Fluctuation data analysis in Fusion Relevant Plasmas

N. Vianello

June 17, 2011

Motivation & Outline

Diagnostics provides information with different time and spatial resolution

1. Measurements coming from a single point
2. Spatially distributed arrays of measurements (resolving portion of the plasma or entire torus)
3. line integrated measurements (single Line of Sight (LoS))
4. Arrays of LoS (examples are tomographic reconstruction)
5. We will focus on analysis technique suitable for single-point/multi point measurements, extracting information on spatial/temporal dynamics

Motivation & Outline

Diagnostics provides information with different time and spatial resolution

1. Measurements coming from a single point
2. Spatially distributed arrays of measurements (resolving portion of the plasma or entire torus)
3. line integrated measurements (single Line of Sight (LoS))
4. Arrays of LoS (examples are tomographic reconstruction)
5. We will focus on analysis technique suitable for single-point/multi point measurements, extracting information on spatial/temporal dynamics

The needs for the Fourier transform

- Generally theories are developed in the frequency/wavevector domains $(\omega, \mathbf{k})$ rather than in time/space (t, r)

The needs for the Fourier transform

- Generally theories are developed in the frequency/wavevector domains $(\omega, \mathbf{k})$ rather than in time/space (t, r)
- The reason for that resides principally on the fact that *Fourier transform* of governing equation allows an easier treatment of typical mathematical operator

$$\frac{\partial}{\partial t} \Longrightarrow i\omega; \qquad \frac{\partial f}{\partial x_i} \Longrightarrow ik_i\hat{f}(\mathbf{k}, t)$$

The needs for the Fourier transform

- Generally theories are developed in the frequency/wavevector domains $(\omega, \mathbf{k})$ rather than in time/space (t, r)
- The reason for that resides principally on the fact that *Fourier transform* of governing equation allows an easier treatment of typical mathematical operator

$$\frac{\partial}{\partial t} \Longrightarrow i\omega; \qquad \frac{\partial f}{\partial x_i} \Longrightarrow ik_i\hat{f}(\mathbf{k}, t)$$

- Some remarks on basic Fourier Transform and its discrete counterpart the Discrete Fourier Transform are mandatory

Continuous and Discrete Fourier transform 2

- The Direct and Inverse fourier transform of a generic function of time $x(t)$ is defined as :

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-2\pi i f t} \mathrm{d}t \qquad x(t) = \int_{-\infty}^{+\infty} X(f) e^{2\pi i f t} \mathrm{d}f$$

Continuous and Discrete Fourier transform 2

- The Direct and Inverse fourier transform of a generic function of time $x(t)$ is defined as :

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-2\pi i f t} \mathrm{d}t \qquad x(t) = \int_{-\infty}^{+\infty} X(f) e^{2\pi i f t} \mathrm{d}f$$

- In the case of discrete signals $x_n = x(n\Delta t)$ with $0 \le n \le N-1$, sampled at frequency $f_c = \frac{1}{\Delta t}$ we have the corresponding Direct and Inverse Discrete Fourier transform

$$X_n = \frac{1}{N} \sum_{k=0}^{N-1} x_k e^{-2\pi kn/N} \qquad x_k = \frac{1}{N} \sum_{n=-N/2}^{N/2} X_n e^{2\pi kn/N}$$

Continuous and Discrete Fourier transform 2

- The Direct and Inverse fourier transform of a generic function of time $x(t)$ is defined as :

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-2\pi i f t} \mathrm{d}t \qquad x(t) = \int_{-\infty}^{+\infty} X(f) e^{2\pi i f t} \mathrm{d}f$$

- In the case of discrete signals $x_n = x(n\Delta t)$ with $0 \leq n \leq N-1$, sampled at frequency $f_c = \frac{1}{\Delta t}$ we have the corresponding Direct and Inverse Discrete Fourier transform

$$X_n = \frac{1}{N} \sum_{k=0}^{N-1} x_k e^{-2\pi k n/N} \qquad x_k = \frac{1}{N} \sum_{n=-N/2}^{N/2} X_n e^{2\pi k n/N}$$

- The `Sampling Theorem` (Bracewell 1999) ensure that a function whose Fourier transform is zero for $f > f_c$ is fully specified by values spaced at equal intervals not exceeding $\frac{1}{2} f_c^{-1}$

Continuous and Discrete Fourier transform 2

- The Direct and Inverse fourier transform of a generic function of time $x(t)$ is defined as :

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-2\pi i f t} \mathrm{d}t \qquad x(t) = \int_{-\infty}^{+\infty} X(f) e^{2\pi i f t} \mathrm{d}f$$

- In the case of discrete signals $x_n = x(n\Delta t)$ with $0 \leq n \leq N-1$, sampled at frequency $f_c = \frac{1}{\Delta t}$ we have the corresponding Direct and Inverse Discrete Fourier transform

$$X_n = \frac{1}{N} \sum_{k=0}^{N-1} x_k e^{-2\pi k n/N} \qquad x_k = \frac{1}{N} \sum_{n=-N/2}^{N/2} X_n e^{2\pi k n/N}$$

- The `Nyquist Frequency` $f_N = \frac{1}{2\Delta t}$, defines the maximum frequency which can be properly resolved, or equivalently given the frequency of the system we would like to investigate, we had to sample at least at twice the values of this frequency.

Properties of Fourier transform

- Various theorems may be applied to FT (Bracewell 1999) among which we cite:

Theorem

Convolution theorem: If $x(t)$ and $g(t)$ have FT respectively equal to $X(f)$ and $G(f)$ the convolution of the two function $h(t) = \int_{-\infty}^{+\infty} x(t')g(t-t')\mathrm{d}t'$ is equal to $X(f)G(f)$

Theorem

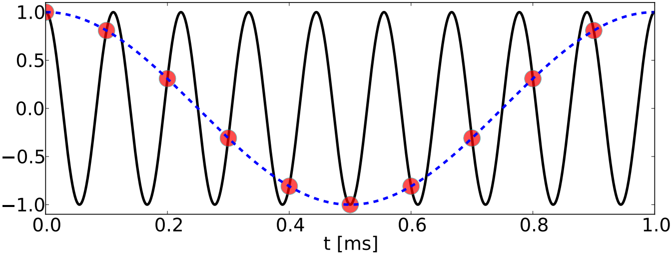
Rayleigh's Theorem The integral of squared modulus of a function is equal to the integral of the squared modulus of its spectrum, i.e:

$$\int_{-\infty}^{+\infty} |f(t)|^2 \mathrm{d}t = \int_{-\infty}^{+\infty} |F(f)|^2 \mathrm{d}f$$

Aliasing, leaking and windowing

CONSORZIO RFX
Ricerca Formazione Innovazione

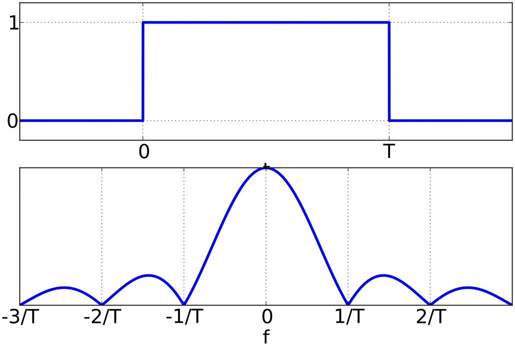
- The presence of frequency higher than the Nyquist frequency may lead to the presence of spurious frequency



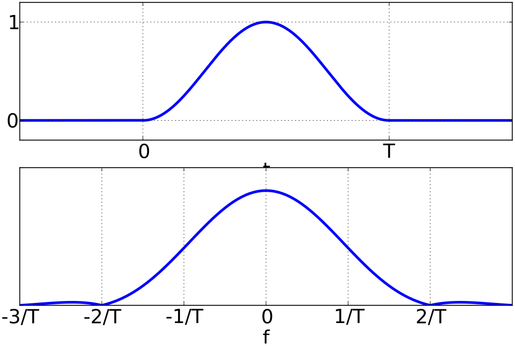
- A 9 kHz cosine if sampled at 10 kHz exhibits a spurious 1 kHz oscillation

Aliasing, leaking and windowing

- Actually signals are acquired for a given period $T \rightarrow$ equivalent to the convolution with a box function $G(t)$ with domain $0 \leq t \leq T$, i.e. $G(t) = 1$ if $0 \leq t \leq T$, 0 otherwise
- In the Fourier space this is equivalent to the multiplication with the Fourier representation of a box function $\rightarrow \mathrm{sinc}(x) = \sin(x)/x \Rightarrow$leaking some power from one frequency bin to the adjacents ones.



- Solution to the leakage: multiplication with an appropriate window function which reduces the lobes as the *Hanning window*: $u_h(t) = \frac{1}{2}(1 - \cos(2\pi t/T))$ for $0 \leq t \leq T$ and 0 otherwise.



Single Point: the autocorrelation function

- A random process $x(t)$ is completely described by its moments, i.e. averages over the probability distribution function

$$E[x(t)] \quad E[x(t_1)x(t_2)] \quad E[x(t_1)x(t_2)x(t_3)] \quad \ldots$$

- We define `Auto-correlation function`, i.e. the second order momentum of the distribution, and the `autocovariance function`

$$R(\tau) = E|x(t)x(t-\tau)| \qquad C(\tau) = E|(x(t)-m)(x(t-\tau)-m)|$$

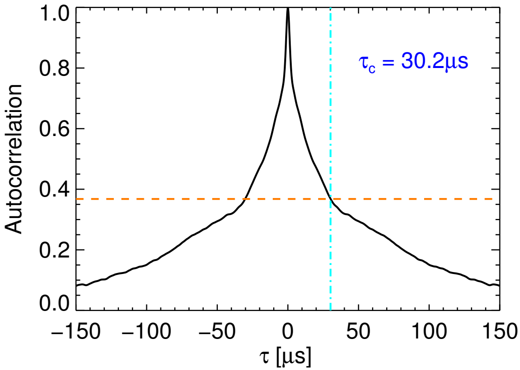
being m the average of $x(t)$

- The `Auto-correlation coefficient factor` is defined as $\rho(\tau) = C(\tau)/C(0)$
- For digitized signals with N samples the estimator of $C(\tau)$ is defined as

$$C_j = \frac{1}{N}\sum_{i=j}^{N-1}(x_i - \overline{x})(x_{i-j} - \overline{x}) \qquad \overline{x} = \frac{1}{N}\sum_{i=0}^{N-1} x_i$$

Auto-correlation: practical use

- Define the `Auto-correlation time` of a turbulent field such as the potential: $R(\tau_c) = \frac{\max(R(\tau))}{e}$



Single point: the power spectrum

- The Power spectrum $S(f)$ is defined as the Fourier transform of the Autocorrelation-function.

Single point: the power spectrum

- The Power spectrum $S(f)$ is defined as the Fourier transform of the Autocorrelation-function.
- It describes the frequency distribution of the *power of the signal.*

Single point: the power spectrum

- The Power spectrum $S(f)$ is defined as the Fourier transform of the Autocorrelation-function.
- It describes the frequency distribution of the *power of the signal.*
- It corresponds to the limit of the `periodogram` of limited signals $x_T(t)$ $\frac{E[FT(x_T(t))^2]}{T} \xrightarrow[T\to\infty]{} S(f)$

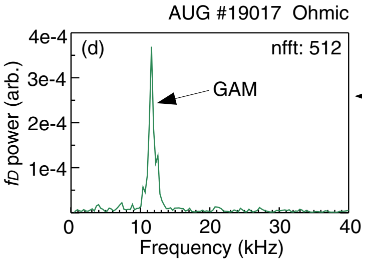
Single point: the power spectrum

- The Power spectrum $S(f)$ is defined as the Fourier transform of the Autocorrelation-function.
- It describes the frequency distribution of the *power of the signal.*
- It corresponds to the limit of the `periodogram` of limited signals $x_T(t)$ $\frac{E[FT(x_T(t))^2]}{T} \xrightarrow[T\to\infty]{} S(f)$
- Numerically, the signal is divided into M slices, treated as independent realizations, and we compute the power spectral estimator $\hat{S}_n$ related to the real power spectrum $S(f_n)$ according to

$$\hat{S}_n = \frac{1}{M}\sum_{k=1}^{M} |X_n^{(k)}|^2; \qquad \hat{S}_n \simeq S(f_n)\Delta f$$

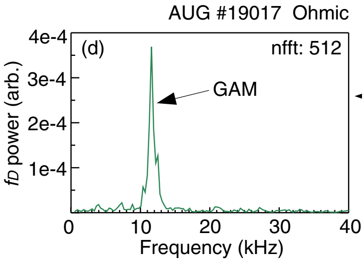
CONSORZIO RFX
Ricerca Formazione Innovazione

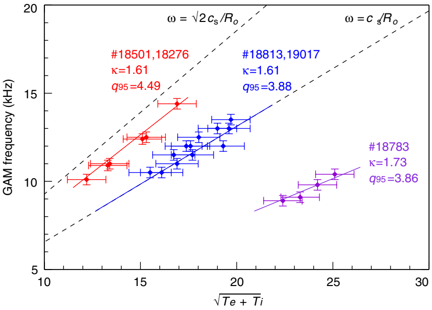
- Mode identification at a given frequency (Conway et al. 2005)



Power spectrum: practical use

- Mode identification at a given frequency (Conway et al. 2005)

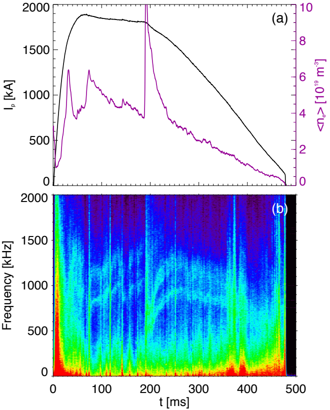




- Information must be completed. In the exampleGeodesic Acoustic Modes identified considering their scaling with c_s

Power spectrum: The spectrogram

- The same information can be also analyzed in time applying the spectrogram technique in the time/frequency space (Spagnolo et al. 2011)



- Alfvénic nature, with ω depenend from the Alfvén velocity $v_A = B/\sqrt{\rho\mu_0}$ revealed by the comparison with the plasma density

Two-points techniques

- Spatially distributed measurements allow access to spatial structure of the fluctuations

Two-points techniques

CONSORZIO RFX
Ricerca Formazione Innovazione

- Spatially distributed measurements allow access to spatial structure of the fluctuations
- The minimum set includes two measurements $x(t)$ and $y(t)$. We can define the Cross-correlation function, The cross-covariance function and the cross-correlation coefficient function

$$\boxed{R_{xt}(\tau) = E[y(t)x(t-\tau)]}$$

$$\boxed{C_{xy}(\tau) = E[(y(t) - \overline{y})(x(t-\tau) - \overline{x})]}$$

$$\boxed{\rho_{yx}(\tau) = \frac{C_{yx}(\tau)}{\sqrt{C_{xx}(0)C_{yy}(0)}}}$$

Two-points techniques

- Spatially distributed measurements allow access to spatial structure of the fluctuations
- The minimum set includes two measurements $x(t)$ and $y(t)$. We can define the `Cross-correlation function`, `The cross-covariance function` and the `cross-correlation coefficient function`

$$R_{xt}(\tau) = E[y(t)x(t-\tau)]$$

$$C_{xy}(\tau) = E[(y(t)-\overline{y})(x(t-\tau)-\overline{x})]$$

$$\rho_{yx}(\tau) = \frac{C_{yx}(\tau)}{\sqrt{C_{xx}(0)C_{yy}(0)}}$$

- In the discrete counterpart of the cross-covariance is defined as

$$C_{yx,j} = \frac{1}{N}\sum_{i=j}^{N-1}(y_i-\overline{y})(x_{i-j}-\overline{x})$$

Two-points technique

- In analogy to the power spectrum we define the Cross-power spectrum $S_{XY}(f)$ as the Fourier transform of the cross-correlation function

Two-points technique

- In analogy to the power spectrum we define the Cross-power spectrum $S_{XY}(f)$ as the Fourier transform of the cross-correlation function
- If $x(t)$ and $y(t)$ are real function, the cross-power spectrum is totally defined by the positive frequency only

Two-points technique

- In analogy to the power spectrum we define the Cross-power spectrum $S_{XY}(f)$ as the Fourier transform of the cross-correlation function
- If $x(t)$ and $y(t)$ are real function, the cross-power spectrum is totally defined by the positive frequency only
- The cross-spectrum is complex-valued $S_{YX}(f) = |S_{YX}(f)|e^{i\Theta_{YX}(f)}$ with $\Theta_{YX}(f)$ equal to the `Phase spectrum`

Two-points technique

- In analogy to the power spectrum we define the Cross-power spectrum $S_{XY}(f)$ as the Fourier transform of the cross-correlation function
- If $x(t)$ and $y(t)$ are real function, the cross-power spectrum is totally defined by the positive frequency only
- The cross-spectrum is complex-valued $S_{YX}(f) = |S_{YX}(f)|e^{i\Theta_{YX}(f)}$ with $\Theta_{YX}(f)$ equal to the `Phase spectrum`
- We define the `coherence` as $\gamma_{YX}(f) = \frac{|S_{YX}(f)|}{\sqrt{S_Y(f)S_X(f)}}$

Two-points technique

- In analogy to the power spectrum we define the Cross-power spectrum $S_{XY}(f)$ as the Fourier transform of the cross-correlation function
- If $x(t)$ and $y(t)$ are real function, the cross-power spectrum is totally defined by the positive frequency only
- The cross-spectrum is complex-valued $S_{YX}(f) = |S_{YX}(f)|e^{i\Theta_{YX}(f)}$ with $\Theta_{YX}(f)$ equal to the `Phase spectrum`
- We define the `coherence` as $\gamma_{YX}(f) = \frac{|S_{YX}(f)|}{\sqrt{S_Y(f)S_X(f)}}$
- In the case of discrete signals with finite temporal length the following definitions hold (in analogy to single point case)

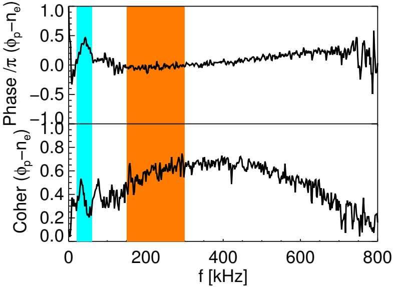
$$\hat{S}_{Y,X,n} = \frac{1}{M}\sum_{k=1}^{M} Y_n^{(k)} X_n^{*(k)} \qquad \hat{S}_{Y,X,n} \simeq S_{YX}(f_n)\Delta f$$

Phase spectrum

- The method can be applied also in the case of two quantities measured on the same location

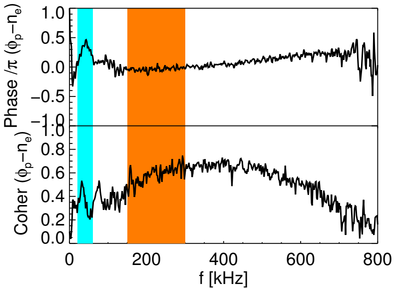
Phase spectrum

- The method can be applied also in the case of two quantities measured on the same location
- In the case of Langmuir probes for example, electron density n_e and plasma potential ϕ_p are know in the same nominal position



Phase spectrum

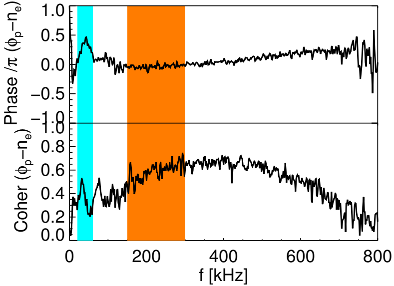
- The method can be applied also in the case of two quantities measured on the same location
- In the case of Langmuir probes for example, electron density n_e and plasma potential ϕ_p are know in the same nominal position



- This allow the possibility to distinguish the frequency where turbulence is Interchange-dominated from that where turbulence is Drift-dominated

Phase spectrum

- The method can be applied also in the case of two quantities measured on the same location
- In the case of Langmuir probes for example, electron density n_e and plasma potential ϕ_p are know in the same nominal position



- This allow the possibility to distinguish the frequency where turbulence is Interchange-dominated from that where turbulence is Drift-dominated
- Other possibility is the determination of the polarization of magnetic fluctuations frequency resolved

Wave-vector estimate

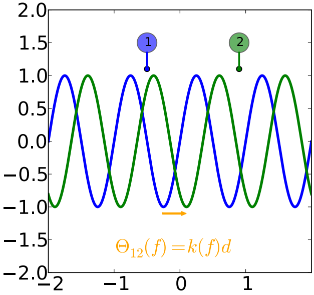
- In the case of a reasonably deterministic dispersion relation between k and f, the phase may be used for the determination of k

Wave-vector estimate

- In the case of a reasonably deterministic dispersion relation between k and f, the phase may be used for the determination of k
- Indeed if $k = k(f)$ then $g(t, \overline{r}) = \int_{-\infty}^{+\infty} G(f) e^{-ik(f)\cdot\overline{r}} e^{i2\pi f} df$

Wave-vector estimate

- In the case of a reasonably deterministic dispersion relation between k and f, the phase may be used for the determination of k
- Indeed if $k = k(f)$ then $g(t, \overline{r}) = \int_{-\infty}^{+\infty} G(f) e^{-ik(f)\cdot\overline{r}} e^{i2\pi f} df$
- `Wave vector` is estimated from `Phase spectrum`



Wave-vector estimate

- In the case of a reasonably deterministic dispersion relation between k and f, the phase may be used for the determination of k
- Indeed if $k = k(f)$ then $g(t, \overline{r}) = \int_{-\infty}^{+\infty} G(f) e^{-ik(f)\cdot\overline{r}} e^{i2\pi f} df$
- If measurements are displaced in the aximuthal direction, i.e. $d = r_p \Delta\Theta$ then the azimuthal mode number is computed as $m = \Theta_{12}/\Delta\Theta$
- The probe distance d must be less than aa wave length, less than a correlation length, but far enough the detect a measurable phase difference

Fluctuation-induced particle transport

- Fluctuations induced particle flux is defined as $\Gamma = E[\tilde{n}(t)\tilde{v}(t)] = E[\tilde{n}(t)\tilde{E}(t)]/B$
- According to previous definitions and properties

$$\Gamma = \frac{1}{B} R_{nE}(\tau = 0) = \frac{2}{B}\int_0^{+\infty} \Re\mathrm{e}[S_{nE}(f)]df$$

- In quasi-static approximation $\tilde{E} = -\nabla\tilde{\phi}$, and considering the finiteness of the measurements we end up with the formula

$$\Gamma(f) = \frac{2}{BT}\Im\mathrm{m}\{E[k(f)N(f)\Phi^*(f)]\}$$

$$\Gamma(f) = \frac{2k(f)}{B}\Im\mathrm{m}\{S_{n\phi}(f)\} \text{ if } k(f) \text{ is deterministic}$$

Fluctuation-induced particle transport 2

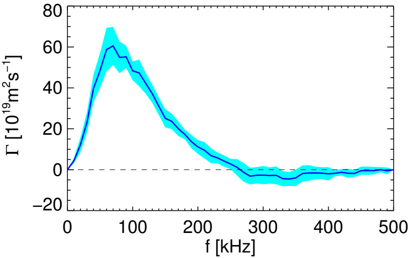
- In practice, considering digitized signals we have (see for example (Antoni et al. 2000))

$$\Gamma(f) = \frac{1}{M}\sum_{k=1}^{M}\Gamma^{k}(f) = \frac{2}{BM}\sum_{k=1}^{M-1}\Im\mathrm{m}\{k^{(k)}(f)N^{(k)}(f)\Phi^{*(k)}(f)\}$$

Fluctuation-induced particle transport 2

- In practice, considering digitized signals we have (see for example (Antoni et al. 2000))

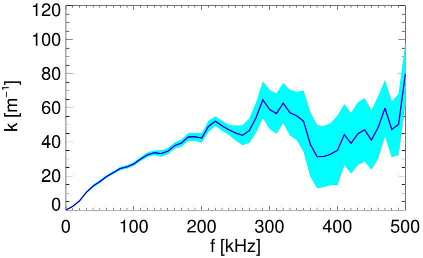
$$\Gamma(f) = \frac{1}{M}\sum_{k=1}^{M}\Gamma^{k}(f) = \frac{2}{BM}\sum_{k=1}^{M-1}\Im\mathrm{m}\{k^{(k)}(f)N^{(k)}(f)\Phi^{*(k)}(f)\}$$



Fluctuation-induced particle transport 2

- In practice, considering digitized signals we have (see for example (Antoni et al. 2000))

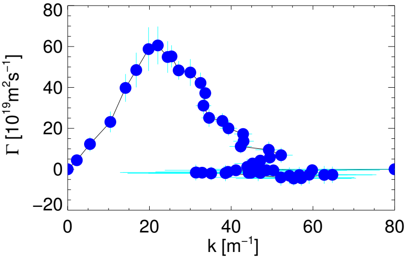
$$\Gamma(f) = \frac{1}{M}\sum_{k=1}^{M}\Gamma^{k}(f) = \frac{2}{BM}\sum_{k=1}^{M-1}\Im\mathrm{m}\{k^{(k)}(f)N^{(k)}(f)\Phi^{*(k)}(f)\}$$



CONSORZIO RFX
Ricerca Formazione Innovazione

- In practice, considering digitized signals we have (see for example (Antoni et al. 2000))

$$\Gamma(f) = \frac{1}{M}\sum_{k=1}^{M}\Gamma^k(f) = \frac{2}{BM}\sum_{k=1}^{M-1}\Im\mathrm{m}\{k^{(k)}(f)N^{(k)}(f)\Phi^{*(k)}(f)\}$$

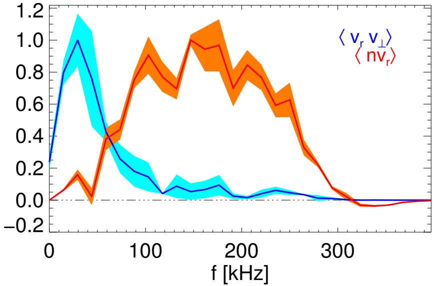


More than transport of particles

- Similar method may be used for the determination of the *Reynolds stress* $\langle \tilde{v}_r \tilde{v}_\perp \rangle$ which play a role in the momentum generation for both Tokamak and RFPs as $\partial_t(V_\phi) \propto -\partial_r \langle \tilde{v}_r \tilde{v}_\phi \rangle + \ldots$ (see e.g. (Vianello et al. 2005a,b, 2006))

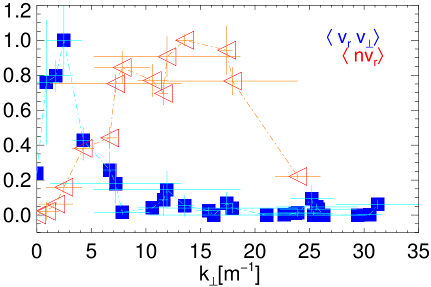
More than transport of particles

- Similar method may be used for the determination of the *Reynolds stress* $\langle \tilde{v}_r \tilde{v}_\perp \rangle$ which play a role in the momentum generation for both Tokamak and RFPs as $\partial_t (V_\phi) \propto -\partial_r \langle \tilde{v}_r \tilde{v}_\phi \rangle + \dots$ (see e.g. (Vianello et al. 2005a,b, 2006))



More than transport of particles

- Similar method may be used for the determination of the *Reynolds stress* $\langle \tilde{v}_r \tilde{v}_\perp \rangle$ which play a role in the momentum generation for both Tokamak and RFPs as $\partial_t(V_\phi) \propto -\partial_r \langle \tilde{v}_r \tilde{v}_\phi \rangle + \ldots$ (see e.g. (Vianello et al. 2005a,b, 2006))



The wavenumber-Frequency Spectrum

- In general, we have a turbulent medium, with different wave numbers corresponding to the same frequency

The wavenumber-Frequency Spectrum

- In general, we have a turbulent medium, with different wave numbers corresponding to the same frequency
- We need to compute the `Wave number-frequency power spectrum` $S(k,\omega)$

The wavenumber-Frequency Spectrum

- In general, we have a turbulent medium, with different wave numbers corresponding to the same frequency
- We need to compute the `Wave number-frequency power spectrum` $S(k, \omega)$
- This is equivalent to the Fourier transform of the space time correlation function $R(\chi, \tau)$

$$R(\chi, \tau) = E[g(x, t)g^*(x - \chi, t - \tau)]$$

$$S(k, \omega) = \iint_{-\infty}^{+\infty} R(\chi, \tau) e^{-i(\omega\tau - k\chi)} d\chi d\tau$$

The wavenumber-Frequency Spectrum

- In general, we have a turbulent medium, with different wave numbers corresponding to the same frequency
- We need to compute the `Wave number-frequency power spectrum` $S(k,\omega)$
- This is equivalent to the Fourier transform of the space time correlation function $R(\chi,\tau)$

$$R(\chi,\tau) = E[g(x,t)g^*(x-\chi,t-\tau)]$$

$$S(k,\omega) = \iint_{-\infty}^{+\infty} R(\chi,\tau)e^{-i(\omega\tau-k\chi)}d\chi d\tau$$

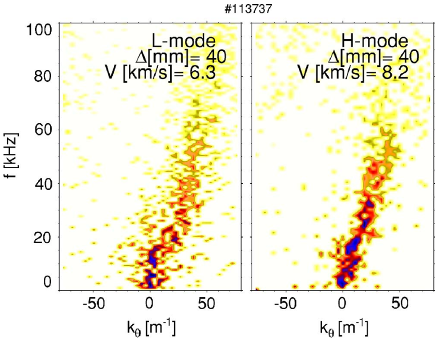
- In the two 2 points cases are available, the spectrum is reconstructed on a statistical basis considering the M different realizations:

$$\hat{S}_L(k,\omega) = \hat{S}_L(p\Delta k, 2\pi n\Delta f) = \frac{1}{M}\sum_{j=1}^{M} S_n^{(j)} I_p[k_n^j]$$

$$I_p[k_n^j] = \begin{cases} 1 \text{ for } (p-1/2)\Delta k < k_n^{(j)} < (p+1/2)\Delta k \\ 0 \text{ elsewhere} \end{cases}$$

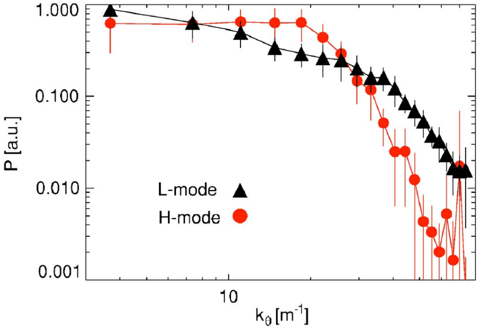
Spectral power density

- Spectral power density from GPI LoS on NSTX (Agostini et al. 2007)



Spectral power density

- We can consequently compute $S(k) = \int_{-\infty}^{+\infty} S(k,\omega)\frac{d\omega}{2\pi}$



Beyond Fourier: Wavelet transform

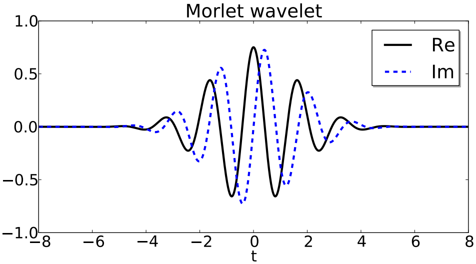
- The Fourier decomposition uses trigonometric functions as orthogonal basis

Beyond Fourier: Wavelet transform

- The Fourier decomposition uses trigonometric functions as orthogonal basis
- These functions oscillates forever, i.e. the information content of a generic function is spread over all the spectral component

Beyond Fourier: Wavelet transform

- The Fourier decomposition uses trigonometric functions as orthogonal basis
- These functions oscillates forever, i.e. the information content of a generic function is spread over all the spectral component
- Thus Fourier decomposition is not suitable for processes highly localized in time/space. We can use `Wavelet Transform`(Farge 1992). Functions well defined in time and scale



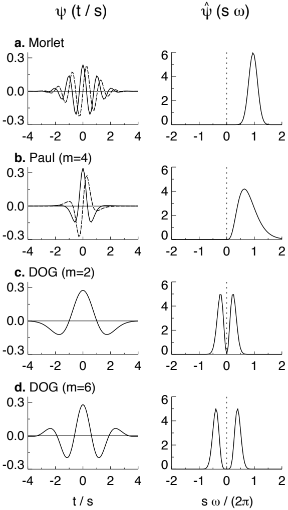
Beyond Fourier: Wavelet transform

- The Fourier decomposition uses trigonometric functions as orthogonal basis
- These functions oscillates forever, i.e. the information content of a generic function is spread over all the spectral component
- The admissibility condition $C_\psi = \int_{-\infty}^{+\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty$, implies that the wavelet ψ is a zero mean function $\int_{\infty}^{\infty} \psi(t) dt = 0$
- Defining time-frequency atoms as $\psi_{s,\tau} = \frac{1}{\sqrt{\tau}} \psi\left(\frac{t-s}{\tau}\right)$ the `Continuous Wavelet Transform` is defined as

$$w(s,\tau) = \frac{1}{\sqrt{\tau}} \int_{-\infty}^{+\infty} f(t) \psi^* \left(\frac{t-s}{\tau}\right) dt$$

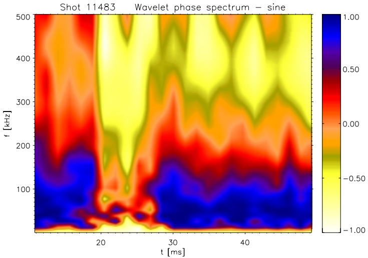
Choice & type of Wavelet

- Example of different types of wavelet (Mallat 1999)



Wavelet application

- In analogy to Fourier we can define Wavelet Cross power spectrum and Corresponding phase spectrum (well localized in time/frequency)



- Phase spectrum between density and potential varies because of variation of the shear → responsible for transport reduction (Antoni et al. 2000)

Wavelet for turbulence analysis

- Wavelet coefficient exhibits similar scaling properties as the fluctuations of the signals at the same

$$\delta_\tau f = f(t+\tau) - f(t) \sim \tau^h \Rightarrow |w(t,\tau)| \sim \tau^{h+1/2}$$

Wavelet for turbulence analysis

- Wavelet coefficient exhibits similar scaling properties as the fluctuations of the signals at the same

$$\delta_\tau f = f(t+\tau) - f(t) \sim \tau^h \Rightarrow |w(t,\tau)| \sim \tau^{h+1/2}$$

- Thus wavelet coefficient may be used for the study of the scaling properties of the fluctuation

Wavelet for turbulence analysis

CONSORZIO RFX
Ricerca Formazione Innovazione

- Wavelet coefficient exhibits similar scaling properties as the fluctuations of the signals at the same

$$\delta_\tau f = f(t+\tau) - f(t) \sim \tau^h \Rightarrow |w(t,\tau)| \sim \tau^{h+1/2}$$

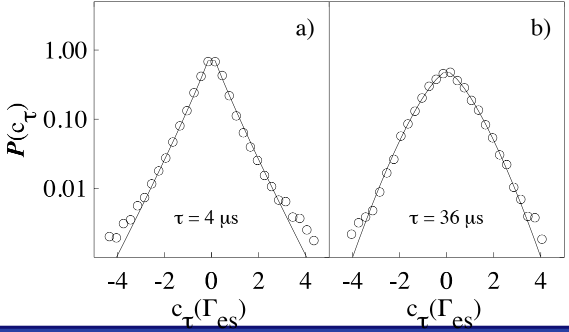
- Thus wavelet coefficient may be used for the study of the scaling properties of the fluctuation
- Easy way to compute the `Probability Distribution Function` of normalized flutuations $C(t,\tau) = \frac{w(t,\tau) - \langle w(t,\tau)\rangle}{\sigma_\tau}$. For self-similar fluctuations, these should collapse to a single form

Wavelet for turbulence analysis

- Wavelet coefficient exhibits similar scaling properties as the fluctuations of the signals at the same

$$\delta_\tau f = f(t+\tau) - f(t) \sim \tau^h \Rightarrow |w(t,\tau)| \sim \tau^{h+1/2}$$

- Thus wavelet coefficient may be used for the study of the scaling properties of the fluctuation
- **Revealing non self-similariy i.e. Intermittency**

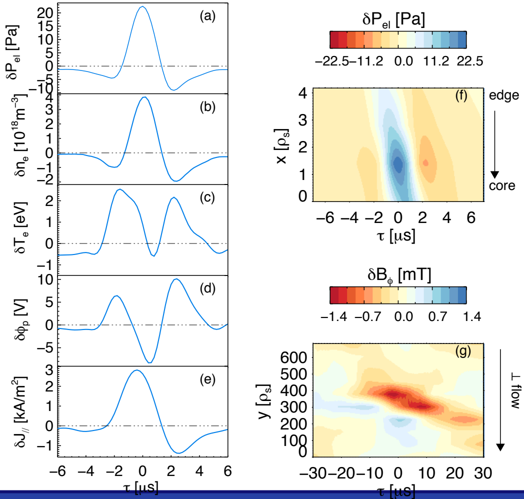


- Intermittency is due to the presence of strong, sporadic fluctuations
- The `Local Intermittency Measurements` is a method, based on wavelet, which identifies in time and scales these fluctuations (Antoni et al. 2001)
- The method distinguish two orthogonal sets :

$$\{w(t,\tau)\} = \{w_e(t,\tau)\} \oplus \{w_g(t,\tau)\} \quad \text{with } F(\tau) = \frac{\langle w_g(t,\tau)^4 \rangle}{[\langle w_g(t,\tau)^2]^2} = 3$$

- The typical fluctuations may be deriven using `Conditional Average Procedure`, i.e. averaging different time windows of the signal, each centered around the occurrence of an Intermittent Events or blobs

- Conditional average may be applied to different signals using the same trigger (Martines et al. 2009; Vianello et al. 2010)



Summary

- High temporal and spatial resolution are needed for better characterization of the plasma. But two points still gives a bunch of information
- Fourier transform allows estimate of quantities directly comparable with theories
- Often localized events (in space or time) require more sophisticated tools which maintain the locality of the information
- As much as possible correlation between different diagnostics are generally needed for an appropriate comprehension

Bibliography I

[1] M Agostini et al. "Study of statistical properties of edge turbulence in the National Spherical Torus Experiment with the gas puff imaging diagnostic". In: *Physics of Plasmas* 14.10 (Jan. 2007), p. 102305.

[2] V Antoni et al. "Electrostatic transport reduction induced by flow shear modification in a reversed field pinch plasma". In: *Plasma Physics and Controlled Fusion* 42.2 (2000), pp. 83–90.

[3] V Antoni et al. "Electrostatic turbulence intermittency and MHD relaxation phenomena in a RFP plasma". In: *EPL (Europhysics Letters)* 54.1 (Jan. 2001), pp. 51–57.

[4] Ronald Bracewell. *The Fourier Transform & Its Applications*. 3rd ed. McGraw-Hill Science/Engineering/Math, June 1999.

[5] G D Conway et al. "Direct measurement of zonal flows and geodesic acoustic mode oscillations in ASDEX Upgrade using Doppler reflectometry". In: *Plasma Physics and Controlled Fusion* 47.8 (July 2005), pp. 1165–1185.

[6] M Farge. "Wavelet Transforms and Their Applications to Turbulence". In: *Annual Review Of Fluid Mechanics* 24 (1992), pp. 395–457.

[7] Stephane Mallat. *A Wavelet Tour of Signal Processing, Second Edition (Wavelet Analysis & Its Applications)*. 2nd ed. Academic Press, Sept. 1999.

Bibliography II

[8] E Martines et al. “Current filaments in turbulent magnetized plasmas”. In: *Plasma Physics and Controlled Fusion* 51 (Dec. 2009), p. 124053.

[9] S Spagnolo et al. “Alfvén Eigenmodes in the RFX-mod reversed-field pinch plasma”. In: *Nuclear Fusion* (2011).

[10] Nicola Vianello et al. “Drift-Alfvén vortex structures in the edge region of a fusion relevant plasma”. In: *Nuclear Fusion* 50.4 (2010), p. 042002.

[11] Nicola Vianello et al. “Reynolds and Maxwell stress measurements in the reversed field pinch experiment Extrap-T2R”. In: *Nuclear Fusion* 45.8 (July 2005), pp. 761–766.

[12] Nicola Vianello et al. “Self-Regulation of E×B Flow Shear via Plasma Turbulence”. In: *Physical Review Letters* 94.13 (Apr. 2005), p. 135001.

[13] Nicola Vianello et al. “Turbulence, flow and transport: hints from reversed field pinch”. In: *Plasma Physics and Controlled Fusion* 48.4 (Mar. 2006), S193–S203.