Homework #8: Exact Cover Problems

October 12, 2017

1 Regular Exact Cover problem

Use backtracking to solve the following exact cover problem, that is: **is there a set of rows containing exactly one 1 in each column?**

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
r_1	0	1	1	1	0	0	1
r_2	0	1	0	1	1	1	0
r_3	0	0	1	0	0	1	1
r_4	1	1	0	1	1	0	1
r_5	0	0	0	0	1	0	0
r_6	1	0	0	0	0	0	0
r_7	1	1	0	1	0	0	0

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
r_1	0	1	1	1	0	0	1
r_2	0	1	0	1	1	1	0
r_3	0	0	1	0	0	1	1
r_4	1	1	0	1	1	0	1
r_5	0	0	0	0	1	0	0
r_6	1	0	0	0	0	0	0
r_7	1	1	0	1	0	0	0

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
r_1	0	1	1	1	0	0	1
r_2	0	1	0	1	1	1	0
r_3	0	0	1	0	0	1	1
r_4	1	1	0	1	1	0	1
r_5	0	0	0	0	1	0	0
r_6	1	0	0	0	0	0	0
r_7	1	1	0	1	0	0	0

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
r_1	0	1	1	1	0	0	1
r_2	0	1	0	1	1	1	0
r_3	0	0	1	0	0	1	1
r_4	1	1	0	1	1	0	1
r_5	0	0	0	0	1	0	0
r_6	1	0	0	0	0	0	0
r_7	1	1	0	1	0	0	0

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
r_1	0	1	1	1	0	0	1
r_2	0	1	0	1	1	1	0
r_3	0	0	1	0	0	1	1
r_4	1	1	0	1	1	0	1
r_5	0	0	0	0	1	0	0
r_6	1	0	0	0	0	0	0
r_7	1	1	0	1	0	0	0

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
r_1	0	1	1	1	0	0	1
r_2	0	1	0	1	1	1	0
r_3	0	0	1	0	0	1	1
r_4	1	1	0	1	1	0	1
r_5	0	0	0	0	1	0	0
r_6	1	0	0	0	0	0	0
r_7	1	1	0	1	0	0	0

2 Generalized Exact Cover problem

Solve this generalized exact cover problem, that is: **Is there a set of rows containing *exactly one* 1 in each primary column (c_1 through c_5), and *at most one* 1 in each secondary column (c_6 and c_7)?**

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
r_1	0	0	1	0	0	1	0
r_2	0	0	0	1	1	0	0
r_3	0	1	0	1	0	0	0
r_4	1	0	1	0	0	0	1
r_5	0	1	0	0	1	1	0
r_6	0	0	0	1	1	0	1
r_7	1	0	1	1	0	0	1

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
r_1	0	0	1	0	0	1	0
r_2	0	0	0	1	1	0	0
r_3	0	1	0	1	0	0	0
r_4	1	0	1	0	0	0	1
r_5	0	1	0	0	1	1	0
r_6	0	0	0	1	1	0	1
r_7	1	0	1	1	0	0	1

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
r_1	0	0	1	0	0	1	0
r_2	0	0	0	1	1	0	0
r_3	0	1	0	1	0	0	0
r_4	1	0	1	0	0	0	1
r_5	0	1	0	0	1	1	0
r_6	0	0	0	1	1	0	1
r_7	1	0	1	1	0	0	1

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
r_1	0	0	1	0	0	1	0
r_2	0	0	0	1	1	0	0
r_3	0	1	0	1	0	0	0
r_4	1	0	1	0	0	0	1
r_5	0	1	0	0	1	1	0
r_6	0	0	0	1	1	0	1
r_7	1	0	1	1	0	0	1

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
r_1	0	0	1	0	0	1	0
r_2	0	0	0	1	1	0	0
r_3	0	1	0	1	0	0	0
r_4	1	0	1	0	0	0	1
r_5	0	1	0	0	1	1	0
r_6	0	0	0	1	1	0	1
r_7	1	0	1	1	0	0	1

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
r_1	0	0	1	0	0	1	0
r_2	0	0	0	1	1	0	0
r_3	0	1	0	1	0	0	0
r_4	1	0	1	0	0	0	1
r_5	0	1	0	0	1	1	0
r_6	0	0	0	1	1	0	1
r_7	1	0	1	1	0	0	1

3 A generalized exact cover problem: 8-Queen Problem

1. Encode the 8-Queen Problem into a matrix of 0s and 1s suitable for a generalized exact cover solution:

- how many rows?

- how many primary columns?

- how many secondary columns?

2. Show that the selection of rows that is a solution to the exact cover problem for your matrix is also a solution for the 8-Queen Problem.