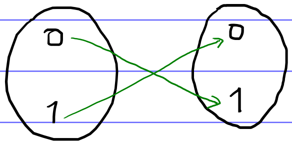
# Algoritmos.

Oráculo (caja negra) → esconde o codifica una función $f$.

$$\boxed{U_f}$$

Problema de Deutsch:

$f_1 : \{0,1\} \to \{0,1\}$



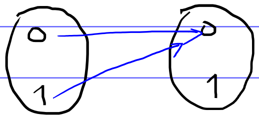

$f(0) = 1$
$f(1) = 0$

Balanceada

$f_3 : \{0,1\} \to \{0,1\}$

$f(0) = 0$
$f(1) = 1$

$f_2 : \{0,1\} \to \{0,1\}$




$f(0) = 0$
$f(1) = 0$

$f_4 : \{0,1\} \to \{0,1\}$

$f(0) = 1$
$f(1) = 1$

Constante

No sirve:

$$\begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 1 & 1 \\ 1 & 0 & 0 \end{array}$$

→ no es unitaria

Problema: Si me dan una de estas 4 funciones dentro de un oráculo quiero clasificarla como constante o balanceada.

input → $\boxed{U_f}$ → $f(\text{input})$

Clasicamente:

1) $0 \longmapsto f(0)$

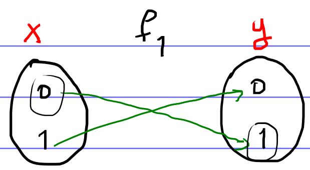
2) $1 \longmapsto f(1)$

$\underline{\underline{2}}$ preguntas

Cómo construir la matriz que codifique nuestra función $f$ como una compuerta → oráculo

**Regla universal** para codificar funciones en matrices unitarias

Ej:




No sirve:

$$\begin{array}{c|cc} & 0 & 1 \\ \hline 0 & & \\ 1 & & \end{array}\quad \begin{array}{cc} & \\ \end{array}$$

$$\begin{array}{ccc} & 0 & 1 \\ 0 & \multicolumn{2}{c}{\multirow{2}{*}{}} \end{array}$$

Regla Universal:

$x \longrightarrow x$

$y \longrightarrow y \oplus f(x)$

$|xy\rangle \rightarrow |x, y \oplus f(x)\rangle$

$|0,0\rangle \xrightarrow{U_f} |0,1\rangle$

$U_f|00\rangle = |01\rangle$

E. inicial — E. final

$x$: $0$, $y$: $0$ → $x$: $0 \rightarrow 0$; $y \oplus f(x)$: $0 \oplus f(0) = 1$, $= 0 \oplus 1$

$x$: $0$, $y$: $1$ → $0$; $1 \oplus f(0) = 1 \oplus 1 = 0$ → $0$, $0$

$x$: $1$, $y$: $0$ → $1$; $0 \oplus f(1) = 0 \oplus 0 = 0$ → $1$, $0$

$x$: $1$, $y$: $1$ → $1$; $1 \oplus f(1)$, $1 \oplus 0 = 1$ → $1$, $1$

salida (e. inicial)

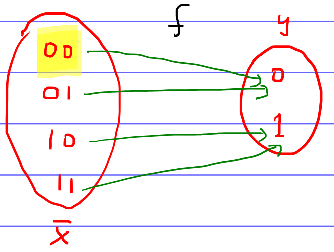
llegada (e. final)

$$\begin{array}{c} \\ 0,0 \\ 0,1 \\ 1,0 \\ 1,1 \end{array}\begin{array}{c} \begin{array}{cccc} 0,0 & 0,1 & 1,0 & 1,1 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} = U_f$$

Deutsch-Jozsa

$2^3$

$f: \{0,1\}^2 \longrightarrow \{0,1\}^1$ → Habrá funciones que no son ni ctes. ni balanceadas




$8 \times 8$

| | 00,0 | 00,1 | 01,0 | 01,1 | 10,0 | 10,1 | 11,0 | 11,1 |
|---|---|---|---|---|---|---|---|---|
| 00,0 | 1 | 0 | | | | | | |
| 00,1 | 0 | 1 | | | | | | |
| 01,0 | 0 | 0 | | | | | | |
| 01,1 | 0 | 0 | | | | | | |
| 10,0 | 0 | 0 | | | | | | |
| 10,1 | 0 | 0 | | | | | | |
| 11,0 | 0 | 0 | | | | | | |
| 11,1 | 0 | 0 | | | | | | |

$\bar{x}$ 0 0 $\quad$ 0 0

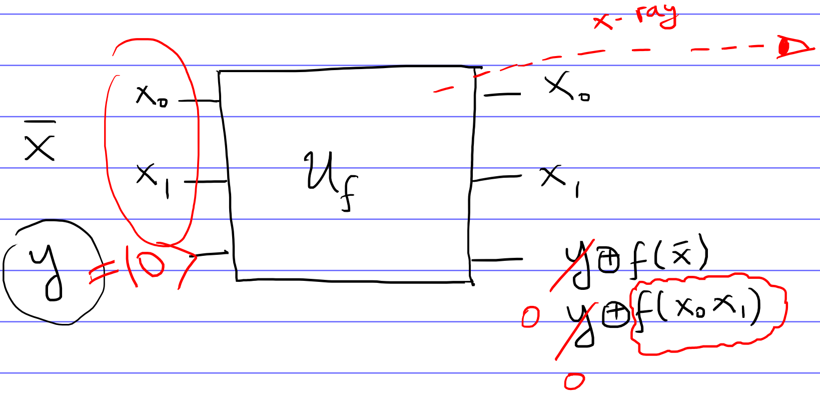
$y$ 0 $\longrightarrow 0 \oplus f(\bar{x})$

$0 \oplus f(00) = 0 \oplus 0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\bar{x}$ 0 0 $\quad$ 0 0

$y$ 1 $\quad 1 \oplus 0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

00 $\quad$ 01 $\quad$ 10 $\quad$ 11



# Dudas quiz:

③ Para cuáles funciones $f:\{0,1\}\to\{0,1\}$ se cumple que $|01\rangle \xrightarrow{U_f} |00\rangle$

$x$ $y$

No se cumple

$x$ $0 \longrightarrow 0$

$y$ $1 \longrightarrow y \oplus f(x) = 1 \oplus f(0) = 1 \oplus 0 = 1$ $\Big\rangle$ $0$, $1$

$0 \longrightarrow 0$

$1 \longrightarrow y \oplus f(x) = 1 \oplus f(0) = 1 \oplus 1 \longrightarrow 0$ $\Big\rangle$ $0$, $0$ sí se cumple

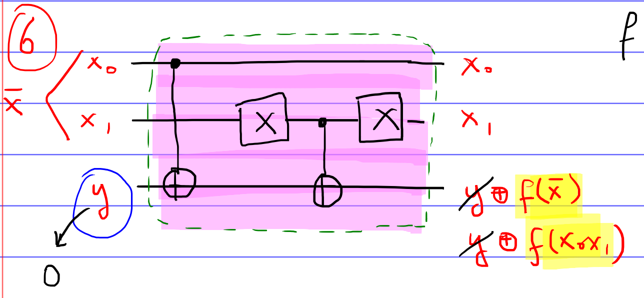
$0 \longrightarrow 0$

$1 \longrightarrow y \oplus f(x) = 1 \oplus \overbrace{f(0)}^{0} = 1$ $\Big\rangle$ $0$, $1$ no

$0 \longrightarrow 0$

$1 \longrightarrow y \oplus f(x) = 1 \oplus \overbrace{f(0)}^{1} = 0$ $\Big\rangle$ $0$, $0$ sí

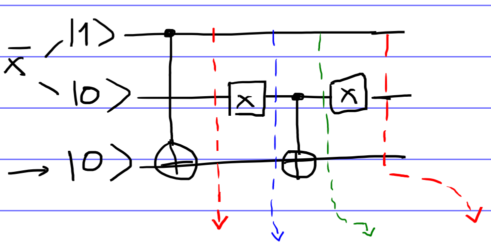
⑥




$f:\{0,1\}^2 \longrightarrow \{0,1\}$

$x$: 00, 01, 10, 11 $\qquad$ $y$: 0, 1

$f(\overset{x_0\, x_1}{10}) = ?$




$\bar{x}\begin{cases} |1\rangle & |1\rangle & |1\rangle & |1\rangle \\ |0\rangle & |1\rangle & |1\rangle & |0\rangle \end{cases}$

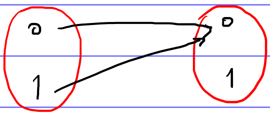
$|1\rangle \quad |1\rangle \quad |0\rangle \quad |0\rangle$

R/. $f(10) = 0$

# Problema de Deutsch

Hallar las matrices correspondientes

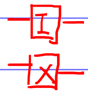
$f\_\emptyset$

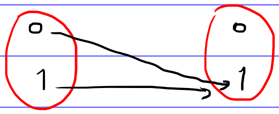



función

$f\_1$

circuito

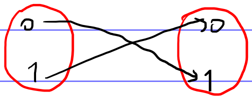


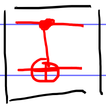

matriz

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$f\_x$

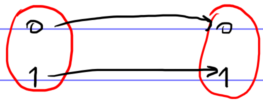



circuito



$f\_id$

función




$f(0)=0$

$f(1)=1$

I

matriz

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

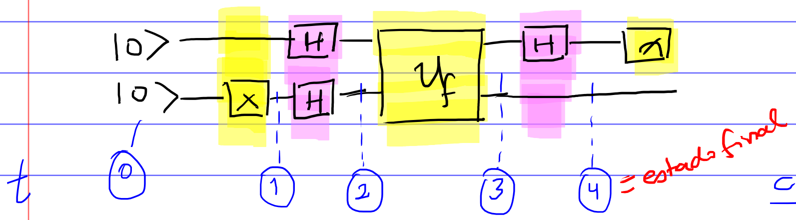
X

Si: $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $X^0 = I \;\|\; X^1 = X$

$$\mathcal{U}_f = \left[\begin{array}{c|c} X^{f(0)} & 0 \\ \hline 0 & X^{f(1)} \end{array}\right]$$

## ¿Cómo analizar un circuito?

Análisis y explicación del circuito para el algoritmo de Deutsch $f:\{0,1\}^1 \to \{0,1\}$ $n=1$




clics / pasos

* Estado en $t=0$: $|0\rangle$, $|0\rangle$ ó $|0\rangle \otimes |0\rangle$

$$|\varphi_0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\begin{bmatrix}1\\0\end{bmatrix}\\ 0\begin{bmatrix}1\\0\end{bmatrix}\end{bmatrix} = \begin{array}{c} \begin{bmatrix}1\\0\\0\\0\end{bmatrix} \end{array}\begin{array}{l} 00\\01\\10\\11\end{array}$$

→ 100% de prob. de colapsar a $|00\rangle$

* Matriz que debo aplicar para llegar a $t=1$

$-\boxed{I}-$
$-\boxed{X}-$ ó

$$I \otimes X = \begin{bmatrix} 1\begin{bmatrix}0&1\\1&0\end{bmatrix} & 0\begin{bmatrix}0&1\\1&0\end{bmatrix} \\ 0\begin{bmatrix}0&1\\1&0\end{bmatrix} & 1\begin{bmatrix}0&1\\1&0\end{bmatrix}\end{bmatrix}$$

$$= \begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$$

- Estado en $t=1$: Acción de la matriz que acabamos de hallar sobre el vector de estado anterior

$$|\varphi_1\rangle = \begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\end{bmatrix} = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}\begin{array}{l} 00\\01\\10\\11\end{array}$$

100% de prob. de colapsar al estado $|01\rangle$

* Matriz que debo aplicar para llegar a $t=2$.

$$-\boxed{H}- \qquad H\otimes H = H^{\otimes 2} \qquad H=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} = \begin{bmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ 1/\sqrt{2} & -1/\sqrt{2}\end{bmatrix}$$
$$-\boxed{H}-$$

$$= \frac{1}{\sqrt{2}}\left[\begin{array}{c|c} 1\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} & 1\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \\ \hline 1\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} & -1\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\end{array}\right]$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{bmatrix}$$

2ª columna de $H\otimes H$

$$|\varphi_2\rangle = \frac{1}{2}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{bmatrix}\begin{bmatrix}0\\1\\0\\0\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1\\-1\\1\\-1\end{bmatrix}$$

$$= \begin{bmatrix}1/2\\-1/2\\1/2\\-1/2\end{bmatrix}\begin{array}{llll} 00 \leftarrow & 25\% & \left((1/2)^2\right)\\ 01 \leftarrow & 25\% & \left((-1/2)^2\right)\\ 10 \leftarrow & 25\% & \left((1/2)^2\right)\\ 11 \leftarrow & 25\% & \left((-1/2)^2\right)\end{array}$$

Paralelismo cuántico

* Matriz que necesito calcular para llegar a $t=3$:

$$U_f = \left[\begin{array}{c|c} X^{f(0)} & 0\\ \hline 0 & X^{f(1)}\end{array}\right]$$

matrices de 2×2

$$|\varphi_3\rangle = \frac{1}{2}\left[\begin{array}{c|c} X^{f(0)} & 0 \\ \hline 0 & X^{f(1)} \end{array}\right]\left[\begin{array}{c} 1 \\ -1 \\ \hline 1 \\ -1 \end{array}\right]$$

$$= \frac{1}{2}\left[\begin{array}{c} X^{f(0)}\begin{bmatrix}1\\-1\end{bmatrix} + \cancel{\begin{bmatrix}0&0\\0&0\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix}} \\ \hline \cancel{\begin{bmatrix}0&0\\0&0\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix}} + X^{f(1)}\begin{bmatrix}1\\-1\end{bmatrix} \end{array}\right]$$

$$= \frac{1}{2}\left[\begin{array}{c} X^{f(0)}\begin{bmatrix}1\\-1\end{bmatrix} \\ \hline X^{f(1)}\begin{bmatrix}1\\-1\end{bmatrix} \end{array}\right]$$

* Hallamos la siguiente matriz: $-\boxed{H}-$ , $-\boxed{I}-$

$$\underline{\underline{H \otimes I}} = \left[\begin{array}{c|c} \frac{1}{\sqrt{2}}\begin{bmatrix}1&0\\0&1\end{bmatrix} & \frac{1}{\sqrt{2}}\begin{bmatrix}1&0\\0&1\end{bmatrix} \\ \hline \frac{1}{\sqrt{2}}\begin{bmatrix}1&0\\0&1\end{bmatrix} & \frac{-1}{\sqrt{2}}\begin{bmatrix}1&0\\0&1\end{bmatrix} \end{array}\right] = \frac{1}{\sqrt{2}}\begin{bmatrix} 1&0&1&0 \\ 0&1&0&1 \\ 1&0&-1&0 \\ 0&1&0&-1 \end{bmatrix}$$

$$= \left[\begin{array}{c|c} \frac{1}{\sqrt{2}} I & \frac{1}{\sqrt{2}} I \\ \hline \frac{1}{\sqrt{2}} I & -\frac{1}{\sqrt{2}} I \end{array}\right]$$

$$= \frac{1}{\sqrt{2}}\left[\begin{array}{c|c} I & I \\ \hline I & -I \end{array}\right]$$

* Encontramos el estado final $(t=4)$

$$|\varphi_4\rangle = \underbrace{\frac{1}{\sqrt{2}}\left[\begin{array}{c|c} I & I \\ \hline I & -I \end{array}\right]}_{\text{matriz}} \cdot \underbrace{\frac{1}{2}\left[\begin{array}{c} X^{f(0)}\begin{bmatrix}1\\-1\end{bmatrix} \\ \hline X^{f(1)}\begin{bmatrix}1\\-1\end{bmatrix}\end{array}\right]}_{\text{vector}}$$

$$= \frac{1}{2\sqrt{2}}\left[\begin{array}{c|c} I & I \\ \hline I & -I \end{array}\right]\left[\begin{array}{c} X^{f(0)}\begin{bmatrix}1\\-1\end{bmatrix} \\ \hline X^{f(1)}\begin{bmatrix}1\\-1\end{bmatrix}\end{array}\right]$$

$$= \frac{1}{2\sqrt{2}}\left[\begin{array}{c} I\cdot X^{f(0)}\begin{bmatrix}1\\-1\end{bmatrix} + I\cdot X^{f(1)}\begin{bmatrix}1\\-1\end{bmatrix} \\ \hline I\cdot X^{f(0)}\begin{bmatrix}1\\-1\end{bmatrix} - I X^{f(1)}\begin{bmatrix}1\\-1\end{bmatrix}\end{array}\right]$$

$$= \frac{1}{2\sqrt{2}}\left[\begin{array}{c} X^{f(0)}\begin{bmatrix}1\\-1\end{bmatrix} + X^{f(1)}\begin{bmatrix}1\\-1\end{bmatrix} \\ \hline X^{f(0)}\begin{bmatrix}1\\-1\end{bmatrix} - X^{f(1)}\begin{bmatrix}1\\-1\end{bmatrix}\end{array}\right]$$

* Analizamos el vector de estado final

$$|\varphi_{final}\rangle = \frac{1}{2\sqrt{2}} \begin{bmatrix} X^{f(0)}\begin{bmatrix}1\\-1\end{bmatrix} + X^{f(1)}\begin{bmatrix}1\\-1\end{bmatrix} \\ \hdashline X^{f(0)}\begin{bmatrix}1\\-1\end{bmatrix} - X^{f(1)}\begin{bmatrix}1\\-1\end{bmatrix} \end{bmatrix}$$

$X\begin{bmatrix}1\\-1\end{bmatrix}$

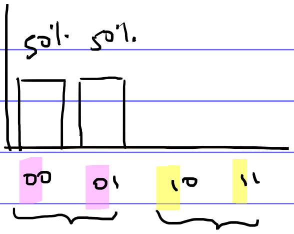
$\begin{bmatrix}0&1\\1&0\end{bmatrix}\cdot\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}-1\\1\end{bmatrix}$

* Si la función f es constante: $X^{\overset{0}{f(0)}} = X^{\overset{0}{f(1)}} = I$

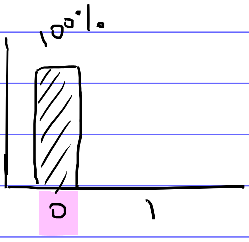
- Si f es la cte. 0:

$$|\varphi_{final}\rangle = \frac{1}{2\sqrt{2}} \begin{bmatrix} \begin{bmatrix}1\\-1\end{bmatrix} + \begin{bmatrix}1\\-1\end{bmatrix} \\ \begin{bmatrix}1\\-1\end{bmatrix} - \begin{bmatrix}1\\-1\end{bmatrix} \end{bmatrix} = \frac{1}{\cancel{2}\sqrt{2}}\begin{bmatrix}\cancel{2}\\-\cancel{2}\\0\\0\end{bmatrix}$$

$$|\varphi_{final}\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\\0\\0\end{bmatrix} = \begin{array}{c|c} \begin{bmatrix}1/\sqrt{2}\\-1/\sqrt{2}\\0\\0\end{bmatrix} & \begin{array}{c}00\\01\\10\\11\end{array}\end{array}$$




$\xrightarrow{\text{colapso con respecto}}$ al primer qubit (medí)

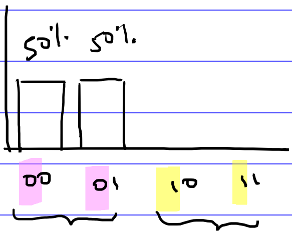


$$X^{f(0)} = X^1 = X \qquad X^{f(1)} = X^1 = X$$

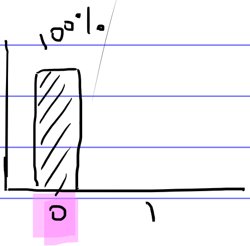
• Si f es la cte. 1:

$$|\varphi_{final}\rangle = \frac{1}{2\sqrt{2}}\begin{bmatrix}\begin{bmatrix}-1\\1\end{bmatrix}+\begin{bmatrix}-1\\1\end{bmatrix}\\[2ex] \begin{bmatrix}-1\\1\end{bmatrix}-\begin{bmatrix}-1\\1\end{bmatrix}\end{bmatrix} = \frac{1}{\cancel{2}\sqrt{2}}\begin{bmatrix}-\cancel{2}\\+\cancel{2}\\0\\0\end{bmatrix}$$

$$|\varphi_{final}\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}-1\\+1\\0\\0\end{bmatrix} = \begin{array}{c} \begin{bmatrix}-1/\sqrt{2}\\+1/\sqrt{2}\\0\\0\end{bmatrix} \end{array}\begin{array}{l}00\\01\\10\\11\end{array}$$




colapso con respecto al primer qubit (medí)



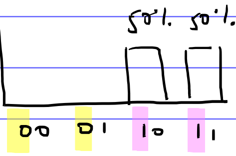

* Si f es balanceada:

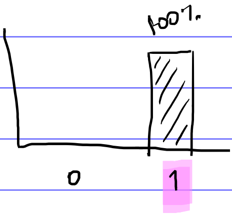
$$|\varphi_{final}\rangle = \frac{1}{2\sqrt{2}}\begin{bmatrix}\overbrace{X^{f(0)}}^{I}\begin{bmatrix}1\\-1\end{bmatrix}+\overbrace{X^{f(1)}}^{X}\begin{bmatrix}1\\-1\end{bmatrix}\\[2ex] \underbrace{X^{f(0)}}_{I}\begin{bmatrix}1\\-1\end{bmatrix}-\underbrace{X^{f(1)}}_{X}\begin{bmatrix}1\\-1\end{bmatrix}\end{bmatrix}$$

• Si f es la identidad $f(0)=0$ y $f(1)=1$

$$|\varphi_{final}\rangle = \frac{1}{2\sqrt{2}}\begin{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix}+\begin{bmatrix}-1\\1\end{bmatrix}\\[2ex] \begin{bmatrix}1\\-1\end{bmatrix}-\begin{bmatrix}-1\\1\end{bmatrix}\end{bmatrix} = \frac{1}{\cancel{2}\sqrt{2}}\begin{bmatrix}0\\0\\\cancel{2}\\-\cancel{2}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\0\\1\\-1\end{bmatrix}$$

$$= \begin{bmatrix}0\\0\\1/\sqrt{2}\\-1/\sqrt{2}\end{bmatrix}\begin{array}{l}00\\01\\10\\11\end{array}$$



De la misma forma podemos ver que si $f$ es la función $f(0)=1$, $f(1)=0$ el circuito nos dará que el qubit de arriba colapsa al estado 1 con 100% de prob.

**Conclusión:** $f:\{0,1\} \to \{0,1\}$

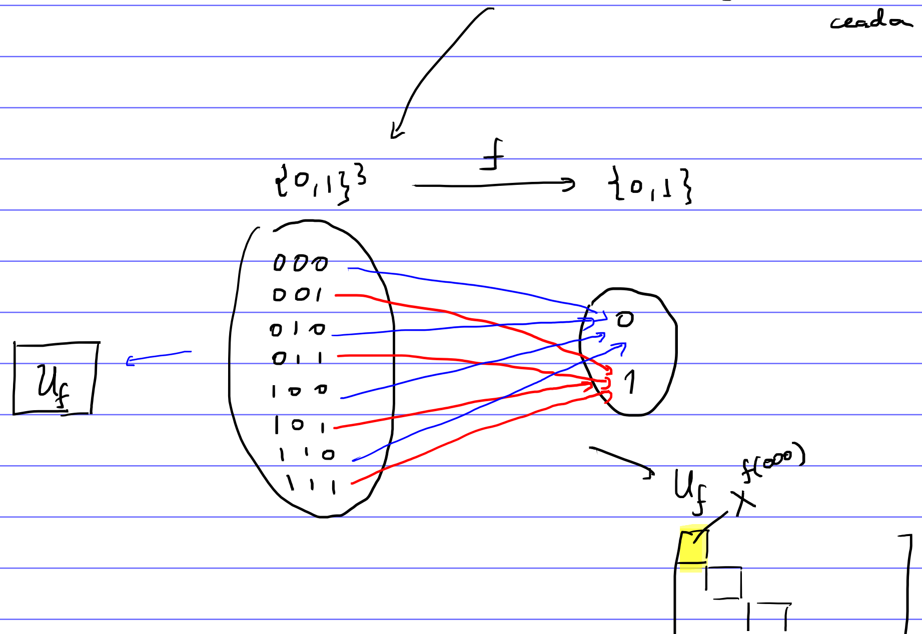
Si el alg. de Deutsch nos da $|0\rangle \to$ cte.

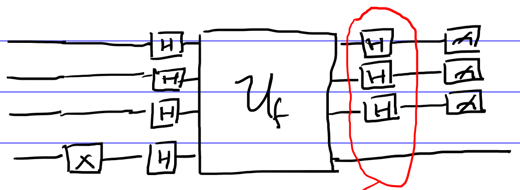
$|1\rangle \to$ balanceada

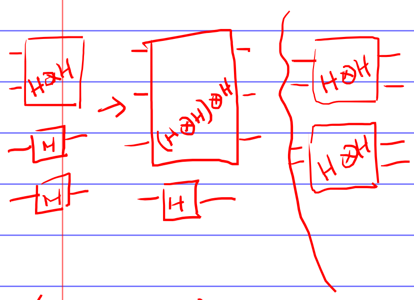
$f:\{0,1\}^n \to \{0,1\}$

Si el alg. de Deutsch-Jozsa da $|\overbrace{00\ldots0}^{n \text{ veces}}\rangle \to$ cte

otro $\to$ balanceada




$(H\otimes I)\otimes(H\otimes H)$

arriba abajo

$(H\otimes H)\otimes(H\otimes I)$

$((H\otimes H)\otimes H)\otimes H$

alg-lin. Multiplicación fila-columna por bloques

Tradicional:

1

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1+0+3+4 \\ 0+2+3-4 \\ 0+2+3+0 \\ 1+0+0-4 \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 5 \\ -3 \end{bmatrix}$$

$4 \times 4$ $\quad$ $4 \times 1$

2 Por bloques

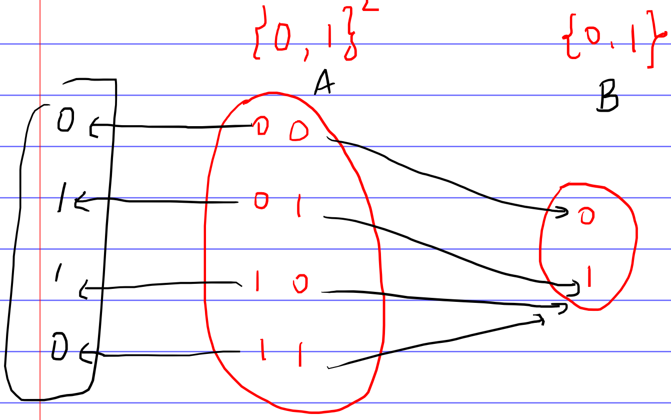
$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ \hline 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \end{array}\right] \left[\begin{array}{c} 1 \\ 2 \\ \hline 3 \\ 4 \end{array}\right]$$

$2 \times 2$ $\quad$ $2 \times 1$

$$\begin{bmatrix} -2 & 3 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} -7 \\ 1 \end{bmatrix} = \begin{bmatrix} (-2)(-7) + (3)(1) \\ (5)(-7) + (4)(1) \end{bmatrix}$$

$$= \begin{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} \\ \\ \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 7 \\ -1 \end{bmatrix} \\ \\ \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ -4 \end{bmatrix} \end{bmatrix} = \left[\begin{array}{c} 8 \\ 1 \\ \hline 5 \\ -3 \end{array}\right]$$




$f: A \to B$

$|B|^{|A|}$

ni cte.

ni balanceada

# funciones en total: $2^4 = 16$

# funciones ctes. $2$

# funciones balanceadas → Reordenamientos de $\underbrace{00}\underbrace{11}$

$$\frac{4!}{2!\,2!} = \frac{24}{4} = 6$$

# ni lo uno ni lo otro: $8$