Introduction to QAOA with Qiskit

For Masters Students
(with basic knowledge in computer science & quantum computing)

TABLE OF CONTENTS

01

Motivation

What is Quantum Computing?

Quantum Superiority

Companies globally are contributing billions of dollars towards research in quantum computing. Quantum computers can enable extreme advances in discovery and analysis with their extremely high processing power.

QAOA & QISKIT

Quantum Approximate Optimization Algorithm

- Harnesses quantum superiority
- Optimization problem

Qiskit

- Python package created by IBM
- Open source
- Simulation and running of quantum computers

QAOA Applications across industries

Pharmaceutical

QAOA is used to for optimization of imaging techniques in the pharmaceutical industry.

Finance

In the finance sector, quantum computing is being used in portfolio optimization and to enable faster trading possibilities.

Logistics

QAOA can be used to solve logistics optimization problems such as the traveling salesman problem.

Timeline

1920s

Quantum mechanics is first developed by Neils Bohr.

1998

Isaac Chuang, Neil Gershenfeld, and Mark Kubinec create the first quantum computer.

2014

QAOA Algorithm is introduced by Fahri et al.

2017

Qiskit package is created for Python.

Future: further implementations of QAOA and quantum computing.

Motivation

Qiskit provides a detailed documentation and comprehensive educational materials. Those materials are sometimes inaccessible to people without a deep background in quantum computing or a degree in physics.

Goal

Introduce motivation and the basic concepts of QAOA via an implementation and usage example. We aim to make QAOA usable by non-physicists for optimization problems in their domain.

02

QAOA Theory

Cost function

$$C(x) = \sum_{i,j=1}^{n} x_i Q_{ij} x_j + \sum_{i=1}^{n} c_i x_i$$

QUBO cost function ⟶ Hamiltonian

Maximize C(x) ⇔ find the ground state* of the Hamiltonian

*System is in its ground state when the corresponding energy is minimal

Adiabatic Theorem

"A physical system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough and if there is a gap between the eigenvalue and the rest of the Hamiltonian's spectrum"

Max Born and Wladimir Fock

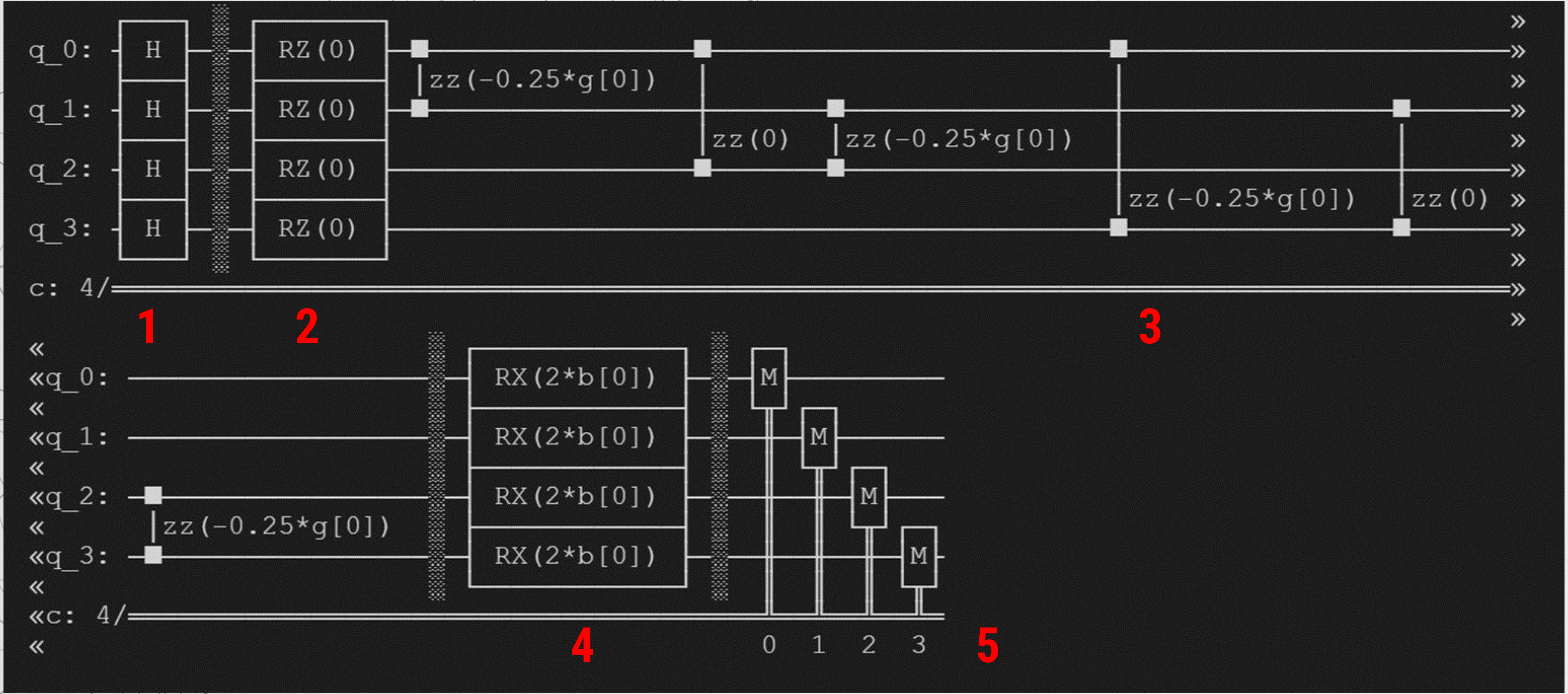
- $H(t) = \frac{t}{T}\mathrm{H_C} + \left(1 - \frac{t}{T}\right)\mathrm{H_M}$
- $x(t) = e^{\frac{-iHt}{\hbar}}\mathrm{x}(0)$
- $e^{-i(A+B)t} \approx \left(e^{-iAt/p}e^{-iBt/p}\right)^p$

$$e^{-iH(T)} \approx e^{-iH_C\gamma_p}e^{-iH_M\beta_p} * \ldots * e^{-iH_C\gamma_1}e^{-iH_M\beta_1}$$

$$\boxed{e^{-iH_C\gamma_1} = \prod_{i=1}^{n} Rz_i(w_i * \gamma) \prod_{i,j=1}^{n} Rz_i z_j(w_{ij} * \gamma)}$$

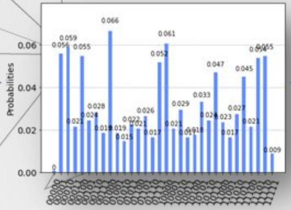
$$\boxed{e^{-iH_M\beta} = \prod_{i=1}^{n} \mathrm{Rx}(2\beta)}$$

QAOA Circuit Structure

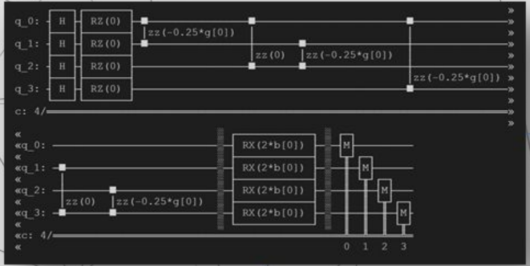


QAOA - An Overview

Measure $|s(\theta)>$

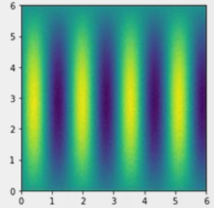


Variational Quantum Circuit



$$E(s) = \sum_{i=1}^{N} \frac{E(s_i) * P(s_i)}{N}$$

Classical optimizer



$$\theta_{i+1} = \theta_i - \lambda \frac{\partial E(\theta)}{\partial \theta}$$

02 Theory continuation (link)
03 Max Cut Problem
04 QAOA using Qiskit (link)

05

QAOA in Practice

Outlook to QAOA applications in the far future

Determining the optimal route for trucks and machines within a production facility is non-trivial and expensive.

Software testing involves searching in a high-dimensional search space, which is very costly in time and money.

Test-vehicles are designed to test new feature combinations. Reducing the number of vehicle designs by efficiently assigning tested features needs.

Current Supply Chains can be approximated by linear programming. Transforming these linear results into lot sizes is complex and slow.

PVC foam for corrosion protection Is applied by multiple robots with multiple tools. Collision free and fast coverage of all seams is only solved approximately today.

Disease spread control has a multitude of dependent parameters which change in real time and require to adjust the “optimal” treatment with limited resources (logistics, multi factor)“.

Additional Resources

Use these for self-learning or further reading

1. Documentation on solving optimization problems using Qiskit
   https://qiskit.org/textbook/ch-applications/qaoa.html

2. Additional applications of QAOA
   https://www.qutac.de/wp-content/uploads/2021/06/QUTAC_Paper.pdf

3. QAOA Algorithm explained - for better understanding of theory
   https://www.mustythoughts.com/quantum-approximate-optimization-algorithm-explained

References

1. Industry Applications: https://www.qutac.de/wp-content/uploads/2021/06/QUTAC_Paper.pdf
2. Quantum Computing: https://www.ibm.com/topics/quantum-computing
3. Quantum Physics: https://www.newscientist.com/definition/quantum-physics/
4. Qiskit textbook: https://qiskit.org/textbook/ch-applications/qaoa.html
5. Outside Industry Applications: https://www.forbes.com/sites/forbestechcouncil/2021/07/30/four-ways-quantum-computing-could-change-the-world/?sh=7344dd9d4602
6. Slide Formatting: https://slidesgo.com/theme/tech-newsletter#search-gray+professional&position-18&results-142

Homework Exercise

Solve an instance of the maxcut problem using QAOA through Qiskit.
Homework Exercise

Solutions provided in Github for lecturer’s reference.