```
Clear[aux, t, x, a1, a2, a3, a4, a5, b1, b2, c1, c2, q1,
  q2, q3, q4, q5, V1, V12, V22, u, x1, x2, y2, y3, z2, z3, m, n, in];


R = 1.3036; θ0 = 4 / 7; θ1 = 4/7;

G0[aux_, t_, in_, k_] = -(-t)^(k-1)/(k-2)! Integrate[e^(-aux*u*t*θ1) u^(k-2) (1-u)^in t^in, {u, 0, 1}];

G1[aux_, t_, in_, k_] =

  Which[k == 2, -2*t^in, k == 3, -3(-t)^(3-2)/(3-3)! Integrate[e^(-aux*u*t*θ1) u^(3-3) (1-u)^in t^in, {u, 0, 1}]];

G2[aux_, t_, in_, k_] = Which[k == 2, aux*θ1*t^in + in*t^(in-1), k == 3, -3*t^in];

G3[aux_, t_, in_, k_] = Which[k == 2, 0, k == 3, aux*θ1*t^in + in*t^(in-1)];

I2[aux_, t_, in_, k_] = Which[k == 0, G0[aux, t, in, 2], k == 1,
   G1[aux, t, in, 2], k == 2, G2[aux, t, in, 2], k == 3, G3[aux, t, in, 2]];

I3[aux_, t_, in_, k_] = Which[k == 0, G0[aux, t, in, 3], k == 1,
   G1[aux, t, in, 3], k == 2, G2[aux, t, in, 3], k == 3, G3[aux, t, in, 3]];

P1[aux_, t_, in_] = aux*θ0*t^in + in*t^(in-1);

PC = Compile[{{m, _Integer}, {n, _Integer}},
   Sum[m!*n!/(l!*(m-l)!*(n-l)!*(m+n)!), {l, 0, Min[m, n]}]];

P00[x_] = x + p02 x (1-x) + p03 x (1-x)^2 + p04 x (1-x)^3 + p05 x (1-x)^4;

P01[x_] = p11 x + p12 x^2 + p13 x^3;

P02[x_] = p21 x + p22 x^2 + p23 x^3;

Q[x_] = q0 + q1 (1-2x) + q2 (1-2x)^3 + (1-q0-q1-q2) (1-2x)^5;

{p1[0], p1[1], p1[2], p1[3], p1[4], p1[5]} = CoefficientList[P00[x], x];

{p2[0], p2[1], p2[2], p2[3]} = CoefficientList[P01[x], x];

{p3[0], p3[1], p3[2], p3[3]} = CoefficientList[P02[x], x];

{q[0], q[1], q[2], q[3], q[4], q[5]} = CoefficientList[Q[x], x];
```

```
i = 1; cc = 0;
While[i < 6 , j = 1;
  While[j < 6, k = 0;
   While[k < 6, l = 0;
    While[l < 6, concon[a_, b_, x_] = P1[a, 1 - x, i] * P1[b, 1 - x, j];
                 concons[a_, b_, x_] = (concon[a, b, x] - e^(-a-b) concon[-b, -a, x])/(a + b);

                 conconq := Derivative[l, k, 0][concons];
                 cc = cc + p1[i] * p1[j] * q[k] * q[l] * (-1)^(1+k)/Θ0 *
        Re[NIntegrate[conconq[-R, -R, x], {x, 0, 1}, Method → {"GlobalAdaptive",
            "MaxErrorIncreases" → 10000, Method → "GaussKronrodRule"}]];
     l = l + 1];
    k = k + 1];
   j = j + 1];
  i = i + 1];
TimeUsed[]
8.389

Clear[i, j, k, l, r, cf2];
```

```
i = 1; cf2 = 0;
While[i < 6, j = 1;
 While[j < 4, k = 0;
  While[k < 6, l = 0;
   While[l < 6, r = 0;
    While[r < 3,
     confen02[a_, b_, x_] = x^r * P1[a, 1 - x Θ1/Θ0, i] * I2[b, 1 - x, j, r];
     confens02[a_, b_, x_] = 1/(a + b) (confen02[a, b, x] - ℯ^(-a-b) confen02[-b, -a, x]);
     confenq02 := Derivative[l, k, 0][confens02];
     cf2 = cf2 + p1[i] * p2[j] * q[k] * q[l] * (-1)^(1+k)/(r! * Θ0) *
       Re[NIntegrate[confenq02[-R, -R, x], {x, 0, 1}, Method → {"GlobalAdaptive",
         "MaxErrorIncreases" → 10000, Method → "GaussKronrodRule"}]];
     r++];
    l = l + 1];
   k = k + 1];
  j = j + 1];
 i = i + 1]
TimeUsed[]
19.92
```

```
Clear[i, j, k, l, r, cf3];
i = 1; cf3 = 0;
While[i < 6, j = 1;
 While[j < 4, k = 0;
  While[k < 6, l = 0;
   While[l < 6, r = 0;
    While[r < 4,
     confen03[a_, b_, x_] = x^r * P1[a, 1 - x θ1/θ0, i] * I3[b, 1 - x, j, r];
     confens03[a_, b_, x_] = 1/(a + b) (confen03[a, b, x] - e^(-a-b) confen03[-b, -a, x]);
     confenq03 := Derivative[l, k, 0][confens03];
     cf3 = cf3 + p1[i] * p3[j] * q[k] * q[l] * (-1)^(1+k)/(r! * θ0) *
       Re[NIntegrate[confenq03[-R, -R, x], {x, 0, 1}, Method → {"GlobalAdaptive",
         "MaxErrorIncreases" → 10000, Method → "GaussKronrodRule"}]];
     r++];
    l = l + 1];
   k = k + 1];
  j = j + 1];
 i = i + 1]
TimeUsed[]
216.764
```

```
Clear[i, j, k, l, r, s, ff22];
i = 1; ff22 = 0;
While[i < 4, j = 1;
 While[j < 4, k = 0;
  While[k < 6, l = 0;
   While[l < 6, r = 0;
    While[r < 3, s = 0;
     While[s < 3,
                fenfen22[a_, b_, x_] = x^(r+s) * I2[a, 1 - x, i, s] * I2[b, 1 - x, j, r];

       fenfens22[a_, b_, x_] = 1/(a + b) (fenfen22[a, b, x] - e^(-a-b) fenfen22[-b, -a, x]);
                fenfenq22 := Derivative[l, k, 0][fenfens22];
                ff22 = ff22 + p2[i] * p2[j] * q[k] * q[l] * (-1)^(1+k)/Θ1 * PC[r, s] *
          Re[NIntegrate[fenfenq22[-R, -R, x], {x, 0, 1}, Method → {"GlobalAdaptive",
              "MaxErrorIncreases" → 10000, Method → "GaussKronrodRule"}]];
      s++];
     r++];
    l = l + 1];
   k = k + 1];
  j = j + 1];
 i = i + 1]
TimeUsed[]
265.264
```

```
Clear[i, j, k, l, r, s, ff23];
i = 1; ff23 = 0;
While[i < 4, j = 1;
 While[j < 4, k = 0;
  While[k < 6, l = 0;
   While[l < 6, r = 0;
    While[r < 4, s = 0;
     While[s < 3,
                fenfen23[a_, b_, x_] = x^(r+s) * I2[a, 1 - x, i, s] * I3[b, 1 - x, j, r];

      fenfens23[a_, b_, x_] = 1/(a + b) (fenfen23[a, b, x] - e^(-a-b) fenfen23[-b, -a, x]);
                fenfenq23 := Derivative[l, k, 0][fenfens23];
                ff23 = ff23 + p2[i] * p3[j] * q[k] * q[l] * (-1)^(1+k)/Θ1 * PC[r, s] *
          Re[NIntegrate[fenfenq23[-R, -R, x], {x, 0, 1}, Method → {"GlobalAdaptive",
              "MaxErrorIncreases" → 10000, Method → "GaussKronrodRule"}]];
      s++];
     r++];
    l = l + 1];
   k = k + 1];
  j = j + 1];
 i = i + 1]
TimeUsed[]
696.764
```

```
Clear[i, j, k, l, r, s, ff33];
i = 1; ff33 = 0;
While[i < 4, j = 1;
 While[j < 4, k = 0;
  While[k < 6, l = 0;
   While[l < 6, r = 0;
    While[r < 4, s = 0;
     While[s < 4,
                fenfen33[a_, b_, x_] = x^(r+s) * I3[a, 1 - x, i, s] * I3[b, 1 - x, j, r];

      fenfens33[a_, b_, x_] = 1/(a + b) (fenfen33[a, b, x] - e^(-a-b) fenfen33[-b, -a, x]);
                fenfenq33 := Derivative[l, k, 0][fenfens33];
                ff33 = ff33 + p3[i] * p3[j] * q[k] * q[l] * (-1)^(1+k)/Θ1 * PC[r, s] *
          Re[NIntegrate[fenfenq33[-R, -R, x], {x, 0, 1}, Method → {"GlobalAdaptive",
              "MaxErrorIncreases" → 10000, Method → "GaussKronrodRule"}]];
      s++];
     r++];
    l = l + 1];
   k = k + 1];
  j = j + 1];
 i = i + 1]
TimeUsed[]
```

1708.2

```
Minimize[cc + 2 (cf2 + cf3 + ff23) + ff22 + ff33,
 {p02, p03, p04, p05, p11, p12, p13, p21, p22, p23, q0, q1, q2, q3}]
```

{2.13745, {p02 → 0.261077, p03 → -1.07101, p04 → -0.23684, p05 → 0.260234,
  p11 → 1.04827, p12 → 1.31991, p13 → -0.940058, p21 → 0.522812, p22 → -0.68651,
  p23 → -0.0499239, q0 → 0.490464, q1 → 0.636852, q2 → -0.159327, q3 → -0.0452241}}

$$1 - \frac{\text{Log}[2.1374544061177065`]}{R}$$

0.417294

```
1 - 0.49104466722644263 - 0.6323866423049164` - (-0.15444126926636118`)
```

0.03101

```
P00[x_] = x + 0.26107655428766874` x (1 - x) - 1.0710070584688371` x (1 - x)^2 -
   0.2368403810100388` x (1 - x)^3 + 0.26023389454818024` x (1 - x)^4 ;
P01[x_] = 1.0482749626606975` x + 1.3199126585889513` x^2 - 0.9400584473859237` x^3;
P02[x_] = 0.5228119879990683` x - 0.6865103565474571` x^2 - 0.0499239413756441` x^3;
Q[x_] = 0.4904641213705433` + 0.6368518935202375` (1 - 2 x) +
   -0.15932727726618862` (1 - 2 x)^3 + 0.032011262375407856` (1 - 2 x)^5;
```

```
P00[1]
```

```
1.
```

```
P00[0]
```

```
0.
```

```
Q[0]
```

```
1.
```

```
FullSimplify[Q[x] + Q[1 - x]]
```

```
0.980928
```

```
Solve[0.9679887376245921` + A == 1]
```

```
{{A → 0.0320113}}
```

```
Q[1]
```

```
0.0581636
```

```
0.41729396200519175`
```