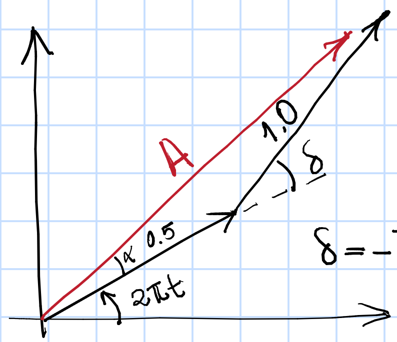
Solución Parcial #1.

1. $y_1 = \text{Sen}(2\pi t + \pi/4) = \text{Cos}(2\pi t + \pi/4 - \pi/2) = \text{Cos}(2\pi t - \pi/4)$

$y_2 = 0.5\,\text{Cos}(2\pi t)$



$\delta = -\pi/4$

- $A^2 = (0.5)^2 + (1.0)^2 - 2(0.5)(1.0)\,\text{Cos}(180 - \delta)$

$\boxed{A = 1.4}$

- $\dfrac{\text{Sen}\,\alpha}{1.0} = \dfrac{\text{Sen}(180-\delta)}{A} \Rightarrow \alpha = \text{Sen}^{-1}\left(\dfrac{\text{Sen}(180+45)}{1.4}\right)$

$\boxed{\alpha = -0.53 \text{ rad} = -30.3^\circ}$

- No es una pulsación, solo existe una frecuencia.

(BONO +1)

• Si los movimientos fueran perpendiculares digamos

$$x = \text{Sen}(2\pi t + \pi/4) = \text{Sen}(2\pi t)\,\text{Cos}(\pi/4) + \text{Sen}(\pi/4)\,\text{Cos}(2\pi t)$$
$$= \frac{1}{\sqrt{2}}\left(\text{Sen}(2\pi t) + \text{Cos}(2\pi t)\right)$$

$$y = \frac{1}{2}\,\text{Cos}(2\pi t)$$

– Elevando al cuadrado ambas funciones tenemos

$$x^2 = \frac{1}{2}\left(\text{sen}^2(2\pi t) + 2\,\text{Sen}(2\pi t)\,\text{Cos}(2\pi t) + \text{Cos}^2(2\pi t)\right)$$
$$= \frac{1}{2}\left(1 + \text{Sen}(4\pi t)\right) =$$

$$y^2 = \frac{1}{4}\,\text{Cos}^2(2\pi t)$$

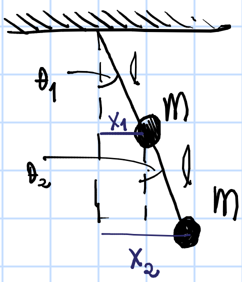
– Multiplicando las funciones $xy = \frac{1}{2\sqrt{2}}\left(\text{Cos}(2\pi t)\,\text{Sen}(2\pi t) + \text{Cos}^2(2\pi t)\right)$

y sumando $x^2 - 2\sqrt{2}\,xy + 4y^2 = \frac{1}{2}\left(1 + \text{Sen}(4\pi t)\right) - \left(\frac{1}{2}\,\text{Sen}(4\pi t) + \text{Cos}^2(2\pi t)\right) + \text{Cos}^2(2\pi t)$

$$= \frac{1}{2}$$

es decir que se tiene $\boxed{2x^2 - 4\sqrt{2}\,xy + 8y^2 = 1}$ lo que representa una elipse rotada.

2.



$$x_1 = l\,\text{Sen}\theta_1$$
$$y_1 = l - l\cos\theta_1$$

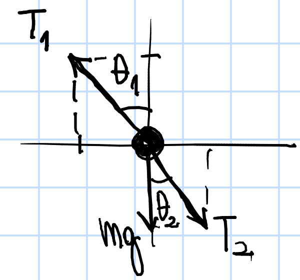
$$x_2 = l\,\text{Sen}\theta_1 + l\,\text{Sen}\theta_2$$
$$y_2 = (l - l\cos\theta_1) + (l - l\,\text{Cos}\theta_2)$$

Para θ's pequeños $\Longrightarrow$

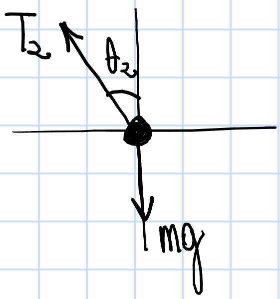
$$x_1 \approx l\theta_1 \longrightarrow \ddot{x}_1 = l\ddot{\theta}_1$$
$$y_1 \approx 0$$

$$x_2 \approx l(\theta_1 + \theta_2) \rightarrow \ddot{x}_2 = l(\ddot{\theta}_1 + \ddot{\theta}_2)$$
$$y_2 \approx 0$$

Por otro lado, para las fuerzas que actuan sobre cada masa



$$\Sigma F_x = -T_1 \operatorname{Sen}\theta_1 + T_2 \operatorname{Sen}\theta_2 = m\ddot{x}_1$$
$$\Sigma F_y = T_1 \operatorname{Cos}\theta_1 - T_2 \operatorname{Cos}\theta_2 - mg \approx 0$$



$$\Sigma F_x = -T_2 \operatorname{Sen}\theta_2 = m\ddot{x}_2$$
$$\Sigma F_y = T_2 \operatorname{Cos}\theta_2 - mg \approx 0$$

Para ángulos pequeños se tiene:

$$-T_1\theta_1 + T_2\theta_2 = m\ddot{x}_1 \qquad ; \qquad -T_2\theta_2 = m\ddot{x}_2$$
$$T_1 - T_2 \approx mg \qquad\qquad T_2 \approx mg$$

y las ecuaciones quedarian

$$-2mg\theta_1 + mg\theta_2 = ml\ddot{\theta}_1$$
$$-mg\theta_2 = ml(\ddot{\theta}_1 + \ddot{\theta}_2) = -2mg\theta_1 + mg\theta_2 + ml\ddot{\theta}_2$$

luego
$$\ddot{\theta}_1 = -2g/l\,\theta_1 + g/l\,\theta_2 = g/l\,(-2\theta_1 + \theta_2)$$
$$\ddot{\theta}_2 = +2g/l\,\theta_1 - 2g/l\,\theta_2 = 2g/l\,(\theta_1 - \theta_2)$$

b) Reescribimos matricialmente

$$\frac{d^2}{dt^2}\begin{pmatrix}\theta_1\\ \theta_2\end{pmatrix} + \begin{pmatrix}2a & -a\\ -2a & 2a\end{pmatrix}\begin{pmatrix}\theta_1\\ \theta_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \quad ; \quad a = g/l$$

$$\begin{vmatrix}2a-\omega & -a\\ -2a & 2a-\omega\end{vmatrix} = (2a-\omega)^2 - 2a^2 = 0 \Rightarrow (2a-\omega) = \pm\sqrt{2}\,a$$

$$\omega_+ = (2+\sqrt{2})a$$
$$\omega_- = (2-\sqrt{2})a$$

y calculamos los vectores propios:

$$\begin{pmatrix} 2a - a(2+\sqrt{2}) & -a \\ -2a & 2a - a(2+\sqrt{2}) \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 1 \\ -\sqrt{2} \end{pmatrix}$$

$$-\sqrt{2}\cancel{a}\, u_1 - \cancel{a} u_2 = 0 \rightarrow u_2 = -\sqrt{2}\, u_1$$

$$\begin{pmatrix} 2a - a(2-\sqrt{2}) & -a \\ -2a & 2a - a(2-\sqrt{2}) \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\vec{V} = \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix}$$

$$\sqrt{2}\, a V_1 - a V_2 = 0 \rightarrow V_2 = \sqrt{2}\, V_1$$

las soluciones para los modos normales son $q_+(t) = A_+ \cos(\omega_+ t + \delta_+)$ y $q_-(t) = A_- \cos(\omega_- t + \delta_-)$ y para las partículas:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -\sqrt{2} & \sqrt{2} \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

entonces se tiene

$$x_1 = A_+ \cos(\omega_+ t + \delta_+) + A_- \cos(\omega_- t + \delta_-)$$

$$x_2 = -\sqrt{2} A_+ \cos(\omega_+ t + \delta_+) + \sqrt{2} A_- \cos(\omega_- t + \delta_-)$$

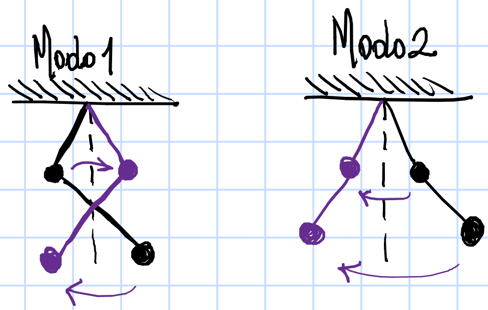
c) $\omega_+ = (2+\sqrt{2})a$

$\omega_- = (2-\sqrt{2})a$

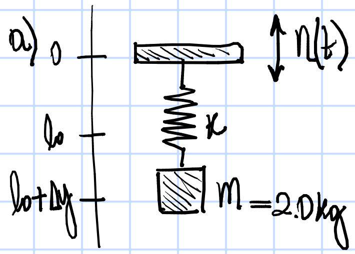
$$\frac{\omega_+}{\omega_-} = \frac{2+\sqrt{2}}{2-\sqrt{2}} \cdot \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{(2+\sqrt{2})^2}{4-2} = \frac{4+4\sqrt{2}+2}{2}$$

$$= 3 + 2\sqrt{2}$$

d) Los modos fundamentales serían



3. a)



Inicialmente $k\Delta y - mg = 0 \Rightarrow k = \dfrac{mg}{\Delta y} = \dfrac{(2.0\,kg)(9.8\,m/s^2)}{2.5\times 10^{-2}\,m}$

luego

$$k = 784\ N/m \quad ; \quad \omega_0 = 14\sqrt{2} = 19.8\ rad/s$$

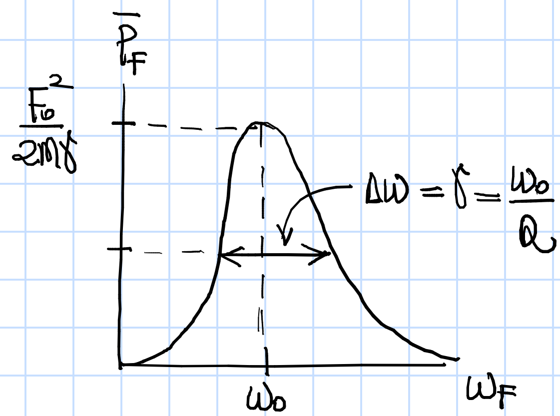
pero por otro lado $Q = \dfrac{\omega_0}{\gamma} \longrightarrow \gamma = \dfrac{\omega_0}{Q} = \dfrac{\sqrt{k/m}}{Q} = \dfrac{14}{15}\sqrt{2} = 1.32\,(m/s)^{-1}$

de otra forma $\omega_1 = \sqrt{\omega_0^2 - (\gamma/2)^2} = \omega_0\sqrt{1 - \dfrac{1}{4Q^2}} = 19.8\,s^{-1} = \dfrac{2\pi}{T}$

es decir $T = 0.317\,s$

b) La potencia absorbida

$$\overline{P_F(\omega_F)} = \frac{F_0^2}{2m\gamma}\cdot\frac{(\gamma\omega_F)^2}{(\omega_0^2-\omega_F^2)^2+(\gamma\omega_F)^2}$$



* El máximo:

$$\overline{P}_F^{\,máx} = \frac{F_0^2}{2m\gamma}\cdot\frac{\omega_0}{\omega_0} = \frac{F_0^2 Q}{2m\omega_0} = 0.19\ Watts$$

* El ancho: $\Delta\omega = \dfrac{\omega_0}{Q} = \dfrac{14\sqrt{2}}{15} = 1.32\ (kg\,m/s)^{-1}$

c) $E(t) = E_0 e^{-\gamma t}$ $\qquad E(t^*) = E_0 e^{-5} = E_0 e^{-\frac{\omega_0}{Q}t^*} \Rightarrow 5 = \dfrac{\omega_0}{Q}t^*$

luego $t^* = \dfrac{5Q}{\omega_0} = \dfrac{5(15)}{14\sqrt{2}} = 3.78\,s$