

RamanujanSum[q_, n_] :=

$$\sum_{a=1}^{\text{Min}[q,n]} \text{If}[\text{Divisible}[q, a] == \text{True} \&\& \text{Divisible}[n, a] == \text{True}, 1, 0] \text{MoebiusMu}\left[\frac{q}{a}\right] a$$

CohenMuSum[q_, n_, b_] :=

$$\sum_{a=1}^{\text{Min}[q,n]} \text{If}[\text{Divisible}[q, a] == \text{True} \&\& \text{Divisible}[n, a^b] == \text{True}, 1, 0] \text{MoebiusMu}\left[\frac{q}{a}\right] a^b$$

$$\text{DivisorBeta}[k_, n_, b_] := \sum_{d=1}^n \text{If}[\text{Divisible}[n, d^b] == \text{True}, 1, 0] d^{b \cdot k}$$

$$\text{CohenMoment}[y_, x_, b_] := \sum_{n=1}^y \sum_{q=1}^x \text{CohenMuSum}[q, n, b]$$

$$\text{CohenMoment2}[y_, x_, b_] := \sum_{n=1}^y \left(\sum_{q=1}^x \text{CohenMuSum}[q, n, b] \right)^2$$

$$\text{DivisorBetaF}[k_, n_, b_] := \sum_{a=1}^n \text{If}[\text{Divisible}[n, a^b] == \text{True}, 1, 0] a^k$$

Explicit2Moment[y_, N_] :=

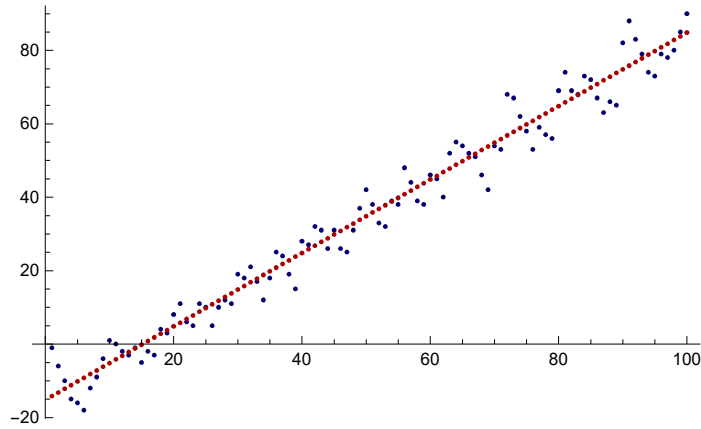
$$\sum_{k=1}^N \left(\text{Zeta}\left[1 + \frac{\text{ZetaZero}[k]}{2}\right] \left(\text{Zeta}\left[\frac{\text{ZetaZero}[k]}{2}\right] \right)^2 \text{Zeta}\left[-1 - \frac{\text{ZetaZero}[k]}{2}\right] \right. \\ \left. \frac{y^{1 + \frac{\text{ZetaZero}[k]}{2}}}{\text{Zeta}'[\text{ZetaZero}[k]] (2 + \text{ZetaZero}[k])} + \right. \\ \left. \text{Zeta}\left[1 + \frac{1 - \text{ZetaZero}[k]}{2}\right] \left(\text{Zeta}\left[\frac{1 - \text{ZetaZero}[k]}{2}\right] \right)^2 \text{Zeta}\left[-1 - \frac{1 - \text{ZetaZero}[k]}{2}\right] \right. \\ \left. \frac{y^{1 + \frac{1 - \text{ZetaZero}[k]}{2}}}{\text{Zeta}'[1 - \text{ZetaZero}[k]] (2 + 1 - \text{ZetaZero}[k])} \right)$$

$$f[\beta_, s_] = \frac{\text{Zeta}\left[1 - \frac{s}{\beta}\right] \text{Zeta}[2 + s - \beta]}{(\text{Zeta}[1 + s])^2} \frac{1}{(1 + s)^2 (\beta - s)};$$

$\beta = 1;$

$x = 10;$

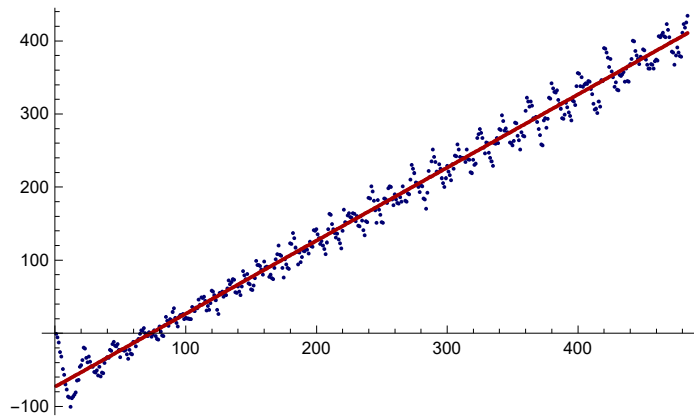
$\text{DiscretePlot}\left[\left\{\text{CohenMoment}[y, x, \beta], y - \frac{x^{1+\beta}}{2(1+\beta)\text{Zeta}[1+\beta]}\right\}, \{y, 1, x^{1+\beta}\},\right.$
 $\left.\text{AxesOrigin} \rightarrow \{0, 0\}, \text{Filling} \rightarrow \text{None}, \text{PlotStyle} \rightarrow \{\text{Darker}[\text{Darker}[\text{Blue}]], \text{Darker}[\text{Red}]\}\right]$



$\beta = 1;$

$x = 22;$

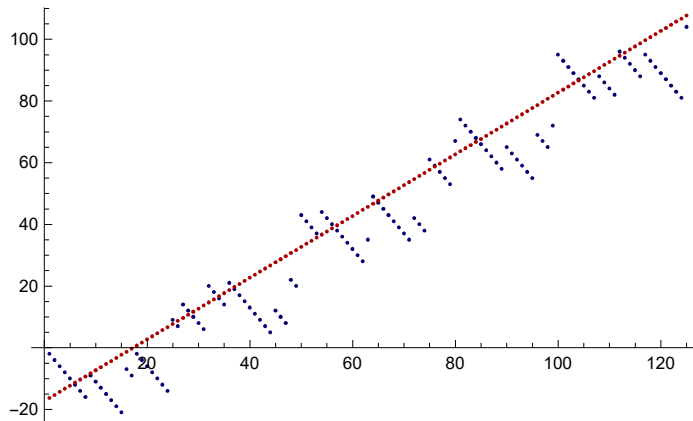
$\text{DiscretePlot}\left[\left\{\text{CohenMoment}[y, x, \beta], y - \frac{x^{1+\beta}}{2(1+\beta)\text{Zeta}[1+\beta]}\right\}, \{y, 1, x^{1+\beta}\},\right.$
 $\left.\text{AxesOrigin} \rightarrow \{0, 0\}, \text{Filling} \rightarrow \text{None}, \text{PlotStyle} \rightarrow \{\text{Darker}[\text{Darker}[\text{Blue}]], \text{Darker}[\text{Red}]\}\right]$



$\beta = 2;$

$x = 5;$

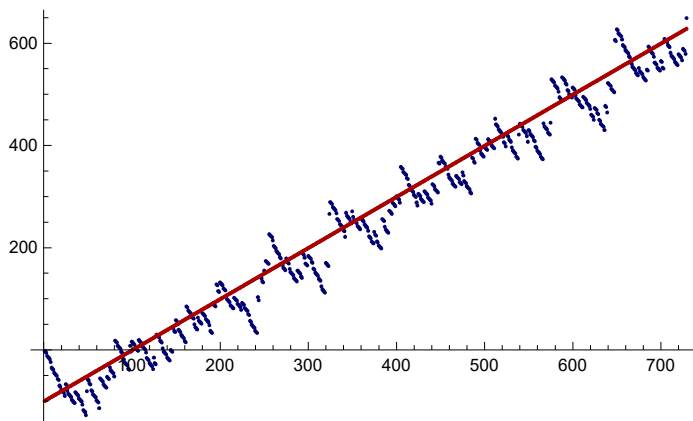
`DiscretePlot` $\left[\left\{ \text{CohenMoment}[y, x, \beta], y - \frac{x^{1+\beta}}{2 (1 + \beta) \text{Zeta}[1 + \beta]} \right\}, \{y, 1, x^{1+\beta}\}, \right.$
`AxesOrigin` $\rightarrow \{0, 0\}$, `Filling` \rightarrow None, `PlotStyle` $\rightarrow \{\text{Darker}[\text{Darker}[\text{Blue}]], \text{Darker}[\text{Red}]\}$ $\left. \right]$



$\beta = 2;$

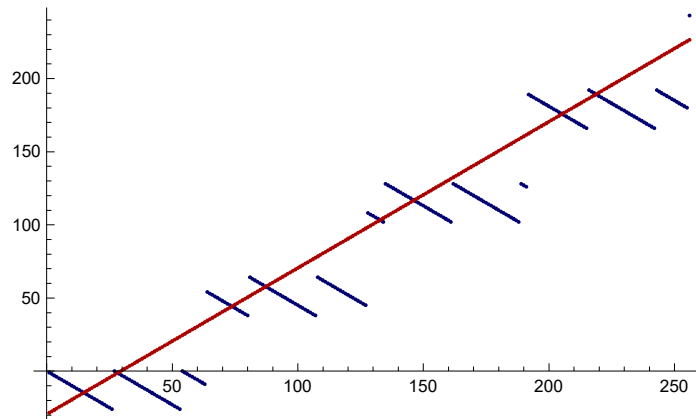
$x = 9;$

`DiscretePlot` $\left[\left\{ \text{CohenMoment}[y, x, \beta], y - \frac{x^{1+\beta}}{2 (1 + \beta) \text{Zeta}[1 + \beta]} \right\}, \{y, 1, x^{1+\beta}\}, \right.$
`AxesOrigin` $\rightarrow \{0, 0\}$, `Filling` \rightarrow None, `PlotStyle` $\rightarrow \{\text{Darker}[\text{Darker}[\text{Blue}]], \text{Darker}[\text{Red}]\}$ $\left. \right]$

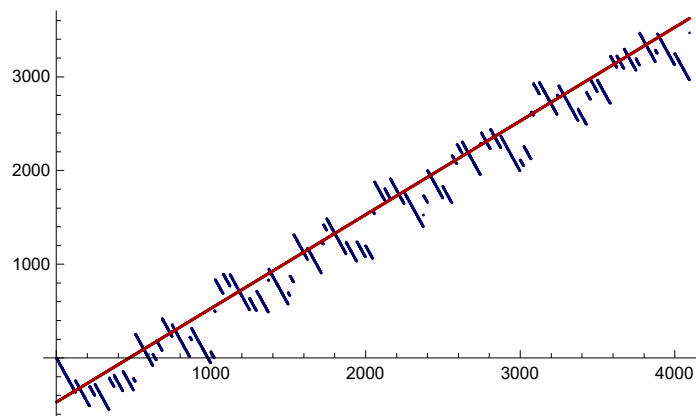


$\beta = 3;$
 $x = 4;$

```
DiscretePlot[ {CohenMoment[y, x,  $\beta$ ],  $y - \frac{x^{1+\beta}}{2(1+\beta) \text{Zeta}[1+\beta]}$ }, {y, 1,  $x^{1+\beta}$ },
  AxesOrigin -> {0, 0}, Filling -> None, PlotStyle -> {Darker[Darker[Blue]], Darker[Red]} ]
```


 $\beta = 3;$
 $x = 8;$

```
DiscretePlot[ {CohenMoment[y, x,  $\beta$ ],  $y - \frac{x^{1+\beta}}{2(1+\beta) \text{Zeta}[1+\beta]}$ }, {y, 1,  $x^{1+\beta}$ },
  AxesOrigin -> {0, 0}, Filling -> None, PlotStyle -> {Darker[Darker[Blue]], Darker[Red]} ]
```

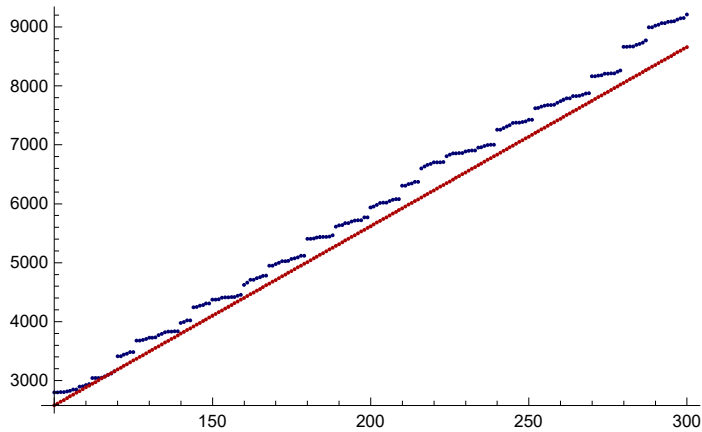


$\beta = 1;$

$x = 10;$

$d = 3;$

`DiscretePlot` $\left[\left\{ \text{CohenMoment2}[y, x, \beta], \frac{y * x^{1+\beta}}{(1+\beta) \text{Zeta}[1+\beta]} - \frac{x^{2+2\beta}}{2 (1+\beta)^2 (\text{Zeta}[1+\beta])^2} \right\}, \{y, x^{2\beta}, d * x^{2\beta}\}, \text{Filling} \rightarrow \text{None}, \text{PlotStyle} \rightarrow \{\text{Darker}[\text{Darker}[\text{Blue}]], \text{Darker}[\text{Red}], \text{Darker}[\text{Green}]\} \right]$

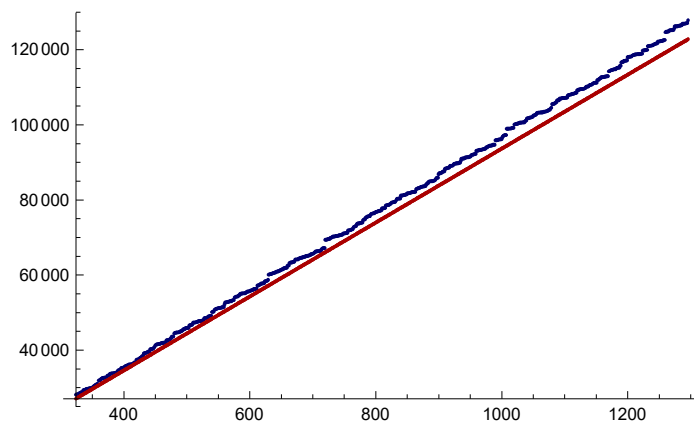


$\beta = 1;$

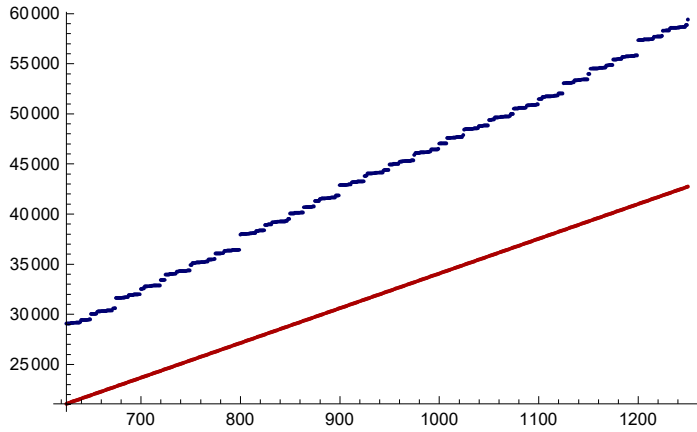
$x = 18;$

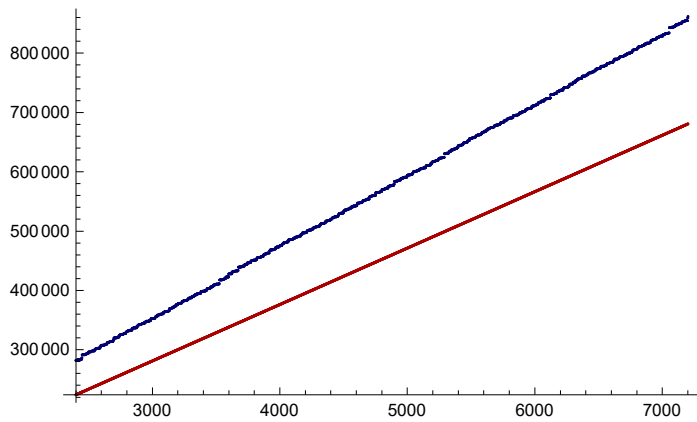
$d = 4;$

`DiscretePlot` $\left[\left\{ \text{CohenMoment2}[y, x, \beta], \frac{y * x^{1+\beta}}{(1+\beta) \text{Zeta}[1+\beta]} - \frac{x^{2+2\beta}}{2 (1+\beta)^2 (\text{Zeta}[1+\beta])^2} \right\}, \{y, x^{2\beta}, d * x^{2\beta}\}, \text{Filling} \rightarrow \text{None}, \text{PlotStyle} \rightarrow \{\text{Darker}[\text{Darker}[\text{Blue}]], \text{Darker}[\text{Red}], \text{Darker}[\text{Green}]\} \right]$

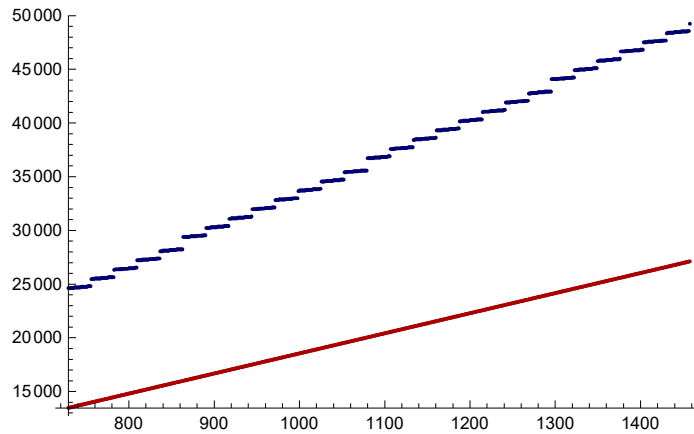


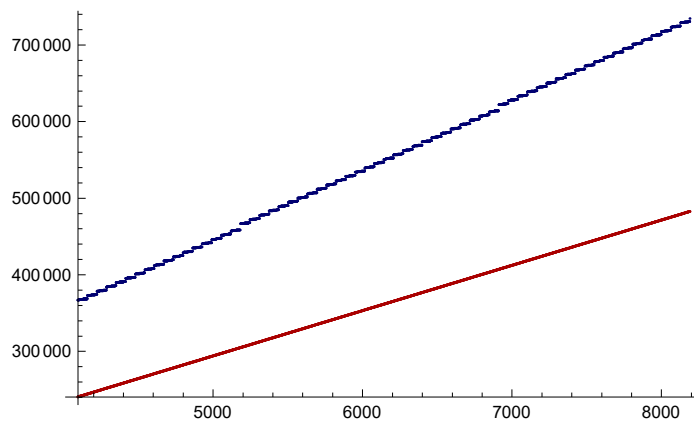
$\beta = 2;$
 $x = 5;$
 $d = 2;$

$$\text{DiscretePlot}\left[\left\{\text{CohenMoment2}[y, x, \beta], \frac{y * x^{1+\beta}}{(1+\beta) \text{Zeta}[1+\beta]} - \frac{x^{2+2\beta}}{2 (1+\beta)^2 (\text{Zeta}[1+\beta])^2}\right\}, \{y, x^{2\beta}, d * x^{2\beta}\}, \text{Filling} \rightarrow \text{None}, \text{PlotStyle} \rightarrow \{\text{Darker}[\text{Darker}[\text{Blue}]], \text{Darker}[\text{Red}], \text{Darker}[\text{Green}]\}\right]$$

 $\beta = 2;$
 $x = 7;$
 $d = 3;$

$$\text{DiscretePlot}\left[\left\{\text{CohenMoment2}[y, x, \beta], \frac{y * x^{1+\beta}}{(1+\beta) \text{Zeta}[1+\beta]} - \frac{x^{2+2\beta}}{2 (1+\beta)^2 (\text{Zeta}[1+\beta])^2}\right\}, \{y, x^{2\beta}, d * x^{2\beta}\}, \text{Filling} \rightarrow \text{None}, \text{PlotStyle} \rightarrow \{\text{Darker}[\text{Darker}[\text{Blue}]], \text{Darker}[\text{Red}], \text{Darker}[\text{Green}]\}\right]$$


$\beta = 3;$
 $x = 3;$
 $d = 2;$

$$\text{DiscretePlot}\left[\left\{\text{CohenMoment2}[y, x, \beta], \frac{y * x^{1+\beta}}{(1+\beta) \text{Zeta}[1+\beta]} - \frac{x^{2+2\beta}}{2 (1+\beta)^2 (\text{Zeta}[1+\beta])^2}\right\}, \{y, x^{2\beta}, d * x^{2\beta}\}, \text{Filling} \rightarrow \text{None}, \text{PlotStyle} \rightarrow \{\text{Darker}[\text{Darker}[\text{Blue}]], \text{Darker}[\text{Red}], \text{Darker}[\text{Green}]\}\right]$$

 $\beta = 3;$
 $x = 4;$
 $d = 2;$

$$\text{DiscretePlot}\left[\left\{\text{CohenMoment2}[y, x, \beta], \frac{y * x^{1+\beta}}{(1+\beta) \text{Zeta}[1+\beta]} - \frac{x^{2+2\beta}}{2 (1+\beta)^2 (\text{Zeta}[1+\beta])^2}\right\}, \{y, x^{2\beta}, d * x^{2\beta}\}, \text{Filling} \rightarrow \text{None}, \text{PlotStyle} \rightarrow \{\text{Darker}[\text{Darker}[\text{Blue}]], \text{Darker}[\text{Red}], \text{Darker}[\text{Green}]\}\right]$$

 $4 * 35^2$

4900

```
 $\beta = 1;$ 
```

```
 $x = 35;$ 
```

```
 $d = 4;$ 
```

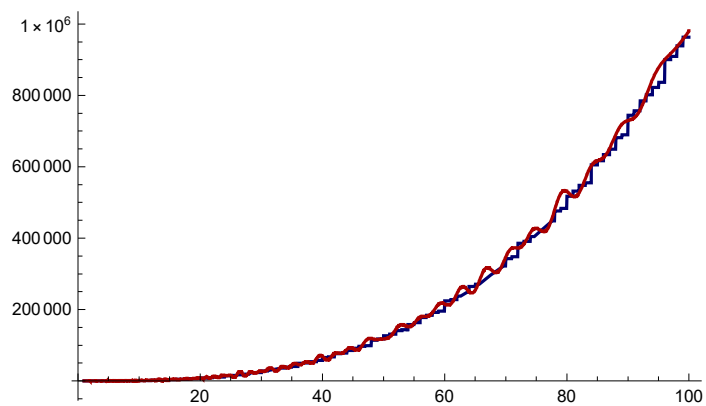
```
DiscretePlot[ {CohenMoment2[y, x,  $\beta$ ],  $\frac{y * x^{1+\beta}}{(1+\beta) \text{Zeta}[1+\beta]} - \frac{x^{2+2\beta}}{2 (1+\beta)^2 (\text{Zeta}[1+\beta])^2}$ }, {y,  $x^{2\beta}$ ,  
d *  $x^{2\beta}$ }, Filling -> None, PlotStyle -> {Darker[Darker[Blue]], Darker[Red], Darker[Green]} ]
```

```
TimeUsed[ ]
```

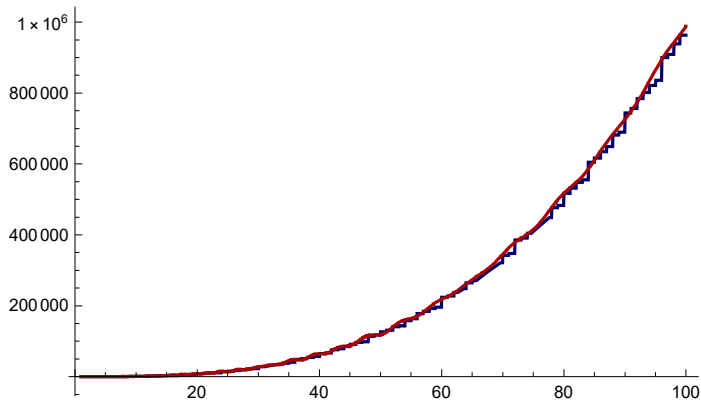
```
$Aborted
```

```
12869.8
```

```
Plot[ {  $\sum_{n=1}^y \text{DivisorBeta}[1, n, 1] \text{DivisorBeta}[1, n, 1]$ ,  
 $\frac{1}{24} y - \frac{\text{EulerGamma}}{2} y^2 - \frac{1}{192} + \frac{5}{6} \text{Zeta}[3] y^3 + \text{Explicit2Moment}[y, 100]$  },  
{y, 1, 100}, PlotStyle -> {Darker[Darker[Blue]], Darker[Red]} ]
```




```
Plot[ { Sum[ DivisorBeta[1, n, 1] DivisorBeta[1, n, 1],
            {n, 1, y}],
      1/24 y - EulerGamma/2 y^2 - 1/192 + 5/6 Zeta[3] y^3 + Explicit2Moment[y, 50] },
      {y, 1, 100}, PlotStyle -> {Darker[Blue], Darker[Red]} ]
```

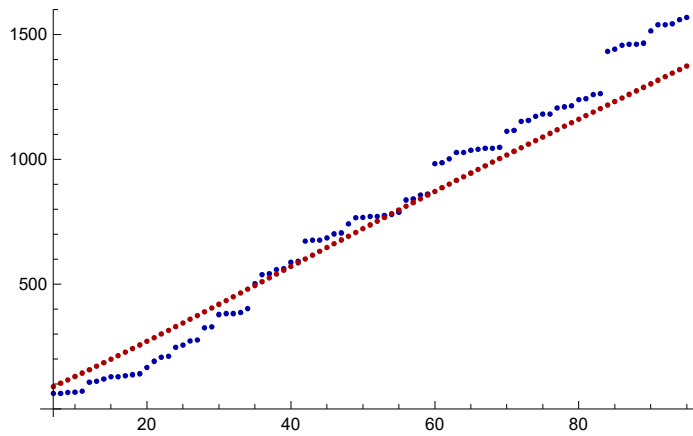


```
x = 7;
```

```
β = 1;
```

```
B = 1;
```

```
DiscretePlot[ { CohenMoment2[y, x, β],
                y * x^(1+β) / ((1+β) Zeta[1+β]) + y * x^2 / Zeta[2] NIntegrate[ 1/(2 π) f[β, i * t] e^(-i * t * y^(1/2) * x^-2), {t, -2, 2} ] },
                {y, x, x^(1+β) (Log[x])^B}, Filling -> None, PlotStyle -> {Darker[Blue], Darker[Red]} ]
```



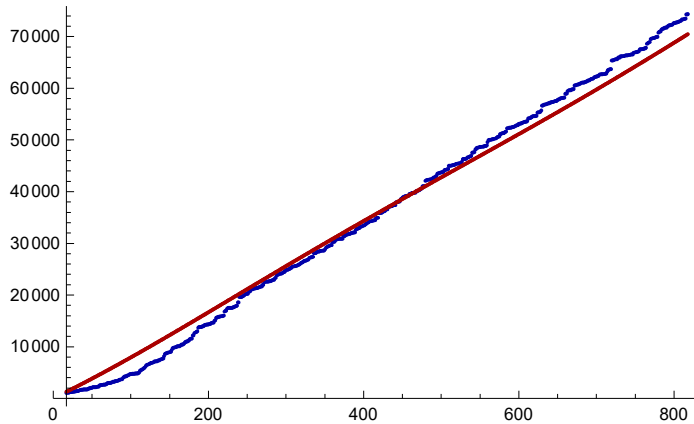
```

x = 17;
β = 1;
B = 1;
DiscretePlot[{CohenMoment2[y, x, β],

$$\frac{y * x^{1+\beta}}{(1+\beta) \text{Zeta}[1+\beta]} + \frac{y * x^2}{\text{Zeta}[2]} \text{Re}\left[\text{NIntegrate}\left[\frac{1}{2\pi} f[\beta, i * t] e^{-i * t * y^{\frac{1}{\beta}} * x^{-2}}, \{t, -2, 2\}\right]\right]},$$

{y, x, x^{1+\beta} (Log[x])^B}], Filling -> None, PlotStyle -> {Darker[Blue], Darker[Red]}]

```



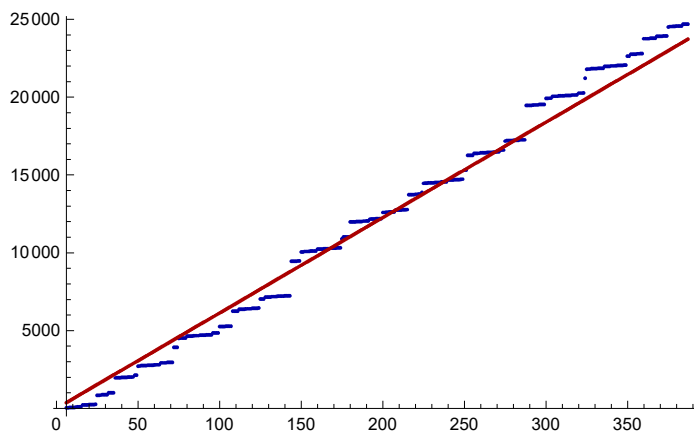
```

x = 6;
β = 2;
B = 1;
DiscretePlot[{CohenMoment2[y, x, β],

$$\frac{y * x^{1+\beta}}{(1+\beta) \text{Zeta}[1+\beta]} + \frac{y * x^2}{\text{Zeta}[2]} \text{Re}\left[\text{NIntegrate}\left[\frac{1}{2\pi} f[\beta, i * t] e^{-i * t * y^{\frac{1}{\beta}} * x^{-2}}, \{t, -2, 2\}\right]\right]},$$

{y, x, x^{1+\beta} (Log[x])^B}], Filling -> None, PlotStyle -> {Darker[Blue], Darker[Red]}]

```



$x = 8;$

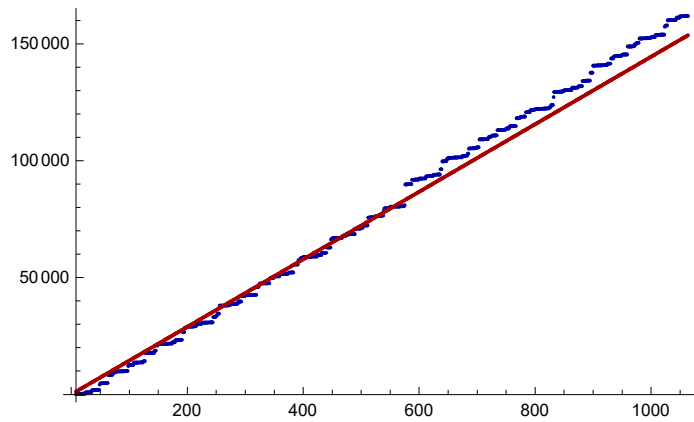
$\beta = 2;$

$B = 1;$

DiscretePlot[{CohenMoment2[y, x, β],

$$\frac{y * x^{1+\beta}}{(1+\beta) \text{Zeta}[1+\beta]} + \frac{y * x^2}{\text{Zeta}[2]} \text{Re}\left[\text{NIntegrate}\left[\frac{1}{2\pi} f[\beta, i * t] e^{-i * t * y^{\frac{1}{\beta}} * x^{-2}}, \{t, -2, 2\}\right]\right]},$$

{y, x, $x^{1+\beta} (\text{Log}[x])^B$ }, Filling → None, PlotStyle → {Darker[Blue], Darker[Red]}]



$x = 3;$

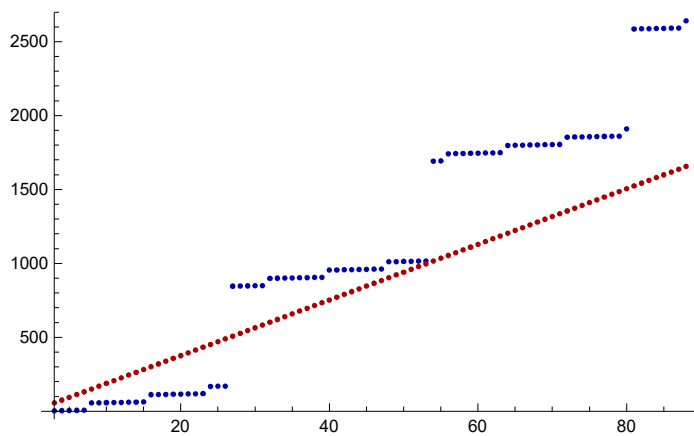
$\beta = 3;$

$B = 1;$

DiscretePlot[{CohenMoment2[y, x, β],

$$\frac{y * x^{1+\beta}}{(1+\beta) \text{Zeta}[1+\beta]} + \frac{y * x^2}{\text{Zeta}[2]} \text{Re}\left[\text{NIntegrate}\left[\frac{1}{2\pi} f[\beta, i * t] e^{-i * t * y^{\frac{1}{\beta}} * x^{-2}}, \{t, -2, 2\}\right]\right]},$$

{y, x, $x^{1+\beta} (\text{Log}[x])^B$ }, Filling → None, PlotStyle → {Darker[Blue], Darker[Red]}]



```

x = 4;
β = 3;
B = 1;
DiscretePlot[ {CohenMoment2[y, x, β],

$$\frac{y * x^{1+\beta}}{(1+\beta) \text{Zeta}[1+\beta]} + \frac{y * x^2}{\text{Zeta}[2]} \text{Re}\left[\text{NIntegrate}\left[\frac{1}{2\pi} f[\beta, i * t] e^{-i * t * y^{\frac{1}{\beta}} * x^{-2}}, \{t, -2, 2\}\right]\right]},$$

{y, x, x^{1+\beta} (Log[x])^B}, Filling -> None, PlotStyle -> {Darker[Blue], Darker[Red]} ]

```

