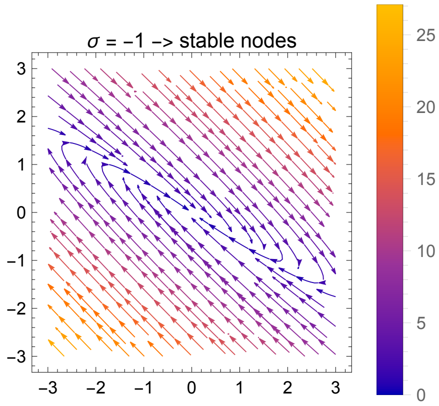
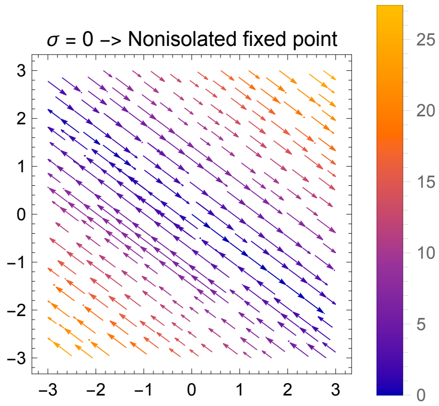
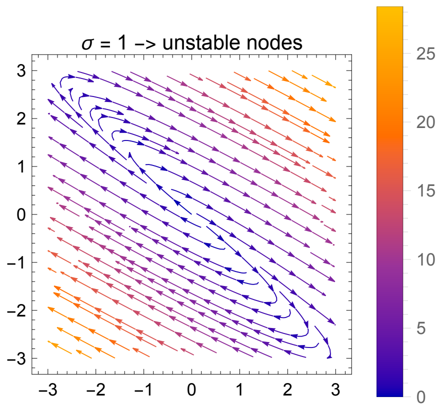
```
(*1.3a*)
(*For σ taking each of the values {-1,0,1} plot a set of
 representative trajectories. Classify the fixed point in each
 of the cases and write your classification in the plots*)
ClearAll["Global`*"];
Remove["Global`*"];

f[x, y, σ] = (σ + 3) * x + 4 * y;
g[x, y, σ] = - (9 / 4) * x + (σ - 3) * y;

p1 = StreamPlot[{(σ + 3) x + 4 y, - (9 / 4) x + (σ - 3) y} /. {σ → -1},
   {x, -3, 3} , {y, -3, 3}, PlotLabel → "σ = -1 -> stable nodes",
   PlotLegends → Automatic, AxesLabel → {x, y}];
p2 = StreamPlot[{(σ + 3) x + 4 y, - (9 / 4) x + (σ - 3) y} /. {σ → 0},
   {x, -3, 3} , {y, -3, 3}, PlotLabel → "σ = 0 -> Nonisolated fixed point",
   PlotLegends → Automatic, AxesLabel → {x, y}];
p3 = StreamPlot[{(σ + 3) x + 4 y, - (9 / 4) x + (σ - 3) y} /. {σ → 1},
   {x, -3, 3} , {y, -3, 3}, PlotLabel → "σ = 1 -> unstable nodes",
   PlotLegends → Automatic, AxesLabel → {"x", "y"}];
p4 = GraphicsColumn[{p1, p2, p3}]
Export["1.3a_plot.png", p4];
```




```
(*1.3b*)
(*Analytically compute the eigenvalues, λ1 and λ2, of Mσ in terms ofσ*)
eqs = Transpose[{{f[x, y, σ]}, {g[x, y, σ]}}];
jacobian = Grad[eqs, {x, y}];
eigenValues = Eigenvalues[jacobian]
```

```
{σ, σ}
```

```
(*1.3c*)
(*Analytically compute all the eigenvectors of Mσ. Remind yourself that
  an eigenvector is always non-zero by definition. For definiteness,
normalise the vectors to one and choose the xx-components positive*)
eigenVectors = Eigenvectors[jacobian];
eigenVectorsNormalized = -Normalize[eigenVectors[[1]]]
```

$\left\{\frac{4}{5}, -\frac{3}{5}\right\}$

```
(*1.3d*)
(*Analytically compute the inverse matrix M^{-1} of Mσ*)
inverseM = Inverse[jacobian[[1]]]
```

$\left\{\left\{\frac{-3+\sigma}{\sigma^2}, -\frac{4}{\sigma^2}\right\}, \left\{\frac{9}{4\,\sigma^2}, \frac{3+\sigma}{\sigma^2}\right\}\right\}$

```
(*1.3e*)
(*Give the value of σ for which Mσ is singular*)
(*The answer on this is zero because Mσ is singular when the inverse is zero,
hence dividing σ=0 results in that the inverse not
   exsisting. One can also calculate det M ==0 and solve for σ*)
dM = Det[jacobian[[1]]];
Solve[dM == 0, σ]
```

$\{\{\sigma \to 0\}, \{\sigma \to 0\}\}$

```
(*1.3f*)
(*Now consider the generalized problem in the description,
find values on d and d to recover the
 dynamical system in the previuos excercises.*)
(*We can se by compare the two equation sets and solve for c & d*)
jacobian1 = {{σ - c * d, d ^ 2}, {-c ^ 2, σ + c * d}};
Solve[jacobian1 == jacobian[[1]] && c > 0, {c, d}]
```

$\left\{\left\{c \to \frac{3}{2}, d \to -2\right\}\right\}$

```
 (*1.3g*)(*Analytically compute the eigenvalues,
λ1 and λ2 of this dynamical system in terms σ*)
eigenValues1 = Eigenvalues[jacobian1]
```

$\{\sigma, \sigma\}$

```
(*1.3h*)
(*Analytically calculate one of the following:the stable direction for σ=-1,
the direction of the line of fixed points for σ=0,
the unstable direction for σ=1*)
eigenVectors1 = Eigenvectors[jacobian1];
eigenVectorsNormalized1 = Normalize[eigenVectors1[[1]]]
```

$$\left\{\frac{d}{c\sqrt{1+\mathrm{Abs}\left[\frac{d}{c}\right]^2}}, \frac{1}{\sqrt{1+\mathrm{Abs}\left[\frac{d}{c}\right]^2}}\right\}$$