Nombre:

Mecánica del Continuo (GEOC2057)

Quiz 1 - 14/02/2017

The number of points attributed to each question is mentioned beside them. The total number of points is 10. *El número de puntos asignado a cada pregunta aparece debajo de cada una de las mismas. El número máximo total de puntos es 10.*

Stress

1. Briefly explain the difference between *Force*, *Traction*, *Stress* and the *Stress tensor*. **2 pts** *Explicar brevemente la differencia entre Fuerza, Traccíon, Esfuerzo y tTensor de esfuerzos.*

Correction:
Force: interaction acting through the entire body (body forces) or on a surface (surface forces). (to the corrector: be easy on this definition) **0.5 pts**
Traction: force per unit area on a surface with a specific orientation **0.5 pts**
Stress: a pair of equal and opposite tractions acting across a surface with a specific orientation **0.5 pts**
Stress tensor: 2nd order tensor defining the state of stress on three mutually orthogonal planes at a given point. (to the corrector: be easy on this definition) **0.5 pts**

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2. In 2D, the normal stress on a plane P is (see figure 1): *En 2D, el esfuerzo normal en un plano P es (ver figura 1):*

$$\sigma_n = \sigma_x \cos^2 \alpha + \tau_{xy} \sin 2\alpha + \sigma_y \sin^2 \alpha$$

And the shear stress on a plane P is: *Y el esfuerzo de corte en un plano P es:*

$$\tau = (\sigma_x - \sigma_y) \sin \alpha \cos \alpha - \tau_{xy}(\cos^2 \alpha - \sin^2 \alpha)$$

   (a) Which stress directions correspond to $\tau = 0$ (shear stresses equal 0)? **1 pt** *Cuáles son las direcciónes de esfuerzos que corresponden a $\tau = 0$ (esfuerzos de corte igual a 0).*

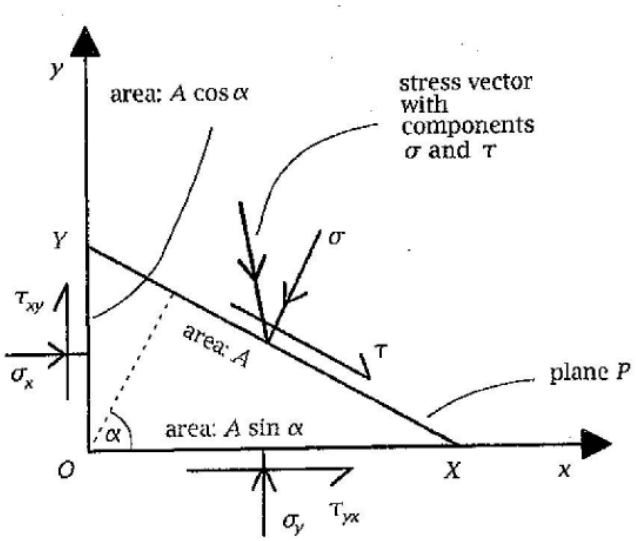

Figure 1: Stress components acting on a plane P whose normal is at an angle α from the x axis. *Componentes de esfuerzos en un plano P del que la normal hace un angulo α con el eje x.*

Correction:
Stress directions corresponding to shear stresses equal 0 are the principal stress directions. **1 pt**

(b) Using the equation for shear stress above, show that these planes are at 90° from each others. **1 pt** *Usando la ecuación para el esfuerzo de corte más arriba, muestre que estos planos son a 90° entre si.*

Correction:
All correct mathematical demonstrations are accepted **1 pt**, here are different examples:
1) Using

$$\tau = (\sigma_x - \sigma_y)\sin\alpha\cos\alpha - \tau_{xy}(\cos^2\alpha - \sin^2\alpha)$$

In the case of principal stresses, we have shear stresses=0 so,

$$\tau = (\sigma_x - \sigma_y)\sin\alpha\cos\alpha - \tau_{xy}(\cos^2\alpha - \sin^2\alpha) = 0$$

$$\frac{\tau_{xy}}{\sigma_x - \sigma_y} = \frac{\tan 2\alpha}{2}$$

The solution of this equation is $\alpha = \alpha_0$ and since $\tan 2\alpha = \tan(2\alpha + 180) = \tan(2(\alpha + 90))$, so the two solutions are α_0 and $\alpha_0 + 90$.

2) Using the formula for shear stress when shear stresses = 0:

$$\tau = (\sigma_x - \sigma_y) \sin\alpha \cos\alpha = 0$$

We see that this equation is = 0 for $\sin\alpha = 0$ or $\cos\alpha = 0$ which corresponds to $\alpha = 0, 90, 180, 360$, corresponding to 2 directions at 90° from each other.
3) Using the formula of shear stress in the case of principal stresses:

$$\tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\alpha = 0$$

which happens for $2\alpha = 0, 180$ and $\alpha = 0, 90$, for 2 planes at 90° from each other.

(c) With this stress tensor: *Con este tensor de esfuerzos:*

$$\sigma = \begin{pmatrix} 20 & 2 \\ 2 & -15 \end{pmatrix}$$

Calculate the normal stress σ_n and the shear stress τ for a plane P with a normal at an angle of 45° from the x direction. **2 pts** *Calcular el esfuerzo normal σ_n y el esfuerzo de corte τ sobre un plano P con una normal haciendo un angulo de 45° con el eje x.*

Correction:

$$\sigma_n = (\sigma_x \cos(\alpha)^2) + (\tau_{xy} \sin(2\alpha)) + (\sigma_y \sin(\alpha)^2)$$
$$\sigma_n = (20 \cos(45)^2) + (2 \sin(90)) + (-15 \sin(45)^2) = 4.5$$

1 pt

$$\tau = ((\sigma_x - \sigma_y) \sin(\alpha) cos(\alpha)) - (\tau_{xy}(\cos(\alpha)^2 - \sin(\alpha)^2))$$
$$\tau = ((20 + 15) \sin(45) \cos(45)) - (2(\cos(45)^2 - \sin(45)^2)) = 17.5$$

1 pt

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3. With this stress tensor given in terms of principal stresses: *Con este tensor de esfuerzo dado en término de los esfuerzos principales:*

$$\sigma = \begin{pmatrix} 15 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

(a) To what stress state is this stress tensor corresponding to? **1 pt** *A cual caso de esfuerzo corresponde este tensor de esfuerzos?*

Correction:
Axial or confined compression **1 pt**

(b) Draw the Mohr circle corresponding to this stress tensor. **1 pt** *Dibuje el circulo de Mohr que corresponde a este tensor de esfuerzos.*

Correction:
See figure 2. **1 pt**

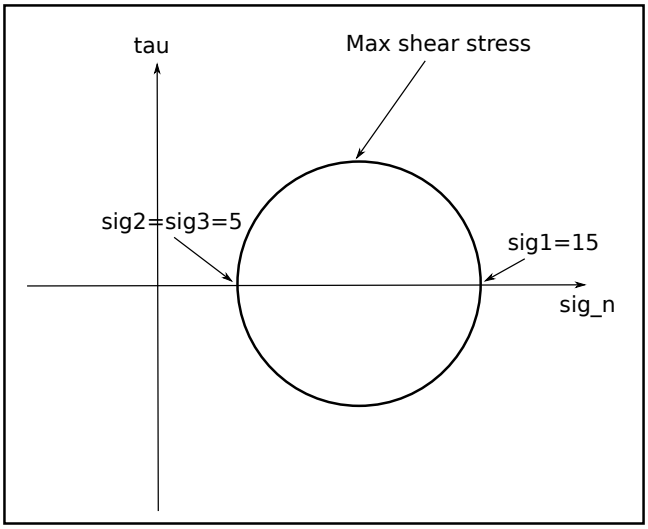

Figure 2: Mohr circle for question 3b

(c) Give the equations of maximum shear stress and mean stress, and give their values for the stress tensor given above. **2 pts** *Dé las ecuaciónes de esfuerzo de corte máximo y esfuerzo medio, y dé los valores correspondiendo al tensor de esfuerzos dado más arriba.*

Correction:
Maximum shear stress:

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{15 - 5}{2} = 5$$

1 pt (0.5 pts for equation and 0.5 pts for value)

Mean stress:

$$\sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = 8.3$$

1 pt (0.5 pts for equation and 0.5 pts for value)