```
(*1.4a*)

(*Analytically compute the eigenvalues, λ1 and λ2,
of Mσ and confirm that they constitute a complex conjugate pair*)
f[x, y, σ] = (σ + 1) * x[t] + 3 * y[t];
g[x, y, σ] = -2 * x[t] + (σ - 1) * y[t];
M = {{σ + 1, 3}, {-2, σ - 1}}; (*from the eqs above*)
eigenVal = Eigenvalues[M]
```

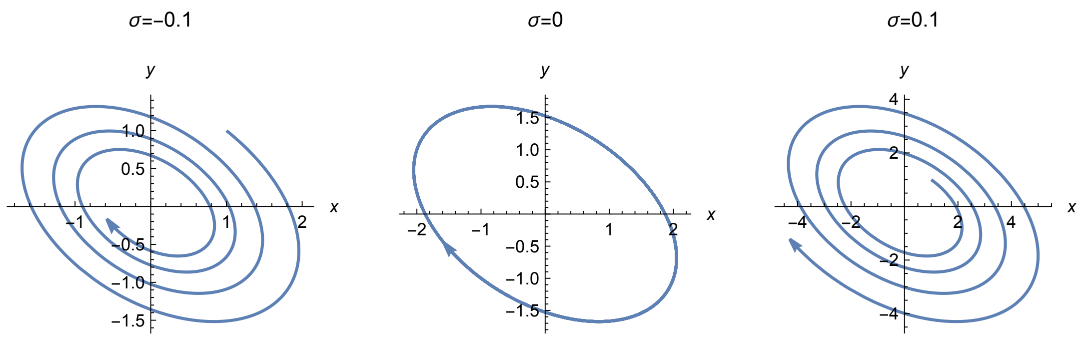
$$\left\{-i\sqrt{5}+\sigma,\ i\sqrt{5}+\sigma\right\}$$

```
(*1.4b*)
(*Solve the dynamical system
  analytically for the initial values x(0)=u and y(0)=v*)

x1[t_] = {x[t], y[t]};
dynS = x1'[t] == M.x1[t];
solution = DSolve[dynS, {x, y}, t];
x1[t_, x0_, y0_, σ_] = {x[t], y[t]} /. solution[[1]] /. {C[1] → u, C[2] → v}
```

$$\left\{\frac{3\, e^{t\sigma}\, v\, \mathrm{Sin}\left[\sqrt{5}\ t\right]}{\sqrt{5}}+\frac{1}{5}\, e^{t\sigma}\, u\left(5\, \mathrm{Cos}\left[\sqrt{5}\ t\right]+\sqrt{5}\ \mathrm{Sin}\left[\sqrt{5}\ t\right]\right),\right.$$
$$\left.-\frac{2\, e^{t\sigma}\, u\, \mathrm{Sin}\left[\sqrt{5}\ t\right]}{\sqrt{5}}+\frac{1}{5}\, e^{t\sigma}\, v\left(5\, \mathrm{Cos}\left[\sqrt{5}\ t\right]-\sqrt{5}\ \mathrm{Sin}\left[\sqrt{5}\ t\right]\right)\right\}$$

```
(*1.4c*)
(*Plot in three figures representative trajectories for σ=-1/10,
σ=0 and σ=1/10*)
f1 = x1[t, 1, 1, -0.1];
f1[[1]];
p1 = ParametricPlot[x1[t, 1, 1, -0.1] /. u → 1 /. v → 1, {t, 0, 10},
    AxesLabel → {x, y}, PlotLabel → "σ=-0.1"] /. Line → Arrow;
p2 = ParametricPlot[x1[t, 1, 1, 0] /. u → 1 /. v → 1,
    {t, 0, 10}, AxesLabel → {x, y}, PlotLabel → "σ=0"] /. Line → Arrow;
p3 = ParametricPlot[x1[t, 1, 1, 0.1] /. u → 1 /. v → 1, {t, 0, 10},
    AxesLabel → {x, y}, PlotLabel → "σ=0.1"] /. Line → Arrow;
GraphicsRow[{p1, p2, p3}]
```



```
(*1.4d*)
(*When σ=0 the system has an invariant orbit in the form of
  an ellipse. Analytically compute the period of the ellips0*)
Solve[x1[0, u, v, 0] == x1[t, u, v, 0], t]
```

$\left\{\left\{t \to \boxed{\frac{2\,\pi\,c_1}{\sqrt{5}} \text{ if } c_1 \in \mathbb{Z}}\right\}\right\}$

```
(*e) Analytically calculate the length ratio
 between the major and minor axes of the ellipse*)
t0 = 1;
xyVal = x1[t, 1, 1, 0] /. u → 1 /. v → 1;
(*Circle equation r^2=x^2+y^2*)
radius = Sqrt[(xyVal[[1]])^2 + (xyVal[[2]])^2];
a = FindMaximum[radius, {t, t0}];
b = FindMinimum[radius, {t, t0}];
ratio = a/b;
ratio[[1, 1]](*The ratio is equivalent with (1+sqrt(5))/2*)
```

```
1.39095
```

```
(*1.4f*)
(*Analytically calculate the direction
 of the major ellipse axis in the (x,y) plane*)
(*As in the previous task where σ=
 0. We get t-position from the major ellipse and normalize.*)
dirMajor = xyVal /. {u → 1, v → 1, σ → 0, a[[2]][[1]]};
Normalize[dirMajor]
```

```
{0.850651, -0.525731}
```