# Fluctuation data analysis in Fusion Relevant Plasmas

## Extracting information on relevant underlying dynamics

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# Motivation & Outline

Diagnostics provides information with different time and spatial resolution

1. Measurements coming from a single point
2. Spatially distributed arrays of measurements (resolving portion of the plasma or entire torus)
3. line integrated measurements (single Line Of Sight (LoS))
4. Arrays of LoS (examples are tomographic reconstruction)

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We will focus on analysis technique suitable for single-point/multi point measurements, extracting information on spatial/temporal dynamics

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Some remarks on basic Fourier Transform and its discrete counterpart the Discrete Fourier Transform are mandatory

# Continuous and Discrete Fourier transform 2

The Direct and Inverse fourier transform of a generic function of time $x(t)$ is defined as :

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-2\pi i f t} \mathrm{d}t \qquad x(t) = \int_{-\infty}^{+\infty} X(f) e^{2\pi i f t} \mathrm{d}f$$

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## Theorem

*Similarity Theorem: If $x(t)$ has the Fourier transform $X(f)$ then $x(at)$ has the Fourier Transform $|a|^{-1}F(f/a)$*

This will be useful in the analysis of scaling properties of the fluctuations

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**Theorem**

*Addition Theorem:* *If $x(t)$ and $g(t)$ have FT respectively $X(f)$ and $G(f)$ then $x(t) + g(t)$ has Fourier transform $X(f) + G(f)$*

The linearity of Fourier transform allows an easy treatment of linear equation in the Fourier domain

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## Theorem

*Shift theorem: If $x(t)$ and $g(t)$ have FT respectively $X(f)$ then $x(t-a)$ has Fourier transform $e^{-2\pi iaf}X(f)$*

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## Theorem

*Convolution theorem: If $x(t)$ and $g(t)$ have FT respectively equal to $X(f)$ and $G(f)$ the convolution of the two function $h(t) = \int_{-\infty}^{+\infty} x(t')g(t-t')\mathrm{d}t'$ is equal to $X(f)G(f)$*

The importance of the convolution equation resides on the fact that it allows treatment of non-linearities as the term $\mathbf{v} \cdot \nabla \mathbf{v}$ in the Navier-Stokes equations

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## Theorem

*Rayleigh's Theorem The integral of squared modulus of a function is equalt to the integral of the squared modulus of its spectrum, i.e:*

$$\int_{-\infty}^{+\infty} |f(t)|^2 \mathrm{d}t = \int_{-\infty}^{+\infty} |F(f)|^2 \mathrm{d}f$$

This is equivalent to an energy conservation law for the time or frequency domain representation of the signal

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- A foundamental results of signal analysis is *Sampling Theorem* which states that A function whose Fourier transform is zero for $f > f_c$ is fully specified by values spaced at equal intervals not exceeding $\frac{1}{2} f_c^{-1}$
- The *Nyquist Frequency* $f_N = \frac{1}{2\Delta t}$, equivalent to half ot the sampling frequency, thus define the maximum frequency which can be properly resolved, or equivalently given the frequency of the system we would like to investigate, we had to sample at least at twice the values of this frequency.

# Discrete Fourier Transform

- Given the discretiness of the sampling, also the corresponding Fourier transform will be discrete in the Fourier space.
- The basic concept of DFT resides on the fact that the algorithm applied on an $N$-sampled data, will produces an information on $N$ frequencies, assuming that the information is conserved
- The Fourier transform of a digitized $N$-sampled data will be of the form $f_n = n\Delta f$ with (if $N$ is even) $-\frac{N}{2} \leq n \leq \frac{N}{2}$. The frequency resolution, in order to span all the allowed frequency range will be of the form $\Delta f = \frac{1}{T} = \frac{1}{\Delta t}$
- The Direct and Inverse Discrete Fourier transform are then defined as

$$X_n = \frac{1}{N} \sum_{k=0}^{N-1} x_k e^{-2\pi kn/N}$$

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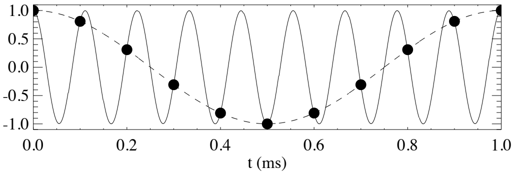
# Aliasing, leaking and windowing

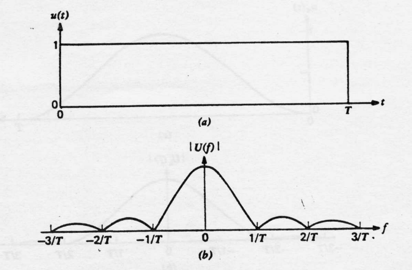
- The presence of frequency higher than the Nyquist frequency may lead to the presence of spurious frequency



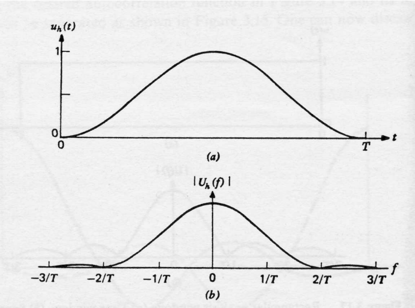

- A 9 kHz sine if sampled at 10 kHz exhibits a spurious 1 kHz oscillation

# Aliasing, leaking and windowing

- The finite extension of the measurements, acquired for a given period $T$ is equivalent to the convolution of the signal with a box function $G(t)$ with domain $0 \leq t \leq T$, i.e. $G(t) = 1 \text{if} 0 \leq t \leq T$ 0 otherwise
- According the theorems for the Fourier transform (which can be applied also for discrete transform) this is equivalent in the Fouirer space to the moltiplication of the of the fourier transform but the Fourier representation of a box function is $\mathrm{sinc}(x) = \sin(x)/x$ function as shown, whichleaking some power from one frequency bin to the adjacents ones.



- Solution to the lakage problem is multiplying data by an appropriate window function which reduces the lobes as the *Hanning window* defined as $u_h(t) = \frac{1}{2}(1 - \cos(2\pi t/T))$ for $0 \leq t \leq T$ and 0 otherwise. Its behavior in real and fourire space is the following



# Single Point: the autocorrelation function

- We know from the statistics a random process $x(t)$ is completely described by its moments, which are the average over the probability distribution function

$$E|x(t)| \quad E|x(t_1)x(t_2)| \quad E|x(t_1)x(t_2)x(t_3)| \quad \ldots$$

- The Auto-correlation function, i.e. the second order momentum of the distribution, or the autocovariance function

$$\boxed{R(\tau) = E|x(t)x(t-\tau)|} \qquad \boxed{C(\tau) = E|(x(t)-m)(x(t-\tau)-m)|}$$

being $m$ the average of $x(t)$

- The Auto-correlation coefficient factor is defined as $\rho(\tau) = C(\tau)/C(0)$
- For digitized signals with $N$ samples the estimator of $C(\tau)$ is defined as

$$C_j = \frac{1}{N}\sum_{i=j}^{N-1}(x_i - \overline{x})(x_{i-j} - \overline{x}) \qquad \overline{x} = \frac{1}{N}\sum_{i=0}^{N-1} x_i$$

# Auto-correlation: practical use

- Define the *Auto-correlation time* of a turbulent field such as the potential
- **Inserisci figura con autocorrelazione di un potenziale**

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- From a practial point of view, we divide signals into $M$ slices, assumed as indipendent realization of the same stochastic process and we compute

$$\hat{S}(f) = \frac{1}{M}\sum_{k=1}^{M} S^{(k)}(f); \qquad S^{(k)}(f) = \frac{1}{T}|X_T^{(k)}(f)|$$

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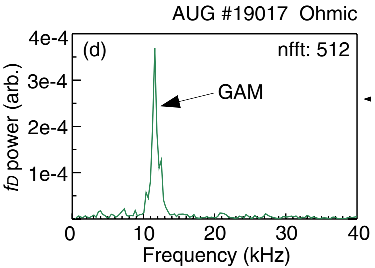
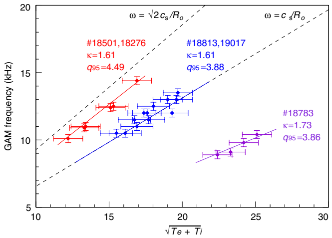
- With digitized signal, the power spectral estimator $\hat{S}_n$ is related to the real power spectrum $S(f_n)$ as

$$\hat{S}_n = \frac{1}{M}\sum_{k=1}^{M} |X_n^{(k)}|^2; \qquad \hat{S}_n \simeq S(f_n)\Delta f$$

# Power spectrum: practical use



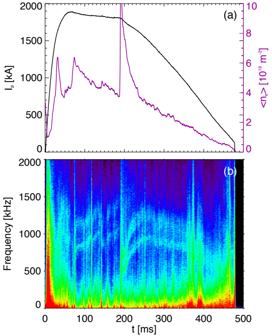

- Mode identification at a given frequency (Conway et al. 2005)




- But information must be completed. In the example, precise identification require scaling of identified mode as a function of ion sound gyroradius

# Power spectrum: The spectrogram

- The same information can be also analyzed in time applying the spectrogram technique which shows how the spectral density of the signal varying in time/frequency space (Spagnolo et al. 2011)




- Again the information must be completed, as in the case proposed where the Alfvénic nature of the observed peaks is revealed by the comparison with the plasma density

# Two-points techniques

- Spatially distributed measurements allow access to spatial structure of the fluctuations

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- The minimum set includes two measurements $x(t)$ and $y(t)$. We can define the Cross-correlation function, The cross-covariance function and the cross-correlation coefficient function

$$R_{xt}(\tau) = E[y(t)x(t-\tau)]$$

$$C_{xy}(\tau) = E[(y(t)-\overline{y})(x(t-\tau)-\overline{x})]$$

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- In the discrete counterpart of the cross-covariance is defined as

$$C_{yx,j} = \frac{1}{N}\sum_{i=j}^{N-1}(y_i-\overline{y})(x_{i-j}-\overline{x})$$

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- We can also defined the `coherence` as $\gamma_{YX}(f) = \frac{|S_{YX}(f)|}{\sqrt{S_Y(f)S_X(f)}}$
- In the case of discrete signals with finite temporal length the following definitions hold (in analogy to single point case)

$$\hat{S}_{Y,X,n} = \frac{1}{M}\sum_{k=1}^{M} Y_n^{(k)} X_n^{*(k)} \qquad \hat{S}_{Y,X,n} \simeq S_{YX}(f_n)\Delta f$$

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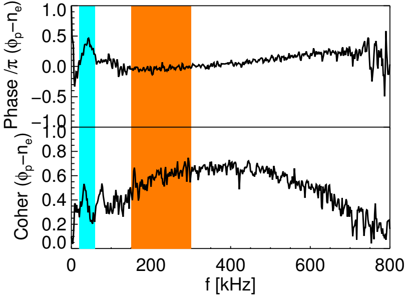

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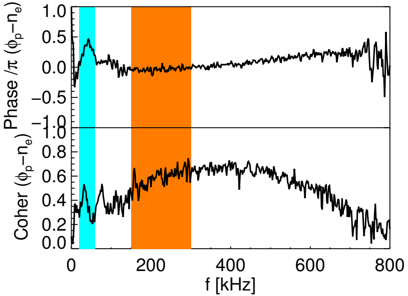


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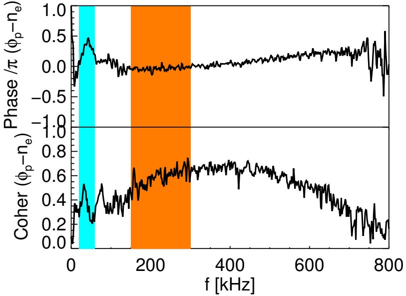


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- Other possibility is the determination of the polarization of magnetic fluctuations frequency resolved

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- If measures exhists at two position $g(t, \overline{r}_1)$ and $g(t, \overline{r}_2)$ separated by $d$ at a given frequency the two signals will be phase shifted of $\Theta_{12}(f) = k(f)d$ where $\Theta_{12}(f)$ is computed trough the cross-power spectrum

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- If measures exhists at two position $g(t, \overline{r}_1)$ and $g(t, \overline{r}_2)$ separated by $d$ at a given frequency the two signals will be phase shifted of $\Theta_{12}(f) = k(f)d$ where $\Theta_{12}(f)$ is computed trough the cross-power spectrum
- If measurements are displaced in the aximuthal direction, i.e. $d = r_p\Delta\Theta$ then the azimuthal mode number is computed as $m = \Theta_{12}/\Delta\Theta$

# Wave-vector estimate

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- If measurements are displaced in the aximuthal direction, i.e. $d = r_p\Delta\Theta$ then the azimuthal mode number is computed as $m = \Theta_{12}/\Delta\Theta$
- The probe distance $d$ must be less than aa wave length, less than a correlation lenght, but far enough the detect a measurable phase difference

# Fluctuation-induced particle transport

- Fluctuations induced particle flux is defined as $\Gamma = E[\tilde{n}(t)\tilde{v}(t)] = E[\tilde{n}(t)\tilde{E}(t)]/B$
- According to previous definitions and properties

$$\Gamma = \frac{1}{B}R_{nE}(\tau = 0) = \frac{1}{B}\int_{-\infty}^{+\infty} S_{nE}(f)e^{i2\pi f\tau}df = \frac{2}{B}\int_{0}^{+\infty} \Re[S_{nE}(f)]df$$

- In quasi-static approximation $\tilde{E} = -\nabla\tilde{\phi}$, and considering the finitness of the measurements we end up with the formula

$$\Gamma(f) = \frac{2}{BT}\Im\{E[k(f)N(f)\Phi^*(f)]\}$$

$$\Gamma(f) = \frac{2k(f)}{B}\Im\{S_{n\phi}(f)\} \text{ if } k(f) \text{ is deterministic}$$

# Fluctuation-induced particle transport 2

- In practice, considering digitzed signals we have (see for example (Antoni et al. 2000))

$$\Gamma(f) = \frac{1}{M}\sum_{k=1}^{M}\Gamma^{k}(f)$$

$$\Gamma^{(k)}(f) = \frac{2}{B}\Im\{k^{(k)}(f)N^{(k)}(f)\Phi^{*(k)}(f)\}0 < f < N/2 - 1$$

$$\Gamma^{(k)}(f) = \frac{1}{B}\Im\{k^{(k)}(f)N^{(k)}(f)\Phi^{*(k)}(f)\}f = 0, f = N/2$$

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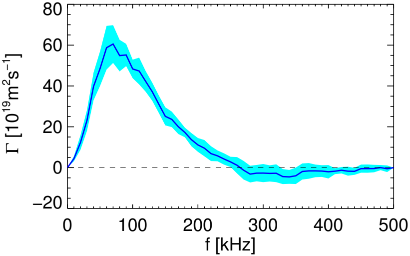

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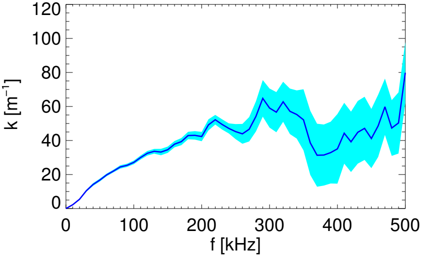

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