Fluctuation data analysis in Fusion Relevant Plasmas

Extracting information on relevant underlying dynamics

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Motivation & Outline

Diagnostics provides information with different time and spatial resolution

1. Measurements coming from a single point
2. Spatially distributed arrays of measurements (resolving portion of the plasma or entire torus)
3. line integrated measurements (single Line Of Sight (LoS))
4. Arrays of LoS (examples are tomographic reconstruction)
5. We will focus on analysis technique suitable for single-point/multi point measurements, extracting information on spatial/temporal dynamics

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- Some remarks on basic Fourier Transform and its discrete counterpart the Discrete Fourier Transform are mandatory

Continuous and Discrete Fourier transform 2

- The Direct and Inverse fourier transform of a generic function of time $x(t)$ is defined as :

$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-2\pi ift}\mathrm{d}t \qquad x(t) = \int_{-\infty}^{+\infty} X(f)e^{2\pi ift}\mathrm{d}f$$

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- In the case of discrete signals $x_n = x(n\Delta t)$ with $0 \leq n \leq N-1$, sampled at frequency $f_c = \frac{1}{\Delta t}$ we have the corresponding Direct and Inverse Discrete Fourier transform, then defined as

$$X_n = \frac{1}{N} \sum_{k=0}^{N-1} x_k e^{-2\pi kn/N} \qquad x_k = \frac{1}{N} \sum_{n=-N/2}^{N/2} X_n e^{2\pi kn/N}$$

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- The `Sampling Theorem` (Bracewell 1999) ensure that A function whose Fourier transform is zero for $f > f_c$ is fully specified by values spaced at equal intervals not exceeding $\frac{1}{2} f_c^{-1}$

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- The `Nyquist Frequency` $f_N = \frac{1}{2\Delta t}$, defines the maximum frequency which can be properly resolved, or equivalently given the frequency of the system we would like to investigate, we had to sample at least at twice the values of this frequency.

Properties of Fourier transform

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Theorem

Convolution theorem: *If $x(t)$ and $g(t)$ have FT respectively equal to $X(f)$ and $G(f)$ the convolution of the two function $h(t) = \int_{-\infty}^{+\infty} x(t')g(t-t')\mathrm{d}t'$ is equal to $X(f)G(f)$*

- The importance of the convolution equation resides on the fact that it allows treatment of non-linearities as the term $\mathbf{v} \cdot \nabla \mathbf{v}$ in the Navier-Stokes equations

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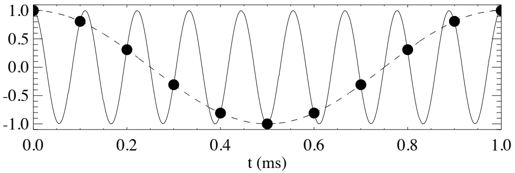
Rayleigh's Theorem The integral of squared modulus of a function is equalt to the integral of the squared modulus of its spectrum, i.e:

$$\int_{-\infty}^{+\infty} |f(t)|^2 \mathrm{d}t = \int_{-\infty}^{+\infty} |F(f)|^2 \mathrm{d}f$$

- This is equivalent to an energy conservation law for the time or frequency domain representation of the signal

Aliasing, leaking and windowing

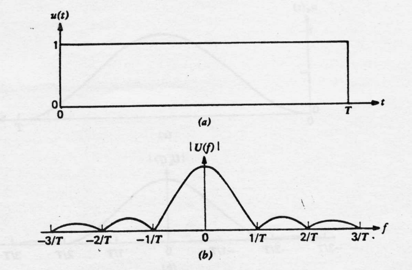
- The presence of frequency higher than the Nyquist frequency may lead to the presence of spurious frequency



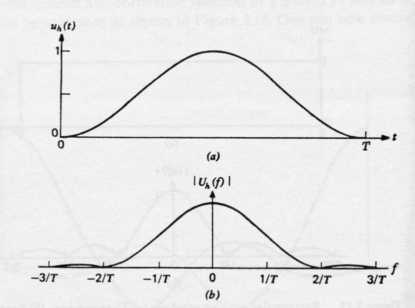
- A 9 kHz sine if sampled at 10 kHz exhibits a spurious 1 kHz oscillation

Aliasing, leaking and windowing

- Actually signals are acquired for a given period $T \rightarrow$ equivalent to the convolution with a box function $G(t)$ with domain $0 \leq t \leq T$, i.e. $G(t) = 1$ if $0 \leq t \leq T$ 0 otherwise
- In the Fouirer space equivalent to the moltiplication with the Fourier representation of a box function $\rightarrow \mathrm{sinc}(x) = \sin(x)/x$ function as shown, which leaking some power from one frequency bin to the adjacents ones.



- Solution to the leakage: moltiplication with an appropriate window function which reduces the lobes as the *Hanning window*: $u_h(t) = \frac{1}{2}(1 - \cos(2\pi t/T))$ for $0 \leq t \leq T$ and 0 otherwise.



Single Point: the autocorrelation function

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- A random process $x(t)$ is completely described by its moments, which are the average over the probability distribution function

$$E[x(t)] \quad E[x(t_1)x(t_2)] \quad E[x(t_1)x(t_2)x(t_3)] \quad \ldots$$

- We definte Auto-correlation function, i.e. the second order momentum of the distribution, or the autocovariance function

$$R(\tau) = E|x(t)x(t-\tau)| \qquad C(\tau) = E|(x(t)-m)(x(t-\tau)-m)|$$

being m the average of $x(t)$

- The Auto-correlation coefficient factor is defined as $\rho(\tau) = C(\tau)/C(0)$
- For digitized signals with N samples the estimator of $C(\tau)$ is defined as

$$C_j = \frac{1}{N}\sum_{i=j}^{N-1}(x_i - \overline{x})(x_{i-j} - \overline{x}) \qquad \overline{x} = \frac{1}{N}\sum_{i=0}^{N-1} x_i$$

Auto-correlation: practical use

- Define the *Auto-correlation time* of a turbulent field such as the potential
- **Inserisci figura con autocorrelazione di un potenziale**

Single point: the power spectrum

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- Practically, we divide signals into M slices, assumed as indipendent realization of the same stochastic process and we compute

$$\hat{S}(f) = \frac{1}{M}\sum_{k=1}^{M} S^{(k)}(f); \qquad S^{(k)}(f) = \frac{1}{T}|X_T^{(k)}(f)|$$

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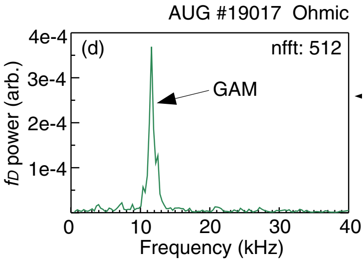
- With digitized signal, the power spectral estimator $\hat{S}_n$ is related to the real power spectrum $S(f_n)$ as

$$\hat{S}_n = \frac{1}{M}\sum_{k=1}^{M} |X_n^{(k)}|^2; \qquad \hat{S}_n \simeq S(f_n)\Delta f$$

Power spectrum: practical use

- Mode identification at a given frequency (Conway et al. 2005)



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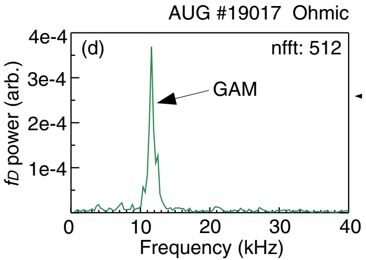
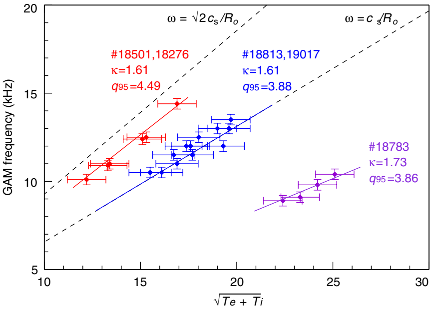
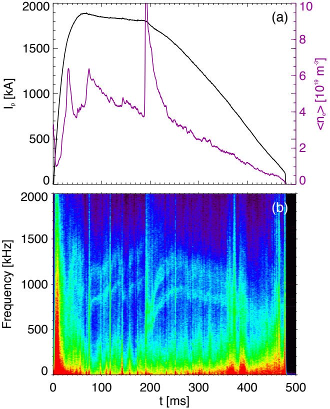




- Information must be completed. In the example scaling of identified mode as a function of ion sound velocities

Power spectrum: The spectrogram

- The same information can be also analyzed in time applying the spectrogram technique which shows how the spectral density of the signal varying in time/frequency space (Spagnolo et al. 2011)



- Alfvénic nature revealed by the comparison with the plasma density

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$$R_{xt}(\tau) = E[y(t)x(t-\tau)]$$

$$C_{xy}(\tau) = E[(y(t)-\overline{y})(x(t-\tau)-\overline{x})]$$

$$\rho_{yx}(\tau) = \frac{C_{yx}(\tau)}{\sqrt{C_{xx}(0)C_{yy}(0)}}$$

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- In the discrete counterpart of the cross-covariance is defined as

$$C_{yx,j} = \frac{1}{N}\sum_{i=j}^{N-1}(y_i-\overline{y})(x_{i-j}-\overline{x})$$

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- In the case of discrete signals with finite temporal length the following definitions hold (in analogy to single point case)

$$\hat{S}_{Y,X,n} = \frac{1}{M}\sum_{k=1}^{M} Y_n^{(k)} X_n^{*(k)} \qquad \hat{S}_{Y,X,n} \simeq S_{YX}(f_n)\Delta f$$

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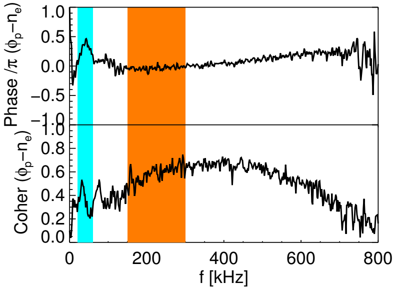

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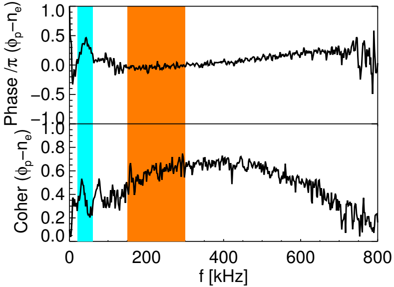

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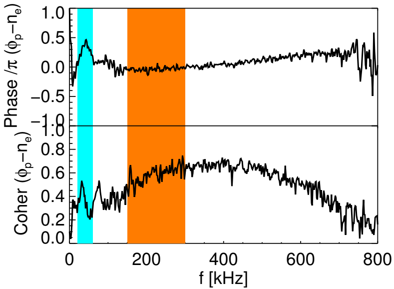

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- Other possibility is the determination of the polarization of magnetic fluctuations frequency resolved

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- If measures exhists at two position $g(t, \overline{r}_1)$ and $g(t, \overline{r}_2)$ separated by d at a given frequency the two signals will be phase shifted of $\Theta_{12}(f) = k(f)d$ where $\Theta_{12}(f)$ is computed trough the cross-power spectrum

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- The probe distance d must be less than aa wave length, less than a correlation lenght, but far enough the detect a measurable phase difference

Fluctuation-induced particle transport

- Fluctuations induced particle flux is defined as $\Gamma = E[\tilde{n}(t)\tilde{v}(t)] = E[\tilde{n}(t)\tilde{E}(t)]/B$
- According to previous definitions and properties

$$\Gamma = \frac{1}{B} R_{nE}(\tau = 0) = \frac{1}{B}\int_{-\infty}^{+\infty} S_{nE}(f) e^{i2\pi f \tau} df = \frac{2}{B}\int_{0}^{+\infty} \Re[S_{nE}(f)] df$$

- In quasi-static approximation $\tilde{E} = -\nabla\tilde{\phi}$, and considering the finitness of the measurements we end up with the formula

$$\Gamma(f) = \frac{2}{BT}\Im\{E[k(f)N(f)\Phi^*(f)]\}$$

$$\Gamma(f) = \frac{2k(f)}{B}\Im\{S_{n\phi}(f)\} \text{ if } k(f) \text{ is deterministic}$$

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- In practice, considering digitzed signals we have (see for example (Antoni et al. 2000))

$$\Gamma(f) = \frac{1}{M}\sum_{k=1}^{M}\Gamma^{k}(f)$$

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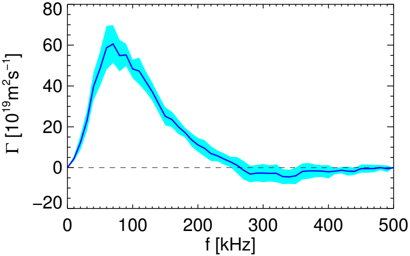

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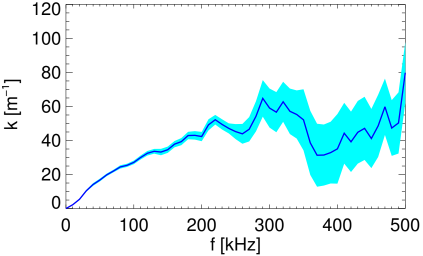

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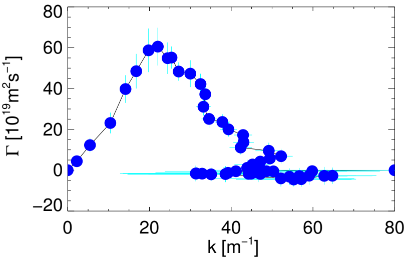

More than transport of particles

- Similar method may be used for the determination of the *Reynolds stress* $\langle \tilde{v}_r \tilde{v}_\perp \rangle$ which play a role in the momentum generation for both Tokamak and RFPs as $\partial_t (V_\phi) \propto -\partial_r \langle \tilde{v}_r \tilde{v}_\phi \rangle + \ldots$ (see e.g. (Vianello et al. 2005a,b, 2006))

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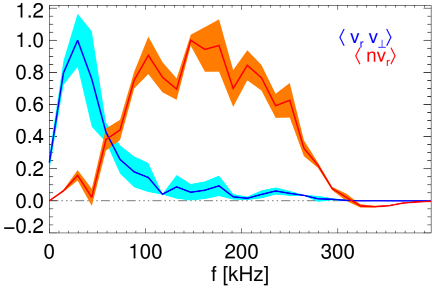

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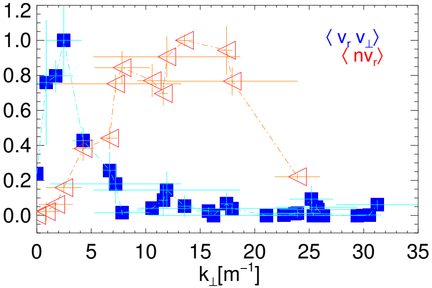

The wavenumber-Frequency Spectrum

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- Again for finite time length T and finite space L we have an estimate

$$\hat{S}(k, \omega) = \frac{1}{LT} E[G_{LT}(k, \omega) G^*_{LT}(k, \omega)]$$

$$\lim_{L,T \to \infty} \hat{S}(k, \omega) = S(k, \omega)$$

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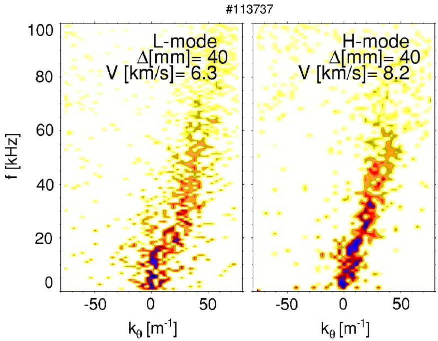
- If, as usual, only 2 points are available, the spectrum is reconstructed on a statistical basis:

$$\hat{S}_L(k, \omega) = \hat{S}_L(p\delta k, 2\pi n \Delta f) = \frac{1}{M} \sum_{j=1}^{M} S_n^{(j)} I_p[k_n^j]$$

$$I_p[k_n^j] = \begin{cases} 1 \text{ for } (p - 1/2)\Delta k < k_n^{(j)} < (p + 1/2)\Delta k \\ 0 \text{ elsewhere} \end{cases}$$

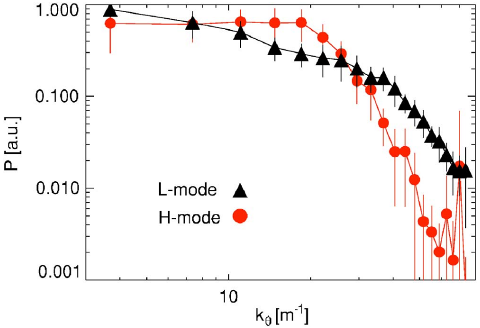
Spectral power density

- Spectral power density from GPI LoS on NSTX (Agostini et al. 2007)



Spectral power density

- We can consequently compute $S(k) = \int_{-\infty}^{+\infty} S(k,\omega)\frac{d\omega}{2\pi}$



Beyond Fourier: Wavelet transform

- The Fourier decomposition uses trigonometric functions as orthogonal basis
- These functions oscillates forever, i.e. the information content of a generic function is spread over all the spectral component
- Thus Fourier decomposition is not suitable for processes highly localized in time/space. We can use `Wavelet Transform`(Farge 1992)
- A Wavelet is a function $\psi \in \mathrm{L}^2(\mathbb{R})$ which satisfies the admissibility condition $C_\psi = \int_{-\infty}^{+\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty$, i.e. ψ is a zero mean function $\int_{\infty}^{\infty} \psi(t)dt = 0$
- Defining time-frequency atoms as $\psi_{s,\tau} = \frac{1}{\sqrt{\tau}}\psi\left(\frac{t-s}{\tau}\right)$ the `Continuous Wavelet Transform` is defined as

$$w(s,\tau) = \frac{1}{\sqrt{\tau}} \int_{-\infty}^{+\infty} f(t)\psi^*\left(\frac{t-s}{\tau}\right) dt$$

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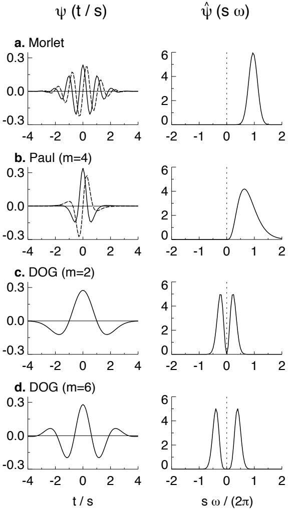
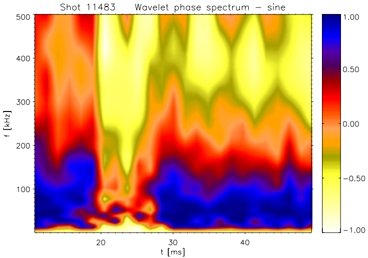
Choice & type of Wavelet

- Example of different types of wavelet (Mallat 1999)



Wavelet application

- In analogy to Fourier we can define Wavelet Cross power spectrum and Corresponding phase spectrum (well localized in time/frequency)



- Phase spectrum between density and potential varies because of varioation of the shear → responsible for transport reduction (Antoni et al. 2000)

Wavelet for turbulence analysis

- Wavelet coefficient exhibits similar scaling properties as the fluctuations of the signals at the same

$$\delta_\tau f = f(t+\tau) - f(t) \sim \tau^h \Rightarrow |w(t,\tau)| \sim \tau^{h+1/2}$$

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- Easy way to compute the `Probability Distribution Function` of normalized flutuations $C(t,\tau) = \frac{w(t,\tau) - \langle w(t,\tau)\rangle}{\sigma_\tau}$. For self-similar fluctuations, these should collapse to a single form

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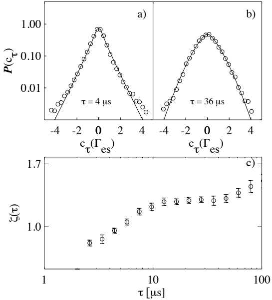
- **Revealing non self-similariy i.e. Intermittency**



Local Intermittency Measurements

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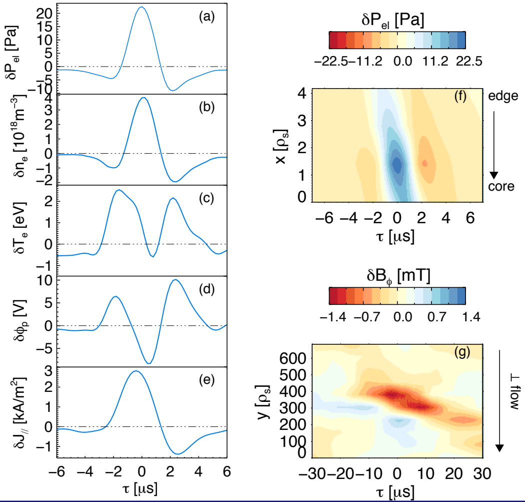
- Intermittency is due to the presence of strong, sporadic fluctuations
- The `Local Intermittency Measurements` is a method, based on wavelet, which identifies in time and scales these fluctuations (Antoni et al. 2001)
- The method is based on the following:

$$\{w(t,\tau)\} = \{w_e(t,\tau)\} \oplus \{w_g(t,\tau)\} \quad \text{with } F(\tau) = \frac{\langle w_g(t,\tau)^4 \rangle}{[\langle w_g(t,\tau)^2]^2} = 3$$

- The typical fluctuations may be deriven using `Conditional Average Procedure`, i.e. averaging different time windows of the signal, each centered around the occurrence of an Intermittent Events or blobs

Example of structures

- Conditional average may be applied to different signals using the same trigger



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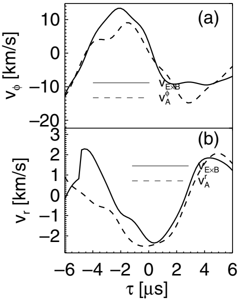
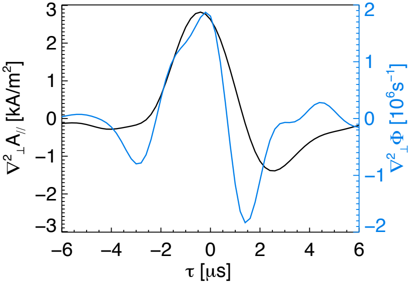
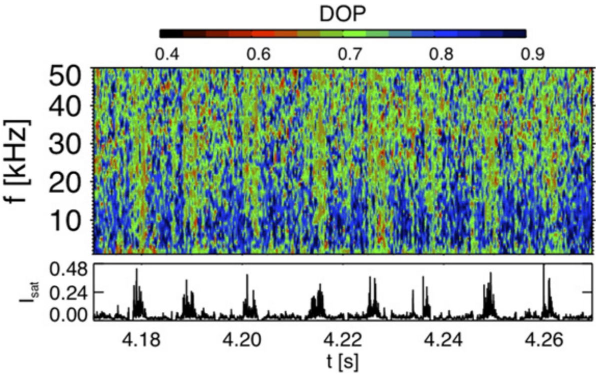




- Example of Drift-Kinetic Alfén vortices (Martines et al. 2009; Vianello et al. 2010)

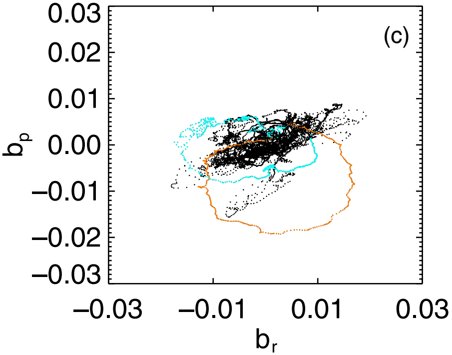
Other sophisticated tools: hints

- Degree of Polarization, which tests, frequency by frequency by frequency, the plane wave ansatz (Vianello et al. 2011)



Other sophisticated tools: hints

- The `hodogram`, which follows the trajectory of fluctuating fields, highlightning the state of polarization (Vianello et al. 2011)



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- High temporal and spatial resolution are needed for better characterization of the plasma. But two points still gives a bunch of information
- Fourier transform allows estimate of quantities directly comparable with theories
- Often localized events (in space or time) require more sophisticated tools which maintain the locality of the information
- As much as possible correlation between different diagnostics are generally needed for an adeguated comprehension

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