A Career Like No One Else Can Offer: On the Conditions for Two-Party Dominance

Nicolas Motz*

Universidad Carlos III de Madrid

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Abstract

The determinants of the number of parties competing in any given first-past-the-post election have been widely studied. Much less clear are the conditions required for two parties to dominate all elections across separate districts and at different levels of government, as is the case in the US for example. In this paper, I propose a novel model of party formation in which politicians use parties as a vehicle to establish a public profile in the early stages of their career, but may later on abandon the party and run as independents. I show that two parties can only dominate all elections if they provide sufficient opportunities for members while limiting the success of defectors. More specifically, I establish three conditions for two-parties dominance: *i*) parties must be divided into a left-wing and a right-wing camp in any two-party equilibrium, *ii*) voters at the national level cannot be too concentrated in the centre relative to the most radical districts, and *iii*) politicians need to be sufficiently motivated by the desire to win elections at higher

*nicolas.motz@uc3m.es, Department of Economics, Universidad Carlos III de Madrid, Calle Madrid 126, Getafe, 28903, Spain. An earlier version of this paper formed a chapter of my doctoral thesis submitted at University College London. I am grateful to my supervisors Ian Preston and Guy Laroque for their support. The paper also benefited greatly from the help of and discussions with Omer Ali, Philipp Denter, Boris Ginzburg, and Antoine Loeper. In addition, I would like to thank Marco Bassetto, Antonio Cabrales, Benoît Crutzen, Anders Jensen, Zoë Kuehn, Gilat Levy, Aureo de Paula, Michael Ting, and Lukas Wenner. Declarations of interest: none.

levels of government. Furthermore, I establish the existence of a specific two-party equilibrium featuring a centre-left and a centre-right party. An extension that introduces regionalism shows that high salience of this second dimension of policy is by itself not enough to rule out two-party equilibria.

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1 Introduction

Duverger's famous law states that first-past-the-post (FPTP) elections in combination with single member districts introduce a tendency towards competition among only two parties. The logic underlying this claim is that parties with no realistic chance of winning are either abandoned by voters or decide to drop out of the race until only two parties remain. However, this line of reasoning applies to a single election, but not to elections held across separate districts or at multiple levels of government (see Cox 1994). Applying Duverger's law to the US, for example, we should expect to see two parties competing for the governorship of California and two parties competing for the presidency, but there is no reason why the same two parties should be competing in both of these elections. Similarly, two parties should be contesting each individual seat in Congress, but more than two parties may be represented in Congress overall. In fact, one does not have to look far to find political systems relying on FPTP elections where more than two parties attract significant vote shares: The Houses of Commons of the UK and Canada each currently feature four parties represented by more than ten MPs. National politics in India, on the other hand, are dominated by two parties, but many more parties enjoy success at the regional level. What are the conditions required to enable two parties to dominate all elections across a country? When do more than two parties emerge?

In this paper, I propose a novel model of party formation and characterise the conditions under which equilibria with two parties exist. The model features elections at different levels of government and imposes no ad-hoc restrictions on the number of parties that compete across these elections. Politicians standing at the beginning of their career join parties in order to signal their policy preferences to voters. Parties enable politicians to do so by allowing only certain types of politicians to join. Parties thus serve as "informative labels" (Snyder & Ting 2002) that provide information about their members to voters. A politician who has won a regional election then

has a chance to become their party's candidate at the national level. In addition, politicians also have the option of contesting elections as independents at any point in their career.

I start by considering a one-dimensional policy space. A key assumption of the model is that there is a minimum amount of heterogeneity in voter preferences across regions. In a two-party equilibrium, this heterogeneity forces parties to adopt a sufficiently broad ideological profile if they want to cater to voters' diverse tastes and prevent entry of additional parties. But if parties allow a range of politicians to join, this creates internal competition for party nominations in the run-up to regional elections. As a consequence, members potentially have an incentive to join smaller parties with a narrower ideological profile. In fact, successful entry of smaller parties is always possible in a two-party equilibrium. To prevent politicians from taking advantage of this possibility, parties need to provide sufficient opportunities for members at the national level while ensuring that the success of defectors does not extend beyond the regional level. A two-party equilibrium thus forces parties to implicitly take on additional roles beyond signalling the ideology of their members: parties need to enable and channel the careers of politicians, for example by coordinating voting behaviour or ensuring that all of their members have a shot at winning nominations. Paradoxically, the need to assume a broad ideological profile implies that parties fulfil their original purpose of revealing information about their members less effectively than they could.

The main results of the paper all follow from the logic laid out in the preceding paragraph. The first result that I present is that parties are divided into a (centre-)left and a (centre-)right camp in any two-party equilibrium. If, for example, both parties allowed left-wing politicians to join, this would create the possibility that both parties nominate such a candidate for the national election. But since both candidates then fall on the same side of the national median voter, this situation creates an opening for a moderate

independent or third-party candidate. Based on this result, I derive necessary conditions for existence.

The first necessary condition for the existence of two-party equilibria follows from the requirement that parties give even their most extreme members a shot at contesting the national election. Such candidates can only win the national election if facing a similarly extreme competitor—a situation which raises the possibility of a centrist independent entering the race. Entry can be prevented if voters continue to vote for the extremists even if entry occurs, perhaps because voting along party lines serves as a focal point. Parties thus implicitly take on the role of coordinating voting behaviour in equilibrium. The ability of parties to achieve coordination on their candidates has limits though. Moderate voters, in particular, have little to lose by voting for a moderate independent. The first necessary condition for the existence of two-party equilibria is therefore that voters in the national election are not too concentrated in the centre of the policy space. A polarisation of the electorate, in contrast, is not a threat to a two-party equilibrium.

The preceding results together imply that the equilibrium parties dominate the national election even when a third candidate enters the race. These parties can thus compensate their members for the intense internal competition that they generate at the regional level with the prospect of success on the national stage. A necessary prerequisite for politicians to be willing to make this trade-off, however, is that they value opportunities at the national level sufficiently strongly relative to winning at the state level. The second necessary condition for the existence of two-party equilibria is therefore that politicians have strong career concerns. The threshold on the strength of career concerns required to support a specific two-party equilibrium depends on how parties nominate candidates. In particular, parties have an incentive to commit to giving members of different factions within the party a shot at winning nominations instead of always favouring the candidates who are most likely to win. Otherwise defections by politicians who are popular in

their region but less well-positioned to win nationally are hard to prevent.

In a subsequent step, I establish the existence of an equilibrium with a centre-left and a centre-right party. This equilibrium exists if the previously established necessary conditions are satisfied, demonstrating that these conditions are also sufficient for the existence of a two-party equilibrium.

I then extend the model by introducing a second dimension of policy intended to capture a concern specific to a subset of regions, like the presence of an ethnic minority or an independence movement. Interestingly, regionalism in itself is not a threat to the existence of two-party equilibria, even if regionalism is the dominant issue for voters. This is the case because the presence of a regionalist party can stabilise a party representing all remaining voters, which would otherwise suffer from defections. However, the possibility of a regionalist candidate running for the national election relaxes the conditions required for a moderate independent to win. In particular, two-party equilibria are unlikely to exist if regionalism is salient and regionalist voters make up a small share of the national electorate. Given that Scottish voters account for less than ten percent of the electorate in UK general elections, the model thus provides an explanation of the existence of the Scottish National Party.

Aldrich & Lee (2016) also highlight the importance of political ambitions in explaining why only two parties exist in the US. To make this point, the authors specify a utility function for politicians and explain how the utility of joining a party that offers the highest probability of winning a state election can be lower than joining a national party as long as the national party offers a sufficiently high probability of winning elections at the federal level. My paper extends the analysis of Aldrich & Lee in several ways. In particular, I provide an explanation for why the chances of winning a state election should be lower as a member of the national party in the first place. In the model presented here, national parties are less attractive due to a higher level of internal competition for nominations, which arises endogenously. In addition,

I present a number of additional requirements for two-party dominance that are not discussed in Aldrich & Lee (2016).

This paper is also related to the literature on political competition with entry (Palfrey 1984, Osborne 1993, 2000, Callander 2005), which analyses the effect that the threat of entry has on the equilibrium behaviour of two parties. Closest to the current paper is Callander (2005), who studies competition between two parties in multiple single-member districts with threat of entry of independent candidates at the district level. Parties, whose formation is not part of the model, are free to choose any platform. Callander (2005) finds that the threat of entry leads to the divergence of party platforms, similar to this paper. A key difference is that Callander establishes conditions that rule out entry at the district level, which would be equivalent to establishing conditions that rule out entry at the regional level in the current paper. As it turns out though, entry at the regional level is always possible in a two-party equilibrium in the current paper. The only reason that entry does not occur in equilibrium is that parties offer potential entrants a more attractive alternative. Eyster & Kittsteiner (2007) also present a model that features multiple districts, but take the number of parties as fixed. Neither of these papers mentions career concerns nor allows for regionalism.

Citizen candidate models as introduced by Osborne & Slivinski (1996) and Besley & Coate (1997) have previously been used to investigate the determinants of the number of parties competing in elections (See, for example, Dickson & Scheve 2010). In these models, parties are identical to individual candidates. The current paper therefore requires a different approach, as parties have to be organisation that span multiple levels of government. Few papers have modelled parties as consisting of multiple politicians while endogenising the number of parties existing in equilibrium (Jackson & Moselle 2002, Levy 2004, Morelli 2004, Osborne & Tourky 2008, Eguia 2011). To the best of my knowledge, I am the first to do so employing the concept of par-

ties as informative labels.[1] Given the need to include multiple elections with separate electorates, affiliation choices of politicians, as well as assumptions about candidate selection, the model is necessarily relatively complex. Nevertheless, the model is tractable and naturally lends itself to the purpose of investigating other questions, such as the interplay between social diversity and electoral rules in determining the number of political parties (Dickson & Scheve 2010, Milazzo et al. 2018). Indeed, my results suggest that a theoretical analysis of the number of parties competing in a particular district may be misleading if linkages across levels of government are not taken into account. The extension introducing regionalism indicates that the model can accommodate multi-dimensional policy spaces.

The rest of the paper is organized as follows: Section 2 explains the details of the model, while Section 3 presents the theoretical results. Section 4 extends the model to allow for regionalism. Robustness of the results to relaxing some of the assumptions is discussed in Section 5. Section 6 concludes.

2 The Model

A federal country consisting of $S \geq 3$ states selects federal and state governments through FPTP elections. Political parties nominate candidates for these elections, but politicians who are not nominated can decide to run as independents. Initially a large number of parties exists, but only those that attract members can compete in elections. The timing is as follows: In the beginning of the game, politicians decide which party to join. Once affiliation decisions have been made, parties nominate candidates in each state and state elections are held. Each winner of a state election then has a chance to become their party's candidate for the federal election. After the federal

[1] In contrast, Snyder & Ting (2002) as well as other contributions building on their approach (Ashworth & Bueno de Mesquita 2008, Bernhardt et al. 2009) consider the behaviour of a given number of parties.

election the game ends. In any of these elections, any politician who has not been nominated by a party can run as an independent candidate. The following sections describe the elements of the model in detail.

2.1 Players

The strategic players of the game are politicians and voters.

2.1.1 Politicians

There are five politicians in each state. Every politician is endowed with a platform p and in each state there is one politician for each of the five possible platforms collected in the set $\mathcal{T} = \{-1, -.5, 0, .5, 1\}$. For brevity, $g_{s,p}$ refers to the politician with platform p from state s. The set of all politicians is given by $\mathcal{G}$ while $\mathcal{G}_s$ is the set of all politicians in state s.

Whenever a politician wins an election, they are committed to implementing their platform. Politicians with platform -1 or 1 are referred to as partisans, while all remaining politicians are labelled as moderates.

Politicians are office-motivated. The utility of a politician who does not win any elections is normalised to zero. In contrast, the winning candidate in an election at the state level receives a payoff of $y_s > 0$, while the utility of the winner of the federal election further increases by $y_f > 0$.

2.1.2 Voters

Each state s contains a set of voters that is large, finite, and odd. Let p_l denote the policy that is implemented in region $l \in \{1, ..., S, f\}$. Given beliefs over the platforms of candidates and the behaviour of other voters, the objective of a voter with ideal policy $i \in \mathbb{R}$ who participates in the election in region l is to maximize

$$\mathbb{E}[u(|p_l - i|)] \ ,$$

where $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly decreasing.[2] The set of voters in state s is described by a measure Λ_s that assigns to any subset of $\mathbb{R}$ the share of voters whose ideal policies lie in this subset. Let m_s denote the ideal policy of the median voter of state s.

A key assumption concerns the distributions of voters across states.

Assumption 1 (Regional heterogeneity)**.** *There exists at least one state such a strict majority of voters in the state strictly prefers the platform -1 over any other platform, at least one state such that a strict majority of voters in the state strictly prefers the platform 0 over any other platform, as well as at least one state such that a strict majority of voters in the state strictly prefers the platform 1 over any other platform.*

As the labels of states are arbitrary, it is without loss of generality to refer to the states listed in Assumption 1 as states 1, 2, and 3, respectively.

All voters vote in the federal election and the federal electorate is described by a corresponding measure Λ_f with $m_f = 0$.

2.2 Political Parties

A party consists of a "shape" that specifies which politicians are allowed to join the party and a set of functions determining how candidates are selected. The shape of a party P is a consecutive list of platforms denoted by I_P and only politicians with these platforms can join. A possible shape of a party is therefore $\{0, .5, 1\}$, while a party of the shape $\{0, 1\}$ is ruled out for example.

Given a pool of potential nominees, the selection of candidates by parties occurs according to fixed probabilities. Denote by $\mathcal{M}_{P,l}$ the set of politicians who are eligible to be nominated for the election in region $l \in \{1, ..., S, f\}$ by party P. The probability that a politician with platform p who belongs

[2]Elections at the state level determine who becomes a candidate at the federal level, but it is assumed that voters do not take this interdependence into account when voting in a state election.

to $\mathcal{M}_{P,l}$ is nominated for the election in region l is given by a function $\eta_{P,l}(p|\mathcal{M}_{P,l})$. The only restrictions placed on this function are that for any $\mathcal{M}_{P,l}$ the nomination probabilities across all members of $\mathcal{M}_{P,l}$ are strictly positive and sum to one.

The set of parties that exists in the beginning of the game is denoted by $\mathcal{P}$. Since any party can only compete in elections if joined by at least one politician, the set $\mathcal{P}$ is referred to as the set of potential parties. Any party that does attract members is referred to as an active party. The set of potential parties $\mathcal{P}$ is "large". In particular, for any possible shape I there exists at least one party $P \in \mathcal{P}$ such that $I_P = I$. $\mathcal{P}(p)$ denotes the set of parties that allow politicians with platform p to join.

2.3 Timing

The game proceeds as follows:

i) All politicians decide whether to join one of the parties in $\mathcal{P}$ or remain independent.

ii) In every state, any party that has been joined by at least one politician in the state nominates a candidate.

iii) After observing the candidates put forward by parties, all politicians who have not been nominated decide simultaneously whether to run as independents for the election in their state. Let $\mathcal{C}_s \subset \mathcal{P} \cup \{d_1, ..., d_5\}$ be a list of candidates running for the election in state s where party candidates are identified by their party affiliation while d_n is a label given to an independent candidate.

iv) Each voter casts a vote at the election in their state and the winner in each state is the candidate who receives the highest number of votes. Ties are broken randomly. Winners of state elections implement their platform as the state policy.

v) Any party that has won at least one state election nominates one of their winning candidates as their candidate for the federal election.

vi) After observing the candidates put forward by parties, all politicians who have not been nominated decide simultaneously whether to run as independents for the federal election. Let $\mathcal{C}_f \subset \mathcal{P} \cup \{d_1, ..., d_{|\mathcal{G}|}\}$ be a list of candidates running for the federal election where d_n is again a label given to an independent candidate.

vii) All voters vote in the federal election. The winner is once more the candidate who receives the highest number of votes, who then implements their platform as the federal policy.[3]

2.4 Information

Voters have limited information about politicians. Specifically, it is assumed that the electorate cannot distinguish between different politicians and initially believes that the platform of a politician is equal to any of the five possible platforms with probability .2. Furthermore, voters can see which parties have nominated a candidate in their state, but not how many politicians have joined each party. Voters do know, however, how candidates are selected. This knowledge combined with a belief about which politicians have joined a particular party allows voters to update their beliefs about the platform of a party's candidate prior to casting their vote at the state-level election.

[3]Even in a presidential system, policymaking often requires passing legislation, which in turn requires a majority in parliament. With more than two parties competing the choice of policy may therefore require a process of coalition formation. I abstract from such issues here. At least the two-party equilibria presented below do not depend on what is assumed about the process of policy formation when no party achieves a majority. This is because voters are allowed to vote strategically, which implies that there always exists a voting equilibrium with one party winning with a strict majority, even off the equilibrium path when a third party has entered.

The winner of a state election implements their platform at the state level, thus revealing it to voters. Voters accordingly have full information about the platform of any candidate for the federal election who has previously won a state election.

Finally, all agents are fully informed about the distribution of voters in all states and at the federal level.

2.5 Equilibrium

The party-formation game described in the previous sections is a dynamic game of incomplete information and requires a corresponding equilibrium concept. I focus on the set of perfect Bayesian equilibria. Without further restrictions, this choice entails a huge number of equilibria due to the fact that voters are allowed to vote strategically instead of assumed to vote sincerely. For example, with three or more voters and two or more candidates it is always an equilibrium that all voters vote for any specific candidate, even if all voters strictly prefer a different candidate. This extreme form of coordination failure makes the existence of equilibria with two parties trivial, since it is always possible to find a voting equilibrium such that any third-party candidate loses. On the other hand, assuming perfect coordination among voters is not realistic either. In addition, one of the roles that parties implicitly take on in equilibrium is to coordinate voting behaviour. To strike the right balance, I impose the following restriction: if a candidate in some election is the unique most preferred option of a strict majority of voters based on their beliefs at the point when the election is held, then a voting equilibrium where this candidate wins the election is selected.[4] While such

[4]When a strict majority of voters favours a particular candidate, the restriction imposed here yields the same winner as sincere voting. However, assuming sincere voting would require exact knowledge of the distribution of voters in order to determine the winner of an election when a strict majority in favour of one candidate does not exist. Under the assumptions made here, in contrast, any candidate can be the winner in such a situation, which actually makes the model more tractable.

an equilibrium always exists under the stated conditions, there are typically additional equilibria where a different candidate wins. Nevertheless, it seems highly plausible that voters are able to coordinate on electing a candidate who is favoured by a strict majority.[5]

The following definition summarises the equilibrium concept:

Definition 1 (Party-Formation Equilibrium)**.** *A party-formation equilibrium is a perfect Bayesian equilibrium of the party-formation game that satisfies the following condition: If a candidate in some election is the unique most preferred option of a strict majority of voters based on their beliefs at the point when the election is held, then this candidate wins the election.*

Denoting by $\Delta(\mathcal{Z})$ the set of all probability distributions over some set $\mathcal{Z}$, a perfect Bayesian equilibrium of the game consists of the following elements:

i) A profile of behavioural strategies $(\tau^P_{s,p}, \tau^s_{s,p}, \tau^f_{s,p})$ for each politician $g_{s,p} \in \mathcal{G}$ consisting of

   a. a choice of party affiliation given by $\tau^P_{s,p} \in \Delta(\mathcal{P}(p) \cup \emptyset)$ where $\emptyset$ denotes the choice to not join a party,

   b. a function $\tau^s_{s,p} : 2^{(\mathcal{G}_s \setminus g_{s,p}) \times \mathcal{P}} \to [0,1]$ specifying the probability that the politician runs as an independent in the election in their state conditional on any possible constellation of candidates nominated by parties that does not include the politician themself, and

   c. a function $\tau^f_{s,p} : 2^{(\mathcal{G} \setminus g_{s,p}) \times \mathcal{P}} \to [0,1]$ specifying the probability that the politician runs as an independent in the federal election conditional on any possible constellation of candidates nominated by parties that does not include the politician themself.

[5] For example, Myatt (2007) models a three-candidate election as global game. While coordination failure is generally a feature of equilibrium in this model, a candidate favoured by a majority of voters wins with certainty.

ii) A profile of behavioural strategies (ν_j^s, ν_j^f) for each voter j from a state s where ν_j^l is a function such that $\nu_j^l(\mathcal{C}_l) \in \Delta(\mathcal{C}_l)$, which specifies the probability with which the individual votes for each candidate in the election in region $l \in \{s, f\}$ under any possible constellation of candidates.

iii) Beliefs common to all voters given by a set of functions $(\sigma_1, ..., \sigma_S, \sigma_f)$ such that $\sigma_l(\mathcal{C}_l) \in \Delta(\mathcal{T}^{|\mathcal{C}_l|})$, assigning probabilities to the possible combinations of platforms of candidates in the election in region $l \in \{1, ..., S, f\}$ under any possible constellation of candidates.[6]

$\mathcal{P}^*$ will denote the set of parties that are active in an equilibrium.

2.6 Discussion

In the following I discuss some of the features of the model. The assumption that politicians cannot choose the policy they implement is not standard. I show in Section 5.1 that the results extend to a setting where politicians care strongly about the policy that they choose once elected, but are otherwise free to implement any platform.

Allowing for five different platforms is the smallest number that enables me to show all of the results. Additional platforms can easily be accommodated.

Next, I would like to highlight how general the assumptions regarding candidate selection are. While they imply that parties commit in advance to nominating specific candidates with fixed probabilities, these probabilities can be arbitrarily close to the case of strategic nomination of candidates without commitment. For example, the aim of maximising the probability of winning the federal election when there is one competing candidate is achieved by nominating the politician located closest to zero. The model

[6]While voters do know the platform of any candidate who has won a state election, they are uncertain about the platform of an independent candidate in the federal election who has not previously held an office.

allows parties to implement this strategy with only a very small probability of mistakes. The results below show that parties do in fact have an incentive to commit to giving all types of politicians a chance at winning nominations in order to avoid defections. Such a commitment can be achieved, for example, by selecting candidates through primary elections or caucuses, which limit the control of the national party committee over the nomination process.

An assumption that may appear strong is that there can be parties that perfectly reveal the platforms of their members. However, the essential feature of the model is not that there can be such "singleton parties", but that there can be parties that reveal different amounts of information about their members. The Tea Party movement in the US (Arceneaux & Nicholson 2012), while not a party of its own, illustrates that it is possible to send a more fine-grained signal to voters than, for instance, the Republican party label does. A separate issue is whether a single politician should be allowed to form a new party. I discuss this question in Section 5.2, where I consider the possibility of allowing for joint deviations by groups of politicians.

Some readers may wonder about the difference between deviating to joining a singleton party and running as an independent. Conceptually, forming a new party should be thought of as a process that requires more time and preparation, while the decision to run as an independent can be made even late in a race.[7] This idea is highlighted by the fact that independent candidates in the model can make their entry decision conditional on the candidates put forward by established parties. It is true, however, that at the state level no politician would ever strictly prefer to run as an independent over forming a singleton party. The choice of allowing independent candidates to enter state elections is thus made merely for completeness. At the federal

[7]In exceptional cases, an already established and popular politician can create a party organisation at surprising speed, as the example of Emmanuel Macron's En Marche movement in France illustrates (Mény 2017). Given that En Marche was constructed entirely around the persona of Macron, this case is, nevertheless, better described as entry of an independent candidate and contrasts sharply with the establishment of parties out of grassroots movements, such as in the case of European green parties (Kaelberer 1998).

level, on the other hand, politicians who have previously been elected in a state are much less dependent on their party. Given that such candidates are already known to voters, running a successful campaign as an independent can be a viable option.

Finally, it would also be possible to introduce an electoral college at the federal level to fit the model more closely to the US. In this case the results go through unchanged if the median voter of the state with the median electoral vote is assumed to be located at zero.[8]

3 Results

The aim of this paper is to investigate conditions under which equilibria with two active parties exist. It is useful to introduce a formal definition of the type of equilibrium that is the main focus.

Definition 2 (Two-Party Equilibrium)**.** *A two-party equilibrium is a party-formation equilibrium such that the number of active parties in equilibrium is equal to two and no independent candidates run in any elections along the equilibrium path.*

A two-party equilibrium thus requires not only that the number of active parties is equal to two, but also that these parties face no competition from independent candidates. Independent candidates are a common occurrence in most democratic countries, but independent candidates with a serious chance of winning are much rarer.

Throughout the following sections, I will illustrate the results by means of a simple example, referred to as Example 1, which is represented graphically

[8]The median electoral vote can be calculated as follows: Create a distribution of electoral votes by assigning the electoral college votes of the state to the ideal policy of the median voter of the state. Then find the median of this distribution. When there are two parties competing at the federal election, the party closest to the median voter of the state with the median electoral vote wins a majority of electoral votes.

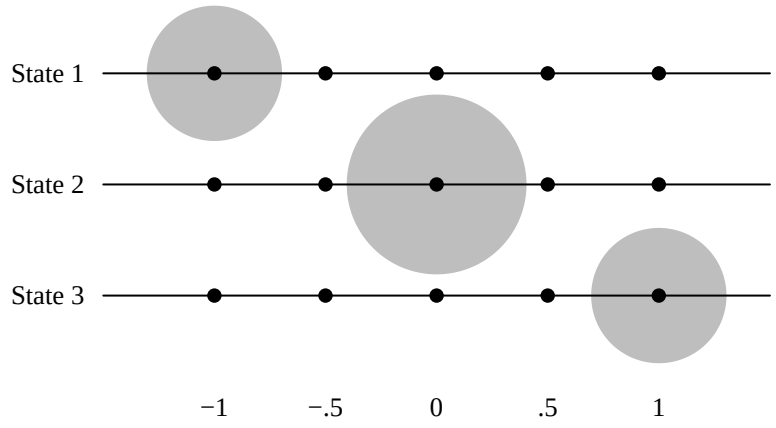

Figure 1: An Illustration of Example 1

Black lines represent the policy space and there is one politician located at each black dot. A grey disc indicates that voters are located at the centre of the disc. The size of each disc is proportional to the share of voters in the federal election at the indicated location.

in Figure 1. In the example, there are three states. All voters in state 1 have the ideal policy -1, all voters in state 2 the ideal policy 0, and all voters in state 3 the ideal policy 1. The populations of states 1 and 3 have equal size and the fraction of voters who are located in state 2 is denoted by λ.

In Example 1, all voters in state 1 prefer the platform -1 over any other platform. If the politician with platform -1 of the state joins a party of shape $\{-1\}$, voters know that a candidate nominated by this party must have the platform -1 since no other politicians are allowed to join. Accordingly, the politician with platform -1 can always win the state election with certainty by joining a party of shape $\{-1\}$. The same logic applies to the politician with platform 0 in state 2 and the politician with platform 1 in state 3. A

minimal requirement that any equilibrium of the game in Example 1 must fulfil is therefore that politicians with platforms -1, 0, and 1 are able to join one of the parties that are active in equilibrium. Otherwise at least one of the aforementioned politicians would win the election in their state with probability zero and therefore have an incentive to join a singleton party.

Even though voter preferences are not quite as homogeneous within states 1, 2, and 3 in the general model compared to Example 1, the same logic applies.

Lemma 1. *In any two-party equilibrium, politicians with platforms* -1*,* 0 *and* 1 *must be allowed to join at least one of the parties that are active along the equilibrium path.*

Proof. See Appendix. □

A second feature of any two-party equilibrium is that any politician who has a chance to join a party that can win the election in the politician's state must do so. A central ingredient of this result is the assumption that voters cannot observe the membership of a party at the state level. A politician joining a party out of equilibrium therefore does not affect voters' expectations regarding the platform of the party's candidate and the outcome of the elections remains unchanged. Even politicians who are not well aligned with the median voter of a state can thus join a party without jeopardising the electoral success of the party.

Lemma 2. *Consider any two-party equilibrium and suppose there is a politician in some state* s *who is eligible to join a party that wins the election in the state with positive probability along the equilibrium path. Then the equilibrium strategy of this politician must place zero probability on not joining a party or on joining a party that loses the state election with certainty.*

Proof. See Appendix. □

A consequence of Lemma 1 is that at least one party must allow multiple types of politicians to join. In combination with the fact that successful parties attract members, this observation implies that a necessary part of any two-party equilibrium is internal competition for nominations at the state level by different factions within a party. In isolation, this factor gives party members an incentive to defect to smaller parties with a narrower ideological profile in order to increase their chances of winning state elections. If there was no federal election in Example 1, for instance, the unique equilibrium would feature three active parties with shapes $\{-1\}$, $\{0\}$, and $\{1\}$. Nevertheless, politicians can be persuaded to stay loyal to a broader party if this party provides a path to winning the federal election. Two parties can therefore only form an equilibrium if they offer their members sufficient success at the federal level while simultaneously limiting that of defectors. The main results presented in the next three sections all follow from this logic.

3.1 Left versus Right

The first main result provides a partial characterisation of the parties that can be active in a two-party equilibrium. Specifically, it must be the case in any two-party equilibrium that there is one active party which does not admit members with positive platforms, while the other party does not admit members with negative platforms.

Proposition 1. *Consider any constellation of two parties A, $B \in \mathcal{P}$. Then a two-party equilibrium such that A and B are the active parties in equilibrium exists only if $I_A \subseteq \{-1, -.5, 0\}$ and $I_B \subseteq \{0, .5, 1\}$.*

Proof. See Appendix. □

Parties must thus be divided into clear ideological camps in any two-party equilibrium. Otherwise, there would be the possibility that both parties nominate candidates for the federal election whose platforms lie on the same side

relative to the median voter, which would enable an independent with platform 0 to enter and win. Note, however, that there is the possibility that parties, for example, both allow politicians with negative platforms to join, but the sorting behaviour of politicians nevertheless prevents that both parties simultaneously nominate such a candidate for the federal election. The proof of Proposition 1 builds on the logic that such sorting behaviour is difficult to maintain in equilibrium. If one party has no members with positive platforms while the other party has no members with negative platforms, then the former party must win the election in state 1 while the latter party must win the election in state 3. But if a party wins a state election with certainty, then all eligible politicians of the state must join the party by Lemma 2. Accordingly, both parties must have members with, for example, negative platforms if both parties allow such politicians to join.

3.2 No Outlier States

The second necessary condition for the existence of two-party equilibria states that voters in the federal election cannot be too concentrated in the centre of the policy space.

Proposition 2. *Consider any constellation of two parties A, $B \in \mathcal{P}$. Then a two-party equilibrium such that A and B are the active parties in equilibrium exists only if there are no more than half of all voters in the federal election located in at least two of the intervals $(-.5, .5)$, $(-.75, .25)$, and $(-.25, .75)$.*

Proof. See Appendix. □

As was already established by Lemma 1, the active parties have to allow partisans to join in any two-party equilibrium. As a consequence, the situation may arise that both parties nominate partisans for the federal election. According to Proposition 1, these candidates will necessarily be drawn from

opposite ends of the political spectrum. Voters in the centre of the policy space would then prefer a moderate candidate. If the share of voters who would prefer a particular type of moderate over any partisan is large, such a candidate can successfully run as an independent in the federal election, upsetting the equilibrium. For example, a politician with platform .5 is strictly preferred over any partisan by all voters in the interval $(-.25, .75)$. Again by Lemma 1, politicians with platform zero must be allowed to join one of the active parties in any two-party equilibrium. The same must be true of politicians with at least one of the platforms $-.5$ and .5. If the condition of Proposition 2 is violated, at least one of these types of politicians has in incentive to run as an independent when both parties nominate partisans.

Proposition 1 highlights the role that parties need to play in coordinating voter behaviour in any two-party equilibrium. In a situation where one party has nominated a candidate with platform -1 for the federal election while the other party has nominated a candidate with platform 1 and an independent with platform 0 joins the race, one may argue that even radical voters have an incentive to vote for the moderate to prevent the opposite camp from winning. Collectively this is certainly true. If voting along party lines is a focal point, however, partisan voters will likely prefer not to voter for the moderate for fear of splitting the vote. Voters in the centre who dislike both types of partisans equally, on the other hand, have little to lose by voting for the independent candidate. In the model, coordinating on electing the moderate candidate is only successful when the number of centrist voters is sufficiently high as specified by Proposition 2.

Importantly, the condition of Proposition 2 is only necessary for the existence of two-party equilibria due to the presence of the relatively extreme voters of states 1 and 3. Without such states, there would be two-party equilibria where the active parties do not allow partisans to join and Proposition 2 loses its bite. The assumption that a strict majority of voters in state 1 strictly prefers the platform -1 over any other platform implies that

$m_1 < -.75$. Equivalently, it must be the case that $m_3 > .75$. Another way of expressing the condition in Proposition 2 is therefore that there cannot be states whose median voters are "outliers" in the distribution of voters overall in the sense that their position is more radical than the ideal policies of more than half of all voters.

In Example 1, moderate politicians can win against two competitors with respective platforms -1 and 1 if an only if the voters of state 2 make up more than half of all voters in the federal election, that is, $\lambda > .5$. Reducing λ, on the other hand, which corresponds to increasing polarisation, poses no risk to the existence of two-party equilibria. A slight modification of Example 1 can be used to illustrate this point more fully: fix $\lambda = 1/3$ and assume the voters of states 1 and 3 are located at $-x$ and x, respectively, instead of at -1 and 1. Increasing x corresponds to an increase in polarisation of the electorate. If $x < .75$, no two-party equilibria exist where both types of partisans join parties in equilibrium since politicians with platform $-.5$ and .5 are both strictly preferred by a strict majority of voters in the federal election over the platforms -1 and 1 and moderates can run as independent candidates in the federal election. If $x > .75$, on the other hand, Lemma 1 applies and no two-party equilibria exist where any partisans are excluded from joining a party in equilibrium. In this example, increasing the polarisation of the electorate thus forces parties to nominate more radical candidates. Nevertheless, two-party equilibria exist even when x is large. This logic may help explain why the two-party system is alive and well in the US despite increasing polarization both of political elites and of the electorate (Iyengar et al. 2019). In 2016, there was a chance that both the Democratic and the Republican Party were going to nominate radical candidates for the presidential election, namely Bernie Sanders and Donald Trump. Anticipating this scenario, it was rumoured that Michael Bloomberg was considering a run as an independent (Burns & Haberman 2016). However, it is far from clear that Bloomberg's campaign would have attracted sufficient support from moderate voters to

gain any kind of momentum. It is perhaps telling that Bloomberg chose to pursue the Democratic ticket for the 2020 election instead.

Two examples from UK politics lend further support to the relevance of the logic underlying Proposition 2. The formation of the Social Democratic Party in 1982 was an attempt to seize the centre ground vacated by other parties that, at least for a while, looked likely to succeed (Crewe & King 1995). The party was founded by four senior members of the Labour party at a time when the leaders of both Labour and the Conservatives were pursuing radical policies. The electoral alliance that the Social Democratic Party entered with the previously marginalised Liberal Party soon found itself topping opinion polls, indicating that the radicalisation of parties was in this case not driven by a radicalisation of the electorate as a whole. It is widely held that it was only the Falklands War that turned the tables in favour of Margaret Thatcher. The entry of the Social Democratic Party could thus be explained by failure of the condition presented in Proposition 2. The recent polarisation in British politics induced or uncovered by Brexit also lead to the formation of a centrist group of breakaway MPs. However, this new party never managed to attract significant support from voters and was dissolved less than a year after its inception. It thus appears that, in this instance, defections were more driven by a marginalisation of moderate members within their parties rather than by the prospect of electoral success as members of a new party.

3.3 Career Concerns

The results introduced in the preceding two sections follow from the necessity of ensuring that the success of defectors does not extend beyond the state level. If these conditions are satisfied, the relatively broad parties that are necessarily part of any two-party equilibrium can compensate their members for the internal competition that they generate with opportunities at the federal level. Politicians need to value these opportunities sufficiently strongly,

otherwise defections cannot be prevented.

Proposition 3. *For any constellation of two parties* $\{A, B\} \subset \mathcal{P}$*, there exists a constant* $\bar{y} > 0$ *such that a two-party equilibrium in which* A *and* B *are the active parties in equilibrium only exists if* $y_f/y_s \geq \bar{y}$*.*

Proof. See Appendix. □

In words, Proposition 3 requires that the ratio between the payoff from winning the federal election and the payoff of winning a state election must exceed a certain threshold for a two-party equilibrium to exist. The level of this threshold depends on the specific equilibrium under consideration, but is generally driven by how likely different politicians are to win elections in equilibrium. These probabilities in turn depend on the candidates nominated by the opposing party and the intensity of internal competition. At the federal level, internal competition increases with the number of state elections won by a party: the greater the electoral success of a party at the sub-national level, the greater the number of party members with a public profile who would make viable candidates for the federal election. Furthermore, the chances of any given politician of securing the federal nomination depend on how their party selects candidates. A party that gives priority to moderates due to their electability is likely to suffer defections by partisans. If the party heavily favours the latter type of candidate, in contrast, moderates may be reconsidering their options. It should be kept in mind, however, that not all politicians are equally able to achieve success in state elections as members of a third party. When deciding the federal nomination, parties thus have an incentive to give priority to members whose platform is especially popular in their state. An alternative way to prevent these politicians from defecting would be to increase their chances of being nominated at the state level, which could be achieved through a decentralisation of the nomination process for example by introducing primaries. The use of primaries and caucuses

by parties in the US may thus contribute to making the two-party system particularly resilient.

While the payoffs y_s and y_f depend on the intrinsic motivations of politicians, other factors such as financial rewards, public visibility, or the competencies and powers associated with an office equally play a role. The strength of career concerns accordingly depends on the setup of the political system in a broad sense.

I conclude this section with a brief comparison between the US and Canada. The former country features two stable parties that dominate elections at all levels of government, while Canada is home to a multitude of parties whose federal and provincial branches are only loosely connected (Johnston 2017). The two countries also differ strongly in the typical career paths of politicians. US politicians start their careers at the local or state level and follow a relatively linear path towards offices at the federal level (Diermeier et al. 2005). Political life in Canada, in contrast, has historically been characterised by parallel career tracks at the federal and the provincial level (Barrie & Gibbins 1989). While politicians in the US thus face strong incentives to stay loyal to established parties, the absence of a clear path for provincial politicians towards federal offices in Canada has possibly contributed to the fracturing of the Canadian party system over time. Alternatively, the separation between provincial and federal careers in Canada may itself be a consequence of a relatively equal appeal of federal offices and opportunities at the provincial level.

3.4 An Example of a Two-Party Equilibrium

The following proposition shows that these necessary conditions introduced the previous sections are also sufficient for the existence of a two-party equilibrium.

Proposition 4. *Consider a constellation of two parties, L and R, with* $I_L = \{-1, -.5, 0\}$ *and* $I_R = \{0, .5, 1\}$ *and suppose that parties use some*

combination of nomination technologies satisfying the assumptions made in Section 2.2. Then an equilibrium such that $\mathcal{P}^* = \{L, R\}$ *exists if*

i) there are no more than half of all voters in the federal election located in each of the intervals $(-.75, .25)$*,* $(-.5, .5)$*, and* $(-.25, .75)$*, and*

ii) $\frac{y_f}{y_s}$ *is larger or equal to some threshold* $\bar{y} > 0$*.*

Proof. See Appendix. □

The equilibrium presented in Proposition 4 features a centre-left and a centre-right party that overlap in the centre of the policy space. This constellation is essentially the one that has been dominating US politics ever since the end of the civil war and also captures the party landscape in Britain in the decades after World War II well. Naturally, the equilibrium features one party that does not allow politicians with positive platforms to join and one party that does not allow politicians with negative platforms to join as required by Proposition 1. Proposition 2 reappears as Part *i*) of Proposition 4. Since all types of moderate politicians are able to join the parties L and R, it must be true that neither of these politicians is preferred by strict majority of voters in the federal election over any partisan. Finally, Proposition 3, which requires sufficiently strong career concerns, is reflected in Part *ii*) of Proposition 4.

To illustrate the inner workings of an equilibrium of the type given in Proposition 4, reconsider Example 1. To make the calculation particularly easy, suppose that candidates are nominated completely randomly in any election, that is, every eligible politician is equally like to win the nomination. Start by considering the choices of party affiliation. Clearly, party L (party R) must win the election in state 1 (state 3) and thus have full membership in the state. In state 2, multiple strategies of the politician with platform 0 can be supported in equilibrium. I will focus on the case where this politician joins party L with certainty and party L thus wins the election in state 2. To complete the description of the equilibrium, the behaviour of voters in the

federal election must be specified. The median voter in the federal election is one of the voters in state 2 with ideal policy 0 and the candidate closer to 0 must therefore win the federal election with certainty. If the candidates in the federal election have platforms equidistant from 0, I assume that the voters in state 2 mix such that either candidate wins with equal probability.

Demonstrating the existence of the equilibrium requires verifying that no politician has an incentive to join a third party. If condition i) of Proposition 4 is satisfied, which in the context of Example 1 requires that no more than half of all voters live in state 2, no independent or third-party candidate can win the federal election. Deviations may thus at most raise the probability that a politician wins a state election. In particular, the politician with platform -1 in state 1, the politician with platform 0 in state 2, as well as the politician with platform 1 in state 3 can all win the election in their state with certainty if they deviate and join a party that allows no other politicians to join, achieving a payoff of y_s. Compare this to the equilibrium payoff of the politician with platform -1 in state 1, for example. This politician is competing against two other party members for the nomination in the state, but once nominated wins the state election with certainty. Given that all three members are equally likely to be nominated for the sate election, the politician with platform -1 wins the election in state 1 with probability one third ex ante. Conditional on winning the state election, this politician competes with the winner of the election in state 2 for the federal nomination. The nominee then runs against the candidate of party R, who is the winner of the election in state 3 and is equally likely to have platform 0, .5, or 1. A candidate of party L with platform -1 wins only when tying against a candidate with platform 1. The probability that the politician with platform -1 from state 1 is nominated for and wins the federal election conditional on having won the state election is thus equal to $\frac{1}{2}\frac{1}{3}\frac{1}{2} = \frac{1}{12}$. Putting everything together, the equilibrium payoff is equal to $\frac{1}{3}(y_s + \frac{1}{12}y_f)$ and the politician has a profitable deviation unless the previous payoff is no smaller than y_s.

This requirement is equivalent to the condition that $\frac{y_f}{y_s} \geq 15.2$, which is in fact also sufficient to rule out profitable deviations by other politicians, who either receive a higher equilibrium payoff or cannot win a state election after deviating. For example, the politician with platform 0 in state 2 has the same probability of being nominated for and winning the election in their state and is also equally likely to be nominated for the federal election as the politician with platform -1 from state 1, but wins the federal election with higher probability once nominated. The politician with platform 1 from state 3 is generally in a very similar situation to the politician with platform -1 from state 1, but benefits from a higher chance of being nominated for the federal election since party R wins fewer state elections.

The example above demonstrates a general feature of the type of equilibrium set out by Proposition 4: When the median voter of a state leans to the left or the right, only the party located close to the median voter can win independent of the choice of the politician with platform 0. The latter politician is therefore essentially forced to join the only party that has a chance of winning the state election. A comparative static generated by the equilibrium is therefore that a shift in the position of the median voter of a state can lead to a change in the affiliation of moderate politicians of the state. If one interprets the realignment of voters in the US South since the 1960s from the Democratic to the Republican Party as such a shift in voter preferences, then the model would predict the occurrence of state politicians changing their affiliation from Democrat to Republican. This is exactly what could be observed in the 1990s and early 2000s, as documented by McKee & Yoshinaka (2015). In addition, the equilibrium generates a distribution of vote margins also reminiscent of US politics. Elections in states with centrist median voters generate relatively equal vote shares while states where the mass of voters is shifted away from the centre can be characterised as "safe seats" where election results heavily favour one party.

While the equilibrium established in Proposition 4 is appealing due to

its similarity to two-party systems observed in reality, any other two-party equilibrium must similarly feature a (centre-)left party and a (centre-)right party. Lemma 1 and Proposition 1 in combination imply that the possible two-party equilibria feature a party of shape $\{-1, -.5, 0\}$ competing against a party of shape $\{0, .5, 1\}$, $\{.5, 1\}$, or $\{1\}$, or a mirror image of any of these constellations. Other types of two-party equilibria can be shown to exist under similar conditions to those of Proposition 4. An exception are constellations of parties where neither party allows politicians with platform $-.5$ or .5 to join, which entail the additional requirement that there is no state where a majority of voters strictly prefer the platform of the excluded type of politician over any other platform.

4 Regionalism

In this section, I introduce a second dimension of policy and derive an additional necessary condition for the existence of two-party equilibria in this extended version of the model. The second dimension of the policy space represents an issue or characteristic specific to some states, such as an independence movement or the presence of an ethnic minority that is concentrated in a subset of states. In line with these examples, the secondary issue is modelled as binary. Accordingly, the policy space is now given by $\mathbb{R} \times \{0, r\}$ with $r > 0$ and the set of possible platforms is $\{-1, -.5, 0, .5, 1\} \times \{0, r\}$. Voters and politicians located at r along the second dimension of policy will be referred to as regionalists and all others as non-regionalists. The terms partisan and moderate continue to describe the position of a politician along the first, ideological dimension of the policy space.

In line with the basic version of the model, I assume that the median voter along the ideological dimension in the federal election is located at zero:

$$\min\{\Lambda_f((-\infty, 0] \times \{0, r\}), \Lambda_f([0, \infty) \times \{0, r\})\} > .5 \ .$$

In addition, $\Lambda_f([-1,1] \times \{0\}) > .5$ such that regionalism is a minority issue. In most states, there are only five politicians, one for each of the platforms $\{-1,-.5,0,.5,1\} \times \{0\}$, and there are no regionalist voters, that is, $\Lambda_s(\mathbb{R} \times \{0\}) = 1$. In a non-empty subset $\mathcal{S}^r$ of states, however, there are 10 politicians, one for each possible platform in $\{-1,-.5,0,.5,1\} \times \{0,r\}$, and the distribution of voters in the state is not restricted to $\mathbb{R} \times \{0\}$. To ensure that the results from the previous sections also apply to the extended version of the model, I assume that there are no regionalist voters in states 1, 2, and 3, that is, $\{1,2,3\} \cap \mathcal{S}^r = \emptyset$.[9] In addition, the assumptions about the distributions of voters in these states are maintained. While regionalism is a minority issue in the federal election, there exists at least one state $s \in \mathcal{S}^r$ such that $\Lambda_s([-1,1] \times \{r\}) > .5$ and regionalist voters form the majority.

Given that the shapes of parties were restricted to contain consecutive platforms in the basic model, a natural generalisation is to require party shapes to be "convex" in the sense that if politicians with two distinct platforms can join a party then the same must be true for any politician located on a straight line between those two platforms.

The aim of a voter with ideal policy (i,j) in an election in some region $l \in \{1,...,S,f\}$ is now to maximise

$$\mathbb{E}[u(|p_{l,1} - i|) + u(|p_{l,2} - j|)] \ ,$$

where $(p_{l,1}, p_{l,2})$ is the policy implemented in region l.

The necessary conditions for the existence of two-party equilibria of Propositions 2 and 3 carry over to the extended model. Intuitively, one might expect that the extended model yields the additional requirement that the salience of regionalism is low enough to prevent entry of a regionalist party. Salience of the regionalist issue is determined by the parameter r. If r is small, the position of a politician along the regionalist dimension has a negligible impact on voters' utility relative to the ideological position. In this case

[9] There are therefore at least four states.

there may even be equilibria in which no active party allows any regionalists to join.[10] If r is large, on the other hand, regionalism becomes the decisive issue for regionalist and non-regionalist voters alike. While the latter case does indeed imply that any equilibrium must feature a regionalist party, the following example illustrates that two-party equilibria can exist no matter how strongly voters care about regionalism.

Proposition 5. *Suppose there are four states with equal-sized populations,* $\mathcal{S}^r = \{4\}$*, and*

$$\begin{aligned} &\Lambda_1([-1,-.75]\times\{0\}) \\ =\ &\Lambda_2((-.25,.25)\times\{0\}) \\ =\ &\Lambda_3([.75,1]\times\{0\}) \\ =\ &1 \end{aligned}$$

while

$$\Lambda_4([-1,-.75]\times\{r\}) \ = \ \Lambda_4([.75,1]\times\{r\}) \ = \ .5 \ .$$

Then there exists a two-party equilibrium such that $\mathcal{P}^* = \{N, R\}$ *with* $I_N = \{-1,-.5,0,.5,1\}\times\{0\}$ *and* $I_R = \{(-1,r)\}$ *if* $r > 2$ *and* y_f/y_s *exceeds some threshold* $\bar{y} > 0$*.*

Proof. See Appendix. □

A specific case satisfying the assumptions made in Proposition 5 is illustrated in Figure 2, where grey discs indicate that voters are located at the centre of the disc. The size of each disc is proportional to the share of voters in the federal election in the specified location and grey numbers indicate which state the voters belong to. Party N is a non-regionalist party with a

[10] Since there are states in which no regionalist politicians are present, the reverse case is not possible.

broad ideological profile, which wins the elections in states 1, 2, and 3. Party R, in contrast, is a regionalist party with a narrow profile, which wins the election in the sole regionalist state and never wins the federal election. The equilibrium requires that the salience of the regionalist issue is sufficiently high to ensure that regionalist voters vote en bloc for the regionalist candidate at the federal election. For lower values of r, a coalition of regionalist and non-regionalist voters would enable entry of a moderate independent in the federal election when both parties nominate candidates located at -1 along the ideological dimension.

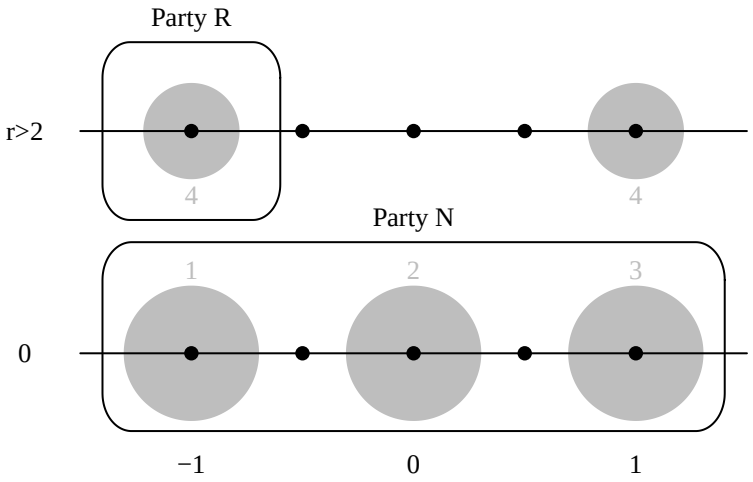

Figure 2: Example of a Two-Party Equilibrium that Exists for Arbitrarily Large Values of r

A grey disc indicates that voters are located at the centre of the disc. The size of each disc is proportional to the share of voters in the federal election at the indicated location. Grey numbers give the state that the voters in question belong to. Rounded rectangles indicate the shapes of the parties that are active in equilibrium, that is, which of the politicians located at black dots are allowed to join each party.

Note that the voters of states 2 and 3 in the example of Proposition 5 would be willing to vote in favour of an independent candidate with platform $(-.5, 0)$ in the federal election if party N nominates a candidate with platform $(-1, 0)$. Similarly, the voters of states 1 and 2 would prefer an independent with platform $(.5, 0)$ over the party candidates if party N nominates a politician with platform $(1, 0)$. If the coalitions of either the voters in states 1 and 2 or of the voters in states 2 and 3 made up a slightly larger share of the federal electorate, one of the aforementioned independents could win the federal election as they would be strictly preferred by a strict majority of voters. This observation forms the basis of the following result.

Proposition 6. *No two-party equilibrium exists if all of the following conditions hold:*

i) $r > 2$,

ii) there exists at least one state where a strict majority of voters strictly prefers a platform $p \in \{-1, -.5, 0, .5, 1\} \times \{r\}$ *over any other platform, and*

iii) the distribution of voters at the federal level satisfies either

$$\Lambda_f((-\infty, .75) \times \{0\}) > .5$$

or

$$\Lambda_f((-.75, \infty) \times \{0\}) > .5 \ .$$

Proof. See Appendix. □

While part *i*) of the preceding Proposition simply states that regionalism is highly salient, part *ii*) requires that there is a state where a strict majority of voters is not only regionalist but also fairly homogeneous in terms of their preferences along the ideological dimension. In combination, these conditions ensures that there must be a party that allows only regionalist politicians

to join in any two-party equilibrium: Due to the salience of regionalism, regionalist candidates cannot win the federal election and are therefore willing to join a narrow party targeted at the preferences of voters in a regionalist state. In a two-party equilibrium, the competing party must then be a broad party that allows at least all non-regionalists to join, which creates the chance that a partisan and a regionalist compete in the federal election. If part *iii*) of Proposition 6 is satisfied, the preceding constellation of candidates creates an opportunity for entry of an independent candidate.

The homogeneous preferences in at least one regionalist district required by part *ii*) of Proposition 6 are most likely to be found in political systems where offices that serve as starting points for a political career are controlled by relatively small electorates. The conditions of part *iii*) of the Proposition, on the other hand, are particularly likely to be satisfied if regionalist voters make up a small share of the federal electorate due to the assumption that the median voter along the ideological dimension of policy is located at 0. The example of Proposition 5 illustrates this logic, where slightly decreasing the number of voters in state 4 would enable entry of independent candidates in the federal election. Intuitively speaking, a larger number of regionalist voters increases the risk that a split in the non-regionalist vote will produce a regionalist winner, thus complicating coordination on a non-regionalist independent.

Note that the conditions that enable entry of a moderate independent in the basic version of the model, as given by Proposition 2, imply that Part *iii*) of Proposition 6 is satisfied, while the reverse is not true. Salience of the regionalist dimension of policy thus relaxes the conditions required to enable successful entry of independent candidates in the federal election.

To what extent do the results presented in this section provide an explanation for the existence of, for example, the Scottish National Party (SNP)? The consequence of the condition on the value of r in Proposition 6 is that non-regionalist voters prefer any non-regionalist over any regionalist, which is

plausibly the case in UK politics. The small electorates of UK parliamentary constituencies make it more likely that Part *ii*) of Proposition 6 is satisfied. In addition, Scottish voters account for less than 10 percent of the electorate in UK general elections, which increases the likelihood that Condition *iii*) of Proposition 6 holds as was argued above. The necessary conditions for entry of a third party provided by the model thus appear to be met in the case of the UK. That the SNP emerged as a political force only in the 1970s may be due to a lack of sufficient voter support for the issue of independence in the previous decades. As Proposition 6 makes clear, however, an up-tick in voter support is by itself not enough to predict entry of a regionalist party.

5 Robustness

The basic model of party formation presented above requires a number of simplifying assumptions for tractability. This section discusses some of these in more detail.

5.1 Policy Choices

The assumption that politicians are committed to implementing their platform is not satisfying. While a number of empirical studies indicates that preferences over policies are the main driver of the choices that politicians make in office (Levitt 1996, Chattopadhyay & Duflo 2004, Lee et al. 2004, Bhalotra & Clots-Figueras 2014), it would be more appealing to see this behaviour emerge as part of an equilibrium rather than imposing it from the outset. Endowing politicians with policy preferences alongside their office motivations introduces two additional difficulties: First, politicians may want to choose a policy equal to or at least closer to zero than their actual ideal policy in an attempt to fool voters. If they succeed, this would increase their chances of winning the federal election. Second, politicians take into account how their choices affect the policies chosen by other politi-

cians. Particularly the latter issue creates difficulties, since it is hard to track how the decision of a politician to join or not to join a party affects events in the federal election. However, it is possible to incorporate the role of state policies as signals of policy preferences without having to deal with the second type of complication. To do so, I follow Snyder & Ting (2002) and Ashworth & Bueno de Mesquita (2008) and assume that politicians only care about policy once elected. The utility of a politician with ideal policy $p \in \{-1, -.5, 0, .5, 1\}$ is now be given by

$$\pi_s[y_s + \alpha\ v(|p_s - p|)]\ +\ \pi_f[y_f + \alpha\ v(|p_f - p|)]\ ,$$

where α measures the relative weight that politicians attach to policy, v is a decreasing function with $v(0) = 0$, and the notation otherwise follows Section 2. Parties then allow only politicians with certain ideal policies to join. In addition, politicians can freely choose the policy they implement at any stage from the set $\{-1, -.5, 0, .5, 1\}$. All other elements of the game remain unchanged.

The following result shows that politicians are always willing to forgo a higher chance of winning the federal election in favour of implementing their own ideal policy at the state level if α is sufficiently large.

Proposition 7. *Suppose $\alpha > -y_f/v(.5)$. Then any politician must implement their own ideal policy at any point of the game in any equilibrium.*

Proof. See Appendix. □

When all politicians always select their ideal point when choosing policy, the utility function considered here simplifies to the one assumed in the benchmark model and all results go through unchanged.

5.2 Coalitional Deviations

Even though the formation of parties has previously been modelled as a non-cooperative game (Morelli 2004, Osborne & Tourky 2008), this is a setting where deviations by coalitions of players are a natural consideration. Members of the same party, in particular, are well placed to coordinate their actions. A prominent contribution to the literature on party formation allows for parties to be broken up by subsets of their membership (Levy 2004). In the following, I will therefore consider the possibility of a group of politicians deviating jointly to joining a third party on the condition that any member of the deviating coalition is made strictly better off by this move. One example where such a joint deviation could matter would be a situation with three parties competing in a state election, where coordination failure among voters gives a group of politicians an incentive to cooperate and join the same party.[11] Since coordination failure does not occur when there are only two candidates—at least under the assumed restrictions on voting behaviour—the results on two-party equilibria are not subject to this issue.

Focusing on two-party equilibria, any such equilibrium requires that no independent candidates can successfully run for the federal election and the same must therefore be true of third party candidates. An exception would be that a coalitional deviation prevents one or both of the equilibrium parties from competing in the federal election. This situation would require, however, that either the politician with platform -1 in state 1 or the politician with platform 1 in state 3 forms part of the deviating coalition. Since partisans can

[11] Considering the basic version of the model, suppose that there is a state where a third of voters is located at each of the policies 1, $-.5$, and 1. If the politicians with the corresponding platforms have each joined a singleton party, neither candidate is favoured by a strict majority of voters and each of them may win the election. Suppose the politician with platform 1 wins. Then the politicians with platforms -1 and $-.5$ would have an incentive to jointly deviate and join a party of shape $\{-1, -.5\}$, since two thirds of voters would strictly prefer any candidate nominated by this party over a candidate with platform 1.

never be preferred by a strict majority of voters in the federal election, these politicians can always be made to lose the federal election off the equilibrium path. Their motivation for participating in a coalitional deviation could therefore derive at most from an increase in the probability of winning the election in their state, but the politician with platform -1 in state 1 and the politician with platform 1 in state 3 can always win the election in their state with certainty by joining a singleton party anyway. These politicians therefore have no incentive to join a coalitional deviation and the equilibrium parties always continue to compete in the federal election, preventing victory of a third-party candidate.

Coalitional deviations may thus at most increase the chances of the participating politicians of winning a state election. However, voters do not generally observe that multiple politicians have changed their party affiliation, but only that an additional party nominates a candidate. It is thus possible to assign voters the same beliefs as in the case of a deviation by a single politician. If a coalition of politicians can join a third party and win the state election this accordingly implies that at least one individual politician could achieve the same through a unilateral deviation. This politician would then prefer the unilateral deviation, since being the unique member of a new party guarantees the nomination for the state election.

While it thus seems unlikely that allowing for coalitional deviations poses a threat to the results presented above, coordination among politicians would be relevant in a slightly modified version of the model. In particular, the assumption that a single politician can form a new party can be dropped if one allows for coalitional deviations while assuming that there are multiple politicians with the same platform in each state. In many situations, all politicians with the same platform in a given state would join the same party in equilibrium[12] and these politicians then form a homogeneous faction within a possibly broader party. The same incentives that drive defections

[12]The only possible exception may occur when there is a tie in a state election.

by individual candidates in the basic version of the model would then apply to such a faction. Suppose, for example, that all politicians with platform -1 in state 1 have joined a party that also has as least some politicians with other platforms as members in the same state. If all politicians with platform -1 in the state jointly deviate and join a party of shape $\{-1\}$, any member of the group would benefit from a reduction in internal competition, while any payoffs received after the state nomination has been decided do not depend on the number of party members in the state anyway. The perhaps unrealistic assumption that a single politician can form a new party could thus be replaced with the assumption that a minimum number of members is required for a new party to compete, at least as long as this number is no larger then the number of politicians with a specific platform in each state. In this sense, the politicians in the original model can be thought of as representing a faction rather than an individual person.

6 Conclusion

Why are the same two parties competing in elections in the US across all levels of government, while more than two parties attract significant vote shares in other countries relying on FPTP such as the UK or Canada? The model of party formation presented in this paper provides a number of potential explanations. Specifically, any two-party equilibrium of the model requires that a number of conditions are satisfied: First, voters in the federal election cannot be too concentrated in the centre of the policy space, as otherwise a centrist independent candidate could successfully enter. Second, politicians need to be sufficiently motivated by career concerns to prevent them from joining parties more targeted at the preferences of voters in specific regions. Finally, an issue that splits regions into two camps, such as ethnic cleavages or an independence movement, must either be less salient than the classical left-right divide or it must be the case that regionalist voters make up a suf-

ficiently large share of the electorate or are divided regarding policies other than regionalism. If any of these conditions fail, only equilibria with three or more parties exist. In this sense, the necessary conditions for the existence of two-party equilibria suggest explanations for the emergence of new parties in different settings. An interesting prediction of the model is that only centrist parties should be expected to be formed top-down by politicians in advanced stages of their career,[13] while parties located in the political wings emerge as grassroots movements at the regional or local level.

In the absence of a regionalist movement and subject to the conditions given above, the model has an equilibrium featuring a centre-left and a centre-right party very much in line with the party system of the US. Even though some politicians have an opportunity to win a state election with higher probability by joining a third party in this equilibrium, they choose not to do so. The reason is that the equilibrium parties offer a more attractive career, since only their members can win the federal election. Parties thus use their dominance of the federal election to also dominate state elections.

Finally, it is worth highlighting that the model introduced in this paper provides a flexible tool for investigating other questions surrounding political parties, such as the strategic choice of the rules that govern the nomination of candidates, for example. While I have argued that parties have an incentive to commit to giving different types of candidates a shot at winning nominations, I assumed the mechanisms used for candidate selection to be exogenously determined. However, endogenising this feature of parties would be feasible. The section on regionalism, though relatively simple, indicates that the model can generate insights even under multidimensional policy spaces.

[13]Strictly speaking, the model does not allow for the formation of parties by politicians who have already moved beyond the state level. However, the model does allow the possibility of independent candidates in the national election. In parliamentary systems, such as the UK, independent candidates would essentially be forced to form a new party since becoming prime minister requires support of a majority in Parliament.

Appendix: Proofs

The following lemma is not presented in the text:

Lemma 3. *The following politicians must win the election in their state with positive probability in any two-party equilibrium: the politician with platform* -1 *in state 1, the politician with platform* 0 *in state 2, and the politician with platform* 1 *in state 3.*

Proof. Suppose that the politician $g_{1,-1}$ joins a party D of shape $I_D = \{-1\}$. Since no other politicians in the state are able to join this party, voters believe that the candidate of party D has platform -1 with certainty, in or out of equilibrium. Since a strict majority of voters in the state strictly prefers the platform -1 over any other platform, the politician with platform -1 thus wins the state election with certainty, receiving a payoff of at least y_s. If this politician was in a situation where they win the election in their state with zero probability, they would receive a payoff of zero and therefore prefer to deviate and join a party of shape $\{-1\}$. An analogous argument applies to the politician $g_{2,0}$ and the politician $g_{3,1}$. □

Proof of Lemma 1. Consider a two-party equilibrium such that $\mathcal{P}^* = \{A, B\}$ and suppose there is a platform $p \in \{-1, 0, 1\}$ such that $p \notin I_A \cup I_B$. This implies that either the politician $g_{1,-1}$, the politician $g_{2,0}$, or the politician $g_{3,1}$ neither joins a party in equilibrium nor runs as an independent and thus wins the state elections with probability zero. This situation cannot be part of two-party equilibrium by Lemma 3. □

Proof of Lemma 2. Recall that voters observe whether a party nominates a candidate for a state election, but not how many politicians have joined a party. Voting behaviour can therefore only be conditional on which parties have nominated candidates. Let party A be a party that wins the election in some state s with positive probability. If party A wins with positive

probability due to the other party not nominating a candidate, then this event does not become any less likely due to additional politicians joining party A. Conditional on facing a competitor, the probability that the candidate of party A wins cannot decrease either due to additional members joining. Either voters cannot detect that a deviation has occurred or the second party has no members left. The probability that party A wins the state election therefore cannot decrease if any politician joins the party with higher probability. Joining party A in state s therefore yields a positive payoff and any politician who has this option can therefore never chose the strategies of remaining passive or joining a party that loses the state election with certainty, given that the latter two strategies lead to a payoff of zero. □

Proof of Proposition 1. Consider a two-party equilibrium with active parties A and B such that both parties allow at least one politician with a platform smaller than zero to join. In equilibrium, the politician $g_{1,-1}$ must win the election in their state with positive probability as a member of either party A or party B by Lemma 3. Without loss of generality, assume that the politician can win the state election as a member of party A.

As a preliminary step, it will be shown that the politician $g_{3,0}$ must win the state election with positive probability. By Lemma 3, the politician with platform 1 in the same state can win the state election with positive probability. One of the equilibrium parties must therefore allow this politician to join while also admitting at least one negative platform. Since parties consist of consecutive platforms, politicians with platform 0 must accordingly also be able to join the party. Lemma 2 therefore implies that the latter politician wins the state election with positive probability.

If party A and party B both nominate politicians with negative platforms for the federal election, a strict majority of voters would strictly prefer a candidate with platform 0, since the federal median voter is located at 0. The same is true if both parties nominate a politician with a positive platform, or if only one party nominates a candidate and this candidate has a platform

other than 0. In these situations, a politician with platform 0 who has won a state election could therefore successfully run as an independent candidate in the federal election and would accordingly have an incentive to do so, contradicting equilibrium. (Recall that voters know the platform of any winner of a state election.) By Lemma 3, the politician $g_{2,0}$ must win the election of the state with positive probability, while it was shown above that the same is true of the politician $g_{3,0}$. Since party A may nominate the politician $g_{1,-1}$ for the federal election, it follows that party B cannot have any members with a negative platform in any state where this party can win the state election other than state 1. Otherwise the party would nominate this politician for the federal election with positive probability, giving the politician with platform 0 from state 2 or state 3 a chance to run as an independent when party A simultaneously nominates the politician $g_{1,-1}$. It further follows that party B does not win any state elections with certainty as it was assumed that party A can win the election in state 1 and party B would be joined by at least one politician with negative platform in any other state where it wins with certainty by Lemma 2.

Since party B does not win any state elections with certainty, it occurs along the equilibrium path that party A wins all state elections and nominates the politician $g_{1,-1}$ as the sole candidate for the federal election. If any moderate has simultaneously won a state election, this situation would enable the latter politician to run as an independent. It must therefore be the case that no moderates join party A in any state other than state 1. The politician with platform 0 of state 2 can win the state election with positive probability in equilibrium and must therefore do so as a member of party B. Suppose the politician with platform 0 was the only member of party B in state 2. Since a strict majority of voters in state 2 strictly prefers the platform 0 over any other platform, Party B would then win the state election with certainty, but this possibility was already ruled out above. Party B must therefore be joined by a politician with positive platform in the state.

Based on the preceding results, it can now be shown that at least one politician has an incentive to run as an independent for the federal election in the potential equilibrium under consideration. Suppose party A is not joined by any politicians with positive platform in state 3. Since party B does not have any members with negative platform, this would imply that party B wins the state election with certainty since the median voter of the state prefers positive platforms over negative platforms and the politician with platform 1 joins party B since this politician must win the state election with positive probability by Lemma 3. However, it was already argued that party B does not win any state elections with certainty. It may thus happen that party A nominates a politician with positive platform from state 3 for the federal election while party B nominates a politician with positive platform from state 2 by the previous paragraph. The politician $g_{1,0}$ therefore cannot win the state election with positive probability, as they would otherwise have an incentive to run as an independent in the federal election. However, party A allows politicians with positive and with negative platforms to join and therefore also politicians with platform 0. Furthermore, party A wins the election in state 1 with positive probability. It therefore must occur that the politician with platform 0 of the state wins the state election by Lemma 2 and subsequently has an incentive to run as an independent candidate in the federal election.

It has thus been shown that it is impossible that there is a two-party equilibrium where both parties allow members with negative platforms to join. By an analogous argument, there is no two-party equilibrium where both parties allow politicians with positive platforms to join. The only remaining constellation of parties that is not consistent with the statement of the proposition is that there is one party that allows all politicians to join while the other party has shape $\{0\}$. Refer to these parties as A and B, respectively. If party B wins any state elections with certainty, then this party also nominates a candidate with platform 0 for the federal election with certainty.

As a consequence, any politicians with a platform other than 0 cannot win the federal election. The politician $g_{1,-1}$ would then prefer to join a singleton party to maximise their chance of winning the state election. If, on the other hand, party B does not win any state election with certainty, then there is the possibility that party A wins all state elections. Since a strict majority of voters in state 2 prefers the platform 0 over any other platform, party A can only win the state election if the politician with platform 0 of the state does not join party B. By Lemma 2, said politician must then join party A instead and wins the state election with positive probability. The politician $g_{1,-1}$, on the other hand, must be able to win the election in their state with positive probability as a member of party A by Lemma 3. It thus occurs with positive probability that party A wins all state elections and nominates a politician with platform -1 for the federal election while a politician with platform 0 has won a state election. The latter politician would then have an incentive to run as an independent in the federal election, contradicting equilibrium. □

Proof of Proposition 2. Suppose three candidates with known platforms are running for the federal election, one with platform -1, one with platform 1, as well as a moderate with platform $p \in \{-.5, 0, .5\}$. Since voters prefer candidates with platforms close to their ideal points, a voter with ideal point i strictly prefers the moderate candidate over any other candidate if and only if $i \in (\frac{-1+p}{2}, \frac{1+p}{2})$. If $\Lambda_f(\frac{-1+p}{2}, \frac{1+p}{2}) > .5$, the moderate candidate must then win the federal election by the definition of party-formation equilibrium.

Now consider the following situation: in a two-party equilibrium, one party has nominated a candidate with platform -1 for the federal election while the other has nominated a candidate with platform 1. At the same time, there is a moderate politician with platform p who has won a state election. Suppose this moderate deviates and runs for the federal election as an independent candidate. Then the independent candidate wins if $\Lambda_f(\frac{-1+p}{2}, \frac{1+p}{2}) > .5$ by the previous paragraph, given that the state election

has resolved any uncertainty about the identity of the politician in question. The deviation is therefore profitable.

To complete the proof, it will be shown that the situation described in the previous paragraph must arise with positive probability in any two-party equilibrium. Proposition 1 and Lemma 1 in combination imply that in any two-party equilibrium one party must allow politicians with platform -1 to join but not politicians with platform greater than zero, while the other party allows politicians with platform 1 to join but not politicians with platform smaller than zero. Refer to these parties as A and B, respectively. Party A (party B) must then win the election in state 1 (state 3) with positive probability with the politician with platform -1 (platform 1) as a member. Otherwise one of these politicians would have a profitable deviation by Lemma 3. As any winner of a state election is nominated by their party with positive probability for the federal election, it thus happens with probability greater than zero that party A nominates a politician with platform -1 while party B nominates a politician with platform 1. At the same time, the politician $g_{2,0}$ must win the state election with positive probability by Lemma 3. Since party shapes consist of consecutive platforms, the same must be true of at least one of the politicians with platforms $-.5$ and $.5$ in the same state. As the identities of state winners are independent of each other conditional on the set of existing parties, it thus occurs with positive probability that a candidate with platform -1 and a candidate with platform 1 are nominated for the federal election while a moderate has won the election in state 2.

A moderate politician therefore has a profitable deviation in any potential two-party equilibrium if there is a strict majority of voters in the federal election located in at least two of the intervals $(-.75, .25)$, $(-.5, .5)$, and $(-.25, .75)$. □

Proof of proposition 3. Consider a two-party equilibrium such that $\mathcal{P}^* = \{A, B\}$. Any politician can only win a state election with certainty in a two-party equilibrium if they are member of a singleton party with certainty.

Otherwise at least one of the parties that the politician joins with positive probability would attract other members with positive probability by Lemma 2. As a consequence, there would be a non-zero chance that the politician misses out on the nomination for the state election. Given that any politician can only win a state election with certainty by joining singleton parties, it is impossible that the politician $g_{1,-1}$, the politician $g_{2,0}$ and the politician $g_{3,1}$ all win the election in their state with certainty, since this would require that three singleton parties are active. Given that no independent candidates run in a two-party equilibrium, at least one of these politicians therefore achieves a payoff in equilibrium of at most

$$\pi(y_s + y_f) \ ,$$

where $0 < \pi < 1$ denotes the probability that the politician wins the state election.

The same politician can achieve a payoff of at least y_s by deviating and joining a singleton party since a strict majority of voters in their state is strictly in favour of their platform. This deviation is profitable if $y_s > \pi(y_s + y_f)$, which can be rewritten as $y_f/y_s < (1-\pi)/\pi$ where $(1-\pi)/\pi > 0$. A necessary condition for the existence of the equilibrium is therefore $y_f/y_s \geq (1-\pi)/\pi > 0$. □

Proof of Proposition 4. Suppose $\mathcal{P}^* = \{L, R\}$, every politician joins one of the two active parties, and no politician ever runs as an independent. Assume that the politician with platform 0 in each state mixes over joining party L or party R, potentially joining one of these parties with certainty. It will be shown below that at least one such strategy exists that is consistent with equilibrium. However, when the ideal policy of the median voter of a state is no larger than $-.25$ and voters believe that the politician with platform 0 of the state has joined party L, then the median voter at least weakly prefers any potential candidate of party L over any potential candidate of

party R. Party L would then win the state election and it is thus consistent with equilibrium that the politician with platform 0 joins party L in such states. By an analogous argument, it is consistent with equilibrium that the politician with platform 0 joins party R and party R wins the state election in any state such that the ideal policy of the median voter is no smaller than .25. Party L thus wins the election in state 1 and party R the election in state 3, so both parties compete in the federal election and one of the candidates closest to the federal median voter located at zero wins. Assume indifferent voters mix such that either candidate wins the federal election with equal probability when two candidates are equidistant from zero.

It needs to be verified that no politician has an incentive to deviate. Part i) of the statement of the proposition is constructed to ensure that no independent candidate can be preferred by a strict majority of voters over the candidates of both parties. Accordingly, the equilibrium can be constructed such that independent candidates in the federal election do no win. Next consider a politician who decides to run as an independent in a state election. After this out-of-equilibrium event, it is possible to assign voters the belief that the politician running as an independent has a platform furthest from the location of the state median voter. This is possible since voters observe only which parties nominate candidates, but not the affiliation choices of individual politicians, thus putting no restrictions on voter beliefs. The equilibrium can thus be constructed such that any independent candidate in a state election does not win.

Finally, it needs to be shown that no politician wants to change their party affiliation. Only politicians with platform 0 can switch between equilibrium parties. Holding all other behaviour constant and assigning continuation payoffs accordingly, the choice between party L and party R of the politician with platform 0 in a specific state can be seen as a finite game of incomplete information between this politician and the median voter of the state. An

equilibrium of this game is guaranteed to exist.[14] In any such equilibrium the politician with platform 0 must win the state election with positive probability by Lemma 2. While the equilibrium can thus be constructed such that no politician wants to change their affiliation from party L to party R or vice versa, no politician prefers not to join a party either since this option gives a payoff of zero. In order to demonstrate that no politician wants to join a previously passive party, it is sufficient to show that no politician wants to join a singleton party. This is true as it is always possible to assign voters the same belief after a politician has deviated and joined a party that allows more than one type to join as in the case of a deviation to a singleton party. Suppose thus that a politician deviates and joins a singleton party instead of party $P \in \{L, R\}$. Start with politicians who do not win a state election in equilibrium. Consider a politician with platform -1 or $-.5$ in a state where party R wins, which implies that the median voter of the state has an ideal policy no smaller than $-.25$. After the deviation, it is possible to assign voters the belief that the candidate of party R has the platform 0 while the candidate of party L has the platform -1 or $-.5$ (depending on the identity of the deviating politician) since the information set reached is off the equilibrium path. The median voter and any voter with a larger ideal policy thus prefer the candidate of party R and the equilibrium can be constructed such that the deviating politician loses the state election and the deviation is not profitable. An analogous argument applies to politicians with platform .5 or 1 who do not win a state election, while politicians with platform 0 always win the state election with positive probability.

Consider thus a politician who wins a state election with positive probability who deviates and joins a singleton party. If the politician wins the state election and faces two competitors for the federal election, Part i) of

[14] It may be necessary that the politician with platform 0 plays a mixed strategy since it is possible that the act of joining a party affects nomination probabilities in such a way that the median voter always prefers the party *not* joined by the politician with platform 0.

the statement of the proposition again ensures that the deviating politician can be made to lose the federal election. A special case is that a deviation has the effect that either party A or party B does not nominate a candidate for the federal election, as may happen when state 1 is the unique state won by party L or state 3 is the unique state won by party R. Without loss of generality, focus on the latter case. If any politician of state 3 other than the politician with platform 1 joins a singleton party, voters may believe that the candidate of party R has the platform 1 and the entrant party loses the election. The politician with platform 1, on the other hand, cannot be strictly preferred by a strict majority of voters in the federal election even when competing against only one other candidate and may thus at most win the state election by joining a singleton party. Any politician who can win a state election in equilibrium can therefore achieve a payoff no higher than y_s by deviating. In equilibrium, on the other hand, such a politician wins the federal election with positive probability. This is true by the assumption that each member of the candidate pool for each election is nominated with non-zero probability and because each candidate wins the federal election with equal probability when two partisans compete. For a particular politician, who is nominated for and wins their state elections with probability $0 < \pi_s < 1$ and, conditional on doing so, is nominated for and wins the federal election with probability $0 < \pi_f < 1$, deviating to joining a singleton party is therefore not profitable if

$$\pi_s(y_s \ + \ \pi_f \ y_f) \geq y_s \Leftrightarrow \frac{y_f}{y_s} \geq \frac{1-\pi_s}{\pi_s \pi_f} > 0 \ .$$

There thus exists a threshold $\bar{y} > 0$ such that no politician has a profitable deviation if $y_f/y_s \geq \bar{y}$. □

Lemma 4. *If $r > 2$, any voter with ideal point (i, j) with $i \in [-1, 1]$ and $j \in \{0, r\}$ strictly prefers any candidate with platform (p, j) over any candidate with platform $(p', r - j)$.*

Proof. The statement of the lemma is true since

$$u(|p-i|)+u(0) \geq u(2)+u(0) > u(r)+u(0) \geq u(|p'-i|)+u(r)\ . \quad \square$$

Proof of Proposition 5. To complete the description of the equilibrium, assume that all eligible politicians have joined party N or party R, while all remaining politicians remain passive. Since all voters are located in the interval $[-1,1]$ along the ideological dimension and $r > 2$, any non-regionalist voter prefers any non-regionalist candidate with platform $(p,0)$ over any regionalist candidate with platform (p',r) and vice versa by Lemma 4. Accordingly, party N wins the elections in states 1, 2, and 3, party R wins the election in state 4, and any candidate of party N wins the federal election. The politician with platform $(-1,r)$ in state 4 thus achieves a payoff of y_s and cannot improve on this through any deviation as long as party N nominates a candidate for the federal election.

Consider the possibility of any independent or third-party candidates other than the politician with platform $(-1,r)$ from state 4 running in the federal election. If a non-regionalist enters the race, a fourth of all voters at least weakly prefers the candidate of party R over the entrant while another fourth of voters at least weakly prefers the candidate of party N. If a regionalist enters, all voters in states 1, 2, and 3 strictly prefer the candidate of party N. Therefore, no strict majority strictly prefers any third candidate and a voting equilibrium can be constructed such that these candidates do not win the federal election. In state elections, on the other hand, beliefs over the platform of any independent candidates can be assigned such that these candidates do not win.

It remains to be verified that no politicians can gain from changing their party affiliation. Deviating to not joining a party is never profitable. In state 4, at least half of all voters strictly prefer the candidate of party R over any other candidate and no additional party can enter successfully. Suppose a politician in any other state could win the state election by joining a third

party. This deviation achieves a payoff of at most y_s, since third party candidates cannot win the federal election by the argument above. In equilibrium, in contrast, each of the politicians in states 1, 2, and 3 wins the federal election with positive probability, receiving a payoff of $\pi_s(y_s + \pi_f y_f)$ where π_s and π_f are positive probabilities. The deviation is therefore not profitable if $y_s \leq \pi_s(y_s + \pi_f y_f)$, which can be rewritten as

$$\frac{y_f}{y_s} \geq \frac{1 - \pi_s}{\pi_s \pi_f} \ . \tag{1}$$

The right-hand side of the preceding inequality is strictly greater than zero since none of the politicians in states 1, 2, or 3 are nominated for the state election with certainty. □

Proof of Proposition 6. Assume $r > 2$ and let state s be a state such that a strict majority of voters strictly prefers a policy belonging to the set $\{-1, -.5, 0, .5, 1\} \times \{r\}$ over any other policy. The politician of the state with the corresponding platform is unable to win the federal election since a strict majority of voters is located in $[-1, 1] \times \{0\}$ and all of these voters strictly prefer any non-regionalist over any regionalist by Lemma 4. Therefore, a regionalist always loses against a non-regionalist and even if all parties nominate a regionalist candidate, an independent from a non-regionalist state (who is certain to be located at 0 along the regionalist dimension) could successfully enter the race. The politician from state s who is favoured by a strict majority of voters there must therefore choose a party affiliation that maximises their chances of winning the state election, which is achieved by joining a party that allows no other types of politicians to join. Denote this party as party A.

Let B be the party competing with party A in a two-party equilibrium. By Lemma 3, the politician with platform $(-1, 0)$ of state 1 and the politician with platform $(1, 0)$ must win the elections in their respective states with positive probability as members of party B in equilibrium. By convexity of

party shapes, party B must allow all types on non-regionalist politicians to join. Since party A cannot have any members in state 2, party B must win the state election there with certainty, which implies that all politicians of the state join the party by Lemma 2. Given that all eligible politicians are nominated with positive probability, it therefore occurs along the equilibrium path that party B nominates the politician with platform $(-1, 0)$ from state 1 for the federal election while party A nominates a regionalist and the politician with platform $(-.5, 0)$ from state 2 has won the state election there. If $\Lambda_f((-.75, \infty)) > .5$, the latter politician would be strictly preferred by a strict majority of voters in the federal election over the remaining candidates by Lemma 4 since $r > 2$. The winner of the election in state 2 could therefore successfully run as an independent. Similarly, it occurs with positive probability along the equilibrium path that party B nominates the politician with platform $(1, 0)$ from state 3 for the federal election while party A nominates a regionalist and the politician with platform $(.5, 0)$ from state 2 has won the state election there, giving the latter politician the opportunity to run as an independent if $\Lambda_f((-\infty, .75)) > .5$. □

Proof of Proposition 7. Consider a politician who has won the election in their state. If they implement their own ideal policy, this could imply at worst that they win the federal election with probability 0. The corresponding continuation payoff would be 0. In contrast, choosing any other policy at best yields a payoff of $\alpha v(.5) + y_f$, which would be the case if the politician subsequently wins the federal election with certainty and implements their own ideal policy federally. Choosing the own ideal policy at the state level leads to a strictly greater payoff if $\alpha > -y_f/v(.5)$, taking into account that $v(.5) < v(0) = 0$. □

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