```
Clear[u, a, b, d, S, B, t]
```

$$B[z\_] := 1 + 2\left(\frac{\text{Sin}[\pi z]}{\pi}\right)^2\left(\frac{1}{z} - \sum_{n=1}^{\infty}\frac{1}{(n+z)^2}\right);$$

$$S1[d\_, b\_, z\_] := -\frac{1}{2}\left(B[d\,(-b-z)] + B[d\,(z-b)]\right);$$

$$S2[d\_, b\_, z\_] := \frac{1}{2}\left(B[d\,(b+z)] + B[d\,(b-z)]\right);$$

```
char[b_, x_] := If[-b < x < b, 1, 0];

S1hat[d_, b_, x_] := NIntegrate[S1[d, b, t] Cos[2 π t x], {t, -200, 200}];

S2hat[d_, b_, x_] := NIntegrate[S2[d, b, t] Cos[2 π t x], {t, -200, 200}];
```

$$x[n\_] := \frac{\text{Log}[n]}{2\pi};$$

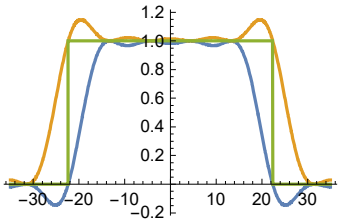
```
id[b_, d_] := If[S1hat[d, b, x[5]] ≥ 0 , 1.0 , -1.0];
```

$$\text{Shat0S1}[b\_, d\_] := 2b - \frac{1}{d};$$

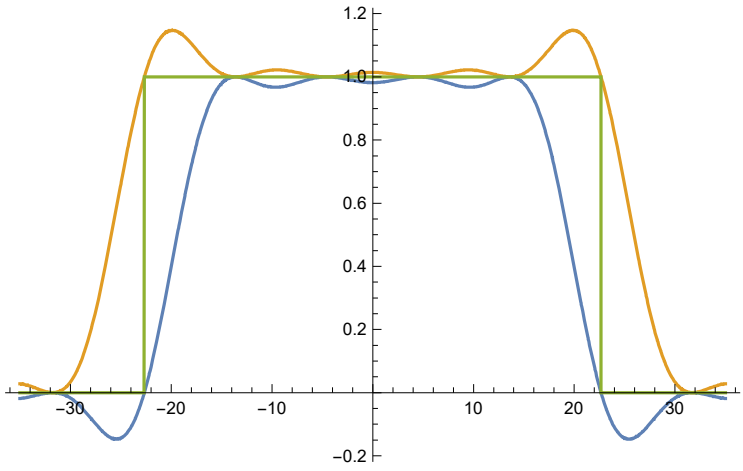
$$\text{Shat0S2}[b\_, d\_] := 2b + \frac{1}{d};$$

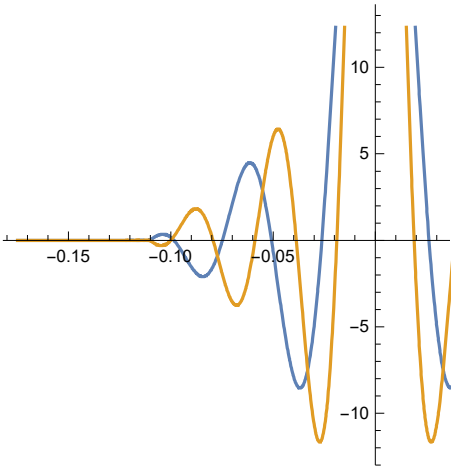
```
digammaintegralS1[b_, d_, x_, y_] :=
  Re[NIntegrate[Gamma'[1/4 + i r/2 + x + y i]/Gamma[1/4 + i r/2 + x + y i] S1[d, b, r], {r, -200, 200}]] -
   Log[π] Shat0S1[b, d];

digammaintegralS2[b_, d_, x_, y_] :=
  Re[NIntegrate[Gamma'[1/4 + i r/2 + x + y i]/Gamma[1/4 + i r/2 + x + y i] S2[d, b, r], {r, -200, 200}]] -
   Log[π] Shat0S2[b, d];
eta[b_, d_] := digammaintegralS1[b, d, 0, 0];



delta = x[2];
beta = 22.36;
```

```
p1 = Plot[{S1[delta, beta, x], S2[delta, beta, x], char[beta, x]}, {x, -35, 35}];
p2 = Plot[{S1hat[delta, beta, x], S2hat[delta, beta, x]},
    {x, -Log[3]/(2 π), Log[3]/(2 π)}, PlotRange → {{-0.15, 0.15}, {-12, 40}}];
GraphicsRow[
 {p1,
  p2}]
$Aborted
```



p2





16 830.3

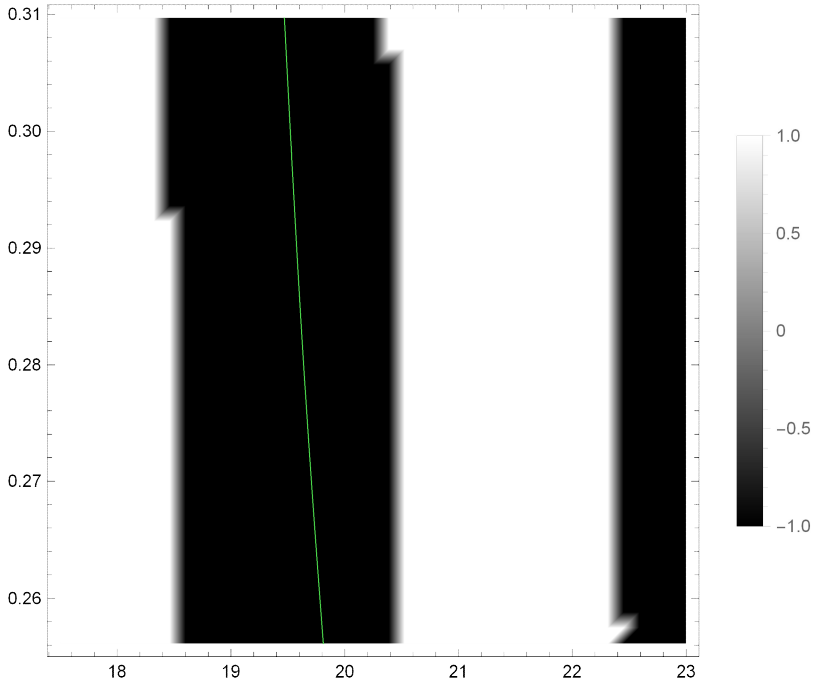
```
x[n_] := Log[n]/(2 π);

id[b_, d_] := If[S1hat[d, b, x[5]] ≥ 0, 1.0, -1.0];

Shat0S1[b_, d_] := 2 b - 1/d;

Shat0S2[b_, d_] := 2 b + 1/d;

digammaintegralS1[b_, d_, x_, y_] :=
  Re[NIntegrate[Gamma'[1/4 + i r/2 + x + y i]/Gamma[1/4 + i r/2 + x + y i] S1[d, b, r], {r, -200, 200}]] -
   Log[π] Shat0S1[b, d];

digammaintegralS2[b_, d_, x_, y_] :=
  Re[NIntegrate[Gamma'[1/4 + i r/2 + x + y i]/Gamma[1/4 + i r/2 + x + y i] S2[d, b, r], {r, -200, 200}]] -
   Log[π] Shat0S2[b, d];

ListDensityPlot[Table[id[b, d], {d, Log[5]/(2 π), Log[7]/(2 π), (Log[7]/(2 π) - Log[5]/(2 π))/40},
  {b, 17.5, 23, (23 - 17.5)/40}], DataRange → {{17.5, 23}, {Log[5]/(2 π), Log[7]/(2 π)}},
 ColorFunction → GrayLevel, PlotLegends → Automatic,
 MeshFunctions → {#1 * #2 - 2.5 &, Function[{x, y, z}, digammaintegralS1[x, y, 0, 0]]},
 MeshStyle → {Lighter[Red], Lighter[Green]}, Mesh → {{0.}}]

TimeUsed[]
```

NIntegrate::ncvb : NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in t near {t} = {195.752}. NIntegrate obtained –0.0026902 and 5.86596964123524`*^-8 for the integral and error estimates. »

NIntegrate::ncvb : NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in t near {t} = {175.976}. NIntegrate obtained 0.05822969734657432` and 5.9188438108225765`*^-8 for the integral and error estimates. »

NIntegrate::ncvb : NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in r near {r} = {-121.778}. NIntegrate obtained 41.019 + 0.0000779768 *i* and 0.000161417136238252` for the integral and error estimates. »

General::stop : Further output of NIntegrate::ncvb will be suppressed during this calculation. »



15 157.1

```
id[20.1, x[7]]
id[20.5, x[7]]
```

```
-1.
```

```
1.
```

```
S1hat[x[7], 20.32, x[5]]
```

```
-0.0568374
```

```
S1hat[x[7], 20.5, x[5]]
```

```
0.106918
```

```
|
```

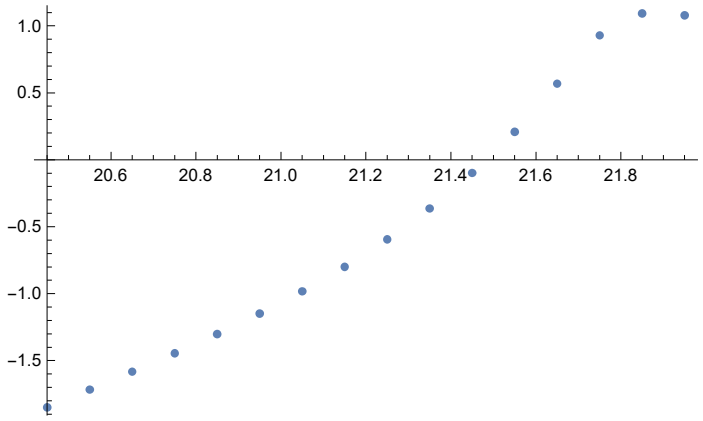
```
eta[b_, d_] := digammaintegralS1[b, d, 0, 0];
```

```
eta[20.0, x[7]]
eta[20.5, x[7]]
```

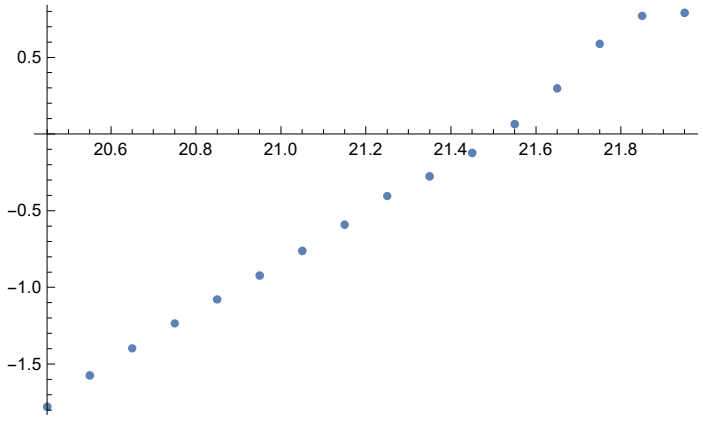
```
1.12694
```

```
2.21808
```

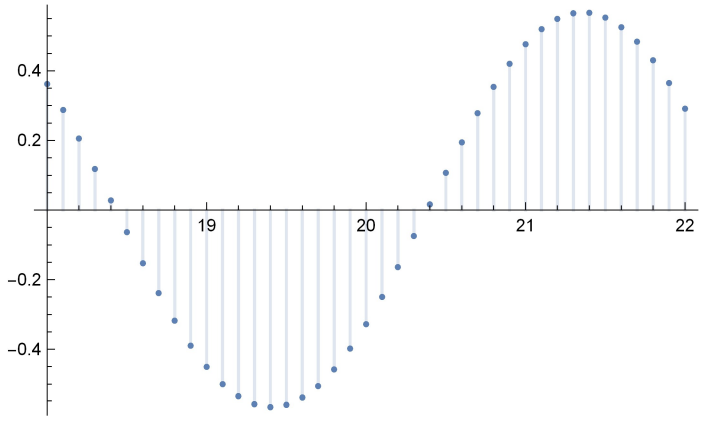
```
Stot[t_] := 1/(S1hat[x[7], 20.0, x[5]] - S1hat[x[7], 20.32, x[5]]
    (-S1hat[x[7], 20.32, x[5]] S1[x[7], 20.5, t] +
      S1hat[x[7], 20.5, x[5]] S1[x[7], 20.32, t]);

Stotfourier[bmin_, b_, t_] :=
  1/(S1hat[x[7], b, x[5]] - S1hat[x[7], bmin, x[5]]) (-S1hat[x[7], bmin, x[5]]
      S1hat[x[7], b, t] + S1hat[x[7], b, x[5]] S1hat[x[7], bmin, t]);

etatot[bmin_, b_] := 1/(S1hat[x[7], b, x[5]] - S1hat[x[7], bmin, x[5]]
    (-S1hat[x[7], bmin, x[5]] eta[b, x[7]] + S1hat[x[7], b, x[5]] eta[bmin, x[7]]);

DiscretePlot[etatot[19.5, b] - Sqrt[2] * Log[2] Abs[Stotfourier[19.5, b, x[2]]] -
  2 * Log[3]/Sqrt[3] Abs[Stotfourier[19.5, b, x[3]]] -
  2 * Log[2]/Sqrt[4] Abs[Stotfourier[19.5, b, x[4]]], {b, 20.45, 22.0, 0.1}, Filling → None]
```



```
DiscretePlot[etatot[19, b] - Sqrt[2] * Log[2] Abs[Stotfourier[19, b, x[2]]] -
   2 * Log[3]/Sqrt[3] Abs[Stotfourier[19, b, x[3]]] - 2 * Log[2]/Sqrt[4] Abs[Stotfourier[19, b, x[4]]],
 {b, 20.45, 22.0, 0.1}, Filling → None]
```



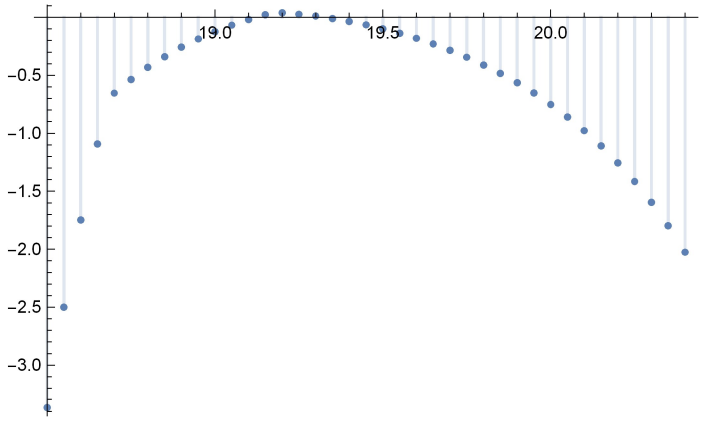

```
value[bmin_, b_] := etatot[bmin, b] - Sqrt[2] * Log[2] Abs[Stotfourier[bmin, b, x[2]]] -
   2 * Log[3]/Sqrt[3] Abs[Stotfourier[bmin, b, x[3]]] -
   2 * Log[2]/Sqrt[4] Abs[Stotfourier[bmin, b, x[4]]];
DiscretePlot[S1hat[x[7], b, x[5]], {b, 18, 22, 0.1}]
```




```
value[19.2, 21.44]
```

```
0.0105457
```

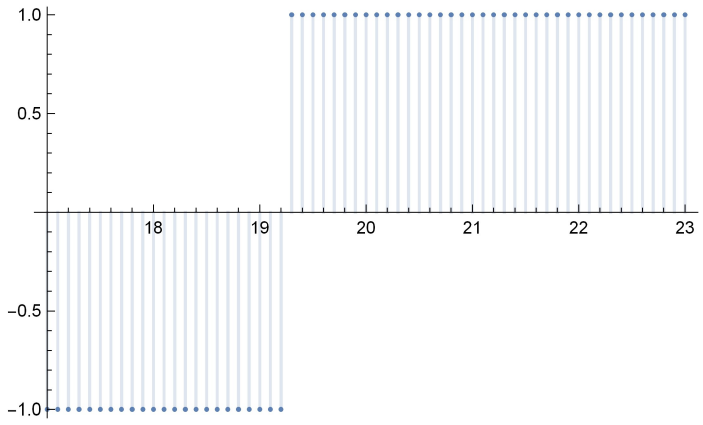
```
DiscretePlot[value[b, 21.45], {b, 18.5, 20.4, 0.05}]
```



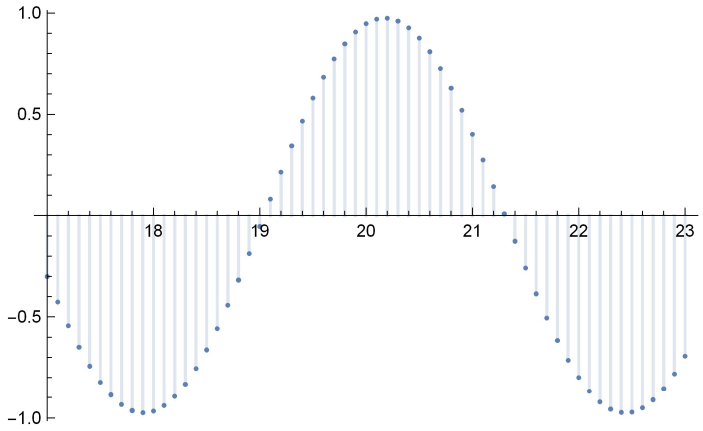
NIntegrate::ncvb : NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in t near {t} = {115.234}. NIntegrate obtained 1.8649690217981135`*^−7 and 2.3079272712208743`*^−11 for the integral and error estimates. ≫

With log(2)/2pi:

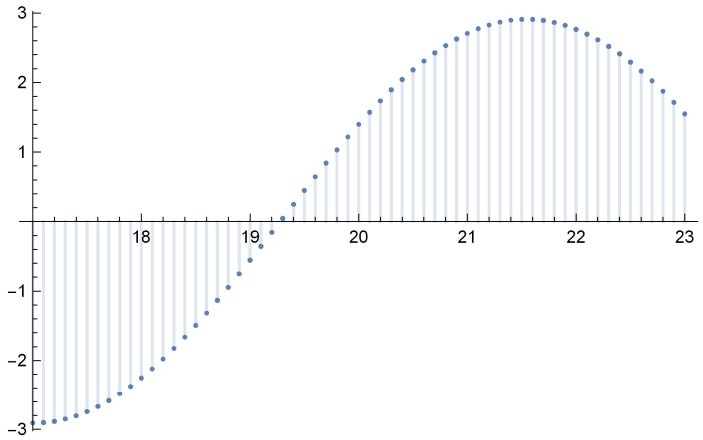
```
id2[b_, d_] := If[S1hat[d, b, x[2]] ≥ 0, 1.0, -1.0];

DiscretePlot[id2[b, x[7]], {b, 17, 23, 0.1}]
```



```
DiscretePlot[S1hat[x[7], b, x[4]], {b, 17, 23, 0.1}]
```

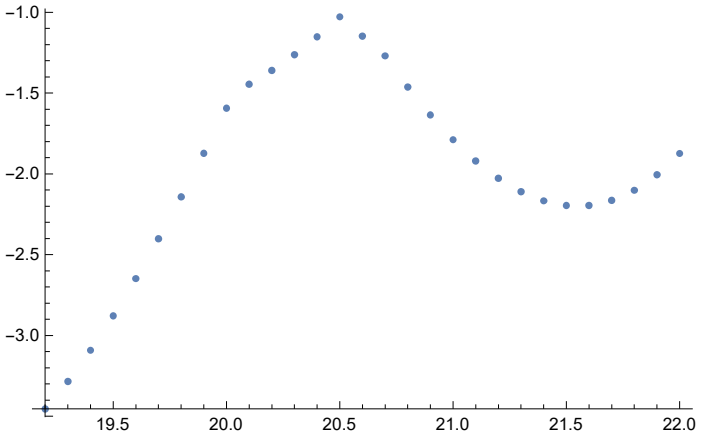



```
DiscretePlot[S1hat[x[7], b, x[2]], {b, 17, 23, 0.1}]
```




```
Stotfourier[bmin_, b_, t_] :=
  1/(S1hat[x[7], b, x[2]] - S1hat[x[7], bmin, x[2]]) (-S1hat[x[7], bmin, x[2]]
      S1hat[x[7], b, t] + S1hat[x[7], b, x[2]] S1hat[x[7], bmin, t]);

etatot[bmin_, b_] := 1/(S1hat[x[7], b, x[2]] - S1hat[x[7], bmin, x[2]])
   (-S1hat[x[7], bmin, x[2]] eta[b, x[7]] + S1hat[x[7], b, x[2]] eta[bmin, x[7]]);
```

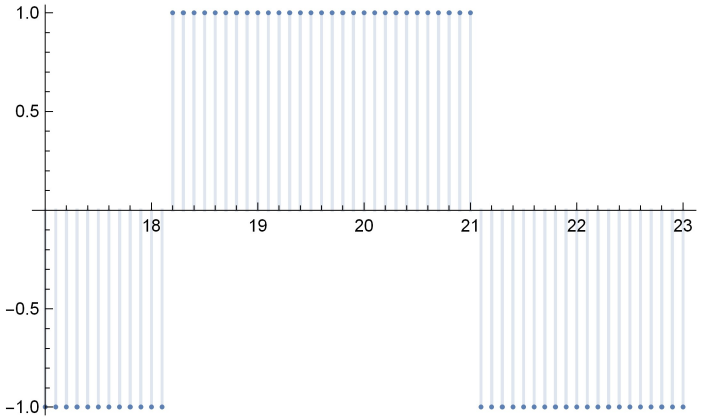
```
DiscretePlot[etatot[17.5, b] - 2 * Log[3]/Sqrt[3] Abs[Stotfourier[17.5, b, x[3]]] -
  2 * Log[2]/Sqrt[4] Abs[Stotfourier[17.5, b, x[4]]] -
  2 * Log[5]/Sqrt[5] Abs[Stotfourier[17.5, b, x[5]]],
 {b, 19.2, 22.0, 0.1}, Filling → None, AxesOrigin → None]
```



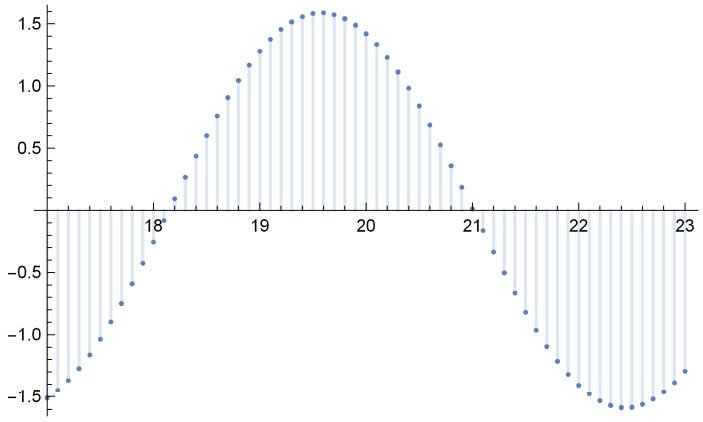
With log(3)/2pi:

```
id3[b_, d_] := If[S1hat[d, b, x[3]] ≥ 0, 1.0, -1.0];
```

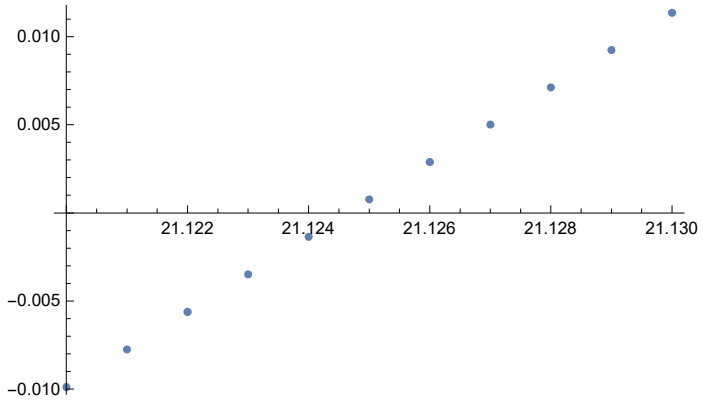
```
DiscretePlot[id3[b, x[7]], {b, 17, 23, 0.1}]
```



```
DiscretePlot[S1hat[x[7], b, x[3]], {b, 17, 23, 0.1}]
```




```
Stotfourier[bmin_, b_, t_] :=
  1/(S1hat[x[7], bmin, x[3]] - S1hat[x[7], b, x[3]]) (S1hat[x[7], bmin, x[3]]
     S1hat[x[7], b, t] - S1hat[x[7], b, x[3]] S1hat[x[7], bmin, t]);

etatot[bmin_, b_] := 1/(S1hat[x[7], bmin, x[3]] - S1hat[x[7], b, x[3]])
   (S1hat[x[7], bmin, x[3]] eta[b, x[7]] - S1hat[x[7], b, x[3]] eta[bmin, x[7]]);

etaplot[bmin_, b_] := etatot[bmin, b] - 2 * Log[2]/Sqrt[2] Abs[Stotfourier[bmin, b, x[2]]] - 2 *
     Log[2]/Sqrt[4] Abs[Stotfourier[bmin, b, x[4]]] - 2 * Log[5]/Sqrt[5] Abs[Stotfourier[bmin, b, x[5]]];

DiscretePlot[etaplot[18.87, b],
 {b, 21.12, 21.13, 0.001}, Filling → None, AxesOrigin → None ]
```

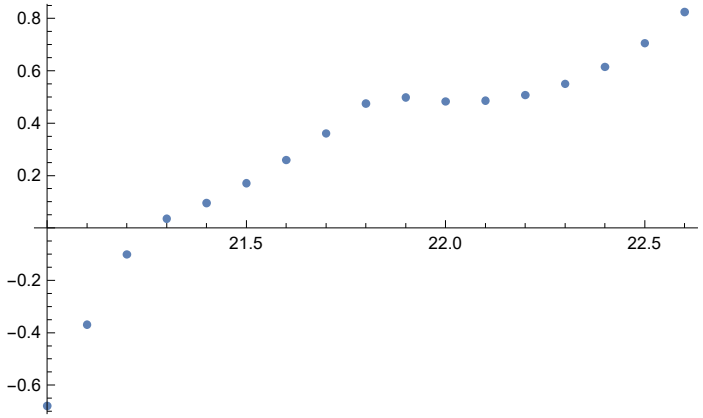


```
etaplot[18.87, 21.125]
```

```
0.000764272
```

```
etaplotKS[bmin_, b_] :=
  etatot[bmin, b] - 2 * Log[2]/Sqrt[2] 2^(7 / 64) Abs[Stotfourier[bmin, b, x[2]]] -
   2 * Log[2]/Sqrt[4] 2^(14 / 64) Abs[Stotfourier[bmin, b, x[4]]] -
   2 * Log[5]/Sqrt[5] 5^(7 / 64) Abs[Stotfourier[bmin, b, x[5]]];
```

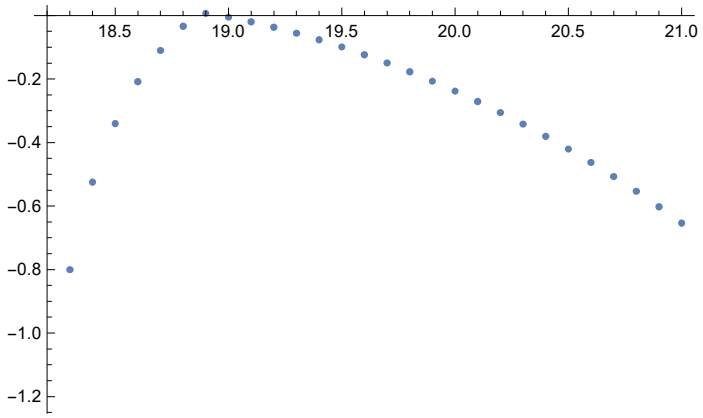
```
DiscretePlot[etaplotKS[18.87, b], {b, 21, 22.6, 0.1}, Filling → None, AxesOrigin → None ]
```



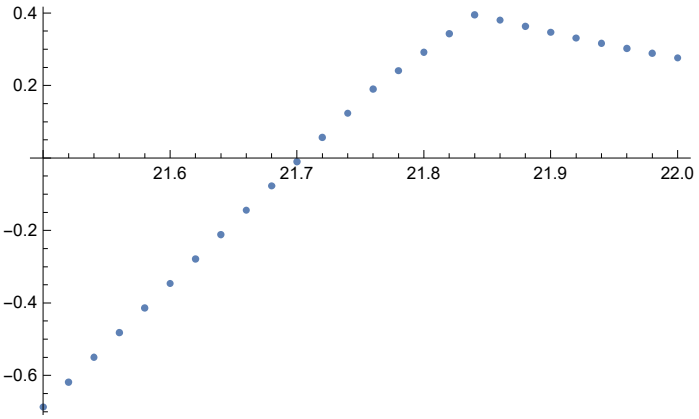

```
etaplotKS[18.87, 21.245]
```

```
0.00962825
```

```
DiscretePlot[etaplotKS[bmin, 21.245],
 {bmin, 18.2, 21, 0.1}, Filling → None, AxesOrigin → None ]
```



```
etaplotUN[bmin_, b_] :=
  etatot[bmin, b] - 2 * Log[2]/Sqrt[2] 2^(1 / 2) Abs[Stotfourier[bmin, b, x[2]]] -
   2 * Log[2]/Sqrt[4] 2 Abs[Stotfourier[bmin, b, x[4]]] -
   2 * Log[5]/Sqrt[5] 5^(1 / 2) Abs[Stotfourier[bmin, b, x[5]]];
DiscretePlot[etaplotUN[19.7, b], {b, 21.5, 22, 0.02}, Filling → None, AxesOrigin → None ]
```




```
etaplotUN[19.7, 21.705]
```

0.00655263

New approach with putting together p = 2 and p = 4:

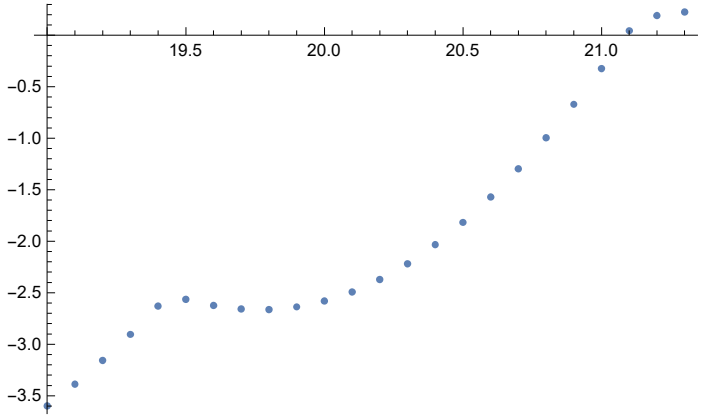
```
pol[d_, b_, y_] := 2 * Log[2]/Sqrt[4] [S1hat[d, b, x[2]] Sqrt[2] y + S1hat[d, b, x[4]] y^2 ];
gain[d_, b_] := S1hat[d, b, x[2]]^2/(2 S1hat[d, b, x[4]]) 2 * Log[2]/Sqrt[4];
loss[d_, b_] := MaxValue[
   {2 * Log[2]/Sqrt[4] S1hat[d, b, x[4]] * (Cos[t] + S1hat[d, b, x[2]]/(Sqrt[2] S1hat[d, b, x[4]])) ^2, -π < t ≤ π}, t];
gain[x[7], 21.08] - loss[x[7], 21.08]
```

−2.9148

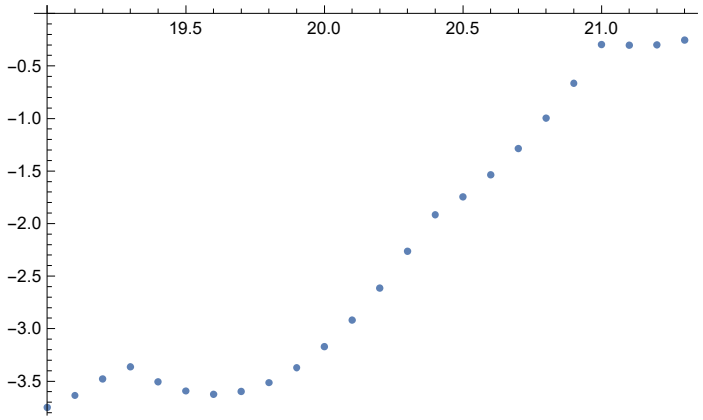
```
-2 * Log[2]/Sqrt[4] (Abs[S1hat[x[7], 21.08, x[2]]] Sqrt[2] + Abs[S1hat[x[7], 21.08, x[4]]])
```

```
-2.9148
```

```
DiscretePlot[eta[b, x[5]] - 2 * Log[3]/Sqrt[3] Abs[S1hat[x[5], b, x[3]]] +
  gain[x[5], b] - loss[x[5], b] , {b, 19.0, 21.3, 0.1}, Filling → None]
```




```
DiscretePlot[
  eta[b, x[7]] - 2 * Log[3]/Sqrt[3] Abs[S1hat[x[7], b, x[3]]] + gain[x[7], b] - loss[x[7], b] -
  2 * Log[5]/Sqrt[5] Abs[S1hat[x[7], b, x[5]]], {b, 19.0, 21.3, 0.1}, Filling → None]
```

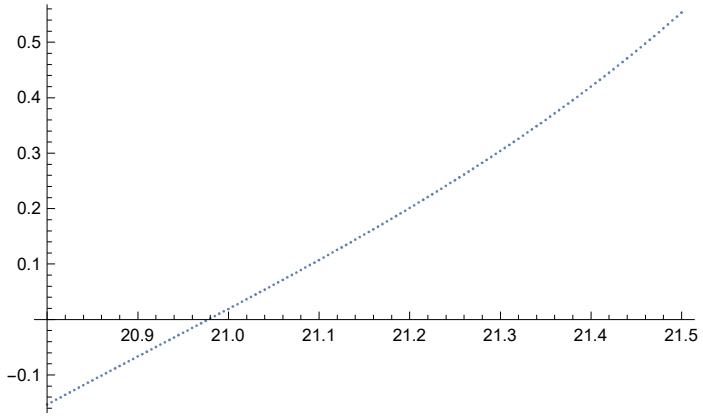


```
Stotfourier[bmin_, b_, delta_, t_] :=
  1/(S1hat[x[7], bmin, delta] - S1hat[x[7], b, delta]) (S1hat[x[7], bmin, delta]
     S1hat[x[7], b, t] - S1hat[x[7], b, delta] S1hat[x[7], bmin, t]);

etatot[bmin_, b_, delta_] := 1/(S1hat[x[7], bmin, delta] - S1hat[x[7], b, delta])
   (S1hat[x[7], bmin, delta] eta[b, x[7]] - S1hat[x[7], b, delta] eta[bmin, x[7]]);

gaincomb[bmin_, b_, delta_] := Stotfourier[bmin, b, delta, x[2]]^2/(2 Stotfourier[bmin, b, delta, x[4]]) 2 * Log[2]/Sqrt[4];

losscomb[bmin_, b_, delta_] := MaxValue[{2 * Log[2]/Sqrt[4] Stotfourier[bmin, b, delta, x[4]] *
      (Cos[t] + Stotfourier[bmin, b, delta, x[2]]/(Sqrt[2] Stotfourier[bmin, b, delta, x[4]]))^2, -π < t ≤ π}, t];


etaplot5[bmin_, b_, delta_] :=
  etatot[bmin, b, delta] - 2 * Log[5]/Sqrt[5] Abs[Stotfourier[bmin, b, delta, x[5]]] +
   gaincomb[bmin, b, delta] - losscomb[bmin, b, delta];


etaplot3[bmin_, b_, delta_] :=
  etatot[bmin, b, delta] - 2 * Log[3]/Sqrt[3] Abs[Stotfourier[bmin, b, delta, x[3]]] +
   gaincomb[bmin, b, delta] - losscomb[bmin, b, delta];

S1hat[x[7], 18.87, x[5]]
S1hat[x[7], 21, x[5]]
-0.369112

0.475977

etaplot3[18.87, 20.979, x[5]]
0.000730003

gaincomb[18.87, 21, x[5]] - losscomb[18.87, 21, x[5]]
-0.74355

losscomb[18.87, 21, x[5]]
4.60784
```

```
2 * Log[2]/Sqrt[2] Abs[Stotfourier[18.87, 21, x[5], x[2]]] +
 2 * Log[2]/Sqrt[4] Abs[Stotfourier[18.87, 21, x[5], x[4]]]
```

```
0.74355
```

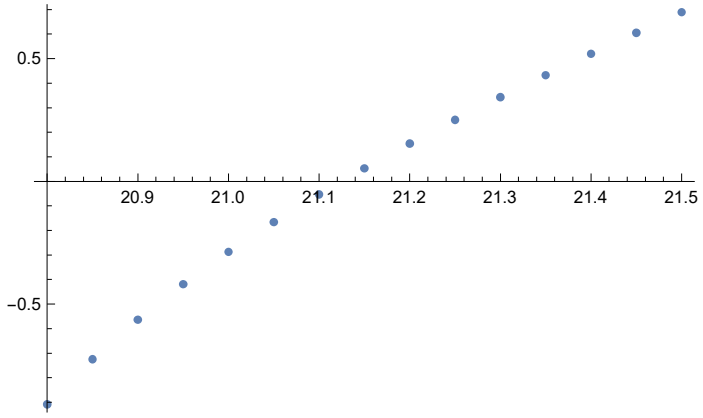
```
DiscretePlot[etaplot3[18.87, b, x[5]], {b, 20.8, 21.5, 0.005}, Filling → None]
```




```
etaplot3[18.87, 20.979, x[5]]
```

```
0.000730003
```

```
DiscretePlot[etaplot5[18.87, b, x[3]], {b, 20.8, 21.5, 0.05}, Filling → None]
```



```
DiscretePlot[etaplot[18.87, b], {b, 20.8, 21.5, 0.05}, Filling → None]
```