Function to determine the shortest path to fulfill an order

LP model to determine the shortest path

To determine the shortest path to fulfill an order, we can implement a linear programming model which takes the order items that need to be obtained, and determines the shortest path to collect them all. The model assumes starting from the packaging area, collecting all items, and returning to the packaging area. The model is described below.

Sets, parameters, variables

I : The set of items in the order, as well as the packaging section (labelled as 0). Note, we can call the order items the nodes to hit.

d_{ij} : The distances between each node $i, j \in I$.

n : The number of items in the order $(I/\{0\})$.

x_{ij} : The decision variable, it equals 1 if you travel from node i to node j and 0 if not.

y_i : A dummy decision variable which tracks the number of nodes visited before node i, this is used to ensure we do not revisit the packaging station prior to collecting all the items.

Model

Min

$$\sum_{i \in I} \sum_{j \in J} x_{ij} d_{ij}$$

Subject to

$$\sum_{i \in I} x_{ij} = 1 \quad \forall j \in I \qquad (1)$$

$$\sum_{j \in I} x_{ij} = 1 \quad \forall i \in I \qquad (2)$$

$$x_{ii} = 0 \quad \forall i \in I \qquad (3)$$

$$x_{ij} + x_{ji} \leq 1 \quad \forall i \in I, j \in I \quad (4)$$

$$u_i - u_j + nx_{ij} \leq n - 1 \quad \forall i \neq j \in I/\{0\} \quad (5)$$

$$x_{ij} \in \{0, 1\}$$

$$y_{ij} \in \mathbb{Z}^+$$

Note - constraint (1) ensures that there is one arc into every node, and constraint (2) ensures that there is one arc out of every node. Constraint (3) ensures that you don't go from node i to node i. Constraint (4) ensures that you cannot return back to the node you just came from. Constraint (5) ensures that you don't return to packaging until all nodes have been hit.

The final constraint - constraint (5) was taken from an mathematical article entitled "Integer Programming Formulation of Traveling Salesman Problems", which discusses the formulation of Travelling Salesman Problem when it is required that there are no sub-networks. Reference - Miller, C, A Tucker, and R Zemlin. “Integer Programming Formulation of Traveling Salesman Problems.” Journal of the ACM 7.4 (1960): 326–329. Web.

Reference link - https://dl-acm-org.eux.idm.oclc.org/doi/pdf/10.1145/321043.321046 (got it from online library).

Implementation of shortest path function

We can define this model as a function which takes in the order, and returns the shortest path. This has been implemented below. Note that we firstly load in all the distances, and define the sets, parameters, and variables before adding constraints, the objective, and producing the output.

In [1]:
```
# Import packages
import numpy as np
import xpress as xp
import pandas as pd
import time
```

In [2]:
```
def shortest_path(order, distances_data):
    '''
    A function which takes as input an order which is a list of item numb

    Returns the shortest path to collect all items, when starting and end
    total distance to fulfill the order.
    '''

    # SORTING SETS, PARAMETERS, VARIABLES

    # Sort the order into ascending order
    order.sort()
```

```
    # Drop the 0 values
    order = [i for i in order if i != 0]

    # Obtain the length of n
    n = len(order)

    # Create a copy to generate the I set
    order_2 = order.copy()

    # Define I to be the order with 0 attached
    order_2.insert(0, 0)
    I = order_2

    # Insert "Packaging into the order list so we can obtain the desired
    order.insert(0,"Packaging")

    # Drop the first column of indices
    d_dat = distances_data.drop(columns = "Index")

    # Select desired rows and columns
    d_data = distances_data.loc[distances_data["Index"].isin(order)] # Fi
    d_data = d_data[order] # Filter columns

    # Rename the packaging column to 0
    d_data = d_data.rename(columns = {"Packaging" : "0"})

    # Select and drop the final row
    first_r = d_data.loc[d_data["0"] == 0]
    d_data = d_data.drop(index = [96])

    # Concat the dataframes so final row is first row
    d = pd.concat([first_r, d_data])

    # Turn the data into an array for use
    d = d.to_numpy()

    # Generate a list of indices for the set
    I_ind = list(range(0, len(I)))

    # Define the x variable
    x = np.array([xp.var(vartype = xp.binary, name = 'x_{0}_{1}'.format(i
                dtype = xp.npvar).reshape(len(I), len(I))

    # Define the y variable
    y = np.array([xp.var(vartype = xp.integer, name = 'y_{0}'.format(i))
                dtype = xp.npvar)



    # DEFINE THE PROBLEM, DECLARE VARIABLES AND CONSTRAINTS

    # Set the problem
    prob = xp.problem(name = "Prob")

    # Add the decision variable to the problem
    prob.addVariable(x)
    prob.addVariable(y)
```

```
# Add the constraints:

# only one arc into each node
prob.addConstraint(
    xp.Sum(x[i, j] for i in I_ind) == 1 for j in I_ind
)

# only one arc out of each node
prob.addConstraint(
    xp.Sum(x[i, j] for j in I_ind) == 1 for i in I_ind
)

# have to go to a different node
prob.addConstraint(
    x[i, i] == 0 for i in I_ind
)

# can't go back to the node you just came from, unless there is only
if len(I_ind) != 2:
    prob.addConstraint(
        x[i, j] + x[j, i] <= 1 for i in I_ind for j in I_ind
    )

# no sub-networks
for i in I_ind :
    for j in I_ind :
        if i != j and i != 0 and j != 0:
            prob.addConstraint(
                y[i] - y[j] + n*x[i, j] <= n - 1
            )


# DEFINE AND ADD OBJECTIVE

# Define the objective function
obj = xp.Sum(xp.Sum(x[i, j]*d[i, j] for i in I_ind) for j in I_ind)

# Set the problems objective function
prob.setObjective(obj, sense = xp.minimize)


# WRITE AND SOLVE PROBLEM
# Write and solve the problem
prob.write("problem","lp") # Used to look for cause of infeasibility
prob.solve()


# DEFINE OUTPUTS

# Obtain optimal x values, and objective value
soln = prob.getSolution(x)
total_distance = prob.getObjVal()

# Set an empty array for arcs
arcs = []
```

```
    # Determine the arcs
    for i in I_ind :
        for j in I_ind :
            if soln[i, j] == 1 :
                arcs.append([I[i], I[j]])

    # Obtain the y solution
    y_soln = prob.getSolution(y)

    # Create a dataframe of the nodes and the y values
    df = pd.DataFrame({"I" : I, 'y' : y_soln})

    # Sort based on the y values
    df2 = df.sort_values(by = ["y"])

    # The order of collection
    sequence = df2['I'].values.tolist()

    return total_distance, sequence
```

Test Functions

To ensure the function is running correctly, define 5 test orders and check output.

In [3]:
```
# Import data file of distances
distances_data = pd.read_excel('DistanceMatrix.xlsx', sheet_name = "Dista
```

In [4]:
```
# Test 1
total_distance, sequence = shortest_path([6, 0, 0, 0, 0], distances_data)
print(f"The total distance is {total_distance}m")
print(f"The sequence is {sequence}")
```

```
Using the license file found in your Xpress installation. If you want to
use this license and no longer want to see this message, use the followin
g code before using the xpress module:
  xpress.init('/Applications/FICO Xpress/xpressmp/bin/xpauth.xpr')
FICO Xpress v9.2.2, Hyper, solve started 12:38:20, Mar 20, 2024
Heap usage: 390KB (peak 422KB, 120KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
         6 rows            6 cols           10 elements          6 entitie
s
Presolved problem has:
         0 rows            0 cols            0 elements          0 entitie
s
LP relaxation tightened
Presolve finished in 0 seconds
Heap usage: 394KB (peak 422KB, 120KB system)
Will try to keep branch and bound tree memory usage below 7.5GB
Starting concurrent solve with dual (1 thread)
```

```
 Concurrent-Solve,   0s
            Dual
    objective   dual inf
 D  42.000000   .0000000
------- optimal --------
Concurrent statistics:
      Dual: 0 simplex iterations, 0.00s
Optimal solution found

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
     0         42.000000      D      0     0        .000000     0
Dual solved problem
  0 simplex iterations in 0.00 seconds at time 0

Final objective                        : 4.200000000000000e+01
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0

Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del     Gap     GInf
Time
*           42.000000    42.000000      1                 0.00%       0
0
 *** Search completed ***
Uncrunching matrix
Final MIP objective                    : 4.200000000000000e+01
Final MIP bound                        : 4.200000000000000e+01
  Solution time / primaldual integral :      0.00s/ 73.931843%
  Number of solutions found / nodes   :         1 /         1
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
The total distance is 42.0m
The sequence is [0, 6]
```

In [5]:
```python
# Test 2
total_distance, sequence = shortest_path([50, 30, 0, 0, 0], distances_dat
print(f"The total distance is {total_distance}m")
print(f"The sequence is {sequence}")
```

```
FICO Xpress v9.2.2, Hyper, solve started 12:38:20, Mar 20, 2024
Heap usage: 396KB (peak 428KB, 123KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        20 rows           12 cols           42 elements        12 entitie
s
Presolved problem has:
         0 rows            0 cols            0 elements         0 entitie
s
Presolve finished in 0 seconds
Heap usage: 401KB (peak 428KB, 123KB system)
Will try to keep branch and bound tree memory usage below 7.5GB
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
           Dual
    objective   dual inf
 D  66.000000   .0000000
------- optimal --------
Concurrent statistics:
      Dual: 0 simplex iterations, 0.00s
Optimal solution found

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
     0         66.000000      D      0     0        .000000     0
Dual solved problem
  0 simplex iterations in 0.00 seconds at time 0

Final objective                       : 6.600000000000000e+01
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0

Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del     Gap     GInf
Time
*           66.000000    66.000000      1                  0.00%       0
0
 *** Search completed ***
Uncrunching matrix
Final MIP objective                   : 6.600000000000000e+01
Final MIP bound                       : 6.600000000000000e+01
  Solution time / primaldual integral :      0.02s/ 55.990721%
  Number of solutions found / nodes   :         1 /         1
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
The total distance is 66.0m
The sequence is [0, 30, 50]
```

In [6]:
```
# Test 3
total_distance, sequence = shortest_path([49, 18, 76, 0, 0], distances_da
print(f"The total distance is {total_distance}m")
print(f"The sequence is {sequence}")
```

```
FICO Xpress v9.2.2, Hyper, solve started 12:38:20, Mar 20, 2024
Heap usage: 400KB (peak 432KB, 126KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        34 rows           20 cols           82 elements        20 entitie
s
Presolved problem has:
        20 rows           15 cols           54 elements        15 entitie
s
Presolve finished in 0 seconds
Heap usage: 435KB (peak 435KB, 126KB system)

Coefficient range                    original                    solved
  Coefficients   [min,max] : [ 1.00e+00,  3.00e+00] / [ 5.00e-01,  1.50e+
00]
  RHS and bounds [min,max] : [ 1.00e+00,  2.00e+00] / [ 1.00e+00,  1.00e+
00]
  Objective      [min,max] : [ 9.00e+00,  7.50e+01] / [ 9.00e+00,  7.50e+
01]
Autoscaling applied standard scaling

Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   150.000000   Time:   0.01    Heuristic: e ***
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
            Dual
    objective   dual inf
 D  150.00000   .0000000
-------- cutoff --------
Concurrent statistics:
      Dual: 9 simplex iterations, 0.00s
Problem is cut off
 *** Search completed ***
Uncrunching matrix
Final MIP objective                   : 1.500000000000000e+02
Final MIP bound                       : 1.500000000000000e+02
  Solution time / primaldual integral :      0.01s/ 100.000000%
  Number of solutions found / nodes   :         1 /         0
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
The total distance is 150.0m
The sequence is [0, 49, 76, 18]
```

In [7]:
```
# Test 4
total_distance, sequence = shortest_path([88, 70, 35, 2, 0], distances_da
print(f"The total distance is {total_distance}m")
print(f"The sequence is {sequence}")
```

```
FICO Xpress v9.2.2, Hyper, solve started 12:38:20, Mar 20, 2024
Heap usage: 418KB (peak 451KB, 128KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        52 rows           30 cols          136 elements        30 entitie
s
Presolved problem has:
        32 rows           24 cols           96 elements        24 entitie
s
Presolve finished in 0 seconds
Heap usage: 455KB (peak 480KB, 128KB system)

Coefficient range                    original                    solved
  Coefficients   [min,max] : [ 1.00e+00,  4.00e+00] / [ 2.50e-01,  1.00e+
00]
  RHS and bounds [min,max] : [ 1.00e+00,  3.00e+00] / [ 7.50e-01,  1.00e+
00]
  Objective      [min,max] : [ 9.00e+00,  8.10e+01] / [ 9.00e+00,  8.10e+
01]
Autoscaling applied standard scaling

Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   186.000000   Time:   0.00    Heuristic: e ***
 *** Solution found:   168.000000   Time:   0.00    Heuristic: k ***
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
           Dual
    objective   dual inf
 D  168.00000   .0000000
-------- cutoff --------
Concurrent statistics:
      Dual: 12 simplex iterations, 0.00s
Problem is cut off
 *** Search completed ***
Uncrunching matrix
Final MIP objective                  : 1.680000000000000e+02
Final MIP bound                      : 1.680000000000000e+02
  Solution time / primaldual integral :      0.00s/ 100.000000%
  Number of solutions found / nodes   :         2 /         0
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
The total distance is 168.0m
The sequence is [0, 88, 70, 35, 2]
```

In [8]:
```
# Test 5
total_distance, sequence = shortest_path([39, 18, 29, 83, 49], distances_
print(f"The total distance is {total_distance}m")
print(f"The sequence is {sequence}")
```

```
FICO Xpress v9.2.2, Hyper, solve started 12:38:20, Mar 20, 2024
Heap usage: 427KB (peak 459KB, 131KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
```

```
OUTPUTLOG = 1
Original problem has:
        74 rows           42 cols          204 elements        42 entitie
s
Presolved problem has:
        47 rows           35 cols          150 elements        35 entitie
s
Presolve finished in 0 seconds
Heap usage: 465KB (peak 490KB, 131KB system)

Coefficient range                    original                    solved
  Coefficients   [min,max] : [ 1.00e+00,  5.00e+00] / [ 2.50e-01,  1.25e+
00]
  RHS and bounds [min,max] : [ 1.00e+00,  4.00e+00] / [ 1.00e+00,  1.00e+
00]
  Objective      [min,max] : [ 9.00e+00,  8.10e+01] / [ 9.00e+00,  8.10e+
01]
Autoscaling applied standard scaling

Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   264.000000   Time:   0.01    Heuristic: e ***
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
            Dual
    objective   dual inf
 D  156.00000   .0000000
------- optimal --------
Concurrent statistics:
      Dual: 18 simplex iterations, 0.00s
Optimal solution found

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
    18        156.000000      D      0     0        .000000     0
Dual solved problem
  18 simplex iterations in 0.00 seconds at time 0

Final objective                        : 1.560000000000000e+02
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0

Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del     Gap     GInf
Time
   1  K    264.000000   156.000000      1     34      0   40.91%      12
0
   2  K    264.000000   156.000000      1      9     29   40.91%       6
0
   3  K    264.000000   156.000000      1      3      8   40.91%       6
0
   4  K    264.000000   156.000000      1      8      4   40.91%       7
0
   5  K    264.000000   156.000000      1      3      7   40.91%       7
0
```

```
   6  K    264.000000   156.000000      1      1      4   40.91%        8
0
   7  K    264.000000   156.000000      1      1      0   40.91%        8
0
   8  K    264.000000   156.000000      1      3      2   40.91%        9
0
   9  K    264.000000   156.000000      1      2      2   40.91%        9
0
b          252.000000   156.000000      2                 38.10%        0
0
  10  K    252.000000   156.000000      2      1      2   38.10%        8
0
  11  K    252.000000   156.000000      2      0      1   38.10%        7
0
  12  G    252.000000   156.000000      2      7      0   38.10%        7
0
  13  G    252.000000   156.000000      2      6     13   38.10%        7
0
Heuristic search 'R' started
Heuristic search 'R' stopped

Cuts in the matrix         : 6
Cut elements in the matrix : 75

Starting tree search.
Deterministic mode with up to 4 running threads and up to 8 tasks.
Heap usage: 3431KB (peak 5920KB, 1055KB system)

    Node     BestSoln    BestBound   Sols Active  Depth     Gap     GInf
Time
       1   252.000000   163.200000      2      2      1   35.24%        7
0
a      2   168.000000   163.200000      3      2      3    2.86%        0
0
       4   168.000000   163.200000      3      2      3    2.86%        7
0
       7   168.000000   163.200000      3      1      3    2.86%       13
0
 *** Search completed ***
Uncrunching matrix
Final MIP objective                   : 1.680000000000000e+02
Final MIP bound                       : 1.680000000000000e+02
  Solution time / primaldual integral :      0.13s/ 47.550132%
  Number of solutions found / nodes   :         3 /         7
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
The total distance is 168.0m
The sequence is [0, 49, 83, 18, 39, 29]
```

Function to determine the total distance

We can now create a function `distance()` which takes a list of orders, calls `shortest_path()` and returns the minimum distance required to process all orders.

This has been implemented below, by firstly loading in the order data, and then defining and running the function.

In [9]:
```
# Load in the data file of orders
orders = pd.read_excel('OrderList.xlsx', sheet_name = "Orders")

# Drop the order numbers
orders = orders.drop(columns = "Order No.")

# Display a snippet of the data frame
orders
```

Out[9]:

	Position 1	Position 2	Position 3	Position 4	Position 5
0	50	30	0	0	0
1	49	18	76	0	0
2	72	52	51	41	35
3	50	4	0	0	0
4	76	19	26	80	6
...	...	...	...	...	...
1995	60	46	35	0	0
1996	8	43	70	77	31
1997	46	0	0	0	0
1998	90	23	64	35	0
1999	4	35	0	0	0

2000 rows × 5 columns

In [10]:
```
def distance(orders, distances_data):
    '''
    A function which takes as input a data frame of orders and a distance

    Returns the total distance required to fulfill all orders.

    '''

    # Define the intital total distance as 0
    tot_distance = 0

    # Loop over the number of orders
    for i in range(0, len(orders)):

        # Obtain the order information from the data frame
        order = orders.iloc[i].values.tolist()

        # Run the shortest path function and obtain the distance required
        total_distance, sequence = shortest_path(order, distances_data)

        # Add this distance to the running total
        tot_distance += total_distance

    # Return the total distance
    return tot_distance
```

In [11]:
```
# Define start time
start = time.time()

# Run the function for the above order list (only first 20 for computatio
tot_distance = distance(orders.head(20), distances_data)

# Define end time
end = time.time()

# Print outcome
print()
print(f"The total distance is {tot_distance} metres")
print(f"The run time of the function is {round(end - start, 4)} seconds")
```

```
FICO Xpress v9.2.2, Hyper, solve started 12:38:20, Mar 20, 2024
Heap usage: 396KB (peak 428KB, 134KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        20 rows           12 cols           42 elements        12 entitie
s
Presolved problem has:
         0 rows            0 cols            0 elements         0 entitie
s
Presolve finished in 0 seconds
Heap usage: 401KB (peak 428KB, 134KB system)
Will try to keep branch and bound tree memory usage below 7.5GB
Starting concurrent solve with dual (1 thread)
```

```
 Concurrent-Solve,   0s
            Dual
    objective   dual inf
 D  66.000000   .0000000
------- optimal --------
Concurrent statistics:
      Dual: 0 simplex iterations, 0.00s
Optimal solution found

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
     0         66.000000      D      0     0        .000000     0
Dual solved problem
  0 simplex iterations in 0.00 seconds at time 0

Final objective                        : 6.600000000000000e+01
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0

Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del     Gap     GInf
Time
*           66.000000    66.000000      1                  0.00%       0
0
 *** Search completed ***
Uncrunching matrix
Final MIP objective                    : 6.600000000000000e+01
Final MIP bound                        : 6.600000000000000e+01
  Solution time / primaldual integral :      0.00s/ 65.188978%
  Number of solutions found / nodes   :         1 /         1
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:20, Mar 20, 2024
Heap usage: 400KB (peak 432KB, 136KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        34 rows           20 cols           82 elements        20 entitie
s
Presolved problem has:
        20 rows           15 cols           54 elements        15 entitie
s
Presolve finished in 0 seconds
Heap usage: 435KB (peak 435KB, 136KB system)

Coefficient range                   original                  solved
  Coefficients   [min,max] : [ 1.00e+00,  3.00e+00] / [ 5.00e-01,  1.50e+
00]
  RHS and bounds [min,max] : [ 1.00e+00,  2.00e+00] / [ 1.00e+00,  1.00e+
00]
  Objective      [min,max] : [ 9.00e+00,  7.50e+01] / [ 9.00e+00,  7.50e+
01]
Autoscaling applied standard scaling
```

```
Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   150.000000   Time:   0.00    Heuristic: e ***
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
           Dual
    objective   dual inf
 D  150.00000   .0000000
-------- cutoff --------
Concurrent statistics:
      Dual: 9 simplex iterations, 0.00s
Problem is cut off
 *** Search completed ***
Uncrunching matrix
Final MIP objective                   : 1.500000000000000e+02
Final MIP bound                       : 1.500000000000000e+02
  Solution time / primaldual integral :      0.00s/ 100.000000%
  Number of solutions found / nodes   :         1 /         0
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:20, Mar 20, 2024
Heap usage: 427KB (peak 459KB, 139KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        74 rows           42 cols          204 elements        42 entitie
s
Presolved problem has:
        47 rows           35 cols          150 elements        35 entitie
s
Presolve finished in 0 seconds
Heap usage: 465KB (peak 490KB, 138KB system)

Coefficient range                    original                  solved
  Coefficients   [min,max] : [ 1.00e+00,  5.00e+00] / [ 2.50e-01,  1.25e+
00]
  RHS and bounds [min,max] : [ 1.00e+00,  4.00e+00] / [ 1.00e+00,  1.00e+
00]
  Objective      [min,max] : [ 3.00e+00,  7.80e+01] / [ 3.00e+00,  7.80e+
01]
Autoscaling applied standard scaling

Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   306.000000   Time:   0.00    Heuristic: e ***
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
           Dual
    objective   dual inf
 D  144.00000   .0000000
------- optimal --------
Concurrent statistics:
      Dual: 17 simplex iterations, 0.00s
Optimal solution found

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
```

```
    17        144.000000      D      0     0        .000000     0
Dual solved problem
  17 simplex iterations in 0.00 seconds at time 0

Final objective                        : 1.440000000000000e+02
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0

Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del      Gap      GInf
Time
   1  K    306.000000   144.000000      1     32      0   52.94%       12
0
   2  K    306.000000   144.000000      1      9     27   52.94%        6
0
   3  K    306.000000   144.000000      1      2      8   52.94%        6
0
   4  K    306.000000   144.000000      1      8      3   52.94%        7
0
   5  K    306.000000   144.000000      1      2      7   52.94%        7
0
   6  K    306.000000   144.000000      1      1      3   52.94%        8
0
   7  K    306.000000   144.000000      1      1      0   52.94%        8
0
b          246.000000   144.000000      2                  41.46%        0
0
   8  K    246.000000   144.000000      2      3      2   41.46%        9
0
   9  K    246.000000   144.000000      2      2      2   41.46%        9
0
b          234.000000   144.000000      3                  38.46%        0
0
  10  K    234.000000   144.000000      3      1      2   38.46%        8
0
  11  K    234.000000   144.000000      3      0      1   38.46%        7
0
  12  G    234.000000   144.000000      3      7      0   38.46%        7
0
  13  G    234.000000   144.000000      3      7     13   38.46%        7
0
Heuristic search 'R' started
Heuristic search 'R' stopped
M          210.000000   144.000000      4                  31.43%        0
0

Cuts in the matrix         : 7
Cut elements in the matrix : 90

Starting tree search.
Deterministic mode with up to 4 running threads and up to 8 tasks.
Heap usage: 3434KB (peak 5807KB, 1062KB system)

    Node     BestSoln    BestBound   Sols Active  Depth      Gap      GInf
```

```
Time
       1   210.000000   148.800000      4      2      1   29.14%        7
0
       2   210.000000   148.800000      4      2      3   29.14%       14
0
       3   210.000000   148.800000      4      2      3   29.14%       16
0
       4   210.000000   148.800000      4      2      3   29.14%       12
0
b      4   204.000000   148.800000      5      2      3   27.06%        0
0
       5   204.000000   148.800000      5      2      3   27.06%        1
0
b      6   168.000000   148.800000      6      1      4   11.43%        0
0
       8   168.000000   148.800000      6      1      4   11.43%       16
0
       9   168.000000   165.000005      6      0      1    1.79%        2
0
 *** Search completed ***
Uncrunching matrix
Final MIP objective                   : 1.680000000000000e+02
Final MIP bound                       : 1.680000000000000e+02
  Solution time / primaldual integral :      0.09s/ 41.756008%
  Number of solutions found / nodes   :         6 /         9
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:21, Mar 20, 2024
Heap usage: 396KB (peak 428KB, 141KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        20 rows           12 cols           42 elements        12 entitie
s
Presolved problem has:
         0 rows            0 cols            0 elements         0 entitie
s
Presolve finished in 0 seconds
Heap usage: 401KB (peak 428KB, 141KB system)
Will try to keep branch and bound tree memory usage below 7.5GB
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
            Dual
    objective   dual inf
 D  54.000000   .0000000
------- optimal --------
Concurrent statistics:
      Dual: 0 simplex iterations, 0.00s
Optimal solution found

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
     0         54.000000      D      0     0        .000000     0
Dual solved problem
  0 simplex iterations in 0.00 seconds at time 0
```

```
Final objective                       : 5.400000000000000e+01
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0

Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del     Gap     GInf
Time
*           54.000000    54.000000      1                  0.00%       0
0
 *** Search completed ***
Uncrunching matrix
Final MIP objective                   : 5.400000000000000e+01
Final MIP bound                       : 5.400000000000000e+01
  Solution time / primaldual integral :      0.00s/ 61.425465%
  Number of solutions found / nodes   :         1 /         1
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:21, Mar 20, 2024
Heap usage: 427KB (peak 459KB, 143KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        74 rows           42 cols          204 elements        42 entitie
s
Presolved problem has:
        47 rows           35 cols          150 elements        35 entitie
s
Presolve finished in 0 seconds
Heap usage: 465KB (peak 490KB, 143KB system)

Coefficient range                    original                  solved
  Coefficients   [min,max] : [ 1.00e+00,  5.00e+00] / [ 2.50e-01,  1.25e+
00]
  RHS and bounds [min,max] : [ 1.00e+00,  4.00e+00] / [ 1.00e+00,  1.00e+
00]
  Objective      [min,max] : [ 9.00e+00,  7.80e+01] / [ 9.00e+00,  7.80e+
01]
Autoscaling applied standard scaling

Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   222.000000   Time:   0.00    Heuristic: e ***
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
           Dual
    objective   dual inf
 D  162.00000   .0000000
------- optimal --------
Concurrent statistics:
      Dual: 18 simplex iterations, 0.00s
Optimal solution found

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
```

```
    18        162.000000      D      0     0        .000000     0
Dual solved problem
  18 simplex iterations in 0.00 seconds at time 0

Final objective                        : 1.620000000000000e+02
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0

Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del      Gap      GInf
Time
   1  K    222.000000   162.000000      1     34      0   27.03%       12
0
   2  K    222.000000   162.000000      1      9     29   27.03%        6
0
   3  K    222.000000   162.000000      1      3      8   27.03%        6
0
   4  K    222.000000   162.000000      1     10      4   27.03%        7
0
   5  K    222.000000   162.000000      1      3      9   27.03%        7
0
   6  K    222.000000   162.000000      1      1      4   27.03%        8
0
   7  K    222.000000   162.000000      1      1      0   27.03%        8
0
   8  K    222.000000   162.000000      1      3      2   27.03%        9
0
   9  K    222.000000   162.000000      1      2      2   27.03%        9
0
  10  K    222.000000   162.000000      1      1      2   27.03%        8
0
  11  K    222.000000   162.000000      1      0      1   27.03%        7
0
  12  G    222.000000   162.000000      1      7      0   27.03%        7
0
  13  G    222.000000   162.000000      1      6     11   27.03%        7
0
Heuristic search 'R' started
Heuristic search 'R' stopped

Cuts in the matrix         : 8
Cut elements in the matrix : 106

Starting tree search.
Deterministic mode with up to 4 running threads and up to 8 tasks.
Heap usage: 3431KB (peak 5819KB, 1067KB system)

    Node     BestSoln    BestBound   Sols Active  Depth      Gap      GInf
Time
       1   222.000000   165.600000      1      2      1   25.41%        7
0
a      2   168.000000   165.600000      2      2      3    1.43%        0
0
       4   168.000000   165.600000      2      2      3    1.43%        8
```

```
0
       7   168.000000   165.600000      2      1      3    1.43%      13
0
 *** Search completed ***
Uncrunching matrix
Final MIP objective                   : 1.680000000000000e+02
Final MIP bound                       : 1.680000000000000e+02
  Solution time / primaldual integral :      0.04s/ 35.470151%
  Number of solutions found / nodes   :         2 /         7
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:21, Mar 20, 2024
Heap usage: 427KB (peak 459KB, 145KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        74 rows           42 cols          204 elements        42 entitie
s
Presolved problem has:
        47 rows           35 cols          150 elements        35 entitie
s
Presolve finished in 0 seconds
Heap usage: 465KB (peak 490KB, 145KB system)

Coefficient range                    original                    solved
  Coefficients   [min,max] : [ 1.00e+00,  5.00e+00] / [ 2.50e-01,  1.25e+
00]
  RHS and bounds [min,max] : [ 1.00e+00,  4.00e+00] / [ 1.00e+00,  1.00e+
00]
  Objective      [min,max] : [ 3.00e+00,  7.50e+01] / [ 3.00e+00,  7.50e+
01]
Autoscaling applied standard scaling

Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   156.000000   Time:   0.00    Heuristic: e ***
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
           Dual
    objective   sum inf
 P  114.00000   .0000000
------- optimal --------
Concurrent statistics:
      Dual: 22 simplex iterations, 0.00s
Optimal solution found

   Its         Obj Value      S   Ninf  Nneg        Sum Inf  Time
    22        114.000000      P      0     0        .000000     0
Dual solved problem
  22 simplex iterations in 0.00 seconds at time 0

Final objective                       : 1.140000000000000e+02
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0
```

```
Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del     Gap     GInf
Time
   1  K    156.000000   119.400000      1     20      0   23.46%       6
0
b          150.000000   150.000000      2                  0.00%       0
0
 *** Search completed ***
Uncrunching matrix
Final MIP objective                   : 1.500000000000000e+02
Final MIP bound                       : 1.500000000000000e+02
  Solution time / primaldual integral :      0.01s/ 82.777262%
  Number of solutions found / nodes   :         2 /         1
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:21, Mar 20, 2024
Heap usage: 427KB (peak 459KB, 148KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        74 rows           42 cols          204 elements        42 entitie
s
Presolved problem has:
        47 rows           35 cols          150 elements        35 entitie
s
Presolve finished in 0 seconds
Heap usage: 465KB (peak 490KB, 148KB system)

Coefficient range                    original                 solved
  Coefficients   [min,max] : [ 1.00e+00,  5.00e+00] / [ 2.50e-01,  1.25e+
00]
  RHS and bounds [min,max] : [ 1.00e+00,  4.00e+00] / [ 1.00e+00,  1.00e+
00]
  Objective      [min,max] : [ 9.00e+00,  8.10e+01] / [ 9.00e+00,  8.10e+
01]
Autoscaling applied standard scaling

Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   264.000000   Time:   0.01    Heuristic: e ***
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
           Dual
    objective   dual inf
 D  156.00000   .0000000
------- optimal --------
Concurrent statistics:
      Dual: 18 simplex iterations, 0.00s
Optimal solution found

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
    18        156.000000      D      0     0        .000000     0
Dual solved problem
  18 simplex iterations in 0.00 seconds at time 0
```

```
Final objective                        : 1.560000000000000e+02
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0

Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del     Gap     GInf
Time
   1  K    264.000000   156.000000      1     34      0   40.91%       12
0
   2  K    264.000000   156.000000      1      9     29   40.91%        6
0
   3  K    264.000000   156.000000      1      3      8   40.91%        6
0
   4  K    264.000000   156.000000      1      8      4   40.91%        7
0
   5  K    264.000000   156.000000      1      3      7   40.91%        7
0
   6  K    264.000000   156.000000      1      1      4   40.91%        8
0
   7  K    264.000000   156.000000      1      1      0   40.91%        8
0
   8  K    264.000000   156.000000      1      3      2   40.91%        9
0
   9  K    264.000000   156.000000      1      2      2   40.91%        9
0
b          252.000000   156.000000      2                 38.10%        0
0
  10  K    252.000000   156.000000      2      1      2   38.10%        8
0
  11  K    252.000000   156.000000      2      0      1   38.10%        7
0
  12  G    252.000000   156.000000      2      7      0   38.10%        7
0
  13  G    252.000000   156.000000      2      6     13   38.10%        7
0
Heuristic search 'R' started
Heuristic search 'R' stopped

Cuts in the matrix         : 6
Cut elements in the matrix : 75

Starting tree search.
Deterministic mode with up to 4 running threads and up to 8 tasks.
Heap usage: 3431KB (peak 5920KB, 1071KB system)

    Node     BestSoln    BestBound   Sols Active  Depth     Gap     GInf
Time
       1   252.000000   163.200000      2      2      1   35.24%        7
0
a      2   168.000000   163.200000      3      2      3    2.86%        0
0
       4   168.000000   163.200000      3      2      3    2.86%        7
0
```

```
       7   168.000000   163.200000      3      1      3    2.86%      13
0
 *** Search completed ***
Uncrunching matrix
Final MIP objective                   : 1.680000000000000e+02
Final MIP bound                       : 1.680000000000000e+02
  Solution time / primaldual integral :      0.07s/ 41.709116%
  Number of solutions found / nodes   :         3 /         7
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:21, Mar 20, 2024
Heap usage: 396KB (peak 428KB, 150KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        20 rows           12 cols           42 elements        12 entitie
s
Presolved problem has:
         0 rows            0 cols            0 elements         0 entitie
s
Presolve finished in 0 seconds
Heap usage: 401KB (peak 428KB, 150KB system)
Will try to keep branch and bound tree memory usage below 7.5GB
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
            Dual
    objective   dual inf
 D  126.00000   .0000000
------- optimal --------
Concurrent statistics:
      Dual: 0 simplex iterations, 0.00s
Optimal solution found

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
     0        126.000000      D      0     0        .000000     0
Dual solved problem
  0 simplex iterations in 0.00 seconds at time 0

Final objective                       : 1.260000000000000e+02
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0

Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del     Gap     GInf
Time
*          126.000000   126.000000      1                  0.00%       0
0
 *** Search completed ***
Uncrunching matrix
Final MIP objective                   : 1.260000000000000e+02
Final MIP bound                       : 1.260000000000000e+02
  Solution time / primaldual integral :      0.00s/ 75.393781%
```

```
  Number of solutions found / nodes   :         1 /         1
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:21, Mar 20, 2024
Heap usage: 396KB (peak 428KB, 152KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        20 rows           12 cols           42 elements        12 entitie
s
Presolved problem has:
         0 rows            0 cols            0 elements         0 entitie
s
Presolve finished in 0 seconds
Heap usage: 401KB (peak 428KB, 152KB system)
Will try to keep branch and bound tree memory usage below 7.5GB
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
           Dual
    objective   dual inf
 D  168.00000   .0000000
------- optimal --------
Concurrent statistics:
      Dual: 0 simplex iterations, 0.00s
Optimal solution found

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
     0        168.000000      D      0     0        .000000     0
Dual solved problem
  0 simplex iterations in 0.00 seconds at time 0

Final objective                       : 1.680000000000000e+02
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0

Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del     Gap     GInf
Time
*          168.000000   168.000000      1                  0.00%       0
0
 *** Search completed ***
Uncrunching matrix
Final MIP objective                   : 1.680000000000000e+02
Final MIP bound                       : 1.680000000000000e+02
  Solution time / primaldual integral :      0.01s/ 51.001526%
  Number of solutions found / nodes   :         1 /         1
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:21, Mar 20, 2024
Heap usage: 427KB (peak 459KB, 155KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
```

```
OUTPUTLOG = 1
Original problem has:
        74 rows           42 cols          204 elements        42 entitie
s
Presolved problem has:
        47 rows           35 cols          150 elements        35 entitie
s
Presolve finished in 0 seconds
Heap usage: 465KB (peak 490KB, 155KB system)

Coefficient range                    original                    solved
  Coefficients   [min,max] : [ 1.00e+00,  5.00e+00] / [ 2.50e-01,  1.25e+
00]
  RHS and bounds [min,max] : [ 1.00e+00,  4.00e+00] / [ 1.00e+00,  1.00e+
00]
  Objective      [min,max] : [ 3.00e+00,  5.70e+01] / [ 3.00e+00,  5.70e+
01]
Autoscaling applied standard scaling

Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   168.000000   Time:   0.00    Heuristic: e ***
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
            Dual
    objective   dual inf
 D  96.000000   .0000000
------- optimal --------
Concurrent statistics:
      Dual: 17 simplex iterations, 0.00s
Optimal solution found

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
    17         96.000000      D      0     0        .000000     0
Dual solved problem
  17 simplex iterations in 0.00 seconds at time 0

Final objective                        : 9.599999999999999e+01
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0

Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del     Gap     GInf
Time
   1  K    168.000000    96.000000      1     32      0   42.86%      12
0
   2  K    168.000000    96.000000      1      9     27   42.86%       6
0
   3  K    168.000000    96.000000      1      2      8   42.86%       6
0
b          150.000000    96.000000      2                  36.00%       0
0
   4  K    150.000000    96.000000      2      8      3   36.00%       7
0
```

```
   5  K    150.000000    96.000000      2      6      7   36.00%       7
0
b          144.000000    96.000000      3                 33.33%       0
0
   6  K    144.000000    96.000000      3      4      7   33.33%       8
0
   7  K    144.000000    96.000000      3      0      4   33.33%       8
0
   8  G    144.000000    96.000000      3      7      0   33.33%       8
0
   9  G    144.000000    98.057143      3      8      9   31.90%       9
0
  10  G    144.000000   114.000000      3     11      7   20.83%      12
0
  11  G    144.000000   114.000000      3     19     21   20.83%      14
0
g          132.000000   114.000000      4                 13.64%       0
0
q          114.000000   114.000000      5                 -0.00%       0
0
 *** Search completed ***
Uncrunching matrix
Final MIP objective                   : 1.140000000000000e+02
Final MIP bound                       : 1.140000000000000e+02
  Solution time / primaldual integral :      0.03s/ 47.556261%
  Number of solutions found / nodes   :         5 /         1
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:21, Mar 20, 2024
Heap usage: 427KB (peak 459KB, 157KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        74 rows           42 cols          204 elements        42 entitie
s
Presolved problem has:
        47 rows           35 cols          150 elements        35 entitie
s
Presolve finished in 0 seconds
Heap usage: 465KB (peak 490KB, 157KB system)

Coefficient range                    original                 solved
  Coefficients   [min,max] : [ 1.00e+00,  5.00e+00] / [ 2.50e-01,  1.25e+
00]
  RHS and bounds [min,max] : [ 1.00e+00,  4.00e+00] / [ 1.00e+00,  1.00e+
00]
  Objective      [min,max] : [ 3.00e+00,  8.10e+01] / [ 3.00e+00,  8.10e+
01]
Autoscaling applied standard scaling

Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   252.000000   Time:   0.01    Heuristic: e ***
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
           Dual
```

```
    objective   dual inf
 D  168.00000   .0000000
------- optimal --------
Concurrent statistics:
      Dual: 21 simplex iterations, 0.00s
Optimal solution found

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
    21        168.000000      D      0     0        .000000     0
Dual solved problem
  21 simplex iterations in 0.00 seconds at time 0

Final objective                        : 1.680000000000000e+02
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0

Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del     Gap     GInf
Time
a          168.000000   168.000000      2                  0.00%       0
0
 *** Search completed ***
Uncrunching matrix
Final MIP objective                    : 1.680000000000000e+02
Final MIP bound                        : 1.680000000000000e+02
  Solution time / primaldual integral :      0.01s/ 75.426206%
  Number of solutions found / nodes   :         2 /         1
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:21, Mar 20, 2024
Heap usage: 427KB (peak 459KB, 160KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        74 rows           42 cols          204 elements        42 entitie
s
Presolved problem has:
        47 rows           35 cols          150 elements        35 entitie
s
Presolve finished in 0 seconds
Heap usage: 465KB (peak 490KB, 160KB system)

Coefficient range                    original                 solved
  Coefficients   [min,max] : [ 1.00e+00,  5.00e+00] / [ 2.50e-01,  1.25e+
00]
  RHS and bounds [min,max] : [ 1.00e+00,  4.00e+00] / [ 1.00e+00,  1.00e+
00]
  Objective      [min,max] : [ 3.00e+00,  8.40e+01] / [ 3.00e+00,  8.40e+
01]
Autoscaling applied standard scaling

Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   216.000000   Time:   0.00    Heuristic: e ***
```

```
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
           Dual
    objective   dual inf
 D  156.00000   .0000000
------- optimal --------
Concurrent statistics:
      Dual: 21 simplex iterations, 0.00s
Optimal solution found

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
    21        156.000000      D      0     0        .000000     0
Dual solved problem
  21 simplex iterations in 0.00 seconds at time 0

Final objective                         : 1.560000000000000e+02
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0

Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del     Gap     GInf
Time
   1  K    216.000000   156.000000      1     20      0   27.78%       6
0
b          204.000000   156.000000      2                 23.53%       0
0
   2  K    204.000000   156.000000      2     10     18   23.53%       6
0
   3  K    204.000000   156.000000      2      4      8   23.53%       7
0
   4  K    204.000000   156.000000      2      4      4   23.53%       7
0
   5  K    204.000000   156.000000      2      7      5   23.53%       9
0
   6  K    204.000000   156.000000      2      4      6   23.53%       9
0
   7  K    204.000000   156.000000      2      4      4   23.53%       8
0
   8  K    204.000000   156.000000      2      4      5   23.53%       9
0
   9  K    204.000000   156.000000      2      4      4   23.53%       9
0
  10  K    204.000000   156.000000      2      1      3   23.53%       7
0
  11  K    204.000000   156.000000      2      1      1   23.53%       6
0
  12  K    204.000000   156.000000      2      5      3   23.53%       8
0
  13  K    204.000000   156.000000      2      6      4   23.53%       8
0
  14  K    204.000000   156.000000      2      5      5   23.53%       8
0
  15  K    204.000000   156.000000      2     11     16   23.53%       8
```

```
0
  16  G    204.000000   156.000000      2      7      0   23.53%       8
0
  17  G    204.000000   156.000000      2     11     18   23.53%       7
0
Heuristic search 'R' started
Heuristic search 'R' stopped
M          174.000000   156.000000      3                 10.34%       0
0

Cuts in the matrix         : 4
Cut elements in the matrix : 79

Performing root presolve...

Reduced problem has:      30 rows      17 columns       103 elements
Presolve dropped   :      21 rows      18 columns       126 elements
Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   168.000000   Time:   0.06    Heuristic: e ***
Crash basis containing 4 structural columns created

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
    80        156.000000      D      0     0        .000000     0
Optimal solution found
Dual solved problem
  80 simplex iterations in 0.00 seconds at time 0

Final objective                       : 1.560000000000000e+02
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0

Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del     Gap     GInf
Time
   1  K    168.000000   156.000000      4      8      4    7.14%       6
0
   2  K    168.000000   156.000000      4      3     10    7.14%      10
0
Heuristic search 'R' started
Heuristic search 'R' stopped

Cuts in the matrix         : 1
Cut elements in the matrix : 3
 *** Search completed ***
Uncrunching matrix
Final MIP objective                   : 1.680000000000000e+02
Final MIP bound                       : 1.680000000000000e+02
  Solution time / primaldual integral :      0.08s/ 31.034047%
  Number of solutions found / nodes   :         4 /         1
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:21, Mar 20, 2024
Heap usage: 418KB (peak 451KB, 163KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
```

```
these control settings:
OUTPUTLOG = 1
Original problem has:
        52 rows           30 cols          136 elements        30 entitie
s
Presolved problem has:
        32 rows           24 cols           96 elements        24 entitie
s
Presolve finished in 0 seconds
Heap usage: 455KB (peak 480KB, 163KB system)

Coefficient range                    original                 solved
  Coefficients   [min,max] : [ 1.00e+00,  4.00e+00] / [ 2.50e-01,  1.00e+
00]
  RHS and bounds [min,max] : [ 1.00e+00,  3.00e+00] / [ 7.50e-01,  1.00e+
00]
  Objective      [min,max] : [ 9.00e+00,  8.10e+01] / [ 9.00e+00,  8.10e+
01]
Autoscaling applied standard scaling

Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   186.000000   Time:   0.01    Heuristic: e ***
 *** Solution found:   168.000000   Time:   0.01    Heuristic: k ***
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
           Dual
    objective   dual inf
 D  168.00000   .0000000
-------- cutoff --------
Concurrent statistics:
      Dual: 12 simplex iterations, 0.00s
Problem is cut off
 *** Search completed ***
Uncrunching matrix
Final MIP objective                  : 1.680000000000000e+02
Final MIP bound                      : 1.680000000000000e+02
  Solution time / primaldual integral :      0.01s/ 100.000000%
  Number of solutions found / nodes   :         2 /         0
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:21, Mar 20, 2024
Heap usage: 427KB (peak 459KB, 165KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        74 rows           42 cols          204 elements        42 entitie
s
Presolved problem has:
        47 rows           35 cols          150 elements        35 entitie
s
Presolve finished in 0 seconds
Heap usage: 465KB (peak 490KB, 165KB system)

Coefficient range                    original                 solved
  Coefficients   [min,max] : [ 1.00e+00,  5.00e+00] / [ 2.50e-01,  1.25e+
```

```
00]
  RHS and bounds [min,max] : [ 1.00e+00,  4.00e+00] / [ 1.00e+00,  1.00e+
00]
  Objective      [min,max] : [ 3.00e+00,  7.50e+01] / [ 3.00e+00,  7.50e+
01]
Autoscaling applied standard scaling

Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   222.000000   Time:   0.01    Heuristic: e ***
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
            Dual
    objective   dual inf
 D  168.00000   .0000000
------- optimal --------
Concurrent statistics:
      Dual: 20 simplex iterations, 0.00s
Optimal solution found

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
    20        168.000000      D      0     0        .000000     0
Dual solved problem
  20 simplex iterations in 0.00 seconds at time 0

Final objective                        : 1.680000000000000e+02
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0

Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del     Gap     GInf
Time
a          168.000000   168.000000      2                  0.00%       0
0
 *** Search completed ***
Uncrunching matrix
Final MIP objective                    : 1.680000000000000e+02
Final MIP bound                        : 1.680000000000000e+02
  Solution time / primaldual integral :      0.01s/ 74.925200%
  Number of solutions found / nodes   :         2 /         1
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:21, Mar 20, 2024
Heap usage: 418KB (peak 451KB, 167KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        52 rows           30 cols          136 elements        30 entitie
s
Presolved problem has:
        32 rows           24 cols           96 elements        24 entitie
s
Presolve finished in 0 seconds
```

```
Heap usage: 455KB (peak 480KB, 167KB system)

Coefficient range                    original                    solved
  Coefficients   [min,max] : [ 1.00e+00,  4.00e+00] / [ 2.50e-01,  1.00e+
00]
  RHS and bounds [min,max] : [ 1.00e+00,  3.00e+00] / [ 7.50e-01,  1.00e+
00]
  Objective      [min,max] : [ 3.00e+00,  8.10e+01] / [ 3.00e+00,  8.10e+
01]
Autoscaling applied standard scaling

Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   222.000000   Time:   0.00    Heuristic: e ***
 *** Solution found:   168.000000   Time:   0.00    Heuristic: k ***
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
           Dual
    objective   dual inf
 D  168.00000   .0000000
-------- cutoff --------
Concurrent statistics:
      Dual: 12 simplex iterations, 0.00s
Problem is cut off
 *** Search completed ***
Uncrunching matrix
Final MIP objective                  : 1.6800000000000e+02
Final MIP bound                      : 1.6800000000000e+02
  Solution time / primaldual integral :      0.00s/ 100.000000%
  Number of solutions found / nodes   :         2 /         0
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:21, Mar 20, 2024
Heap usage: 400KB (peak 432KB, 169KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        34 rows           20 cols           82 elements        20 entitie
s
Presolved problem has:
        20 rows           15 cols           54 elements        15 entitie
s
Presolve finished in 0 seconds
Heap usage: 435KB (peak 435KB, 169KB system)

Coefficient range                    original                    solved
  Coefficients   [min,max] : [ 1.00e+00,  3.00e+00] / [ 5.00e-01,  1.50e+
00]
  RHS and bounds [min,max] : [ 1.00e+00,  2.00e+00] / [ 1.00e+00,  1.00e+
00]
  Objective      [min,max] : [ 2.10e+01,  7.20e+01] / [ 2.10e+01,  7.20e+
01]
Autoscaling applied standard scaling

Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   210.000000   Time:   0.00    Heuristic: e ***
```

```
 *** Solution found:   168.000000   Time:   0.00    Heuristic: k ***
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
           Dual
    objective   dual inf
 D  168.00000   .0000000
-------- cutoff --------
Concurrent statistics:
      Dual: 10 simplex iterations, 0.00s
Problem is cut off
 *** Search completed ***
Uncrunching matrix
Final MIP objective                   : 1.680000000000000e+02
Final MIP bound                       : 1.680000000000000e+02
  Solution time / primaldual integral :      0.00s/ 100.000000%
  Number of solutions found / nodes   :         2 /         0
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:21, Mar 20, 2024
Heap usage: 400KB (peak 432KB, 172KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        34 rows           20 cols           82 elements        20 entitie
s
Presolved problem has:
        20 rows           15 cols           54 elements        15 entitie
s
Presolve finished in 0 seconds
Heap usage: 435KB (peak 435KB, 172KB system)

Coefficient range                    original                  solved
  Coefficients   [min,max] : [ 1.00e+00,  3.00e+00] / [ 5.00e-01,  1.50e+
00]
  RHS and bounds [min,max] : [ 1.00e+00,  2.00e+00] / [ 1.00e+00,  1.00e+
00]
  Objective      [min,max] : [ 9.00e+00,  4.50e+01] / [ 9.00e+00,  4.50e+
01]
Autoscaling applied standard scaling

Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:    90.000000   Time:   0.01    Heuristic: e ***
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
           Dual
    objective   dual inf
 D  90.000000   .0000000
-------- cutoff --------
Concurrent statistics:
      Dual: 10 simplex iterations, 0.00s
Problem is cut off
 *** Search completed ***
Uncrunching matrix
Final MIP objective                    : 9.000000000000000e+01
```

```
Final MIP bound                        : 9.000000000000000e+01
  Solution time / primaldual integral :      0.01s/ 100.000000%
  Number of solutions found / nodes   :         1 /         0
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:21, Mar 20, 2024
Heap usage: 396KB (peak 428KB, 174KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        20 rows           12 cols           42 elements        12 entitie
s
Presolved problem has:
         0 rows            0 cols            0 elements         0 entitie
s
Presolve finished in 0 seconds
Heap usage: 401KB (peak 428KB, 174KB system)
Will try to keep branch and bound tree memory usage below 7.5GB
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
            Dual
    objective   dual inf
 D  168.00000   .0000000
------- optimal --------
Concurrent statistics:
      Dual: 0 simplex iterations, 0.00s
Optimal solution found

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
     0        168.000000      D      0     0        .000000     0
Dual solved problem
  0 simplex iterations in 0.00 seconds at time 0

Final objective                        : 1.680000000000000e+02
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0

Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del     Gap     GInf
Time
*          168.000000   168.000000      1                  0.00%       0
0
 *** Search completed ***
Uncrunching matrix
Final MIP objective                    : 1.680000000000000e+02
Final MIP bound                        : 1.680000000000000e+02
  Solution time / primaldual integral :      0.00s/ 61.307762%
  Number of solutions found / nodes   :         1 /         1
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:21, Mar 20, 2024
Heap usage: 427KB (peak 459KB, 177KB system)
```

```
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        74 rows           42 cols          204 elements        42 entitie
s
Presolved problem has:
        47 rows           35 cols          150 elements        35 entitie
s
Presolve finished in 0 seconds
Heap usage: 465KB (peak 490KB, 177KB system)

Coefficient range                   original                   solved
  Coefficients   [min,max] : [ 1.00e+00,  5.00e+00] / [ 2.50e-01,  1.25e+
00]
  RHS and bounds [min,max] : [ 1.00e+00,  4.00e+00] / [ 1.00e+00,  1.00e+
00]
  Objective      [min,max] : [ 6.00e+00,  8.40e+01] / [ 6.00e+00,  8.40e+
01]
Autoscaling applied standard scaling

Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   264.000000   Time:   0.01    Heuristic: e ***
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
            Dual
    objective   dual inf
 D  156.00000   .0000000
------- optimal --------
Concurrent statistics:
      Dual: 17 simplex iterations, 0.00s
Optimal solution found

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
    17        156.000000      D      0     0        .000000     0
Dual solved problem
  17 simplex iterations in 0.00 seconds at time 0

Final objective                        : 1.560000000000000e+02
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0

Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del     Gap     GInf
Time
   1  K    264.000000   156.000000      1     34      0   40.91%      12
0
   2  K    264.000000   156.000000      1     10     29   40.91%       6
0
   3  K    264.000000   156.000000      1      4      9   40.91%       6
0
   4  K    264.000000   156.000000      1     10      5   40.91%       7
0
```

```
   5  K    264.000000   156.000000      1      4      9   40.91%        7
0
   6  K    264.000000   156.000000      1      1      5   40.91%        8
0
   7  K    264.000000   156.000000      1      1      0   40.91%        8
0
b          210.000000   156.000000      2                  25.71%        0
0
   8  K    210.000000   156.000000      2      3      2   25.71%        9
0
   9  K    210.000000   156.000000      2      2      2   25.71%        9
0
b          180.000000   156.000000      3                  13.33%        0
0
  10  K    180.000000   156.000000      3      1      2   13.33%        8
0

Cuts in the matrix         : 7
Cut elements in the matrix : 58

Performing root presolve...

Reduced problem has:      26 rows      13 columns        83 elements
Presolve dropped   :      28 rows      22 columns       125 elements
Presolve tightened :         2 elements
Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   168.000000   Time:   0.04    Heuristic: e ***
Crash basis containing 4 structural columns created

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
    43        156.000000      D      0     0        .000000     0
Optimal solution found
Dual solved problem
  43 simplex iterations in 0.00 seconds at time 0

Final objective                       : 1.560000000000000e+02
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0

Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del     Gap     GInf
Time
   1  K    168.000000   156.000000      4     12      7    7.14%        5
0
   2  K    168.000000   156.000000      4      3      8    7.14%        6
0
   3  K    168.000000   156.000000      4      3      4    7.14%        9
0
   4  K    168.000000   156.000000      4      5      3    7.14%        7
0
   5  K    168.000000   156.000000      4      3     10    7.14%        7
0
Heuristic search 'R' started
Heuristic search 'R' stopped
```

```
Cuts in the matrix         : 1
Cut elements in the matrix : 2
 *** Search completed ***
Uncrunching matrix
Final MIP objective                    : 1.6800000000000000e+02
Final MIP bound                        : 1.6800000000000000e+02
  Solution time / primaldual integral :      0.07s/ 33.593342%
  Number of solutions found / nodes   :         4 /         1
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0
FICO Xpress v9.2.2, Hyper, solve started 12:38:21, Mar 20, 2024
Heap usage: 427KB (peak 459KB, 179KB system)
Minimizing MILP Prob using up to 4 threads and up to 8192MB memory, with
these control settings:
OUTPUTLOG = 1
Original problem has:
        74 rows           42 cols          204 elements        42 entitie
s
Presolved problem has:
        47 rows           35 cols          150 elements        35 entitie
s
Presolve finished in 0 seconds
Heap usage: 465KB (peak 490KB, 179KB system)

Coefficient range                    original                    solved
  Coefficients   [min,max] : [ 1.00e+00,  5.00e+00] / [ 2.50e-01,  1.25e+
00]
  RHS and bounds [min,max] : [ 1.00e+00,  4.00e+00] / [ 1.00e+00,  1.00e+
00]
  Objective      [min,max] : [ 3.00e+00,  7.80e+01] / [ 3.00e+00,  7.80e+
01]
Autoscaling applied standard scaling

Will try to keep branch and bound tree memory usage below 7.5GB
 *** Solution found:   234.000000   Time:   0.00    Heuristic: e ***
Starting concurrent solve with dual (1 thread)

 Concurrent-Solve,   0s
           Dual
    objective   dual inf
 D  120.00000   .0000000
------- optimal --------
Concurrent statistics:
      Dual: 16 simplex iterations, 0.00s
Optimal solution found

   Its         Obj Value      S   Ninf  Nneg   Sum Dual Inf  Time
    16        120.000000      D      0     0        .000000     0
Dual solved problem
  16 simplex iterations in 0.00 seconds at time 0

Final objective                        : 1.2000000000000000e+02
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max dual violation        (abs/rel) :       0.0 /       0.0
  Max complementarity viol. (abs/rel) :       0.0 /       0.0
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Starting root cutting & heuristics
Deterministic mode with up to 1 additional thread

 Its Type    BestSoln    BestBound   Sols    Add    Del     Gap     GInf
Time
   1  K    234.000000   120.000000      1     34      0   48.72%       12
0
   2  K    234.000000   120.000000      1      9     29   48.72%        6
0
   3  K    234.000000   120.000000      1      3      8   48.72%        6
0
   4  K    234.000000   120.000000      1      9      4   48.72%        7
0
   5  K    234.000000   120.000000      1      1      8   48.72%        7
0
b          198.000000   120.000000      2             39.39%        0
0
   6  K    198.000000   120.000000      2      1      2   39.39%        8
0
   7  K    198.000000   120.000000      2      1      0   39.39%        8
0
   8  K    198.000000   120.000000      2      3      2   39.39%        9
0
   9  K    198.000000   120.000000      2      2      2   39.39%        9
0
  10  K    198.000000   120.000000      2      1      2   39.39%        8
0
  11  K    198.000000   120.000000      2      1      2   39.39%        8
0
  12  K    198.000000   120.000000      2      4      0   39.39%        8
0
  13  K    198.000000   120.000000      2      2      4   39.39%        8
0
  14  K    198.000000   120.000000      2      2      2   39.39%        6
0
  15  K    198.000000   120.000000      2      0      3   39.39%        8
0
  16  G    198.000000   120.000000      2      8      0   39.39%        8
0
  17  G    198.000000   120.000000      2      9     15   39.39%        9
0
Heuristic search 'R' started
Heuristic search 'R' stopped

Cuts in the matrix         : 7
Cut elements in the matrix : 104

Starting tree search.
Deterministic mode with up to 4 running threads and up to 8 tasks.
Heap usage: 3436KB (peak 5922KB, 1102KB system)

    Node     BestSoln    BestBound   Sols Active  Depth     Gap     GInf
Time
       1   198.000000   129.600000      2      2      1   34.55%        6
0
       2   198.000000   129.600000      2      2      3   34.55%        8
0
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       3   198.000000   129.600000      2      2      3   34.55%        4
0
a      3   174.000000   129.600000      3      2      3   25.52%        0
0
       4   174.000000   129.600000      3      2      3   25.52%        1
0
       5   174.000000   129.600000      3      1      1   25.52%        2
0
       6   174.000000   129.600000      3      1      4   25.52%        4
0
       7   174.000000   129.600000      3      1      4   25.52%        1
0
b      7   168.000000   168.000000      4      2      4    0.00%        0
0
STOPPING - MIPRELSTOP target reached (MIPRELSTOP=0.0001  gap=0).
       8   168.000000   168.000000      4      2      4    0.00%        1
0
STOPPING - MIPRELSTOP target reached (MIPRELSTOP=0.0001  gap=0).
 *** Search completed ***
Uncrunching matrix
Final MIP objective                   : 1.6800000000000000e+02
Final MIP bound                       : 1.6800000000000000e+02
  Solution time / primaldual integral :      0.08s/ 46.959974%
  Number of solutions found / nodes   :         4 /         8
  Max primal violation      (abs/rel) :       0.0 /       0.0
  Max integer violation     (abs    ) :       0.0

The total distance is 2934.0 metres
The run time of the function is 0.8349 seconds
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In [ ]: