```
Clear[n, m, x1, x2, x3, x4, x5, x6, x7, x8,
 data1, data2, data3, data4, data5, data6, data7, data8]

Mu2[n_] := DirichletConvolve[MoebiusMu[x1], MoebiusMu[x1], x1, n]

Mu3[n_] := DirichletConvolve[MoebiusMu[x2], Mu2[x2], x2, n]

Mu4[n_] := DirichletConvolve[MoebiusMu[x3], Mu3[x3], x3, n]

LogC2[n_] := DirichletConvolve[Log[x6], Log[x6], x6, n]

LogC3[n_] := DirichletConvolve[Log[x7], LogC2[x7], x7, n]

Feng3[n_] := Sum[If[d0 * d1 * d2 * d3 == n, 1, 0] * MoebiusMu[d0] MangoldtLambda[d1]
    MangoldtLambda[d2] MangoldtLambda[d3], {d0, 1, n}, {d1, 1, n}, {d2, 1, n}, {d3, 1, n}]

data1 = Table[(MoebiusMu[n])^2 * DirichletConvolve[Mu2[m], (Log[m]), m, n], {n, 1, 1000}];

data2 = Table[DirichletConvolve[Mu2[m], (Log[m]), m, n], {n, 1, 1000}];

ListPlot[{data1, data2}, Filling → None,
 PlotStyle → {Directive[Yellow, PointSize[Large]], Directive[Blue]}]
```
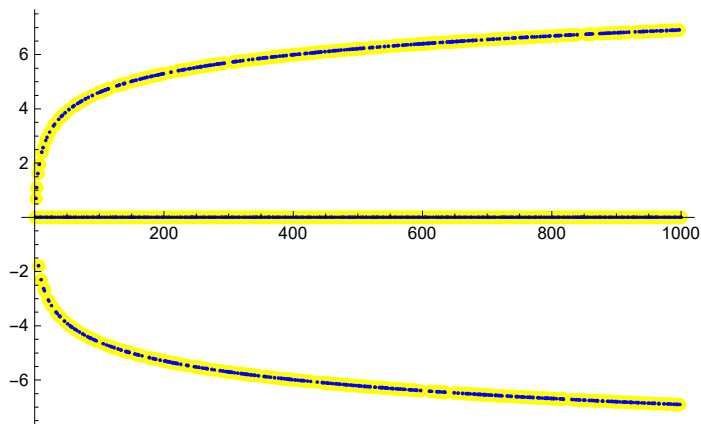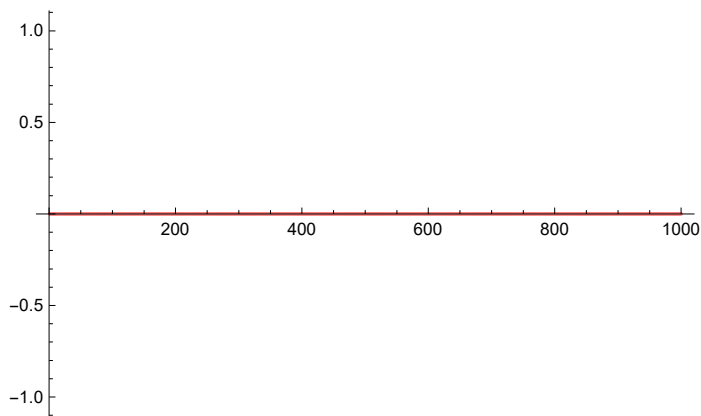


```
ListPlot[{data2 - data1}, Filling → Axis, PlotStyle → {Directive[Pink]}]
```
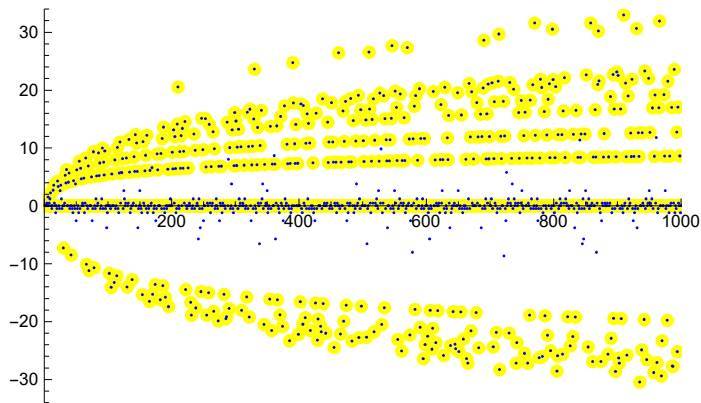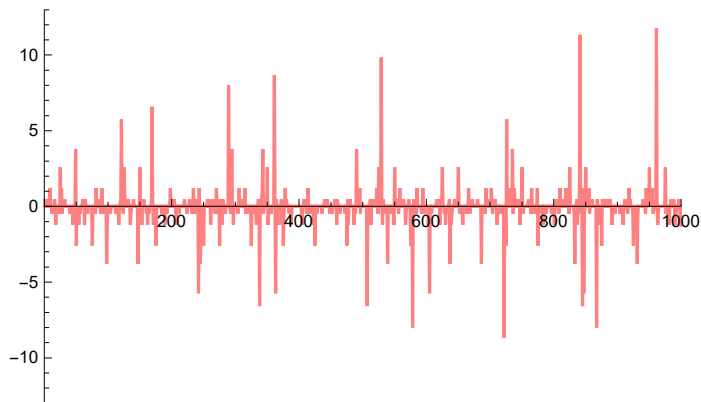


```
data3 = Table[(MoebiusMu[n])^2 * DirichletConvolve[Mu3[m], LogC2[m], m, n], {n, 1, 1000}];

data4 = Table[DirichletConvolve[Mu3[m], LogC2[m], m, n], {n, 1, 1000}];
```

```
ListPlot[{data3, data4}, Filling → None,
 PlotStyle → {Directive[Yellow, PointSize[Large]], Directive[Blue]},
 PlotRange → {{0, 1000}, {-34, 34}}]
```



```
ListLinePlot[{data4 - data3},
 PlotStyle → {Directive[Pink]}, PlotRange → {{0, 1000}, {-13, 13}}]
```



```
Clear[X, Y, f1, f2, f3, f4, a, b, n1, n2]

X[n1_] := DirichletConvolve[f1[a], f2[a], a, n1]

Y[n2_] := DirichletConvolve[X[b], f3[b], b, n2]

Z[n3_] := DirichletConvolve[Y[c], f4[c], c, n3]

Z[100] // Expand

Z[8]

f1[a] := MoebiusMu[a]

f2[a] := MangoldtLambda[a]

f3[b] := MangoldtLambda[b]

f4[c] := MangoldtLambda[c]

Table[Z[n], {n, 1, 10}]
```

$\{0, 0, 0, 0, 0, 0, 0, \text{Log}[2]^3, 0, 0\}$

```
Table[Z[n], {n, 11, 20}]
```

$\{0, 3 \text{Log}[2]^2 \text{Log}[3], 0, 0, 0, 2 \text{Log}[2]^3, 0, 3 \text{Log}[2] \text{Log}[3]^2, 0, 3 \text{Log}[2]^2 \text{Log}[5]\}$

```
Table[Z[n], {n, 21, 30}]
```

$\{0, 0, 0, 3 \text{Log}[2]^2 \text{Log}[3] - \text{Log}[2] (-\text{Log}[2] \text{Log}[3] + \text{Log}[2] \text{Log}[6]),$
$0, 0, \text{Log}[3]^3, 3 \text{Log}[2]^2 \text{Log}[7], 0, 6 \text{Log}[2] \text{Log}[3] \text{Log}[5]\}$

```
Table[Z[n], {n, 31, 40}]
```

$\{0, 3 \text{Log}[2]^3, 0, 0, 0, \text{Log}[2]^2 \text{Log}[3] + \text{Log}[2] \text{Log}[3]^2 -$
  $\text{Log}[3] (-\text{Log}[2] \text{Log}[3] + \text{Log}[2] \text{Log}[6]) - \text{Log}[2] (-\text{Log}[2] \text{Log}[3] + \text{Log}[3] \text{Log}[6]),$
$0, 0, 0, 3 \text{Log}[2]^2 \text{Log}[5] - \text{Log}[2] (-\text{Log}[2] \text{Log}[5] + \text{Log}[2] \text{Log}[10])\}$

```
Table[Z[n], {n, 41, 50}]
```

$\{0, 6 \text{Log}[2] \text{Log}[3] \text{Log}[7], 0, 3 \text{Log}[2]^2 \text{Log}[11], 3 \text{Log}[3]^2 \text{Log}[5], 0, 0,$
 $3 \text{Log}[2]^2 \text{Log}[3] - 2 \text{Log}[2] (-\text{Log}[2] \text{Log}[3] + \text{Log}[2] \text{Log}[6]), 0, 3 \text{Log}[2] \text{Log}[5]^2\}$
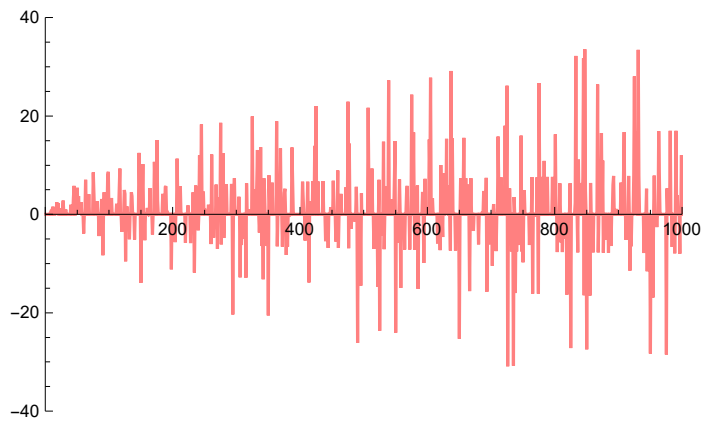
```
data5 = Table[Z[n], {n, 1, 1000}];
```

```
data6 = Table[MoebiusMu[n]^2 Z[n], {n, 1, 1000}];
```

```
ListPlot[{data6, data5}, Filling → None,
 PlotStyle → {Directive[Yellow, PointSize[Large]], Directive[Blue]},
 PlotRange → {{0, 1000}, {-100, 80}}]
```
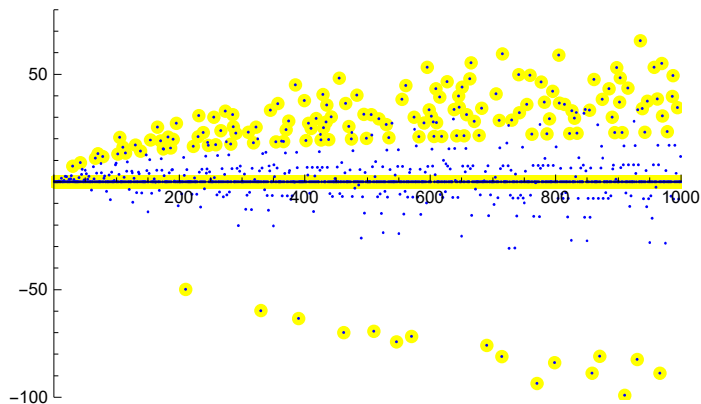
```
ListLinePlot[{data5 - data6},
 PlotStyle → {Directive[Pink]}, PlotRange → {{0, 1000}, {-40, 40}}]
```



```
Lambda2[n_] := DirichletConvolve[MangoldtLambda[m], MangoldtLambda[m], m, n];

F[n_] := DirichletConvolve[MoebiusMu[a], Lambda2[a], a, n]

G[n_] := (MoebiusMu[n])² F[n];
```
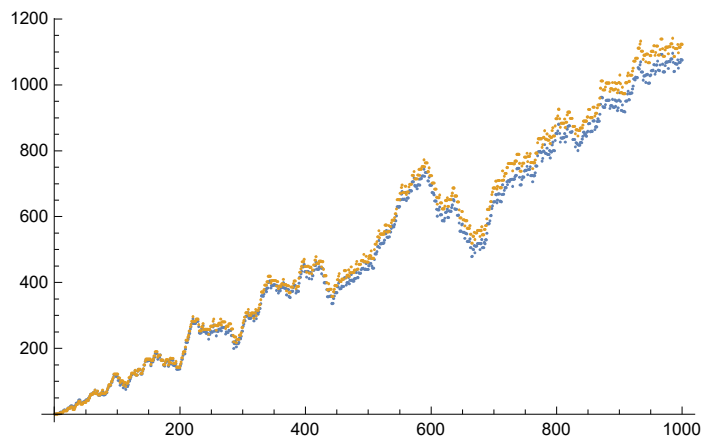
$$\text{data00} = \text{Table}\left[\sum_{n=1}^{x} F[n], \{x, 1, 1000\}\right];$$

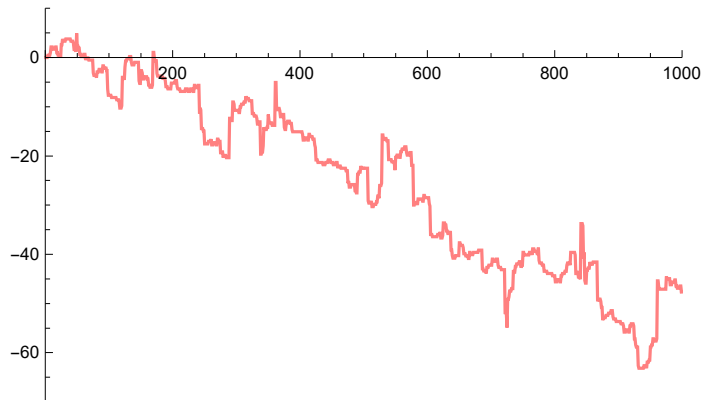$$\text{data01} = \text{Table}\left[\sum_{n=1}^{x} G[n], \{x, 1, 1000\}\right];$$

```
data00
```

```
data01
```
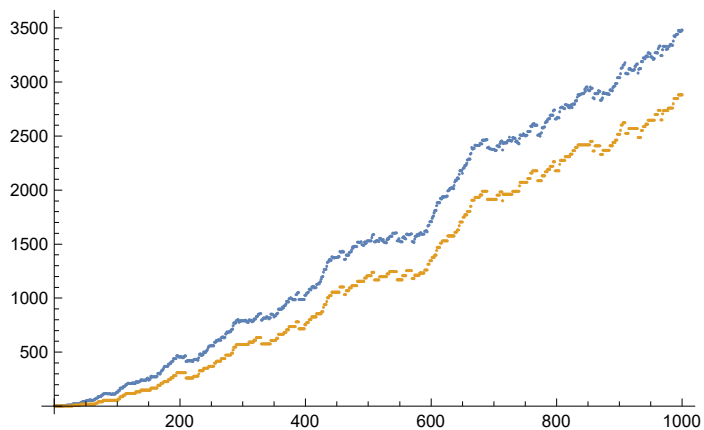
```
ListPlot[{data00, data01}, Filling → None]
```

```
ListLinePlot[{data00 - data01}, Filling → None,
 PlotStyle → {Directive[Pink]}, PlotRange → {{0, 1000}, {-70, 10}}]
```



```
data7 = Table[∑_{n=1}^{x} Z[n], {x, 1, 1000}];
```

$$\text{data7} = \text{Table}\left[\sum_{n=1}^{x} Z[n], \{x, 1, 1000\}\right];$$

$$\text{data8} = \text{Table}\left[\sum_{n=1}^{x} \text{If}\left[\text{MoebiusMu}[n] == 0, 0, (\text{MoebiusMu}[n])^2 Z[n]\right], \{x, 1, 1000\}\right];$$

```
ListPlot[{data7, data8}, Filling → None]
```

```
ListLinePlot[{data7 - data8}, Filling → None,
 PlotStyle → {Directive[Pink]}, PlotRange → {{0, 1000}, {0, 700}}]
```