Nombre: ______________________________

Mecánica del Continuo (GEOC2057)

Partial 1 - 23/02/2017

The number of points attributed to each question is mentioned beside them. The total number of points is 17. *El número de puntos asignado a cada pregunta aparece debajo de cada una de las mismas. El número máximo total de puntos es 17.*

Stress 6.5 pts

1. In the following we have a stress tensor given in terms of principal stresses in MPa. First find the maximum shear stress corresponding to this stress state, and then draw the Mohr circle corresponding to it in the grid below. Be as precise as possible. **2 pts** *A continuación tenemos un tensor de esfuerzos dado en término de los esfuerzos principales en MPa. Primero, dé el esfuerzo de corte maximo correspondiendo a este estado de esfuerzo, y despues dibuje el circulo de Mohr correspondiendo a este estado en la grilla abajo.*

$$\sigma = \begin{bmatrix} 50 & 0 \\ 0 & -10 \end{bmatrix}$$

Correction:
The maximum shear stress is:
$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = 30$ MPa. (0.5 pts for equation and 0.5 pts for results).
See figure 1 for drawing (1 pt). Remove 0.5 pts if the axes have no labels (τ, σ_n) or the positions of σ_1, σ_3 are no indicated.

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2. Considering a fault plane at an angle of -50° from the σ_1 direction, draw it on the Mohr circle. Then give (read on the Mohr circle or calculated) the values of normal and shear stresses acting on this plane. **1.5 pts** *Tenemos un plano de falla haciendo un angulo de -50° con la dirección de σ_1, dibujelo en el circulo de Mohr. Despues, dé (leido en el circulo de Mohr o calculado) los valores de los esfuerzos normal y de corte sobre este plano de falla.*

Correction:
See figure 1 for the drawing (0.5 pts).
The values for normal σ_n and shear τ stresses are:

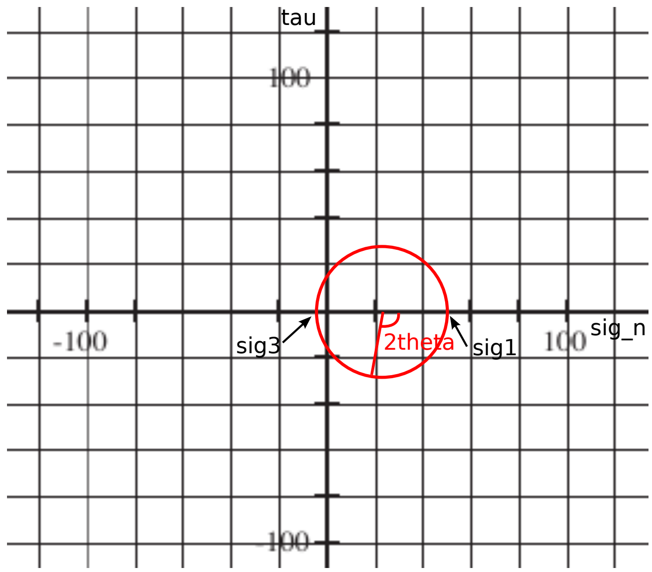

Figure 1: Grid for questions 1 and 2

$\sigma_n = \frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cos 2\theta = 14.8$ MPa (0.5 pts for value, values +/- 3 MPa accepted).
$\tau = \frac{\sigma_1-\sigma_3}{2}\sin 2\theta = -29.5$ MPa (0.5 pts for value, values +/- 3 MPa accepted).

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3. Draw the configuration given by the stress tensor, the fault plane and its normal and shear stresses in real space. **2 pts** *Dibuje la configuración dado por el tensor de esfuerzos, el plano de falla y sus esfuerzos normal y de corte en el espacio real.*

Correction:
See figure 2 for the drawing (0.5 pts for position of normal vector to the right of σ_1, 0.5 pts for position of σ_3 and fault plane, 0.5 pts for position of normal stress σ_n vector, 0.5 pts for shear stress τ vector).

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4. Assuming we measured this stress tensor in a continental area, to what depth is this stress tensor approximately corresponding to? **1 pt** *Suponiendo que medimos este tensor de esfuerzos en una área continental, a cual profundidad corresponde este tensor approxidamente?*

Correction:
In order to know to which depth this tensor might correspond to we need to calculate the lithostatic stress of this tensor. This corresponds to the

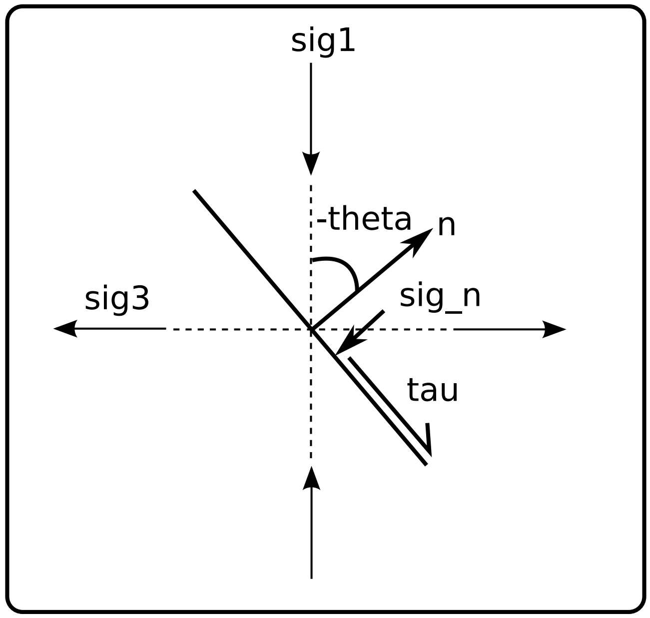

Figure 2: Drawing for question 3.

mean stress given by:
$\sigma_m = \frac{\sigma_1+\sigma_3}{2} = 20$ MPa (0.25 pts).
This stress can be equated to the equation of the lithostatic stress to obtain the depth:
$P_{litho} = \rho g h$, with ρ the density in kg/m3, g the gravity acceleration in m/s2, and h the depth in m (0.25 pts).
So we get: $h = \frac{P_{litho}}{\rho g} = \frac{20\times10^6}{2700\times9.81} = 755$ m (0.5 pts). We here use the average density of continents of approx 2700 kg/m3. Densities in the range 2600-2900 are OK.

Strain 10.5 pts

1. Define what are pure shear and simple shear deformations. **2 pts** *Define que es una deformación en pure shear y en simple shear.*

Correction:
Pure shear: flattening of a body obtained by shortening along one direction and elongating along 1 or 2 other perpendicular directions. It involves no rotation and no translation (1 pt).
Simple shear: displacement along a single direction. Simple shear involves both rotational and shear (strain) components but no translation (1 pt).

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2. Give the equation for shear strains as a function of the shear angle. **1 pt** *Dé la ecuación de la deformación de corte en función del angulo de corte.*

Correction:
Shear strain γ as a function of the shear angle Φ:
$\gamma = \tan \Phi$.

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3. Finite strain Starting with a unit square, it is then elongated by 100 % vertically and shortened by 50 % horizontally. Calculate the elongation ε, the stretch S, and the quadratic elongation λ along these two axes. **1.5 pts** *Deformación finita Empezando con un cuadrado unitario, este cuadrado esta deformado con una elongación de 100 % en el vertical y un acortamiento de 50 % en el horizontal. Calcule la elongación ε, el estiramiento S, y la elongación cuadrática λ según esos dos ejes.*

Correction:
Vertically:
Elongation $\varepsilon_1 = \frac{finallength - originallength}{originallength} = \frac{2-1}{1} = 1$ (0.25 pts).
Stretch $S_1 = \frac{finallength}{originallength} = \frac{2}{1} = 2$ (0.25 pts).
Quadratic elongation $\lambda_1 = S^2 = 4$ (0.25 pts).
Horizontally:
Elongation $\varepsilon_2 = \frac{0.5-1}{1} = -0.5$ (0.25 pts).
Stretch $S_2 = \frac{0.5}{1} = 0.5$ (0.25 pts).
Quadratic elongation $\lambda_2 = S^2 = 0.25$ (0.25 pts).

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4. Draw the Mohr circle for finite strain using the results of the previous question in the following grid. **1 pt** *Dibuje el circulo de Mohr para la deformación finita usando los resultados de la pregunta anterior en la grilla abajo.*

Correction:
To draw the Mohr circle for finite strain we need to calculate $1/\lambda$ in both directions which are the principal strain directions:
$\lambda'_1 = 0.25$ and $\lambda'_2 = 4$ (0.25 pts)
See figure 3 for the drawing (0.75 pts). Remove 0.5 pts if the axes have no labels (λ', γ') or the positions of λ'_1, λ'_2 are no indicated.

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5. Indicate where are the shear strains and the volumetric strains in the following strain tensor: **1 pt** *Indica donde se encuentran las deformaciónes*

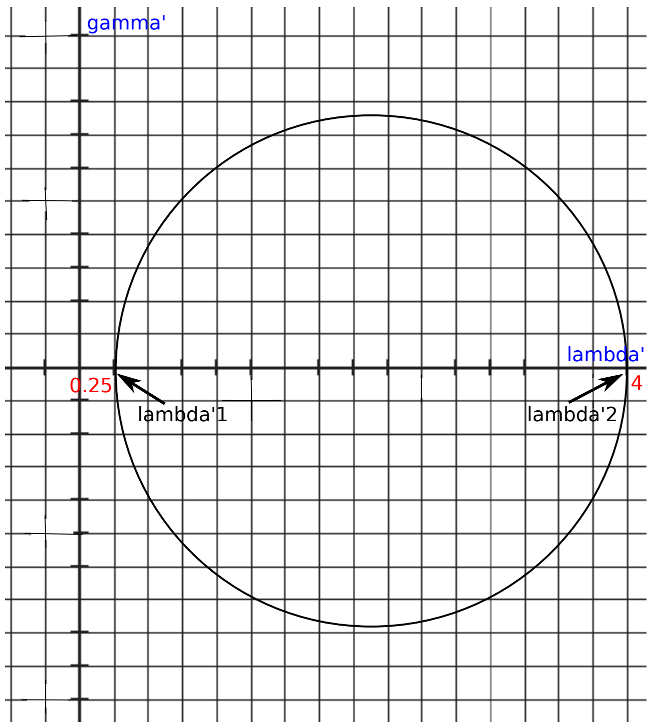

Figure 3: Grid for question 4.

de volumen y de corte en el tensor de deformación abajo

$$\epsilon_{ij} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) & \frac{\partial w}{\partial z} \end{bmatrix}$$

Correction:
Volume strains are corresponding to the diagonal elements of the strain tensor (0.5 pts).
Shear strains are corresponding to the off-diagonal elements of the strain tensor (0.5 pts).

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6. Infinitesimal strain Starting with a unit cube, we have a shortening of 1 % along X, an elongation of 0.5 % along Y and Z, and shear strains parallel to Z (negative shear strain from X axis in the Z axis direction) of -2 %. Give, as a function of the undeformed coordinates (X, Y, Z), the set of equations for the deformed coordinates (x, y, z), the

set of equations for the displacements (u, v, w), and the strain tensor. **4 pts** *Infinitesimal deformación Empezando con un cuadrado unitario, este cuadrado esta deformado con un acortamiento de 1 % según el eje X, una elongación de 0.5 % según los ejes Y y Z, y una deformación de corte paralela al eje Z de -2 % (deformación de corte negativa desde el eje X en la dirección Z). Dé, en función de las coordenadas no deformadas (X, Y, Z), las ecuaciónes de las coordenadas deformadas (x, y, z), las ecuaciónes de los desplazamientos (u, v, w), y el tensor de deformación.*

Correction:

Deformed coordinates (0.5 pts per equation line):

$$\begin{aligned} x &= 0.99X + 0Y + 0Z \\ y &= 0X + 1.005Y + 0Z \\ z &= -0.02Z + 0Y + 1.005Z \end{aligned}$$

Displacement equations (0.5 pts per equation line):
u is in the x direction, v is in the y direction and w is in the z direction.

$$\begin{aligned} u &= -0.01X + 0Y + 0Z \\ v &= 0X + 0.005Y + 0Z \\ w &= -0.02Z + 0Y + 0.005Z \end{aligned}$$

Strain tensor (1 pt):

$$\epsilon_{ij} = \begin{bmatrix} -0.01 & 0 & -0.01 \\ 0 & 0.005 & 0 \\ -0.01 & 0 & 0.005 \end{bmatrix}$$
