase in female representation in India leads to a greater provision of public services typically utilised by women. The results of Lee et al. (2004) go further: According to their estimates the voting behaviour of individual members of the US House of Representatives is independent of their electoral odds. This seems to indicate that the preferences of politicians not only influence but largely determine policy choices. Otherwise one would expect legislators in close election to alter their voting behaviour in an attempt to cater to voter tastes. Similar results are obtained by Levitt (1996) for the US Senate, who additionally controls for a potential role of party discipline imposed on legislators.

A second strand of evidence relates to the nature of political parties. Casual observation suggests that in many countries the vast majority of votes is cast for two parties, one located left and one located right of the political centre, which tend to alternate in power. In view of the studies cited above, the interpretation of the statement that a party is centre-left would have to be that the politicians belonging to such a party prefer centre-left policies, at least on average. This is also formally confirmed by research estimating the ideological positions of politicians. Poole & Rosenthal (1997, 2001) do so using voting records from the US Congress. According to their results the membership of both the Republican and the Democratic Party spans a wide range of positions, but with very little overlap between them. Barberá (2014) finds very similar results using data from the social network Twitter. He also applies his method to five European countries, and again a similar picture emerges.<sup>footnote</sup>As the author remarks, the reason that he finds somewhat more overlap between parties in Europe may be due to the possibility that variation in political preferences is less well captured by a single dimension.

All of this is consistent with the view of parties as collections of similar-minded politicians. In this case voters can learn something about a politician's views from observing which party she is a member of, even if they cannot observe preferences of politicians directly. This is the idea formalised by Snyder & Ting (2002) and applied in this paper. Snyder & Ting also demonstrate empirically that voters' knowledge of an individual politician's position is almost entirely captured by her party affiliation. They use estimates of these positions to predict how voters place candidates on an ideological scale. As it turns out, a simple dummy for party affiliation does just as well in explaining the variation in voter

knowledge.

## 4 The Model

A federal state consisting of  $S \geq 4$  states selects federal and state governments through plurality rule elections. Candidates for these elections are nominated by political parties. I divide the game into two main stages: A party formation stage and an election stage. Each stage will be described in more detail below once some of the basic elements of the game have been introduced. Note that the model features no “states of nature” and the word state therefore always refers to a geographical unit.

### 4.1 Voters, Politicians, and Parties

Each state  $s$  has an infinite set of citizens and each citizen votes in two elections: The election for the government of state  $s$  and the election for the federal government. Let  $p_s$  and  $p_f$  denote the policies that are implemented in state  $s$  and at the federal level, respectively. The objective of voters in election  $l \in \{s, f\}$  is to maximize

$$u(|p_l - i|) ,$$

where  $u : \mathbb{R} \rightarrow \mathbb{R}$  is a decreasing and strictly concave function while  $i \in \mathbb{R}$  is the ideal policy of the voter. As will become clear later, the outcomes of state elections may affect events at the federal level, but it is assumed that voters do not take this interdependence into account when voting at the state level. This is done simply to make the proofs presented in subsequent sections more compact, but is otherwise not necessary.

Each state also has a finite set of politicians. Ideally, I would like to equip these politicians with preferences that depend both on policies as well as their own electoral success. As discussed in section 3, preferences over policies appear to be the main driver of the choices that politicians make in office. However, the decision I want to highlight in the model is which party to join. As it turns out, career concerns are often crucial in this regard. Unfortunately, including both motivations would make the model very hard to solve. I will therefore focus on office motivations and simply assume that every politician is associated with a policy that she has to implement if elected to any state or federal office. Section 6 will discuss to what extent this assumption can be relaxed. In order to avoid

confusion with preference parameters of voters I refer to the policy of a politician as her platform. A second simplification that is required for tractability is that politicians have only three possible platform, namely -1, 0, and 1. The number of cases to consider increases rapidly in the number of possible platforms and is already large with three platforms. A possible interpretation of this assumption is that voters have a coarse perception of the policies chosen by politicians. Each state has three politicians and none of them share the same platform. Put differently, there is one politician located at each of the possible policies -1, 0, and 1. I will apply the labels centrist, extremist, rightist, and leftist to politicians in the obvious manner.

For an election at level  $l \in \{s, f\}$  the winning candidate receives a payoff of  $y_l > 0$ . A politician who has won a state election but does not win the nomination of her party for the federal election nevertheless receives a payoff of  $y_P > 0$  if her party wins the federal election. This payoff represents opportunities to move upwards in the party hierarchy that arise when a party wins the federal election or the chance of becoming a member of the federal government. A second interpretation of this payoff is that career opportunities in the private sector become more valuable if a politician is well connected within the party in power. In order to clearly define the utility of a politician, let  $\pi_s$  be the probability that a politician is nominated for and wins the state election in her state. Conditional on doing so, let  $\pi_n$  give the probability that a politician is nominated for the federal election.  $\pi_f$  is then the likelihood of winning conditional on receiving the nomination, while  $\pi_P$  is the probability that the party wins conditional on some other candidate having received the nomination. All of these probabilities will later be determined in equilibrium. The expected utility of a politician who has joined a party is given by

$$\pi_s(y_s + \pi_n\pi_f y_f + (1 - \pi_n)\pi_P y_P) ,$$

while a politician who is not part of a party receives a payoff of zero. It is assumed that  $y_f > 2y_P$ .

A political party is basically a subset of the policy space and only politicians whose platforms fall within this subset can join. This idea is based on Snyder & Ting (2002), where the leadership of a party chooses a platform and politicians pay a cost for joining the party that depends on the distance between this platform and their own ideal policy. Two interpretations of this cost are given: First, politicians could find it costly to be members of an organisation

that pursues goals that differ from their personal views. Second, parties could be actively screening their members and only promote those who agree with the party line. As a result, only politicians with an ideal policy belonging to an interval centred around the party platform join. The size of the interval depends on the membership cost in the first interpretation, or the effectiveness of screening in the second interpretation. I simplify things by giving parties full control over the size of the interval that represents the party. Given that the space of platforms consists of integers, parties will be given by “integer intervals”: For  $a, b \in \{-1, 0, 1\}$  define

$$[a..b] \equiv \{p \in \{-1, 0, 1\} : a \leq p \leq b\} .$$

If  $a$  equals  $b$  I simply write  $[a]$ . The set of all possible shapes a particular party can have is

$$\mathcal{I} = \{[-1], [0], [1], [-1..0], [0..1]\} .$$

Note that parties that allow all types of politicians to join are not included in this set. Such parties would be unlikely to be stable, due to internal conflict between different factions. Section 6 will discuss the consequences of also admitting parties of shape  $[-1..1]$ .

Parties are organized nationally, meaning that the interval that represents the party is the same in all states. The set of politicians that joins a party does not have to be the same across states, however, as politicians in different states might face different incentives. Individual parties will be denoted by capital letters. For any such party  $P$  the shape of the party is given by  $I_P \in \mathcal{I}$ . Multiple parties are allowed to have the same shape.

## 4.2 The Election Stage

In order to describe the election stage, suppose that there are already  $N$  existing parties, collected in the set  $\mathcal{P}$ . Let  $\mathcal{P}(p)$  denote the possibly empty set of existing parties that include the policy  $p$ . The strategy set of a politician with platform  $p$  in this subgame is then given by  $\mathcal{P}(p)$ . Note that this means that a politician can join at most one party and that a politician who has the ability to join a party must do so. The latter assumption is made for convenience and could easily be replaced with a small payoff that a politician receives once she joins a party.

The election stage starts with politicians making their affiliation decisions,

followed by simultaneous state elections, which in turn are followed by the federal election. Immediately prior to each election every party nominates a candidate, who is randomly drawn from the candidate pool of the party for the election in question. For a particular state the candidate pool of a party consists of all politicians of that state who have joined this party. Each winner of a state election then becomes a member of the candidate pool of their party for the federal election.

The policy that is implemented in a state is equal to the platform of the politician elected in the state election, just like the policy at the federal level is equal to the platform of the politician elected in the federal election. The winner of each election is the candidate that achieves the highest number of votes with ties resolved randomly.

### 4.3 The Party-Formation Stage

Parties are formed by “founders”. Founders, of which there is an infinite number, can choose to propose a party or remain passive. Accordingly, the action space of a founder is given by  $\mathcal{I} \cup \emptyset$ , where  $\emptyset$  stands for the decision to not propose a party. Once a founder has proposed a party, I will also refer to this founder as a party leader.

As discussed in the introduction, one of the forces generating the results is the threat of entry of additional parties. Incorporating entry into the model, however, is not straightforward. One possibility would be to choose the following timing for the party-formation stage: As the first step, all founders simultaneously decide whether to propose a party. Subsequently, all founders who previously remained passive are given a second opportunity to propose a party. The founders who moved first would then have to take into account that changing the shape of their party may induce entry. However, proceeding this way turns out to have stronger consequences than intended. In order to explain why, let me introduce an additional concept. Consider a set of  $N$  parties with shapes collected in the set  $\mathcal{J} \in \mathcal{I}^N$ . For example, if there are two parties  $A$  and  $B$  with shapes  $I_A$  and  $I_B$ , then  $\mathcal{J} = \{I_A, I_B\}$ . I say that a set of shapes of parties  $\mathcal{J}$  deters entry if for any party  $D$  with shape  $I_D \in \mathcal{I}$  there exists a perfect Bayesian equilibrium of the election subgame reached after the set of parties  $\mathcal{J} \cup \{I_D\}$  has been proposed such that party  $D$  does not win any elections.

Now suppose there exist two parties  $A$  and  $B$  with shapes  $\{I_A, I_B\}$  such that this set deters entry and that there exists a shape  $I'_A$  such that the set

$\{I'_A, I_B\}$  also deters entry. Furthermore, suppose that in any equilibrium of the election subgame reached once party  $A$  adopts shape  $I'_A$ , party  $A$  wins all elections against party  $B$ , while this is not true as long as party  $A$  maintains shape  $I_A$ . Nevertheless, there then exists an equilibrium of the game as a whole where the set of parties  $\{I_A, I_B\}$  is formed. Why can the founder of party  $A$  not deviate to  $I'_A$ ? If she did so, then a third founder could subsequently enter with a party  $D$  of the same shape as  $A$  had originally. Because the set  $\{I_A, I_B\}$  deters entry, there exists an equilibrium of the election subgame after party  $D$  has entered such that party  $A$  does not win any elections. By this logic any set of parties that deters entry is automatically part of a perfect Bayesian equilibrium of the game as a whole, unless some founder would be better off remaining passive. To avoid this problem, I will incorporate the threat of entry through the payoffs of founders, rather than making it part of the game tree. This also allows me to keep the timing of the party-formation stage simple: All founders simultaneously decide to stay passive or propose parties.

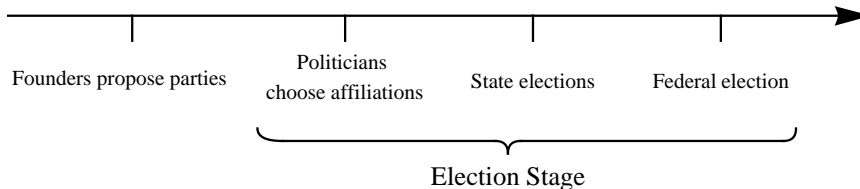
The payoffs of founders then depend on whether the resulting set of parties deters entry or not. In the former case, each founder pays a cost  $c > 0$  for proposing a party, while she receives a payoff of  $x_w > 0$  for every state election that her party wins, as well as a payoff of  $x_f > 0$  if her party wins the federal election. Denoting by  $\rho_r$  with  $r \in \{1, \dots, S, f\}$  the equilibrium probability that the party of a founder wins election  $r$ , the expected utility of a founder who proposes a party when entry is deterred is given by

$$\sum_{s=1}^S \rho_s x_w + \rho_f x_f - c .$$

If the set of formed parties does not deter entry, on the other hand, each active founder simply receives a payoff of  $-c$ . Passive founders always achieve a utility of zero. I assume  $c < x_w$ , so that a founder whose party wins at least one state election does not want to deviate to remaining passive.

Including the threat of entry through payoffs in this way admits more deviations than in the case described above, where entry is a part of the game tree. It also seems reasonable to assume that entry will indeed fail as long as the existing constellation of parties deters entry. One may nevertheless imagine that there could be situations in which repositioning a party could be beneficial even if this entails entry of a modestly successful party. Allowing for this would require somehow specifying general rules for when an entrant should achieve

Figure 3: Timing



success and when not. I feel that such rules would complicate the model without addressing all possible cases appropriately. I will, however, comment when a particular form of entry seems intuitive.

#### 4.4 Information

A crucial feature of the concept of political parties employed here is that voters have limited information about politicians. At the beginning of the game, the electorate cannot distinguish between different politicians, but knows how their platforms are distributed. In contrast, politicians and founders observe platforms. Everything apart from platforms is common knowledge. In particular, voters know which parties have been proposed and how many politicians have joined each one of them in each state. Knowing that a candidate belongs to a certain party therefore allows voters to update their beliefs about this politician's platform prior to casting their vote for the state-level election. The winner of the election then implements her platform at the state level, thus revealing it to voters. Voters accordingly have full information about candidates at the federal level. All agents are also fully informed about the distribution of voters in all states and at the federal level.

#### 4.5 Equilibrium

The timing of the game is summarised in figure 3. Given that the game features incomplete information, the appropriate equilibrium concept is perfect Bayesian equilibrium. By itself, this would entail the possibility of a huge number of

equilibria that exist when voters are allowed to vote strategically. Many papers in the literature on party formation assume sincere voting to avoid this problem. I generally allow for strategic voting, but impose three plausible restrictions: First of all, I consider only equilibria in weakly undominated strategies. The exclusion of weakly dominated strategies is the standard way of refining voting equilibria and excludes the possibility that voters vote for their least preferred candidate. The second restriction reads as follows: If a candidate is the unique most preferred option of a strict majority of voters, then this candidate wins the election. In general there may exist voting equilibria where a different candidate gets elected in this situation, but it nevertheless seems likely that voters will be able to solve the coordination problem in this case. The third restriction is akin to a tie breaking rule: I assume that all candidates receive an equal number of votes if all voters are indifferent between all candidates, but only if the election takes place along the equilibrium path. Imposing this restriction along the equilibrium path only can be interpreted as “party loyalty”: Indifferent voters may for some reason prefer to vote for the same party that they would have voted for in equilibrium.

The following definition summarises the equilibrium concept:

**Definition 1.** *A party-formation equilibrium is a perfect Bayesian equilibrium of the party-formation game that satisfies the following conditions:*

- i) No player uses a weakly dominated strategy.*
- ii) If a candidate in some election is the unique most preferred option of a strict majority of voters, then this candidate wins the election.*
- iii) Along the equilibrium path all candidates receive an equal share of votes if all voters are indifferent between all candidates.*

I will use stars to denote equilibrium objects. In particular,  $\mathcal{P}^*$  will denote the set of parties formed in equilibrium, while  $N^* \equiv |\mathcal{P}^*|$ . When verifying the existence of an equilibrium, the expected utility of a politician from joining some party  $A$  often depends on the number of state elections won by this party. I will therefore denote by  $w_A$  the number of state elections won by party  $A$  in case the politician under consideration decides to join. To illustrate, the choice of a politician between two parties  $A$  and  $B$  may in some cases determine which of these parties wins in the state that the politician belongs to. In this case it either holds that  $w_A + w_B = S + 1$  if joining makes either party the winner, or that  $w_A + w_B = S - 1$  if joining makes either party the loser.



## 4.6 Voter Distributions

A crucial input of the model is the set of voters. I will only make relatively weak assumptions in this regard. More specifically, the results in subsequent sections require a minimum amount of heterogeneity in voter tastes.

Before stating my assumptions, I need to take a step towards analysing behaviour in the model. Suppose there are two parties,  $A$  and  $B$ , contesting a state election. Party  $A$  has the politician with platform -1 as the unique member, while the remaining two politicians have joined party  $B$ . Assume voters are aware of this. Each voter then knows that the candidate nominated by party  $A$  has platform -1. The candidate of party  $B$ , on the other hand, is equally likely to have either platform 0 or 1. Let  $p^-$  be the unique real number such that a voter with ideal policy equal to  $p^-$  is indifferent between the candidate of either party, that is  $p^-$  solves

$$u(|-1 - p^-|) = \frac{1}{2}[u(|-p^-|) + u(|1 - p^-|)] .$$

As  $u$  was assumed to be decreasing and strictly concave, it must be the case that  $p^- \in (-0.25, 0)$ . Next, consider the situation that would result if the politician with platform 0 were to switch from party  $B$  to party  $A$ . In this case a voter with ideal policy  $|p^-|$  would be indifferent between voting for either party. Denote this policy by  $p^+$ .

I assume that the set of voters in state  $s$  can be described by a density function  $V_s$  over possible ideal policies. Let  $m_s$  denote the median associated with this density. Similarly, let  $V_f$  be the density of voters at the federal election with median  $m_f$ . It is assumed that  $m_f$  is equal to zero. The first more substantial assumption regarding voter preferences specifies that there is some minimum amount of heterogeneity in voter distributions across states: Let there be at least one state  $s$  such that  $m_s < -0.5$ , at least one state  $s$  such that  $m_s \in (p^-, p^+)$ , and at least one state  $s$  such that  $m_s > 0.5$ . Note that -0.5 (0.5) is the ideal policy at which a voter is indifferent between the platforms -1 and 0 (0 and 1). I will refer to states with median voter below -0.5 or above 0.5 as extremist states, while states with median voter between  $p^-$  and  $p^+$  are called centrist states. Purely for convenience, I will also assume that there is no state with median voter located at  $p^-$  or  $p^+$ .

The second assumption on voter distributions says that voters at the federal level are not too concentrated in the centre of the policy space:  $V_f([-0.5, 0.5]) \leq$

0.5. This requirement would be satisfied, for example, if  $V_f$  was the density of a uniform distribution with support on an interval of length at least equal to two, or the density of a normal distribution with variance slightly above one-half. While this says nothing about the empirical relevance of this assumption, the data presented in section 2 seems to indicate that actual preference heterogeneity is often substantial as well.<sup>4</sup>

## 5 Results

The model described in the previous section has many equilibria. This should come as no surprise: After all, it features two coordination problems—one between politicians and one between voters—as well as unrestricted out-of-equilibrium beliefs that can be freely chosen to support a specific equilibrium. In particular, voters may believe that a politician who deviates has a platform that the median voter of the state dislikes, which makes it unlikely that the deviation is successful.<sup>5</sup>

Given the multiplicity of equilibria, I will proceed as follows: As a natural starting point, I will consider the possibility that only one party is formed in the next subsection. Given their empirical relevance, my main interest is in equilibria with two parties. It turns out that this class can be fully characterized, as I discuss in section 5.2. This section contains the main results of the paper. It is only with three or more parties that the issue of multiplicity becomes endemic. Section 5.3 will discuss this in more detail and suggest a refinement.

### 5.1 One Party

An equilibrium where only one party is formed would be in stark contrast to the competitive nature of federal elections observed in reality. The following proposition shows that such equilibria do not exist.

**Proposition 1.** *There is no equilibrium such that  $N^* = 1$ .*

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<sup>4</sup>The preference heterogeneity across states visible in figure 1 does not imply the same amount of heterogeneity in the distribution of voters as a whole if states with large populations are home to mostly moderate voters. However, in the US and Germany, for example, some of the largest states are also the most partisan (that is Texas, Bavaria, and Baden-Württemberg).

<sup>5</sup>Such beliefs are not entirely unrestricted though. Consider for example the case where all politicians in some state have joined a party that allows any politician to join. If one politician deviates and joins a party with shape  $[-1..0]$ , voters find themselves at an information set that has two nodes; one for the case in which politician -1 has deviated and one for the case in which politician 0 has deviated. Voters may assign arbitrary weights to either node. Naturally, they may not attach any weight to the possibility that politician 1 has deviated.

*Proof.* Suppose only one party exists and call this party  $A$ . As party  $A$  cannot cover all platforms, there must be some policy  $p$  such that politicians with this platform are not allowed to join party  $A$ . Then a party  $D$  that accepts only politicians with platform  $p$  could enter successfully: By assumption, there exists a state with a median voter who strictly prefers platform  $p$  over any other platform. In this state party  $D$  would accordingly win the state election, as there are only two parties nominating candidates. This contradicts that only one party exists.  $\square$

If there was only one party, this party would not be able to allow all politicians to join. This assumption was justified by arguing that such a party would create too much internal conflict. But if some politicians are unable to join any party, a new party that allows just these politicians to join could easily recruit them. Due to the assumption on heterogeneity in voter preferences across states, this party would then also win at least one state election.

The result that no single party can fend off entry would continue to hold even if parties were allowed to cover all policies. The reason is that such a party would create intense competition for nominations within the party, making at least some politicians willing to join a second party. This kind of “internal competition” will also be a driving force behind other results presented below.

## 5.2 Two Parties

Given that no equilibria with only one party exist, I will now consider equilibria with two parties. Following the discussion in section 3, a natural starting point is a situation with a centre-left and a centre-right party. The most obvious formalisation of this would be an equilibrium where the set of proposed parties is equal to  $\{L, R\}$  with  $I_L = [-1..0]$  and  $I_R = [0..1]$ .

Given that parties  $L$  and  $R$  are the only existing parties, how will politicians behave? Those with platform  $-1$  and  $1$  will become members of the unique party available to them by assumption. Note that voters always know who has joined which party in this setting as there are no information sets that contain more than one node. It is then easy to show that party  $R$  can never be more attractive to a voter with ideal policy below  $p^-$  than party  $L$ . As a consequence, party  $L$  gets elected in states  $s$  such that  $m_s < p^-$  whether politician  $0$  has joined or not. As this is her only chance of getting elected, politician  $0$  will therefore always join party  $L$  in such states. Analogously, a politician with platform  $0$  will join party  $R$  in a state where the ideal policy of the median voter is greater than  $p^+$ .

In centrist states, in contrast, politician 0 can make either party the winner of the state election by joining. The probability with which politician 0 is nominated for and wins the state election in such states is thus the same independent of which party politician 0 becomes a member of. Conditional on receiving the nomination at the federal level, the probability of winning is also independent of the choice of party. This is because each party has a moderate and an extremist member in states where it wins and also nominates these with equal probability for the state election. Accordingly, both parties have an equal number of politicians of either type in their federal candidate pool in expectation. This means that the “expected opponent” at the federal election is equally strong no matter which party a politician joins. The only factor affecting the utility of a politician with platform 0 that may differ between parties is the probability of being nominated at the federal level. This probability is decreasing in the number of states elections won by the party. It follows that politicians with platform 0 in centrist states will join the party that wins fewer elections, a force that works towards equalizing the number of states won by each party. Equilibrium is reached if centrist politicians in centrist states have either joined the party that wins fewer elections or are distributed across parties such that the number of state elections won differs by at most one between them. For example, suppose there are  $2n$  states, in  $n$  of which the median voter’s most preferred policy is  $-1$  and party  $L$  accordingly wins the state election. Then party  $R$  must win all other states, whether they are centrist or rightist.

The observations made in the preceding paragraphs are collected in the following lemma.

**Lemma 1.** *Suppose  $\mathcal{P} = \{L, R\}$  with  $I_L = [-1..0]$  and  $I_R = [0..1]$ . Then*

- i) politician -1 (politician 1) joins party L (party R) in every state,*
- ii) politician 0 joins party L (party R) in any state  $s$  such that  $m_s < p^-$  ( $m_s > p^+$ ),*
- iii) in any state  $s$  such that  $m_s \in (p^-, p^+)$  politician 0 joins party L (party R) if  $w_L < w_R$  ( $w_R < w_L$ ) and may be a member of either party if  $w_L = w_R$ ,*
- iv) and in each state the party that politician 2 joins wins.*

The equilibrium of the election subgame given in lemma 1 has an interesting feature: Parties are more extreme in states where the distribution of voters does

not favour them. For example, members of the centre-left party are more left-leaning on average in states where the distribution of voters is strongly skewed towards the right. This is because centrist politicians join party  $R$  in this case as this is the only party that gives them a chance of winning elections. Only politicians with a left-wing platform remain in party  $L$ . This is contrary to the intuition that a left-leaning party should be expected to shift towards the right in a conservative state in an attempt to win votes. In fact, the causality suggested here runs in the opposite direction: The party is less conservative in a conservative state because it struggles to win any elections.

It was explained above that each party nominates a moderate or an extremist politician with equal probability at the federal level, which is true independent of the number of states won by either party. Given the behaviour imposed on completely indifferent voters, it follows that each party wins the federal election with equal probability ex-ante. The reason for this is that both of them are able to recruit moderate politicians that are attractive to the federal median voter. In contrast, one party would gain a substantial advantage if it were able to attract all centrists. As described above, electoral concerns make this impossible. Due to the importance of this result in the context of the paper, I restate it as a proposition.

**Proposition 2.** *Suppose  $\mathcal{P} = \{L, R\}$  with  $I_L = [-1..0]$  and  $I_R = [0..1]$ . Then each party wins the federal election with equal probability.*

It remains to establish that there actually exists an equilibrium of the game as a whole where parties  $L$  and  $R$  are formed and no other parties enter. This is confirmed in the following proposition, subject to a condition on payoffs being satisfied.

**Proposition 3.** *An equilibrium of the party formation game where  $\mathcal{P}^* = \{L, R\}$ , with  $I_L = [-1..0]$  and  $I_R = [0..1]$ , exists if  $y_s \leq \frac{1}{2}y_P$ .*

*Proof.* First, consider deviations by passive founders. It is sufficient to show that conditional on the affiliation behaviour given in lemma 1 no politician wants to deviate to joining some entering party that admits only politicians with a particular platform. Politicians with platform -1 do not gain by joining a party with shape  $[-1]$  if they are in a state with median greater than -0.5 as they would subsequently lose the state election. In a state with median voter below -0.5, on the other hand, a majority of voters would strictly prefer the new party, which would then win the state election by

assumption. In equilibrium a politician with platform -1 in such a state achieves

$$\frac{1}{2}(y_s + \frac{1}{4w_L}y_f + (1 - \frac{1}{w_L})\frac{1}{2}y_P) .$$

If she joins the entering party her payoff is  $y_s$  as she loses the federal election with certainty. This is because there will then be three parties competing at the federal level and a politician with platform -1 can never be strictly preferred over the other two candidates by a strict majority. Accordingly, there always exists a voting equilibrium where some other party wins. As it was assumed that  $y_f > 2y_P$ , the equilibrium utility decreases as  $w_L$  increases, approaching  $(1/2)y_s + (1/4)y_P$ . This is at least as great as  $y_s$  if  $y_s \leq \frac{1}{2}y_P$ , establishing that the condition  $y_s \leq \frac{1}{2}y_P$  is sufficient to prevent this type of deviation. By symmetry, this condition also ensures that politicians with platform 1 have no profitable deviation either.

For politicians with platform 0 the equilibrium payoff is given by

$$\frac{1}{2}(y_s + \frac{3}{4w_j}y_f + (1 - \frac{1}{w_j})\frac{1}{2}y_P) ,$$

with  $j \in \{L, R\}$ . In case such a politician deviates to a party with shape [0] she can at best hope to win the state election. This is because it was assumed that  $V_f([-0.5, 0.5]) \leq 0.5$  and there accordingly exists a voting equilibrium of the federal election where one of the other two parties wins. This is true even if all parties nominate a candidate with platform 0, as the restriction that all candidates receive an equal share of all votes when all voters are indifferent was only imposed on the equilibrium path. The payoff from the deviation is then  $y_s$ , which is smaller than the equilibrium payoff. This follows because the payoff of centrist politicians is greater than the payoff of extremist politicians for a given number of state election won by the party. Accordingly, centrists do not deviate as long as extremists do not deviate.

It remains to check whether any founder has an incentive to reposition their party. Due to the symmetry of the equilibrium it is sufficient to check for deviations by the founder of party  $L$ . Deviating to  $I_L = [-1]$  is clearly not profitable. Party  $L$  would only win in states  $s$  such that  $m_s < p^-$  and also win the federal election with lower probability. If the new shape of the party is either of [0], [1], or [0..1] the resulting constellation of parties does not deter entry, as a party of shape  $[-1]$  could enter and win all state elections in states with median voter below  $p^-$ . The deviation would therefore result in a payoff of  $-c$ .  $\square$

I will refer to the equilibrium in the preceding proposition as the  $L$ - $R$  equilibrium. Under this constellation of parties no third party can successfully enter for reasons easily illustrated in an example: Suppose a party were to enter that admits only politicians with platform -1 as members. Given that such a party

attracts members, it may do well at the state level, but would not be able to win the federal election due to the presence of the two already established parties. Politicians with platform -1 in states with very left-leaning median voters may nevertheless be tempted to join the entering party, as this eliminates competition from centrist politicians for the party nomination at the state level. The incumbent party prevents them from defecting by offering career prospects at the federal level. These include the possibility of becoming the party's nominee for the federal election as well as the payoff  $y_P$ . In the context of many countries this payoff represents positions in the federal government or high up in the party hierarchy. In the US, where the federal cabinet consists mostly of technocrats and seats in parliament are decided at the state level, parties nevertheless control assignments to posts on congressional committees. Such positions give individual legislators greater influence. This influence may then also translate into higher earnings outside of politics. Nevertheless, the condition for existence in the statement of proposition 3 may seem strong. It should be noted that this condition is a sufficient condition that only becomes binding in the limit as the number of state elections won by each party goes to infinity, and the probability of winning the federal election of each individual politician correspondingly goes to zero. As long as the number of states is small and the value of winning the federal election is large, the necessary condition for preventing defections can be much weaker. As will be illustrated below, the equilibrium may even exist for  $y_P$  equal to zero. In addition, parties may be able to prevent extremist politicians in extremist states from defecting by nominating them for the state election with greater probability. This possibility is discussed further in section 6.

While the set of parties  $\{L, R\}$  is robust to entry in many cases, the threat of entry is nevertheless required to support the equilibrium. Either party leader would otherwise have an incentive to exclude extremist politicians from the party, winning at least as many state elections as before and increasing the probability of winning the federal election. With entry this move is not profitable: That a part of the political spectrum is not covered by any party means that entry must occur. A new party can easily recruit the politicians otherwise unable to join a party and win at least some state elections. The equilibrium concept imposes that any deviation that entails entry will not be profitable. In this particular case that certainly seems reasonable: Suppose that after, say, left-wing politicians have been excluded from party  $L$ , a new party enters that allows only these politicians to join. This party then wins at least some states

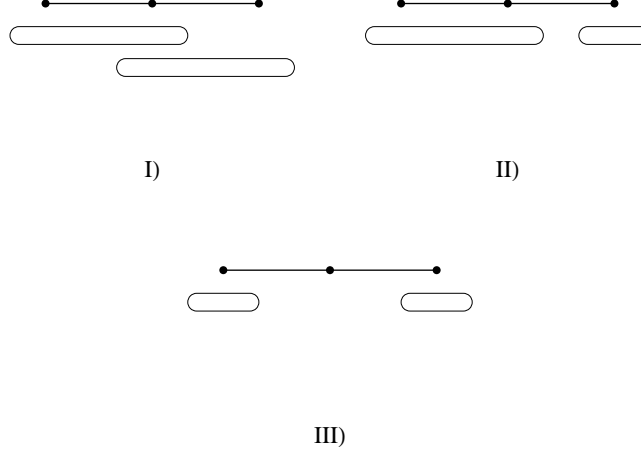
and consequently also nominates a candidate for the general election. With three candidates competing at the federal level there exists a voting equilibrium where the candidate of party  $L$  loses. My interpretation of this is that entry of even an extremist party poses a serious threat because of the danger that the established party loses its core support. Real-life examples of this abound: The UK Conservative Party currently fears the rise of the UK Independence Party (UKIP), not so much because UKIP itself is expected to win many elections, but because the votes lost to UKIP may hand victory in the upcoming general election to the Labour Party. In Germany the market-oriented reforms enacted by chancellor Gerhard Schroeder between 2002 and 2003 alienated the socialist wing of his party and fostered the formation of the left-wing party The Left. The Social Democrats have been trailing the Christian Democrats in federal elections ever since.

Defeat at the general election due to a split in the left-wing vote implies that party  $L$  becomes less attractive and all eligible politicians prefer to join party  $R$ . Even entry of an extremist party would consequently deter the leader of party  $L$  from deviating. There thus exists a mutual dependence: Extremist politicians prefer being members of a more moderate party due to the opportunities that come with being a member of federally powerful party. The party, on the other hand, is happy to offer these opportunities as it benefits from having extremist politicians as members rather than as competitors.

The set of all equilibria with two parties is characterized in appendix A and illustrated in figure 4. The constellation in panel I) is the  $L$ - $R$  equilibrium discussed previously. If career opportunities at the federal level are not sufficiently valuable to prevent extremists from joining smaller parties, this equilibrium fails to exist. There then nevertheless exists a two-party equilibrium, which is exemplified in panel II) of figure 4. Here an extremist left-wing party (right-wing party) faces a centre-right (centre-left) party. I will refer to this equilibrium as the  $M$ - $E$  equilibrium. The moderate party wins the federal election with higher probability than parties in the  $L$ - $R$  equilibrium do, increasing the payoff of its members. This can prevent them from joining a third party, even when the  $L$ - $R$  equilibrium does not exist. On the other hand, failure of the  $L$ - $R$  equilibrium to exist is also a prerequisite for the existence of the  $M$ - $E$  equilibrium. Otherwise the leader of the extremist party would be able to extend her party towards the political centre without losing extremist members to a third party. This would increase both the number of states won as well as the probability of winning the federal election.



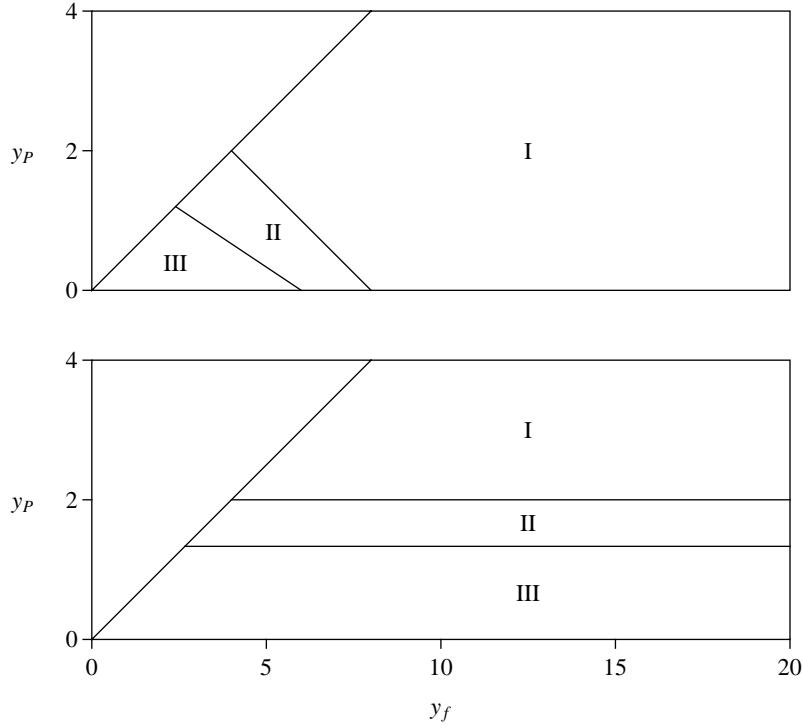
Figure 4: Possible Two-Party Equilibria



If even the  $M$ - $E$  equilibrium is not robust to members of the larger party defecting to a smaller party, the equilibrium in panel III) of figure 4 may exist, but only under special circumstances. I will refer to this equilibrium as the  $E$ - $E$  equilibrium. Existence requires in this case that there are no states in which voters are concentrated in the centre of the political spectrum. Otherwise entry of a centrist party would occur. The reason that neither of the equilibrium parties can shift towards the centre is again the threat of entry. Extending the membership towards centrists, on the other hand, is impossible as long as the  $M$ - $E$  equilibrium fails to exist.

The existence of two-party equilibria is illustrated in figure 5 for  $y_s$  equal to one and two numbers of states. In the upper panel the number of states is equal to four, while the lower panel corresponds to the limit case as the number of states goes towards infinity. Under combinations of values for the payoffs  $y_f$  and  $y_P$  that fall into region I the  $L$ - $M$  equilibrium is the unique two-party equilibrium, while in region II the  $M$ - $E$  equilibrium is the unique equilibrium with two parties. Comparing both panels illustrates how internal competition increases with the number of states. When the number of states is low, each state winner has a high chance of being nominated for the federal election. This prospect alone can be enough to keep politicians from deviating to joining smaller parties, who would offer less competition at the state level.

Figure 5: Existence of Two-Party Equilibria



Notes: Both panels show equilibrium existence in  $y_f$ - $y_P$  space for  $y_s = 1$  and a fixed number of states. In the upper panel the number of states is equal to four. The lower panel presents the limit case as the number of states increases towards infinity.

As the number of states increases, the probability of each individual politician of winning the federal nomination of her party goes to zero. A sufficiently high value for the payoff  $y_P$  is then required in order to prevent defections. In region III of the figure, the equilibrium of proposition 6 is the unique two party equilibrium, but only if the appropriate conditions on voter distributions in states are satisfied. Otherwise no equilibrium with two parties exists for these combinations of parameter values.

### 5.3 More Than Two Parties

As was already mentioned, the number of equilibria with three or more parties is large.<sup>6</sup> What is more, the equilibrium number of parties can be large too as demonstrated by the following example.

**Lemma 2.** *Suppose there are  $S$  states, with two and only two states  $s$  such that  $m_s \in (p^-, p^+)$  and no state  $s$  such that  $V_s([-0.5, 0.5]) > 0.5$ . Then there exists an equilibrium such that  $N^* = S$  if  $8y_s \geq y_P$ .*

*Proof.* Consider the following strategy profile:  $S$  founders propose parties. Two of them propose a party of shape  $[0]$ . Call these centrist parties  $C_1$  and  $C_2$ . All other parties either have the shape  $[-1]$  or  $[1]$ . Call these parties leftist and rightist. The number of parties with shape  $[-1]$  is equal to the number of states  $s$  such that  $m_s < p^-$ . Collect these states in the set  $\mathcal{S}_l$ . The number of parties with shape  $[1]$  equals the number of states  $s$  such that  $m_s > p^+$ . Collect these states in the set  $\mathcal{S}_r$ . Each of the centrist parties wins the election in one of the two states that do not belong to the set  $\mathcal{S}_l \cup \mathcal{S}_r$  and accordingly has a member in that state. Each leftist (rightist) party has a member in one and only one of the states belonging to  $\mathcal{S}_l$  ( $\mathcal{S}_r$ ) and wins the state election in that state. At the federal election the candidates of the centrist parties tie and no other parties receive any votes.

Given that each election features at least three candidates and the assumptions on voter distributions, there always exists a voting equilibrium such that the specified candidate wins. Centrist politicians do not gain from changing their party affiliation as they either continue to lose the state or federal election or simply substitute external competition for internal competition at the federal level. Similarly, extremist politicians do not benefit from switching parties either. No founder wants to deviate to remaining passive, as each founder wins one state election. No founder wants to change the shape of their party. Founders of leftist and rightist parties do not gain from such a move, as they at best continue to win the same state election as in equilibrium and still lose the federal election with certainty. Suppose a founder of a centrist party changes the shape of her party to  $[-1..0]$  or  $[0..1]$ . Without loss of generality, let this be party

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<sup>6</sup>In fact, almost any constellation with three or more parties is trivially an equilibrium under the equilibrium concept employed in the previous sections. This is because deviations by a party leader can be punished by assigning voting equilibria where the party of the deviating founder loses. In particular, it is possible that a party wins the federal election in equilibrium when a specific set of candidates has been nominated, but loses the federal election after a deviation even if the same candidates have been nominated by the same set of parties. This possibility could be eliminated by a suitable extension of the equilibrium concept. In order to avoid additional definitions, I instead write all proofs of existence of an equilibrium as if I was employing the stronger equilibrium concept. This is permissible (if possible) as any equilibrium that exists under stricter requirements is also an equilibrium under weaker requirements.

$C_1$ . Then there exists an equilibrium of the election subgame where all politicians and voters behave the same as in equilibrium with two exceptions: Politician 0 in the state previously won by party  $C_1$  now joins party  $C_2$  and centrist politicians in states belonging to the set  $S_l \cup S_r$  join party  $C_1$ . Deviating back to party  $C_1$  leaves the payoff of politician 0 in the state previously won by party  $C_1$  unchanged. Any extremist politician who now has the option of joining party  $C_1$  does not increase her chance of winning the state election by doing so. Either she already wins the state election, in which case she reduces her probability of being nominated for the state election, or she continues to lose the state election, as she must be in a state where the median voter prefers a politician from the opposite side of the political spectrum. Note that this equilibrium of the election subgame reached after the deviation deters entry, as any additional parties would not win at the federal level due to strategic voting and all politicians who win state elections are already the sole member of their party in their state. The same logic applies to the original equilibrium, which completes the proof.  $\square$

The proof of the preceding proposition illustrates the possibility that a large number of regional parties forms, only a few of which play a role at the federal level. What may be the reason that such an equilibrium is not typically observed in reality? One possibility is that constellations with many parties do not persist because there are incentives for parties to merge. In fact, such mergers are commonly observed in reality. To name just a few examples: The Liberal Democrats of the UK were formed as a fusion of the Liberal Party and the Social Democratic Party. The Conservative Party of Canada came into existence as a merger of the Progressive Conservative Party and the Canadian Alliance. In Australia, the coalition between the Liberal Party and the National Party has existed for so long that it is often treated as a single party.

One gain from a merger may be that an alliance of parties wins more elections than a number of small parties taken together would. As an example, consider a case with three parties. One party, given by the interval  $[0..1]$ , wins most state elections and is the only party that wins the federal election. The other two parties both have the shape  $[-1]$  and win only one state election each. Without any cooperation between party leaders this can be an equilibrium, depending on parameters. If one of the smaller parties expands by letting centrists join as well, it is able to recruit centrist politicians and nominate these for the federal election. Nevertheless, the presence of the second small party remains as an obstacle towards greater success at the national level: Voters on the right of the

political spectrum continue to vote for the centre-right party, while left-wing voters fail to coordinate on giving enough votes to the candidate of the centre-left party. However, if the leaders of the small parties are able to cooperate and form only one party of shape  $[-1..0]$ , they substantially increase their electoral fortunes as should be clear from the previous section.

A second potential reason for a merger of parties lies in synergies such as reduced administrative costs, more effective fund-raising and advertising campaigns, and greater recognition by voters in states where the party does not have a strong presence. In the model, such an incentive for a merger comes from the presence of the fixed cost  $c$  that founders pay when forming a party. Avoiding this cost can make a merger profitable even if the newly formed party does not enjoy greater success.

Note that any merger that happens for reasons of increased electoral success also leads to a reduction in fixed costs, while the converse is not true. In trying to integrate mergers into the model I will therefore focus on fixed costs as the driving force, which is also simpler to implement than the case where electoral motivations play a role. Doing so requires that leaders are somehow able to share the benefits of their joint ventures. I allow for this by giving founders the ability of committing to transfers to other founders in the beginning of the game that are conditional on whether or not the founder receiving the transfer proposes a party. In reality these transfers will often take the form of powerful positions in the party hierarchy or in government, which are pledged to party leaders who agree to enter into an alliance. The ability to credibly commit to these transfers, on the other hand, can be justified by reputational concerns. Formally such a promise takes the shape of a map  $\{0, 1\} \rightarrow \mathcal{R}_+$ , which assigns an amount of utility to be transferred to a founder to an indicator for whether this founder has proposed a party or not. Accordingly, the strategy space of each founder is extended to allow for a choice of an infinite-dimensional vector of such maps. Let  $\bar{t}$  be the sum of all transfers a founder receives net of any transfers the founder carries out. The utility of this founder is then given by

$$\sum_{s=1}^S \pi_s x_w + \pi_f x_f - c + \bar{t} ,$$

using the same notation as in section 4. Call this version of the game the extended party-formation game.

Beyond transfers, mergers also require the possibility of joint deviations by

groups of founders. One way to allow for this would be to look for equilibria that are coalition-proof in the sense of Bernheim et al. (1987). However, a much simpler approach that turns out to be equivalent in this particular case is to look for equilibria that are Pareto efficient among founders in the extended party-formation game. I say that such an equilibrium is robust to party mergers.

The first result regarding the extended party-formation game says that transfers cannot be used to maintain formations of parties that are not stable in the original game. This would require sufficiently large transfers to all passive founders in order to prevent them from taking the opportunity of forming a successful party. In reality, one would expect it to be impossible to pay off all individuals in a position to initiate the formation of a party. The same is true here due to the assumption that the number of founders is infinite. The result would also hold for a finite but sufficiently larger number of founders.

**Lemma 3.** *Any constellation of parties  $\mathcal{P}^*$  that is part of a party-formation equilibrium of the extended game must deter entry.*

*Proof.* Suppose there was an equilibrium of the extended game such that the constellation of parties formed along the equilibrium path does not deter entry. This would require that any passive founder receives a net transfer at least as large as the utility she could achieve by forming a party. This utility would be at least as great as  $x_w - c$ , as any party that successfully enters must win at least one state election. The total sum of transfers from party leaders to passive founders must therefore be infinite, while the total utility of all party leaders is no larger than  $Sx_w + x_f$  and therefore finite. Consequently, at least one party leader would be achieving negative utility and prefer to remain passive.  $\square$

The second result shows that the requirement of robustness to party mergers selects those party-formation equilibria with the lowest number of parties, but does not discriminate between equilibria within that class.

**Lemma 4.** *A party-formation equilibrium is robust to party mergers if and only if there exists no other equilibrium in which a smaller number of parties is formed.*

*Proof.* Assume there is a party-formation equilibrium  $E$  such that  $N^* = k$  and there is a potentially empty set  $\mathcal{F}_T$  of founders who receive transfers from other founders. Let  $\mathcal{F}_E$  be the set of founders who propose parties in this equilibrium. All founders who make transfers must belong to the set  $\mathcal{F}_E$ . Any founder not belonging to this set could lower the transfers she makes to zero. This would not affect the transfers she

receives as these can depend only on whether or not a founder proposes a party. The total utility of all members of the set  $\mathcal{F}_E \cup \mathcal{F}_T$  is therefore equal to  $Sx_w + x_f - kc$ .

To show necessity, suppose that there exists a second party-formation equilibrium  $E'$  in which a number of parties  $k' < k$  is formed. Now consider the following equilibrium: In the first step  $k'$  members of the set  $\mathcal{F}_E$  promise transfers such that any founder belonging to  $\mathcal{F}_E \cup \mathcal{F}_T$  achieves a strictly greater utility than in the equilibrium  $E$ . This is possible because the total utility of all founders is higher in the equilibrium  $E'$  than in the equilibrium  $E$  by the amount  $(k - k')c$ . The same  $k'$  founders subsequently propose the set of parties that exists along the equilibrium path of  $E'$ . If any deviation occurs at the first stage, the same set of parties as in the equilibrium  $E$  is formed. This ensures that no founder can gain from such deviations. Deviations by passive founders to proposing a party are not profitable as the set of parties proposed in any subgame deters entry by lemma 3. This shows that the equilibrium  $E$  is not robust to party mergers.

For sufficiency, note that any equilibrium  $E'$  in which the number of parties is equal to or greater than  $k$  generates a total utility that is no greater than the total utility achieved in the equilibrium  $E$ . It follows immediately that it is impossible that the equilibrium  $E'$  Pareto dominates the equilibrium  $E$ .  $\square$

Is then possible to fully characterize the number of parties formed in party-formation equilibria that are robust to party mergers.

**Proposition 4.** *The number of parties in any party-formation equilibrium that is robust to party mergers is*

- i) no lower than two and no greater than three,*
- ii) equal to two whenever a party-formation equilibrium exists in which two parties are formed.*

*Proof.* By proposition 1 no single party can deter entry. Lemma 3 then implies that there must be at least two parties in any party-formation equilibrium that is robust to party mergers. Combined with lemma 4 this establishes claim *ii*). In order to show that there cannot be more than three parties it needs to be demonstrated that there exists a party-formation equilibrium of the extended game in which three parties are formed whenever no equilibrium with two parties exists. This result is established in proposition 8 in the appendix.  $\square$

A second look at figure 5 illustrates the set of equilibria that are robust to party merger. In regions I the  $L$ - $R$  equilibrium is the unique equilibrium with two parties, while the  $M$ - $E$  equilibrium is the unique equilibrium with two

parties in region II. Accordingly, these equilibria are also the unique equilibria that are robust to party mergers in their respective regions of the parameter space. In region III a unique equilibrium with two parties may exist, but only if there is no state with a large share of moderate voters. Otherwise the only equilibria that are robust to party mergers that exist in this area feature three parties. The equilibrium used to establish this result is one where all existing parties allow only one type of politician to join, there is one such party for each possible platform, and the candidates of the two extremist parties tie at the federal election. Other three-party equilibria exist as well. An interesting possibility are equilibria which are “almost identical” to one of the two-party equilibria. As an example, consider the  $M$ - $E$  equilibrium with  $I_M = [-1..0]$  and  $I_E = [1]$ . A comparison of the upper and the lower panel of figure 5 shows that this equilibrium sometimes exists only if the number of states is sufficiently low. Otherwise too much internal competition makes extremist members of party  $M$  willing to defect to a newly formed party of shape  $[-1]$ . This suggests the existence of an equilibrium where this smaller party forms in addition to the parties of the  $M$ - $E$  equilibrium and voters at the federal level behave as if the smaller party did not exist, the presence of which nevertheless reduces internal competition among members of party  $M$  sufficiently to prevent further deviations.

## 6 Robustness

The basic model of party formation presented here requires a number of simplifying assumptions for tractability. This section will discuss some of these in more detail.

### 6.1 Comprehensive Parties

Parties that allow all politicians to join were ruled out on the basis of the argument that infighting between opposing factions would make such parties inherently unstable. It may nevertheless be of interest to consider what additional equilibria exist if parties of shape  $[-1..1]$  were included in the action space of founders. As was already argued above, even a party that allows all politicians to join would not be able to deter entry of additional parties due to intense internal competition. However, focusing on the class of two-party equilibria, two additional stable constellations of parties emerge. One features either the



equilibrium set of parties  $\{[-1..1], [-1..0]\}$  or  $\{[-1..1], [0..1]\}$ . This constellation can actually be essentially equivalent to the  $M$ - $E$  equilibrium. This is the case when all eligible politicians join the smaller party in all states. Out-of-equilibrium beliefs of voters then prevent any deviations by politicians towards joining the larger party.

The second case is that an equilibrium with two parties of shape  $[-1..1]$  forms. This is possible because out-of-equilibrium beliefs can be used to make both parties have only one member in any state, as in the previous paragraph. It seems unlikely though that a party could maintain such widely varying ideological profiles across different states. The possibility of infighting also seems particularly relevant in this situation. If both parties had three members in most states, on the other hand, entry of a party of shape  $[0]$  would be possible. This is because the larger parties then create too much internal competition. Furthermore, a centrist party could also do well federally, as it is relatively likely that both of the larger parties nominate a candidate with platform -1 or that both of them nominate a candidate with platform 1. In both cases a strict majority of voters would prefer a centrist candidate, who would accordingly win by assumption.

## 6.2 A Greater Number Of Politicians

Allowing for a greater number of politicians is difficult, as this leads to a greater number of cases to consider. It also increases the scope for coordination failure at the stage at which party affiliations are chosen and thus the number of equilibria. Nevertheless, one effect of a higher number of politicians populating each state is clear: More politicians imply greater internal competition for nominations at the state level. This would make existence of the two-party equilibria discussed above less likely, as the magnitude of the payoffs  $y_s$  and  $y_P$  required in order to keep politicians from joining smaller parties increases proportionally with the number of party members. This may not be too much of a concern: The politicians in the model should be thought of as those who have already achieved some prominence within state parties and are therefore in a position to be considered for nominations. At any given point in time the number of such individuals will be limited. In addition, an emerging party focused on issues already covered by an existing party would find it hard to achieve credibility if it fails to attract any of the more prominent members of that party. In fact, the formation of new parties is typically the product of a whole faction of an

existing party defecting jointly. It is possible to interpret each politician in the model as representing factions who coordinate their actions.

### 6.3 Policy Choices

The assumption that politicians are committed to implementing their platform is not satisfying. While the empirical literature presented in section 3 seems to suggest that policy preferences of politicians are the main driver of their choices in office, it would be more appealing to see this behaviour emerge as part of an equilibrium rather than imposing it from the outset. In the model, extremist politicians can often increase their chances of winning the federal election by pretending to be a centrist when choosing state policies. To address this concern I will consider a more general utility function for politicians that includes both career concerns and policy preferences. For a politician with ideal policy  $i$  let the utility function now be given by

$$\pi_s(y_s + \pi_f y_f + \pi_p y_P) - \alpha \sum_{l \in \{s, f\}} (p_l - i)^2 ,$$

using the same notation as in section 4. In addition, assume that politicians can freely choose the policy they implement at any stage. All other elements of the game remain unchanged. This more general version of the model is challenging to solve in its entirety. I will present results for the election subgame reached after parties  $L$  and  $R$  have been proposed.

Proceeding by backwards induction it is clear that any politician elected at the federal level will implement her ideal policy. All other stages are less straightforward. I will start by asking under what conditions a separating equilibrium exists where politicians implement their ideal policy at the state level and otherwise behave as in lemma 1. In this case a politician with ideal policy -1 achieves a continuation utility of

$$\frac{1}{w_L} \left[ \frac{1}{4} y_f + \frac{1}{4} (-\alpha) + \frac{1}{2} (-\alpha) \right] + \left( 1 - \frac{1}{w_L} \right) U_n$$

after winning a state election, where the term  $U_n$  captures the utility in case the politician is not nominated for the federal election. Choosing the policy 0 after the state election results in a utility of

$$-\alpha + \frac{1}{w_L} \left[ \frac{3}{4} y_f + \frac{1}{4} (-\alpha) \right] + \left( 1 - \frac{1}{w_L} \right) U_n .$$

The politician now incurs a cost for a suboptimal policy choice but increases her chance of winning the federal election, simultaneously reducing the probability that an extremist from party  $R$  gets elected. Note that voters do not observe that a deviation has taken place as the politician elected in the state has ideal policy 0 with positive probability ex-ante. It can then be shown that the separating equilibrium exists as long as the ratio  $\alpha/y_f$  is no smaller than  $\max_{P \in \{L,R\}} 2/(4w_P - 5)$ .<sup>7</sup> The greater the number of elections won by a party, the lower the probability that any given politician will get nominated for the federal election, which in turn makes extremist politicians less likely to benefit from pretending to be a centrist.

Proceeding as above it can be shown that if the ratio  $\alpha/y_f$  is smaller than  $\min_{P \in \{L,R\}} 2/(4w_P - 5)$  then there only exists a pooling equilibrium where all politicians implement the policy 0 at the state level.<sup>8</sup> Behaviour nevertheless remains very close to the one given in lemma 1. As all politicians implement the same policy at the state level, voters in state elections will base their choice on considerations regarding federal policies. These are determined by the winner of the federal election, who is effectively picked at random from among state winners. State voters thus benefit from adding politicians to this pool that have similar preferences to them. Voters at the federal election are split at zero between both parties as they cannot distinguish between candidates exactly, but know that the candidate of either party may be a centrist or an extremist with equal probability. For example, a voter with ideal policy -1 in a state where politician 0 has joined party  $L$  would not vote for party  $R$  as a victory by party  $R$  increases the probability that a politician with platform 1 wins the federal election. As long as politicians in both parties pool, the only difference between them is the degree of internal competition for the federal nomination, at least from the perspective of a centrist politician in a centrist state. As before, these politicians will therefore tend to sort into the party that wins fewer state elections.

The empirical evidence is in favour of the separating equilibrium, where all politicians implement their own ideal policies. Nevertheless, the discussion above suggests that the overall results do not necessarily change much even if extremist politicians try to pass off as centrists.

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<sup>7</sup>This condition applies as long as each party wins at least two state elections, which implies that  $4w_P - 5$  is positive.

<sup>8</sup>Cases where politicians in one party separate and politicians in the other party pool are more complicated and will not be considered here.

## 6.4 Candidate Selection

Parties are assumed to nominate candidates randomly in the basic version of the model. This hurts the chances of either party of winning the federal election. However, if parties were to nominate a centrist politician for the federal election whenever one is available, this would reduce the expected utility of extremist party members and potentially induce them to join a third party. In order to prevent this, the party leader may have an incentive to commit ex-ante to nominating extremists with a sufficiently high probability. In practice this could be achieved through choosing a particular mechanism for candidate selection, such as primaries or nomination through voting by delegates at the party convention. As it turns out, the requirement to keep extremist party members satisfied may lead to extremists being nominated with even greater probability than in the basic model. This possibility is demonstrated in appendix B, which analyses an extension of the model where the party leader commits to a probability of nominating either type of politician at the federal election.

A second and closely related possibility of keeping the extremist wing of the party satisfied would be to increase the probability with which extremist politicians get nominated in extremist states. To some extent I feel that the assumption that the nomination process is fairly noisy from the perspective of party-outsiders is more appropriate at the state level than at the federal level. After all, state-level candidates are often little known to the public and different factions within the party will be pulling in different directions, with the party leader (the party establishment) certainly favouring moderate candidates due to their electability at the federal election. Nevertheless, nominating candidates through primary elections may be one way to ensure that candidates fit the preferences of the state median voter well. If this is the case, then it becomes even easier to see how national parties maintain regional monopolies: Extremist politicians in extremist states would basically face no internal competition from moderates and the existence conditions for the two-party equilibrium of 3 would be greatly relaxed. It is noteworthy though that even in this case no single party would be able to monopolize all elections. Intense internal competition for the federal nomination would still lead to the successful entry of a second party and proposition 1 continues to hold.

## 7 Conclusion

The aim of this paper was to provide an explanation for why federal elections are typically strongly competitive, while state elections are often dominated by one party. This pattern is a product of the interaction between political demand (voter preferences) and supply (political parties). The model presented here has focused on political parties while taking heterogeneity in voter preferences as given. It was shown that an equilibrium with two parties, one centre-left and one centre-right, can explain the main features of the data. In this equilibrium, states with centrist median voters will display small vote margins while state elections with more extreme median voters are dominated by one party. Such regional monopolies are possible for two reasons: First of all, parties cannot promise to implement policies other than the ones preferred by its members. This makes it impossible for, say, a left-wing party to appeal to right-wing voters. Secondly, and relatedly, a third party would have to attract some of the members of an existing parties in order to successfully contest a regional monopoly. The existing party, however, prevents its members from defecting by offering its members career opportunities (inside the party and at the federal level) that they would not have as members of a smaller regional party.

The federal level, on the other hand, turns out to be competitive as both parties are able to attract a symmetric candidate pool. If, in contrast, all moderate politicians were to sort into one party, this would skew the electoral odds in favour of that party. This is impossible. For example, if all centrists were to join the centre-right party, there would be some states with a very left-leaning electorate where the left-wing party nevertheless wins elections. In such states moderate politicians simply have to join the centre-left party if they want to win elections. An extremist state thus becomes a source of centrist candidates for the party that can win elections there.

It is, however, not necessarily by choice that each party maintains an ideological profile that makes it particularly attractive towards voters in some states. Moderating the party could potentially increase success at the federal level while also raising the number of votes received in some centrist states. If both parties did so, there would be no clear difference between them and no reason why one of them should do much better than the other in any state election. The force that prevents this outcome is the threat of entry of more extreme parties. It was shown that a single state with a voter located relatively far to the left and a single state with median voter located relatively far to the right is sufficient

to make this threat of entry effective. The prospect of winning a single state election is enough for a third party to form, which would then also be able to participate in the federal election. The established parties do not shift towards the political centre due to the risk that the third party that would subsequently enter could attract enough votes to make the deviating party lose the federal election.

The data presented in this paper was taken from a sample of federal states and the model was also framed in this context. I believe that the mechanisms discussed here should nevertheless be useful for thinking about other types of institutional arrangements as well. For example, imagine someone who is considering contesting a seat in the House of Commons in the UK. If this person stands politically between the Labour Party and the Conservative Party, it seems likely that she will base her choice of affiliation on which party is more popular in her district. Having such moderate members will affect how either party is perceived by voters and accordingly also influence election results. Furthermore, all democratic countries hold regional elections. Winning these can serve to raise the profile of a newly formed party, even if this election does not receive quite as much attention as one for a state parliament would. The threat of entry would then operate in a similar way as it was described above.

A more serious shortcoming is that the analysis has focused on plurality rule, while other electoral systems were given no consideration. In particular, it would be interesting to know how well the results extend to proportional representation, where the number of parties achieving a significant share of votes is typically larger. Due to the already complicated nature of the model it would have been a challenge to incorporate the additional complexity inherent in a system of proportional representation. I hope to address this question in future work.

# Appendix

## A Additional Results for the Basic Model

This appendix provides a full characterization of equilibria with two parties beyond the  $L$ - $R$  equilibrium analysed in the main text and demonstrates the existence of an equilibrium with three parties. The following lemma will simplify subsequent proofs.

**Lemma 5.** *Any constellation of parties such that some extremist politician is unable to join a party does not deter entry.*

*Proof.* Without loss of generality, assume that politicians with platform  $-1$  are unable to join any party. If a passive founder were to propose a party  $D$  of shape  $[-1]$ , all politicians with platform  $-1$  would join. In any state  $s$  such that  $m_s < p^-$  a strict majority of voters then strictly prefers the candidate of party  $D$ . As it was assumed that at least one such state exists and due to the restrictions on voting behaviour, party  $D$  wins at least one state election.  $\square$

The propositions below establish existence conditions for additional equilibria with two parties.

**Proposition 5.** *An equilibrium of the party-formation game where  $\mathcal{P}^* = \{M, E\}$  with either  $I_M = [-1..0]$  and  $I_E = [1]$ , or  $I_M = [0..1]$  and  $I_E = [-1]$  exists whenever the  $L$ - $R$  equilibrium fails to exist and  $y_s \leq \frac{3}{4}y_P$ .*

*Proof.* As the equilibria in the statement of the proposition are symmetric to each other, the proof will focus on the case  $I_M = [-1..0]$  and  $I_E = [1]$ . In this case the affiliation behaviour of politicians is trivial. Party  $M$  wins all states  $s$  such that  $m_s < p^+$  and wins the federal election with probability three-fourth. It will first be checked whether any active founders want to deviate.

Suppose the  $L$ - $R$  equilibrium exists. In the current equilibrium party  $E$  wins at most as many state elections as party  $R$  in the  $L$ - $R$  equilibrium, but wins the federal election with lower probability. The founder of party  $R$  would deviate to  $I_R = [0..1]$ . If the  $L$ - $R$  equilibrium does not exist, this deviation would not be profitable as the resulting set of parties does not deter entry. This establishes that it is necessary for the existence of the current equilibrium that the  $L$ - $R$  equilibrium does not exist.

Suppose the founder of party  $M$  deviates to  $I_M = [-1]$ . Now party  $M$  wins all states  $s$  such that  $m_s \leq 0$  and the federal election with probability one-half. The deviation is not profitable.

All other deviations by active founders result in party constellations that leave at least one extremist politician without a party to join and therefore do not deter entry by lemma 5.

It remains to be shown that entry of additional parties is impossible. The equilibrium utility of extremist members of party  $M$  is

$$\frac{1}{2}(y_s + \frac{1}{2w_M}y_f + (1 - \frac{1}{w_M})\frac{3}{4}y_P) .$$

This expression is decreasing in  $w_M$  under the assumption that  $y_f > 2y_P$  and approaches

$$\frac{1}{2}(y_s + \frac{3}{4}y_P) .$$

If this politician joins an entering party of shape  $[-1]$ , she may win the state election, but can be made the certain loser of the federal election. It follows that a sufficient condition for this deviation not to be profitable is  $y_s \leq \frac{3}{4}y_P$ .

Centrist politicians achieve a higher utility in equilibrium than extremist members of party  $M$  as they win the federal election with higher probability. Due to the assumption that  $V_f([-0.5, 0.5]) \leq 0.5$ , centrist candidates of a third party do not necessarily win at the federal level. Their deviation payoff is accordingly also  $y_s$ . This shows that centrist members of party  $M$  do not deviate to joining a third party whenever politicians with platform -1 refrain from doing so.

Finally, members of party  $E$  do not gain from joining a party of shape  $[1]$  as they at best win all elections with the same probability as before.  $\square$

The equilibrium in the preceding proposition will be referred to as the  $M$ - $E$  equilibrium.

**Proposition 6.** *An equilibrium of the party-formation game where  $\mathcal{P}^* = \{A, B\}$  with  $I_A = [-1]$  and  $I_E = [1]$  exists whenever the  $L$ - $R$  equilibrium and the  $M$ - $E$  equilibrium fail to exist and there exists no state  $s$  such that  $V_s([-0.5, 0.5]) > 0.5$ .*

*Proof.* The affiliation behaviour of politicians is trivial. Party  $A$  wins all states  $s$  such that  $m_s < 0$  with certainty, all states  $s$  such that  $m_s = 0$  with probability one-half, and the federal election also with probability one-half. It will first be checked whether any active founders want to deviate.

Suppose the  $L$ - $R$  or the  $M$ - $E$  equilibrium exists. In both cases at least one of the sets of parties  $\{[-1..0], [1]\}$  or  $\{[-1], [0..1]\}$  deters entry. The founder of at least one party would then have an incentive to deviate accordingly. If the  $L$ - $R$  and the  $M$ - $E$  equilibrium fail to exist, this deviation would not be profitable as the resulting set of parties does not deter entry. This establishes that it is necessary for the existence of the current equilibrium that neither the  $L$ - $R$  nor the  $M$ - $E$  equilibrium exists.

All other deviations by active founders result in party constellations that leave at



least one extremist politician without a party to join and therefore do not deter entry by lemma 5.

It remains to be shown that entry of additional parties is impossible. If an third party were to nominate a candidate for the federal election, there exists a voting equilibrium such that this candidate does not win. Accordingly extremist politicians would not gain from joining any entering party, as they either already win the state election with certainty or would not win even if they joined a third party. Centrist politicians would join a party of shape  $[0]$ . If the voter distribution of a state  $s$  is such that  $V_s([-0.5, 0.5]) > 0.5$ , then this party would win the state election. Otherwise there exists a voting equilibrium where either party  $A$  or party  $B$  wins. If this is true for all states, then entry does not occur.  $\square$

Any other constellation of two parties not considered so far is never part of an equilibrium.

**Proposition 7.** *No constellation of two parties other than  $\{-1..0\}, [0..1]$ ,  $\{-1..0\}, [1]$ ,  $\{-1\}, [0..1]$ , and  $\{-1\}, [1]$  is part of an equilibrium.*

*Proof.* It is straightforward to verify that any constellation of two parties not listed in the statement of the proposition leaves at least some extremist politicians without a party to join. The claim then follows from lemma 5.  $\square$

Finally, it can be shown that an equilibrium with three parties exists whenever no equilibrium with two parties does.

**Proposition 8.** *An equilibrium such that  $\mathcal{P}^* = \{A, B, C\}$  with  $I_A = [-1]$ ,  $I_B = [0]$ , and  $I_C = [1]$  exists whenever the E-M and the L-R equilibrium fail to exist.*

*Proof.* Suppose the set of parties  $\{A, B, C\}$  is formed in equilibrium with  $I_A = [-1]$ ,  $I_B = [0]$ , and  $I_C = [1]$ . Then the affiliation behaviour of politicians is trivial. Assume the candidates of parties  $A$  and  $C$  tie at the national level, party  $B$  wins all centrist states and states  $s$  such that  $V_s([-0.5, 0.5]) > 0.5$ , and party  $A$  (party  $C$ ) wins all state elections in states  $s$  such that  $m_s < p^-$  ( $m_s > p^+$ ) and  $V_s([-0.5, 0.5]) \leq 0.5$ .

Passive founders are unable to enter with a new party. There would be at least three parties competing at the federal level, implying that there exists a voting equilibrium such that the newly formed party loses. Politicians therefore have no incentive to join this new party, as they either already win the election in their state with certainty or would not win even after the deviation.

It remains to check whether any founder has an incentive to reposition their party. The only deviation of the founder of party  $A$  that would not leave politicians with platform -1 without a party to join is  $I_A = [-1, 0]$ . If party  $B$  won at least two state elections prior to the deviation, there then exists an equilibrium of the election subgame where centrist politicians join party  $A$  if and only if they reside in states that

party  $A$  wins in the original equilibrium. If one of the members of party  $B$  were to deviate to joining party  $A$ , there must be at least one other state where party  $B$  still wins. The deviation therefore yields

$$\begin{aligned} \frac{1}{2}(y_s + (1 - \frac{1}{w_A})\frac{1}{4}y_P) &< \frac{1}{2}(y_s + (1 - \frac{1}{w^+})\frac{3}{4}y_P) \\ &< \frac{1}{2}(y_s + \frac{1}{4w^+}y_f + (1 - \frac{1}{w^+})\frac{3}{4}y_P) \\ &< y_s , \end{aligned}$$

where  $w^+$  is the number of states  $s$  such that  $m_s \leq p^+$ . The first inequality holds as  $w_A < w^+$  and the third inequality must be true as long as the  $M$ - $E$  or the  $L$ - $R$  equilibrium do not exist. It follows that no centrist politician who wins a state election in equilibrium wants to become a member of party  $A$ . Accordingly, party  $A$  wins the same number of state elections as before, but wins the federal election with smaller probability. If, on the other hand, party  $B$  wins only one state prior to the deviation, then the only equilibrium of the election subgame reached after the deviation may be that all politicians with platform -1 or 0 join party  $A$ . But if this is true and simultaneously the  $M$ - $E$  and the  $L$ - $R$  equilibrium fail to exist, then the constellation of parties reached after the deviation does not deter entry.

It can be shown analogously that the founder of party  $C$  has not profitable deviations. The only deviations of the founder of party  $B$  that would not leave politicians with platform 0 without a party to join are  $I_B = [-1, 0]$  and  $I_B = [0, 1]$ . Without loss of generality, consider the second deviation. There then exists an equilibrium of the election subgame reached where only politicians with platform 1 in states previously won by party  $B$  change their affiliation. As long as party  $C$  wins at least two state elections prior to the deviation, there always exist voting equilibria such that the candidate of party  $B$  loses, even if one additional rightist politician were to join. This deviation would then not be profitable as joining party  $B$  means winning all elections with lower probability for such a politician. The deviation by the founder of party  $B$  then yields no benefit. If party  $C$  wins only one state election prior to the deviation, then the single member of party  $C$  must still prefer party  $C$  over party  $B$  as long as the  $M$ - $E$  and the  $L$ - $R$  equilibrium fail to exist.  $\square$

## B Candidate Selection

Consider a version of the basic model described in section 4, where the founder of a party  $P$  makes an additional strategic choice in committing to a probability  $q_P$ . In any situation where party  $P$  has both extremist and moderate politicians in its candidate pool for the federal election, the candidate for this election will be randomly drawn from among centrists with probability  $q_P$  and from among extremists with

probability  $1 - q_P$ . The choice of  $q_P$  is made simultaneous to the proposal of the party. This appendix will provide a proof for the claim made in section 6.4 that this extended version of the model may have an equilibrium where extremist politicians are nominated with probability greater than one-half. To do so I will derive an equilibrium of this more general model where parties  $L$  and  $R$  as defined in the main text get proposed, no other parties can successfully enter, and  $q_L = q_R = q$  for some probability  $q$ . Throughout this section it will be assumed that there are four states, one with a leftist median voter, one with a rightist median voter, and two with centrist median voters. It will also be assumed that  $y_f > \frac{29}{9}y_P$ .

Start by considering the affiliation behaviour of politicians. Let  $\pi_f(q|p)$  be the probability that a candidate for the federal election with platform  $p$  wins, given that the other party uses the nomination probability  $q$ . In the case where party  $L$  wins the state election in one other state, the utility of a member of party  $L$  with platform -1 in a state where  $L$  wins is

$$\frac{1}{2}\{y_s + (\frac{1}{2}(1-q) + \frac{1}{4}) \pi_f(q|-1)y_f + [\frac{1}{2}q \pi_f(q|0) + \frac{1}{4}\pi_f(q|-1)]y_P\} ,$$

with  $\pi_f(q|-1) = \frac{1}{8} + \frac{1}{2}(1-q)\frac{1}{2}$  and  $\pi_f(q|0) = \frac{3}{8} + \frac{1}{2} - \frac{1}{4}q$ . For  $q = 0$  this expression becomes

$$\frac{1}{2}(y_s + \frac{9}{32}y_f + \frac{3}{32}y_P) .$$

In the state with median voter at or below -0.5 such a candidate can achieve a utility of at most  $y_s$  by deviating to joining a new party. The deviation utility is no greater than the equilibrium utility for  $q = 0$  as

$$\frac{1}{2}(y_s + \frac{9}{32}y_f + \frac{3}{32}y_P) > \frac{1}{2}(y_s + \frac{29}{32}y_P + \frac{3}{32}y_P) = \frac{1}{2}(y_s + y_P) ,$$

where the first inequality holds due to the assumption that  $y_f > \frac{29}{9}y_P$ . Deviating to joining a new party is consequently worse as long as  $y_P \geq y_s$ . It follows from the continuity of payoffs in  $q$  that politicians with platform -1 or 1 either do not want to deviate even if  $q = 1$  or that there exists some threshold  $q^e \in (0, 1)$  such that the deviation is not undertaken for  $q = q^e$ , but occurs for any  $q > q^e$ . In the former case set  $q^e = 1$ .

Politicians with platform 0 in states with centrist median voters have a choice between joining the same or separate parties. In the latter case each achieves a utility of

$$\frac{1}{2}\{y_s + (\frac{1}{2}q + \frac{1}{4}) \pi_f(q|0)y_f + [\frac{1}{2}(1-q) \pi_f(q|-1) + \frac{1}{4}\pi_f(q|0)]y_P\} , \quad (1)$$

with  $\pi_f(q|-1)$  and  $\pi_f(q|0)$  as given above. For  $q = 0$  this simplifies to

$$\frac{1}{2}(y_s + \frac{7}{32}y_f + \frac{13}{32}y_P) .$$

If both politicians join the same party their utility becomes

$$\frac{1}{2}(y_s + (\frac{1}{12} + \frac{1}{2}q\frac{1}{2} + \frac{1}{4}q)\frac{3}{4}y_f + (\frac{1}{8} + \frac{3}{16}q + \frac{3}{16}(1-q))y_P) . \quad (2)$$

Setting  $q$  to zero yields

$$\frac{1}{2}(y_s + \frac{1}{16}y_f + \frac{5}{16}y_P) .$$

Both politicians thus prefer being in separate parties for  $q = 0$ . It follows from the continuity of payoffs in  $q$  that politicians with platform 0 in centrist states either both want to be members of the same party even if  $q = 1$  or that there exists some threshold  $q^c \in (0, 1)$  such that they are indifferent at  $q = q^c$ , but would prefer being members of the same party for any  $q > q^c$ . In the former case set  $q^c = 1$ .

It can be shown that  $q^c > 0.5$ . To do so evaluate expressions (1) and (2) at  $q = 0.5$ . This yields  $\frac{1}{2}(y_s + \frac{3}{8}y_f + \frac{1}{4}y_P)$  and  $\frac{1}{2}(y_s + \frac{1}{4}y_f + \frac{5}{16}y_P)$ , respectively. The first utility is greater than the second utility as long as  $y_f > y_P$ . This shows that centrist politicians in centrist states prefer to be members of the same party for  $q = 0$  and  $q = 0.5$ . It is easy to show that expression (1) is concave in  $q$  as long as  $y_f > y_P$ , while expression (2) is linear in  $q$ . It immediately follows that the former utility must be greater than the latter utility for any  $q \in [0, 0.5]$ . This demonstrates that the threshold  $q^c$  must be greater than 0.5.

Now assume  $y_s = y_P = 1$  and  $y_f = 5$ . In this case it can be calculated that  $q^e \approx 0.38$ . This is below the threshold  $q^c$ , which must be greater than 0.5. Is there an equilibrium such that  $\mathcal{P}^* = \{L, R\}$  and the founder of each party sets  $q$  equal to  $q^e$ ? As long as the affiliation behaviour of politicians does not change, the utility of a founder is increasing in his choice of  $q$ , as centrist politicians win the federal election with higher probability. Accordingly, neither founder would want to deviate to choosing a lower value of  $q$  than the equilibrium one. Increasing the level of  $q$  beyond  $q^e$  would lead to the entry of a new party, as extremist politicians in the party under consideration would then be willing to deviate by the definition of the cut-off  $q^e$ . As in the proof of proposition 1, the entry of such a party reduces the utility of the party leader undertaking the deviation to zero. As  $q^e < q^c$ , centrist politicians will join different parties. A straightforward calculation based on the expressions derived above shows that their utility is greater than  $y_s$ , which is the utility they could achieve by joining a third party.

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