```mathematica
ClearAll;

list[n_] := FactorInteger[n][[All, 1]];

ProductOdd[x_List, n_] := Times @@ (1 - 1/x^2)/(1 - 1/x^(n+1));

ProductEven[x_List, n_] := Times @@ 1/((1 - 1/x^2) (1 + 1/x^(n+1)));

ProductLiouville[x_List] := Times @@ (1 - 1/x^2);

PrefactorOdd[j_] := (-1)^((j+3)/2) (j + 1)!/(12 BernoulliB[j + 1] (2 π)^(j-1));

PrefactorEven[j_] := (-1)^j (3 Zeta[j + 1] (2 (j + 1))!)/(π^4 BernoulliB[2 (j + 1)] (2 π)^(2 j));

COdd[j_, q_] := PrefactorOdd[j] * ProductOdd[list[q], j];
CEven[j_, q_] := PrefactorEven[j] ProductEven[list[q], j];
MuJ[j_, n_] := If[Max[FactorInteger[n][[All, 2]]] >= j + 1, 0, LiouvilleLambda[n]];
LK[k_, n_] := DirichletConvolve[LiouvilleLambda[m], (Log[m])^k, m, n];
VarTheta[k_, x_, q_, a_] :=
  Sum[(If[GCD[a, q] == 1, 1, 0] * If[Divisible[n - a, q] == True, 1, 0] * LK[k, n]), {n, 1, x}];

LambdaJK[j_, k_, n_] := Sum[If[Divisible[n, d], MuJ[j, d] (Log[n/d])^k, 0], {d, 1, n}];

Psi[j_, k_, x_, q_, a_] :=
  Sum[(If[GCD[a, q] == 1, 1, 0] * If[Divisible[n - a, q] == True, 1, 0] * LambdaJK[j, k, n]), {n, 1, x}];

RHS11Odd[j_, k_, x_, q_, a_] := k COdd[j, q]/EulerPhi[q] * x * (Log[x])^(k-1);

RHS11Even[j_, k_, x_, q_, a_] := k CEven[j, q]/EulerPhi[q] * x * (Log[x])^(k-1);

RHS12[k_, x_, q_] := k/EulerPhi[q] * π^2/6 * ProductLiouville[list[q]] * x * (Log[x])^(k-1);
```

```
DiscretePlot[{EulerPhi[q] / COdd[1, q], EulerPhi[q] / COdd[3, q],
  EulerPhi[q] / COdd[5, q], EulerPhi[q] / COdd[7, q]}, {q, 2, 1000}, Filling → None]
```
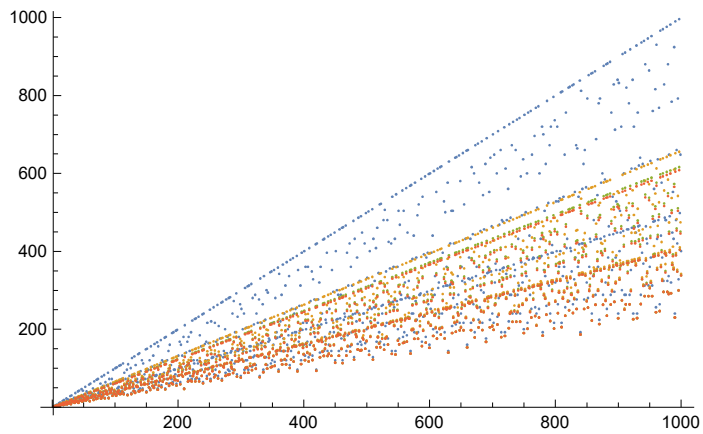


```
DiscretePlot[{EulerPhi[q] / CEven[2, q], EulerPhi[q] / CEven[4, q],
  EulerPhi[q] / CEven[6, q], EulerPhi[q] / CEven[8, q]}, {q, 2, 1000}, Filling → None]
```
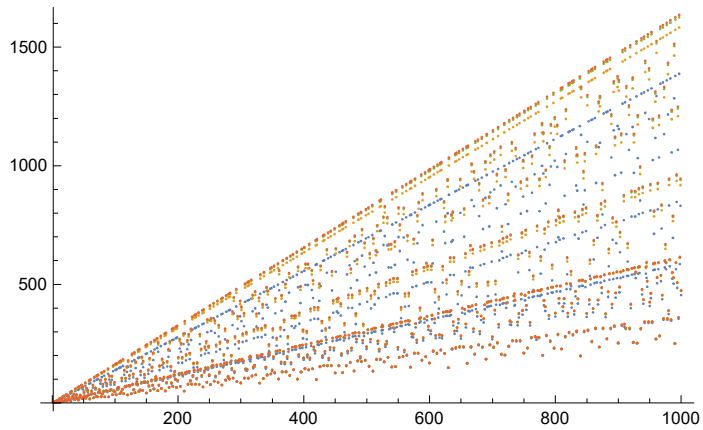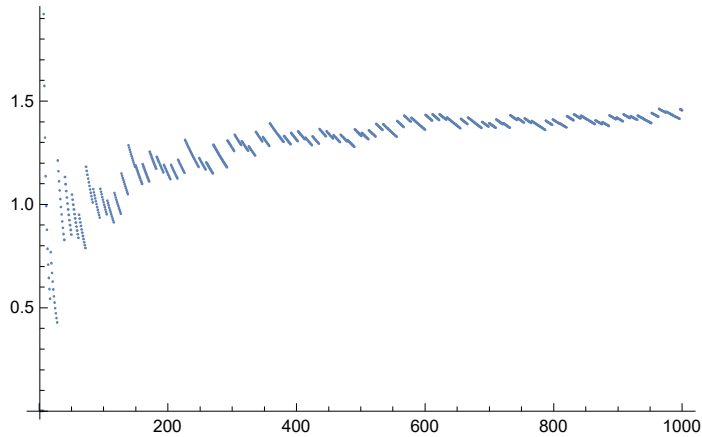
```
j = 2;
k = 2;
q = 11;
a = 7;
DiscretePlot[{Psi[j, k, x, q, a] / RHS11Even[j, k, x, q, a]}, {x, 1, 1000}, Filling → None]
```

⋯ Power: Infinite expression $\frac{1}{0}$ encountered.

⋯ Infinity: Indeterminate expression 0 ComplexInfinity encountered.
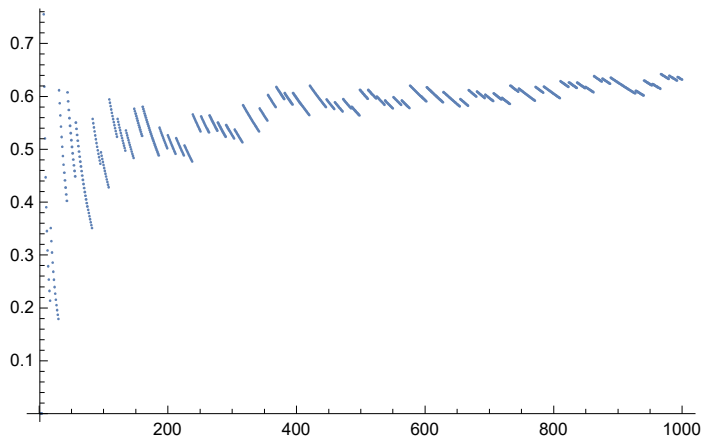


```
j = 3;
k = 2;
q = 13;
a = 5;
DiscretePlot[{Psi[j, k, x, q, a] / RHS11Odd[j, k, x, q, a]}, {x, 1, 1000}, Filling → None]
```

⋯ Power: Infinite expression $\frac{1}{0}$ encountered.

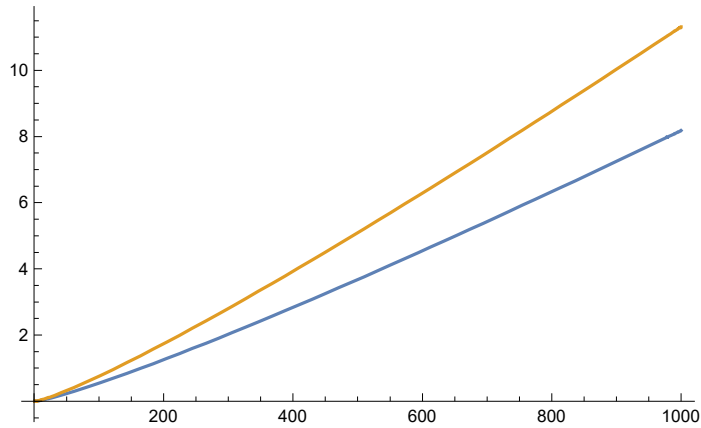⋯ Infinity: Indeterminate expression 0 ComplexInfinity encountered.

```
k = 2;
j = 7;
q = 5000;
a = 5;
```

$$\text{Plot}\Big[\Big\{(\text{RHS11Odd}[j, k, x, q, a]), \Big(\frac{\text{PrefactorOdd}[j]}{\text{EulerPhi}[q]} 2 x \text{Log}[x]\Big)\Big\}, \{x, 1, 1000\}, \text{Filling} \rightarrow \text{None}\Big]$$


```
k = 2;
j = 12;
q = 5000;
a = 7;
```

$$\text{Plot}\Big[\Big\{\text{RHS11Even}[j, k, x, q, a], \frac{\text{PrefactorEven}[j]}{\text{EulerPhi}[q]} 2 x \text{Log}[x]\Big\}, \{x, 1, 1000\}, \text{Filling} \rightarrow \text{None}\Big]$$
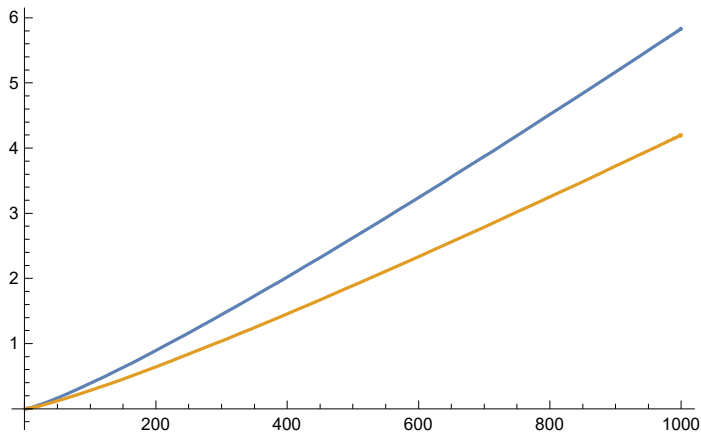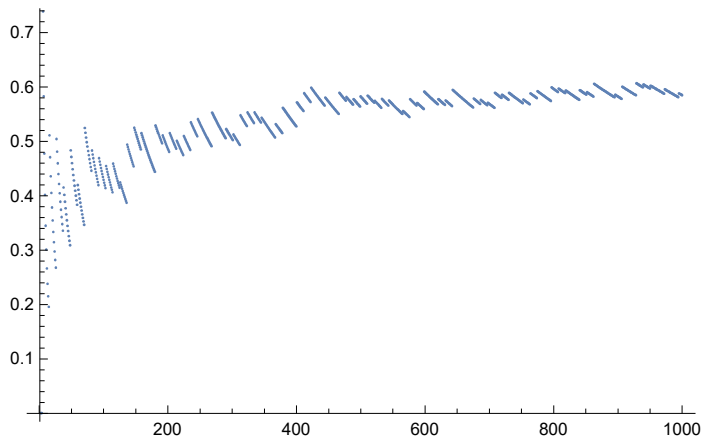
```
k = 2;
q = 11;
a = 5;
DiscretePlot[{VarTheta[k, x, q, a] / RHS12[k, x, q]}, {x, 1, 1000}, Filling → None]
```

⬚⬚⬚ **Power**: Infinite expression $\dfrac{1}{0}$ encountered.

⬚⬚⬚ **Infinity**: Indeterminate expression 0 ComplexInfinity encountered.
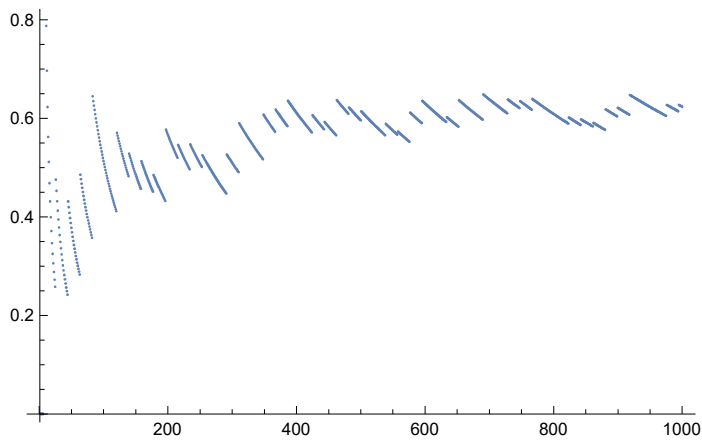


```
k = 2;
q = 19;
a = 7;
DiscretePlot[{VarTheta[k, x, q, a] / RHS12[k, x, q]}, {x, 1, 1000}, Filling → None]
```

⬚⬚⬚ **Power**: Infinite expression $\dfrac{1}{0}$ encountered.

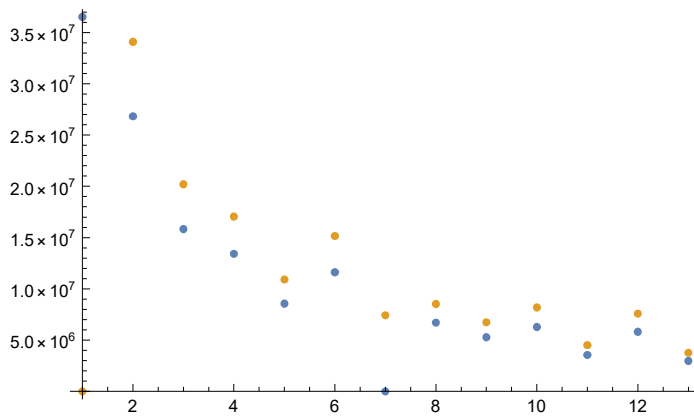⬚⬚⬚ **Infinity**: Indeterminate expression 0 ComplexInfinity encountered.

```
j = 2;
k = 2;
q = 11;
a = 7;
x = 100 000;
DiscretePlot[{Psi[j, k, x, q, a], RHS11Even[j, k, x, q, a]}, {q, 1, Log[x]}, Filling → None]
```

$Aborted

```
k = 2;
a = 7;
x = 1 000 000;
DiscretePlot[{VarTheta[k, x, q, a], RHS12[k, x, q]}, {q, 1, Log[x]}, Filling → None]
```
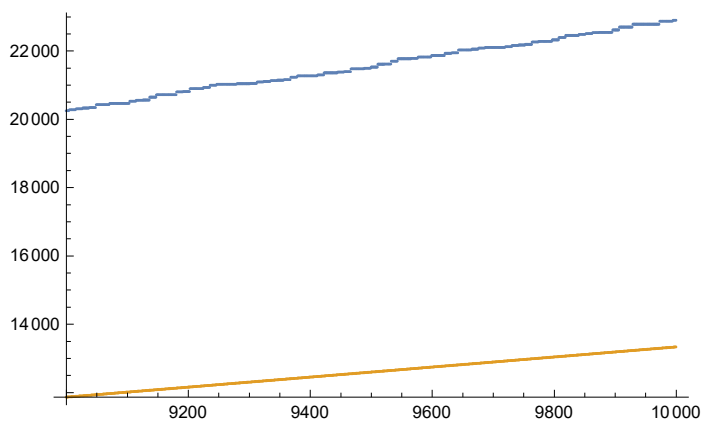


```
j = 2;
k = 2;
q = 11;
a = 7;
DiscretePlot[{Psi[j, k, x, q, a], RHS11Even[j, k, x, q, a]}, {x, 9000, 10 000}, Filling → None]
```
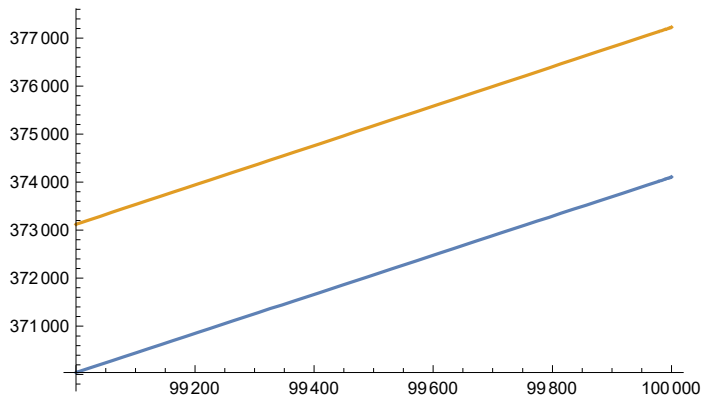
```
k = 2;
j = 7;
q = 11;
a = 5;
Plot[{RHS11Odd[j, k, x, q, a], (PrefactorOdd[j]/EulerPhi[q]) 2 x Log[x]},
  {x, 99000, 100000}, Filling → None]
```
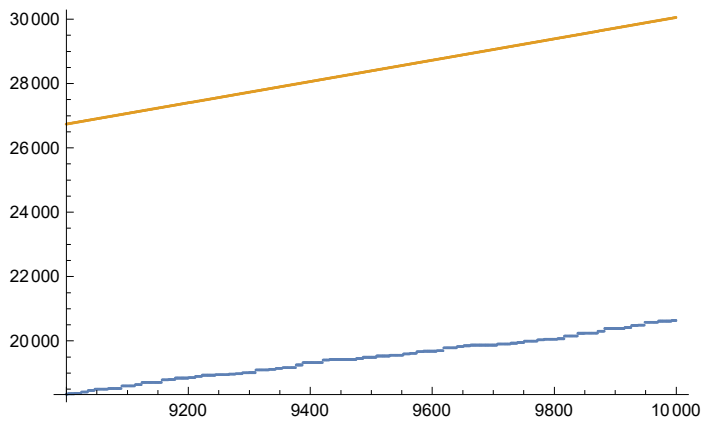


```
k = 2;
q = 11;
a = 5;
DiscretePlot[{VarTheta[k, x, q, a], RHS12[k, x, q]}, {x, 9000, 10000}, Filling → None]
```
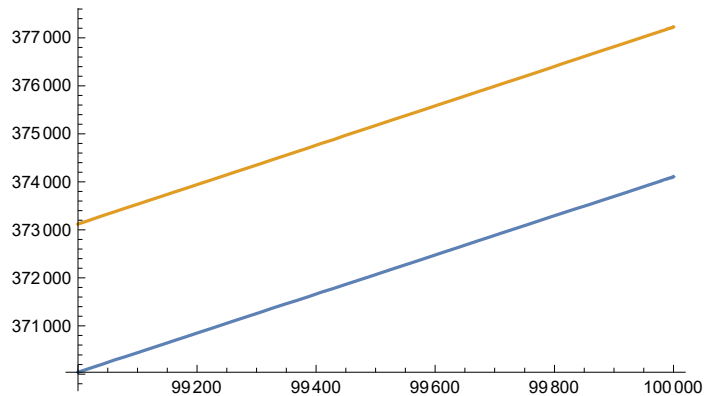
```
k = 2;
j = 7;
q = 5000;
a = 5;
```

$$\text{Plot}\left[\left\{(\text{RHS11Odd}[j, k, x, q, a]), \left(\frac{\text{PrefactorOdd}[j]}{\text{EulerPhi}[q]} 2 x \text{Log}[x]\right)\right\}, \{x, 1, 1000\}, \text{Filling} \rightarrow \text{None}\right]$$


```
k = 2;
j = 12;
q = 13;
a = 7;
```

$$\text{Plot}\left[\left\{\text{RHS11Even}[j, k, x, q, a], \frac{\text{PrefactorEven}[j]}{\text{EulerPhi}[q]} 2 x \text{Log}[x]\right\}, \right.$$
$$\{x, 99000, 100000\}, \text{Filling} \rightarrow \text{None}\Big]$$