A biased random-key genetic algorithm for the Multi-period, Multi-rate and Multi-channels with variable bandwidth Scheduling Problem

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Abstract

This paper introduces a new combinatorial optimization problem that arises from a new technology described in the IEEE 802.11ac standard. The latter enables a higher transmission speed than the previous IEEE standards, because it allows a higher flexibility on how the network frequency spectrum is divided into communication channels. The problem consists in assigning the time slot, channel and bandwidth for a given set of communications links. We propose a biased random-key genetic algorithm for this problem. Computational experiments showed that this heuristic can significantly improve the network throughput by exploiting the flexibility in the channel bandwidth.

1 Introduction

Wireless networks are becoming ubiquitous, present almost everywhere but their demands continue to grow [5]. Given this increasing demand, new techniques and standard are always being developed. The most recently communication standard in Wi-Fi networks is the IEEE 802.11ac [18]. This standard enables a higher transmission speed than the previous IEEE standards because it allows the network frequency spectrum to be divided into communication channels of different bandwidths. The communication channels can have variable bandwidth, varying from 20 MHz to 160 Mhz. This new feature of varying bandwidth channels brings new challenges on how to allocate these channels. Therefore, we address this problem and propose a new algorithm to solve it.

But, scheduling wireless links is not so simple. We have to consider its important physical characteristics. A link in wireless communication networks is a connection between a sender and a receiver device. As the transmission medium is shared, interference may occur when links transfer data at the same time and using overlapping bands of the electromagnetic spectrum. The smaller is the interference level, the larger is the signal quality that reaches the receiver. Furthermore, the larger is the signal quality, the faster is the link transmission speed. On the other hand, the interference level can be high enough that it decreases the signal quality up to the point that it cannot be decoded [11].

Thus, given set of communications links, when scheduling a wireless link, we have to assign a time slot, a channel, and its bandwidth. But, we have to consider that a link in the same time slot and channel might interfere with other link. We call this problem the Multi-period, Multi-rate and Multi-channels with variable bandwidth Scheduling Problem (M3SP).

The motivation of studying M3SP comes from the fact that the IEEE 802.11ac standard is being increasingly adopted in wireless networks, because it allows a better usage of the network frequency spectrum. Despite the many advantages of this standard, the link scheduling problem that arises from this standard is more complex than that of its predecessors, because it allows more flexibility on the usage of the available frequency spectrum for the communication channels. Besides, the realistic SINR model adopted in this study makes the scheduling problem nonlinear, unlike the Unit Graph (UDG) models adopted by other works in the literature [6, 13, 14].

The motivation of using heuristics comes from the fact that the problem is NP-Hard, as shown in this work, and no Integer Linear Programming formulation is known for this problem. We chose a genetic algorithm approach because it is a global optimization method. Besides, Biased Random-key Genetic Algorithm (BRKGA) has been successfully applied to many network optimization problems [2, 3, 17, 19].

In this paper, we present the following contributions: (i) we introduce a novel combinatorial optimization problem that arises from the IEEE 802.11ac standard. We deal with TDMA and FDMA multiplexing, as well as the realistic SNIR model to map the signal quality. Besides, as far as we can tell, this is the first work in the literature that deals with variable bandwidth channels; (ii) We formally define The Multi-period, Multi-rate and Multi-channels with variable bandwidth Scheduling Problem (M3SP); (iii) We prove that this problem is NP-Hard; (iv) We present a Biased Random-key Genetic Algorithm (BRKGA) for solving it and show that this algorithm usually can not generate feasible solutions; Therefore, (v) we present a new heuristic algorithm that modifies BRKGA to include a *reinsertion* procedure, called BRKGA+. (vi) We evaluated BRKGA and BRKGA+ over more than 100 instances and we show that BRKGA+ can significantly improve the network throughput.

This work is organized as follows. First, we describe where this work can be applied and provide some background on the IEEE 802.11ac standard (Section 2). Then, we describe the related work in Section 3. M3SP is formally defined in Section 4. In Section 5, we prove that the problem is NP-Hard. Next, BRKGA and BRKGA+ are proposed in Section 6. Then, computational experiments are presented in Section 7. Finally, concluding remarks are drawn in the last section.

2 Infrastructure Wireless Networks

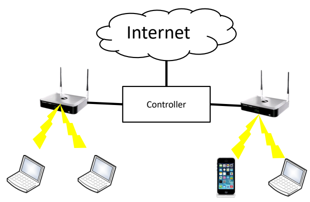

Figure 1: Infrastructure Wireless Networks.

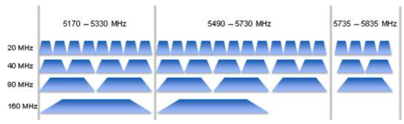

Figure 2: IEEE 802.11ac bandwidth and channel allocation.

Most of today's Wireless Local Area Network (WLAN) are networks in an infrastructure environment. Infrastructure Wireless networks are very common on college campuses, hotels, businesses, and industries. These networks consist of a set of Access Points (AP). APs are devices responsible for transmitting wireless messages. They connect customers (laptops, mobile phones, mobile devices) using a wired network that is connected to the Internet. Figure 1 illustrated the wireless network architecture.

The Access Points can be interconnected, via cable, in a centralizing entity, called controller [15]. This controller can configure the Access Points. The communication between the clients and the Access Point is direct (one single hop) through the IEEE 802.11 ac protocol. Therefore, through a centralizing entity, we can configure access points to operate on a single-hop wireless network. This motivates us to use a centralized approach to solve the problem of configuring a set of access points.

Figure 2 shows the $IEEE802.11ac$ standard bandwidth and channel allocation. It utilizes the 5 GHz spectrum. The x-axis represents the wireless spectrum. The y-axis represents each possible bandwidth allocation. In this case, each link can operate in channels of 20 MHz, 40 MHz, 80 MHz and 160 MHz. That is, any eight contiguous frequency bands of 20 MHz can be used to assign (i) one link to a channel of 160 MHz, (ii) two links to channels of 80 MHz, (iii) four links to channels of 40 MHz, or (iv) eight links to channels of 20 MHz. From the Shannon–Hartley theorem [20], the larger is the channel bandwidth, the larger is the transmission speed of the corresponding link.

Table 1 shows the relation Signal to Noise Ratio (SNR) and Throughput for the $IEEE802.11ac$ standard. It illustrates, for each channel bandwidth, the achieved throughput (in Mbps) given the necessary SNR in decibel (dB), considering a guard interval of 800 ns. We can observe that, the larger is the SNR, the larger is the throughput, which agrees with Shannon–Hartley theorem [20].

20 MHz		40 MHz		80 MHz		160 MHz	
SNR (dB)	Thr. (Mbps)	SNR (dB)	Thr. (Mbps)	SNR (dB)	Thr. (Mbps)	SNR (dB)	Thr. (Mbps)
2,0	6,5	5,0	13,5	8,0	29,3	11,0	58,5
5,0	13,0	8,0	27,0	11,0	58,5	14,0	117,0
9,0	19,5	12,0	40,5	15,0	87,8	18,0	175,5
11,0	26,0	14,0	54,0	17,0	117,0	20,0	234,0
15,0	39,0	18,0	81,0	21,0	175,5	24,0	351,0
18,0	52,0	21,0	108,0	24,0	234,0	27,0	468,0
20,0	58,5	23,0	121,5	26,0	263,3	29,0	526,5
25,0	65,0	28,0	135,0	31,0	292,5	34,0	585,0
29,0	78,0	32,0	162,0	35,0	351,0	38,0	702,0
—	—	34,0	180,0	37,0	390,0	40,0	780,0

Table 1: SNR (dB) and Throughput (Mbps) for each channel bandwidth.

3 Related Work

Some works in the literature that aims at partitioning the link set in order to minimize the network latency. This problem was proven NP-Hard in [10] and exact algorithms were proposed in [21]. The latter was able to solve only instances with up to 24 links. An approximation algorithm was proposed in [9]. The latter consists in partitioning the link set by solving a sequence of maximum weighted independent set problems. Simulation results showed that this algorithm outperformed others algorithms in the literature for many network topologies and sizes.

Wireless communication standards such as IEEE 802.11a/b/g/n enable data transmission at multiple speeds (called data rate), depending on the signal quality that reaches the receiver. This allows transmissions even in the presence of high interference, but at lower transmission rates. There are works in the literature that aims at assigning channels to network links in order to maximize the network throughput. This problem is also NP-Hard [10]. It is shown in [11] that this problem can be approximated by a graph-based model, and that a maximum weighted independent set algorithm applied to this graph provides a constant-factor approximation to the multi-rate scheduling problem. Computational experiments showed that this approach outperforms the previous works in the literature. In [12], they consider that the data rate was not given in the problem's input but was assigned as the output, that is, they studied the problem of data rate assignment and scheduling of wireless transmissions. They presented a parallel approximation algorithm based on the algorithm proposed in [11]. They show that the network throughput can be considerably improved by the use of multiple transmission rates.

Chen & Chen [4] proposed two algorithms based on Simulated Annealing heuristics to solve the channel assignment problem in Wireless networks. The first algorithm has no guarantee that the solutions are always feasible. In the second algorithm, the proposed solutions are always feasible. They compared the developed algorithm with an algorithm based on the tabu search [23].

Ning et al. [16] studied the problem of channel assignment and time-slot scheduling for multi-hop networks. They combined a number of techniques such as link scheduling, spatial reuse, power and rate adaptation and network coding. They utilize a column generation (CG)-based method to resolve the optimization problem and decompose it into a master problem and a pricing problem.

To the best of our knowledge, we are the first to consider variable bandwidth when scheduling wireless links.

4 Problem Definition

M3SP is defined over the following parameters. V is the set of devices, $L = \{(i,j)\}$, where $i, j \in V$, is the set of links, $B = \{20, 40, 80, 160\}$ is the set of bandwidths (in MHz), T is the set of time intervals. Moreover, C is the set of communication channels, i.e. all combinations of frequency bands in channels of 20 MHz, 40 MHz, 80 MHz and 160 MHz that are allowed by the IEEE 802.11ac standard. Besides, we have that l_c gives the bandwidth of channel $c \in C$, and $O \subseteq C \times C$ is the set of the communication channel pairs whose frequency bands overlap. In addition, according to the IEEE 802.11ac standard, the parameter r_{bs} gives the transmission speed of any link that operates on a channel with bandwidth $b \in B$ and has a Signal to Interference plus Noise Ratio (SINR) larger than or equal to s dBm, where $(b, s) \in R$ and R is the set of all (b, s)-pairs supported by this standard.

M3SP consists in assigning a channel $c_{ij} \in C$ and a time interval $t_{ij} \in T$ for each link $(i, j) \in L$. A device may not be the sender or receiver of more than one link at a given time interval, and the resulting interference level must be such that all links can transmit with an speed of at least $\min_{(b,s)\in R} r_{bs}$. The objective is to maximize network *throughput*, i.e. the amount of data transported over the network at every time unit.

M3SP can be formulated with variables $x_{ij}^{tc} \in \{0, 1\}$, where $x_{ij}^{tc} = 1$ if the link $(i, j) \in L$ is online at the time interval $t \in T$ and uses the channel $c \in C$. Otherwise, $x_{ij}^{tc} = 0$. Auxiliary variables I_{ij} and $SINR_{ij}$ store the value of the interference and the SNIR of link $(i, j) \in L$, respectively. Furthermore, auxiliary variables $y_{ij}^{bs} \in \{0, 1\}$ are such that $y_{ij}^{bs} = 1$ if link $(i, j) \in L$ transmits at the speed r_{bs}. Otherwise, $y_{ij}^{bs} = 0$.

$$\text{maximize} \quad F(x) = \frac{1}{|T|} \sum_{(i,j)\in L} \sum_{(b,s)\in R} r_{bs} \cdot y_{ij}^{bs} \quad \text{subject to:} \tag{1}$$

$$\sum_{t\in T} \sum_{c\in C} x_{ij}^{tc} = 1, \quad \forall (i, j) \in L \tag{2}$$

$$\sum_{(b,s)\in R} y_{ij}^{bs} = 1, \quad \forall (i, j) \in L \tag{3}$$

$$I_{ij} = \sum_{(\bar{i},\bar{j})\in L\setminus\{(i,j)\}} \frac{P_{\bar{i}\bar{j}}}{(d_{\bar{i}j})^{\alpha}} \cdot \sum_{t\in T} \sum_{(c,\bar{c})\in O} x_{ij}^{tc} \cdot x_{\bar{i}\bar{j}}^{t\bar{c}}, \quad \forall (i, j) \in L \tag{4}$$

$$SINR_{ij} = \frac{\frac{P_{ij}}{(d_{ij})^{\alpha}}}{I_{ij} + N} \geq s \cdot y_{ij}^{bs}, \quad \forall (i, j) \in L, \forall (b, s) \in R \tag{5}$$

$$y_{ij}^{bs} \leq \sum_{t\in T} \sum_{c\in C \,|\, l_c=b} x_{ij}^{tc}, \quad \forall (i, j) \in L, \forall (b, s) \in R \tag{6}$$

$$\sum_{(i,j)\in L} \sum_{c\in C} x_{ij}^{tc} + \sum_{(j,i)\in L} \sum_{c\in C} x_{ji}^{tc} \leq 1, \forall i \in V, \forall t \in T \tag{7}$$

$$x_{ij}^{tc} \in \{0, 1\}, \quad \forall (i, j) \in L, \forall t \in T, \forall c \in C \tag{8}$$

$$y_{ij}^{bs} \in \{0, 1\}, \quad \forall (i, j) \in L, \forall (b, s) \in R \tag{9}$$

M3SP is defined by the objective function (1) and constraints (2) to (9). The former maximizes the average transmission speeds over the $|T|$ time intervals, i.e. the amount of data transported over the network at every time unit. The equations in (2) ensure that each link only transmits in a single time interval and in a single communication channel, while those in (3) require that each link transmits at a single speed throughout the network operation. The interference level and the SINR of each link $(i, j) \in L$ is respectively computed by the equations in (4) and (5). The constraints (5) and (6), together, guarantee that the link (i, j) can only transmit at the speed r_{bs} if $SINR_{ij} \geq s$ (Equation 5) and if the bandwidth of the corresponding channel is b (Inequality 6). The constraint that a device cannot operate in more than one link in a given time interval is enforced by inequalities (7). In addition, the domain of the variables x and y is defined in (8) and (9), respectively.

5 Computational Complexity

Let $\overline{M3SP}$ be the problem of determining if there is at least one feasible solution for a given instance of M3SP. In this section, we prove that $\overline{M3SP}$ is NP-Complete, and therefore, that M3SP is NP-Hard.
Definition: A k-coloring of an undirected graph $G = (U, E)$ is a function $g : U \rightarrow \{1, 2, \cdots, k\}$ such that $g(u) \neq g(v)$ for every edge $[u, v] \in E$. Thus, the numbers $\{1, 2, \cdots, k\}$ represent the k colors and the adjacent vertices in G have distinct colors. Given the value of k and the graph G, the Graph Coloring Problem (GCP) is to determine if there is a k-coloring to G. This problem is NP-Complete[7].
Lemma 1: *$\overline{M3SP}$ is in NP*
Proof: Given an instance $I = \langle V, L, T, C, O, B, l_c, R, r_{bs}, N, \alpha, P_{ij}, d_{ij} \rangle$ and a certificate $\langle \overline{x}, \overline{y} \rangle$, represented by the characteristic vector of the variables x and y, it is sufficient to prove that there is a polynomial algorithm to verify if $\langle \overline{x}, \overline{y} \rangle$ satisfies (2) to (9) for instance I. The constraint (2) can be checked in $O(|T| \cdot |C||L|)$ and (3) in $O(|R| \cdot |L|)$. The equation (4) can be computed in $O(|T| \cdot |C|^2 \cdot |L|^2)$. In addition, the constraint (5) can be evaluated in $O(|L| \cdot |R|)$ and (6) in $O(|T| \cdot |C| \cdot |L| \cdot |R|)$. Moreover, (7) can be checked in $O(|L| \cdot |C| \cdot |V| \cdot |T|)$. Consequently, there is an algorithm of complexity $O(|V| \cdot |L|^2 \cdot |T| \cdot |C|^2 \cdot |R|)$ to verify if $\langle \overline{x}, \overline{y} \rangle$ satisfies I.

$\square$

Theorem 1: *GCP is reduced to $\overline{M3SP}$.*
Proof: First, we define a function $f : \{\hat{I}\} \rightarrow \{I\}$ that transforms an instance $\hat{I} = \langle G = (U, E), k \rangle$ of GCP into an instance
$I = \langle V, L, T, C, O, B, l_c, R, r_{bs}, N, \alpha, P_{ij}, d_{ij} \rangle$ of M3SP as follows:

- $V = V^s \cup V^r$, where $V^s = \{s_i : i \in U\}$ and $V^r = \{r_i : i \in U\}$, in other words, for each vertex of U, two devices are created in V, one to represent the sender and another to represent the receiver of a connection.
- $L = \{(s_i, r_i) : i \in U\}$, that is, L contains a connection for each vertex of U. In this way, each device is part of a single connection.
- $T = \{1, \cdots, k\}$, in other words, the number of time intervals is equal to the number of colors.
- $C = \{1\}$, that is, all connections transmit on the same communication channel.
- $O = \{(1, 1)\}$. In this way, all the connections that transmit in the same time interval interfere with each other.
- $B = \{20\}$, which means that all channels have the same bandwidth.
- $l_1 = 20$, the bandwidth of the only available channel is equal to 20.
- $R = \{(20, 1)\}$ e $r_{20,1} = 1$, for a connection $(i, j) \in L$ to transmit $SINR_{ij} \geq 1$.
- For convenience, $N = 0$, $\alpha = 1$, and $P_{ij} = 1$ for all $(i, j) \in L$
- Let $\hat{U}(i)$ be a function that returns the vertex of U corresponding to a connection $i \in V$, the value of d_{ij} is defined accordingly to Equation (10).

$$d_{ij} = \begin{cases} 1, & \text{if } \hat{U}(i) = \hat{U}(j), \\ 1/2, & \text{if } (\hat{U}(i), \hat{U}(j)) \in E \text{ e} \\ |L|, & \text{otherwise.} \end{cases} \tag{10}$$

The transformation f is defined in such a way that each vertex in U is associated with a connection in L, and each color is associated with a time interval in T. In addition, since there is only a single channel in C, all connections that transmit in the same time interval interfere with each other. However, the value of d_{ij}, for all $i \in V$ and $j \in V$, is defined so that the connections $(s_u, r_u) \in L$ and $(s_v, r_v) \in L$, where $(u, v) \notin E$, cause insignificant interference with each other, while those whose associated vertices share edges in E cause interference such that the signal quality does not reach the minimum necessary for them to transmit.

It remains to prove that GCP is satisfied for $\hat{I}$, if and only if,$\overline{M3SP}$ is satisfied for I. The values of $\alpha = 1$, $P_{\bar{i},\bar{j}} = 1$ and $d_{\bar{i}\bar{j}} \in \{1/2, |L|\}$ in (4) are such that $I_{ij} > 1$ if there exists a connection $(\bar{i}, \bar{j}) \in L$, such that $(\hat{U}(i), \hat{U}(\bar{i})) \in E$ and $\bar{x}^{t1}_{ij} = \bar{x}^{t1}_{\bar{i}\bar{j}} = 1$, for some $t \in T$. Otherwise $I_{ij} < 1$. The values of $P_{ij} = d_{ij} = s = 1$ in (5) are such that this constraint is only satisfied if $I_{ij} \leq 1$.

$\Longrightarrow$ Given a certificate $\langle \overline{x}, \overline{y} \rangle$ to I, it is possible to create a certificate $\langle \hat{g}(u) = t^{\bar{x}\bar{y}}_{s_u r_u}, \forall u \in V \rangle$ for $\hat{I}$, where $t^{\bar{x}\bar{y}}_{s_u r_u} \in T$ is the time period that the connection $(s_u, r_u) \in L$ transmits according to the certificate $\langle \bar{x}, \bar{y} \rangle$. Since the latter satisfies I, we have $I_{ij} \leq 1$ for all $(i, j) \in L$, and there are no two connections $(s_u, r_u) \in L$ and $(s_v, r_v) \in L$, such that $(u, v) \in E$, transmitting over the same period of time. Consequently, $\langle \hat{g} \rangle$ is a k-coloring of G, since $|T| = k$ and = $\hat{g}(u) = t^{\bar{x}\bar{y}}_{s_u r_u} \neq t^{\bar{x}\bar{y}}_{s_v r_v} = \hat{g}(v)$, for all $(u, v) \in E$.

$\Longleftarrow$ Given a certificate $\langle \hat{g} \rangle$ for $\hat{I}$, it is possible to construct a certificate $\langle \bar{x}, \bar{y} \rangle$ for $\bar{I}$, such that $t^{\bar{x}\bar{y}}_{s_u r_u} = \hat{g}(u)$ and $\overline{y}^{20,1}_{s_u r_u} = 1$, for all $u \in U$. In this way, $\langle \bar{x}, \bar{y} \rangle$ satisfies all constrains (2), (3), (6), and (7) by definition. Moreover, since $\langle \hat{g} \rangle$ satisfies $\hat{I}$, we have that $t^{\bar{x}\bar{y}}_{s_u r_u} = \hat{g}(u) \neq \hat{g}(v) = t^{\bar{x}\bar{y}}_{s_v r_v}$, for all $(u, v) \in E$. As a result,

$$I_{ij} \leq \sum_{(\bar{i},\bar{j}) \in L \setminus \{(i,j)\}} \frac{P_{\bar{i}\bar{j}}}{(d_{\bar{i}\bar{j}})^{\alpha}} = \sum_{(\bar{i},\bar{j}) \in L \setminus \{(i,j)\}} \frac{1}{|L|} = \frac{|L| - 1}{|L|} < 1,$$

for all $(i, j) \in L$. Consequently, the constraint (5) is satisfied for every connection $(i, j) \in L$ and $\langle \bar{x}, \bar{y} \rangle$ to I.

□

Corollary 1: *As GCP reduces to $\overline{M3SP}$ and the latter is in NP, $\overline{M3SP}$ is NP-Complete.*

6 Biased random-key genetic algorithm

The biased random-key genetic algorithm [8] evolves a population of chromosomes that consists of vectors of real numbers (called keys) in the range $[0, 1)$ that are randomly generated in the initial population. The fitness of a chromosome is given by the cost of the solution found by a decoding algorithm that receives the vector of keys as input and outputs a feasible solution with its corresponding cost.

We use the *parameterized uniform crossover* scheme proposed in [22] to combine two parent solutions and produce an offspring solution. In this scheme, the offspring inherits each of its keys from the best fit of the two parents with probability 0.7 and from the least fit parent with probability 0.3. This genetic algorithm does not make use of a standard mutation operator, where parts of the chromosomes are changed with small probability. Instead, the concept of *mutants* is used. In each generation, a fixed number of mutant solutions are introduced in the population. They are generated in the same way as in the initial population. As with mutation, mutants play the role of helping the procedure to escape from local optima.

At each new generation, the population is partitioned into two sets: TOP and $REST$. Consequently, the size of the population is $|TOP| + |REST|$. The best solutions are kept in TOP while the others are placed in $REST$. As illustrated in Figure 3, the chromosomes in TOP are copied, without change, to the population of the next generation. The new mutants are placed in set BOT. The remaining elements of the new population are obtained by crossover with one parent randomly chosen from TOP and the other from $REST$. This distinguishes a biased random-key GA from the random-key GA of Bean [1]. In the latter, both parents are selected at random from the entire population. Since a parent solution can be chosen for crossover more than once in a given generation, elite solutions have a higher probability of passing their keys to the next generation. In this way, $|REST| - |BOT|$ offspring solutions are created. The sizes of sets TOP, $REST$, and BOT are parameters that must be tuned.

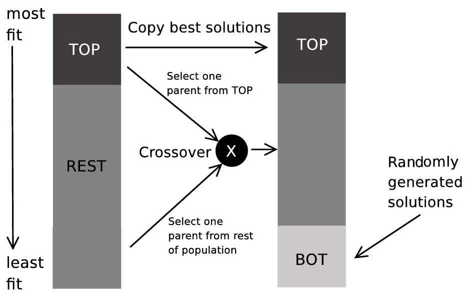

Figure 3: Illustration of the transitional process between consecutive generations of the genetic algorithm with random keys

The biased random-key genetic algorithm for M3SP has two keys associated to each link $(i, j) \in L$, named k^1_{ij} and k^2_{ij}. The decoding algorithm consists in a first fit heuristic with three steps. In each step, the links that are not already scheduled are evaluated in increasing order of their k^1_{ij} values. In the first step, the links are inserted in the first non-overlapping channel of the first time slot whose bandwidth is b_{ij}, where b_{ij} is equal to 20, 40, 80, or 160 MHz if k^2_{ij} is in the range $[0, 0.25)$, $[0.25, 0.5)$, $[0.5, 0.75)$, or $[0.75, 1)$, respectively. In the second step, the links that could not be inserted in the previous step have their values of b_{ij} decreased until they can fit in a non-overlapping channel. The third step starts when all bands of the network frequency spectrum are been used. In this case, links are assigned to an already scheduled channel that has the minimum number of links assigned to it. We point out that the interference is not computed in the third step, because of its high computational cost.

The solution returned by the decoding algorithm can be infeasible, due to the high interference levels in some links. Therefore, we propose and evaluate a *reinsertion* procedure. First, the links whose SINR value is smaller than the minimum necessary to transmit in its current channel are removed. Then, each of these links is reinserted in the first scheduled channel of the first time slot in which it and the other links in the channel have a SINR high enough to transmit. We refer to the BRKGA that runs this procedure after the decoding algorithm as BRKGA+.

7 Computational Experiments

Computational experiments were carried out on a single core of an Intel Xeon 2.4 GHz machine with 32 GB of RAM memory, running Linux Ubuntu 12.04 LTS. BRKGA and BRKGA+ were implemented in C++ and compiled with GNU g++ version 4.8.2. The BRKGA parameters were set to the values suggested in [2, 17], where the size of sets TOP, $REST$ and BOT were set to 25, 75 and 5, respectively, and the algorithm was made to stop after 10 generations without improving the best known solution.

The networks used in the experiments were generated as suggested in [9]. We show two types of topologies: uniform and *clustered* topologies, which is the most challenge [9]. In both cases, they are euclidean networks, where the devices are positioned in a plane field of dimension 1000×1000. For the uniform case, $|V|/2$ receivers were positioned uniformly in the plane. Then, $|V|/2$ senders were placed uniformly around the receivers. For the *clustered* topologies, $n_C = |V|/100$ cluster centers were selected uniformly at random. Then, $|V|/n_c$ sender-receiver pairs were positioned uniformly at random inside discs of radius $r_C = 10$ around each cluster center. We evaluated BRKGA and BRKGA+ over uniform topologies and *clustered* topologies for networks with 100, 200, 400, 800, and 1600 devices. For each network size, 30 networks are generated and the average results are presented. Thus, we evaluated 300 instances.

The size of set T was set to $|T| = |L|/25$ because the IEEE 802.11ac standard has a frequency spectrum of 500 MHz. In this case, every network tested has at least one feasible solution where every link can be assigned to a different channel of 20 MHz. The experiment reported below evaluate if BRKGA and BRKGA+ can find a scheduling with a higher throughput than this solution, referred to as $All20$, by exploiting the flexibility in the channel bandwidth. When referring to the throughput metric, it is already divided by the number of time-slots, as described in the objective function. Since previous related work do not consider variable bandwidth channels, we decided to compare BRKGA and BRKGA+ against $All20$, which is useful as a baseline.

The performance of BRKGA and BRKGA+ is displayed in Table 2.The values of $|V|$, $|L|$, and $|T|$ are given in columns 1 to 3, respectively. Column 4 shows the throughput t^{20} of the solution $All20$. It is worth noticing that this number is the same for all networks, because there is no interference in this solution and the number of time intervals grows linearly with the number of links. The number f of feasible solutions found by BRKGA is reported in Column 5. The average throughput t^{GA} over the f instances where feasible solutions were found is displayed in Column 6. Besides, the percentage deviation of t^{GA} over t^{20} is $(t^{GA} - t^{20})/t^{20}$ and the average running time are given in columns 7 and 8, respectively. The same results are presented for BRKGA+ in columns 9 to 12, respectively.

			All20	BRKGA				BRKGA+			
$\|V\|$	$\|L\|$	$\|T\|$	throughput	f	throughput	%dev	time (s)	f	throughput	%dev	time (s)
100	50	2	1950.00	30	12087.62	519.88%	0.38	30	12333.69	532.50%	0.37
200	100	4	1950.00	30	12049.79	517.94%	1.91	30	12161.57	523.67%	1.18
400	200	8	1950.00	30	11979.59	514.34%	8.21	30	12065.60	518.75%	4.88
800	400	16	1950.00	30	11878.35	509.15%	33.84	30	11933.74	511.99%	22.04
1600	800	32	1950.00	30	11799.82	505.12%	148.70	30	11840.04	507.18%	88.23

Table 2: Performance evaluation of BRKGA and BRKGA+ for uniform topologies.

Table 2 shows that BRKGA and BRKGA+ found feasible solutions for all instances and BRKGA+ was able to improved the average throughput more than BRKGA and $All20$.

The performance of BRKGA and BRKGA+ over *clustered* topologies is displayed in Table 3. This table is similar to Table 2 but we add column 4 to indicate n_C.

				All20	BRKGA				BRKGA+			
$\|V\|$	$\|L\|$	$\|T\|$	n_C	throughput	f	throughput	%dev	time (s)	f	throughput	%dev	time (s)
100	50	2	1	1950.00	21	1911.98	-1.95%	0.85	30	2025.68	3.88%	2.18
200	100	4	2	1950.00	0	—	—	1.61	30	4462.08	128.82%	10.81
400	200	8	4	1950.00	0	—	—	5.66	30	9107.59	367.06%	130.32
800	400	16	8	1950.00	0	—	—	19.48	30	11086.91	468.56%	669.71
1600	800	32	16	1950.00	0	—	—	57.58	30	11910.11	510.77%	2843.78

Table 3: Performance evaluation of BRKGA and BRKGA+ for *clustered* topologies.

One can see from Table 3 that BRKGA was able to find feasible solutions for only 21 out of the thirty networks with 100 devices. The average throughput for these 21 instances were smaller than that of $All20$. Moreover, no feasible solution was found for larger networks. On the other hand, BRKGA+ was able to find feasible solutions for all networks, but with larger computational times. Besides, the average throughput of the solutions provided by BRKGA+ was up to 510.77% larger than that of the solution where all links have channel bandwidth equal to 20 MHz.

Figure 4 shows the performance evaluation of BRKGA+, BRKGA, and $All20$ for *clustered* topologies. Each algorithm has its own color. The top graph represents throughput (in Mbps) and the bottom graph indicate time (in s). The error bars illustrate the standard deviation. We can observe how the quality of the solution, indicated by the throughput solution, improves over CPU time.

Figure 5 illustrates the average distribution of channel bandwidth, plotted as a cumulative distribution function, for BRKGA+ solutions. For $|T| = 2$, most of the links has bandwidth of 20 MHz. For $|T| = 32$, most of the links has bandwidth of 160 MHz. As we increase the number of time-slots, BRKGA+ takes advantage of this and is able to allocate links with larger channel bandwidth.

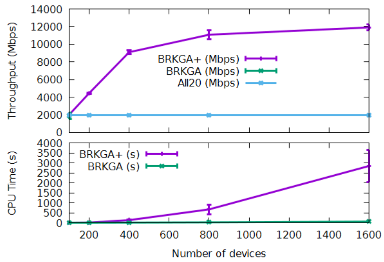

Figure 4: Evaluation of BRKGA and BRKGA+.

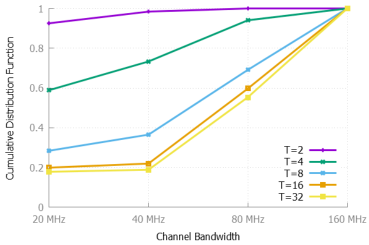

Figure 5: CDFs of bandwidth per time-slots.

8 Concluding remarks

This paper introduced a new combinatorial optimization problem that arises from a new technology described in the IEEE 802.11ac standard. The latter enables a higher transmission speed than the previous IEEE standards, because it allows the network frequency spectrum to be divided into communication channels with different bandwidths. However, as far as we can tell, the optimization models and algorithms in the literature do not deal with variable bandwidth channels. Therefore, we proposed a biased random-key genetic algorithm for this problem. Computational experiments showed that BRKGA was not able to systematically find feasible solutions without a *reinsertion* procedure that has a relatively high computational cost. However, BRKGA+ showed that a significant improvement can be obtained by exploiting the flexibility in the channel bandwidth. As for future works, we suggest the research of other metaheuristics for M3SP, as well as exact algorithms that can be used to assess the performance of these heuristics on small and medium sized networks.

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