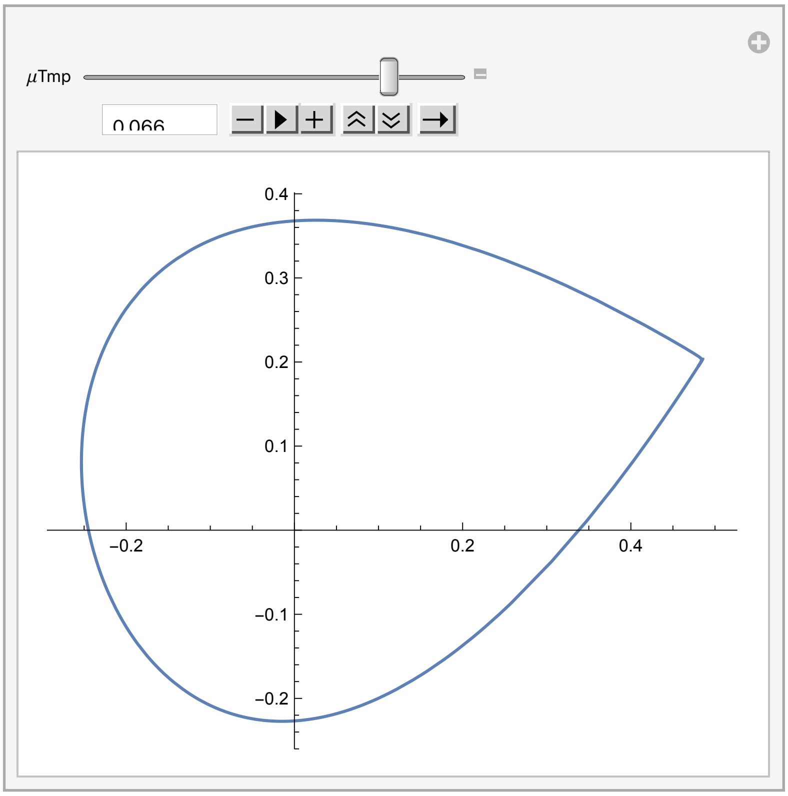
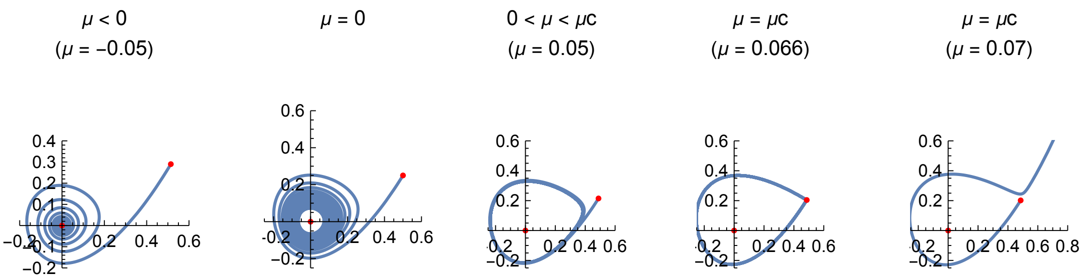
```
(*2.4a*)
(*Using a numerical method of your choice,
find the value μ=μc where the system undergoes a homoclinic
  bifurcation. Give your result with two significant digits.*)
f1[x_, y_] := μ * x + y - x^2;
f2[x_, y_] := -x + μ * y + 2 x^2;
(*Calc. the fixed points*)
fixedP = Solve[{f1[x, y] == 0, f2[x, y] == 0}, {x, y}];
(*Calc. the eigenvalues of the second fixed point*)
jacobian = Grad[{f1[x, y], f2[x, y]}, {x, y}];
jacobian2 = jacobian /. fixedP[[2]];
eigVal = Eigenvalues[jacobian2];
unstableVal = eigVal[[2]];
M = Eigenvectors[jacobian2];
unstableM = M[[2]];
(*Trajectory from saddle in unstable direction*)
tstep = 0.001;
sol[μTmp_, T_] := NDSolve[
   Join[{x'[t] == μ * x[t] + y[t] - x[t]^2, y'[t] == -x[t] + μ * y[t] + 2 x[t]^2}, Thread[
      {x[0], y[0]} == {fixedP[[2]][[1]][[2]], fixedP[[2]][[2]][[2]]} - tstep * unstableM]] /.
    μ → μTmp, {x[t], y[t]}, {t, 0, T}];
(*We can use to vary the value on μ and see where it breaks*)
Manipulate[ParametricPlot[Evaluate[{x[t], y[t]} /. sol[μTmp, 20]], {t, 0, 20}],
 {μTmp, -0.1, 0.1, tstep}]
(*We can see that μc=0.066-which is in agreement with the book*)
```



```
(*2.4b*)
(*Plot the phase portraits that occur for the cases μ<0,
μ=0, 0<μ<μc and μ=μc*)
μc = 0.066;
fp1 = Graphics[{Red, Point[{0, 0}]}];
fp21 = Graphics[{Red, Point[{fixedP[[2]][[1]][[2]], fixedP[[2]][[2]][[2]]} /. μ → -0.05]}];
fp22 = Graphics[{Red, Point[{fixedP[[2]][[1]][[2]], fixedP[[2]][[2]][[2]]} /. μ → 0]}];
fp23 = Graphics[{Red, Point[{fixedP[[2]][[1]][[2]], fixedP[[2]][[2]][[2]]} /. μ → 0.05]}];
fp24 = Graphics[{Red, Point[{fixedP[[2]][[1]][[2]], fixedP[[2]][[2]][[2]]} /. μ → μc]}];
fp25 = Graphics[{Red, Point[{fixedP[[2]][[1]][[2]], fixedP[[2]][[2]][[2]]} /. μ → 0.07]}];

p1 = ParametricPlot[Evaluate[{x[t], y[t]} /. sol[-0.05, 200]], {t, 0, 200},
    PlotRange → {{-0.2, 0.6}, {-0.2, 0.4}}, PlotLabel → "μ < 0\n(μ = -0.05)"];
p2 = ParametricPlot[Evaluate[{x[t], y[t]} /. sol[0, 200]], {t, 0, 200},
    PlotRange → {{-0.25, 0.6}, {-0.25, 0.6}}, PlotLabel → "μ = 0"];
p3 = ParametricPlot[Evaluate[{x[t], y[t]} /. sol[0.05, 200]],
    {t, 0, 200}, PlotRange → {{-0.25, 0.6}, {-0.25, 0.6}},
    PlotLabel → "0 < μ < μc\n(μ = 0.05)"];
p4 = ParametricPlot[Evaluate[{x[t], y[t]} /. sol[μc, 200]], {t, 0, 200},
    PlotRange → {{-0.25, 0.6}, {-0.25, 0.6}}, PlotLabel → "μ = μc\n(μ = 0.066)"];
p5 = ParametricPlot[Evaluate[{x[t], y[t]} /. sol[0.07, 20]], {t, 0, 20},
    PlotRange → {{-0.25, 0.8}, {-0.25, 0.6}}, PlotLabel → "μ = μc\n(μ = 0.07)"];
GraphicsRow[{Show[p1, fp1, fp21], Show[p2, fp1, fp22], Show[p3, fp1, fp23],
   Show[p4, fp1, fp24], Show[p5, fp1, fp25]}, ImageSize → Full]
```




```
(*2.4c*)
(*Find an analytical expression for
  the time t₁ to escape from the saddle to x(t₁)=1*)
solution = DSolve[
   {x'[t] == u * x[t], y'[t] == s * y[t], x[0] == γ, y[0] == 1}, {x[t], y[t]}, t];
Solve[(x[t] /. Part[solution, 1, 1]) == 1, t]
```

$$\left\{\left\{t \to \frac{2\, i\, \pi\, c_1 + \mathrm{Log}\left[\frac{1}{\gamma}\right]}{u} \quad \text{if} \quad c_1 \in \mathbb{Z}\right\}\right\}$$

```
(*t = log(1/γ)/u*)
```

```
(*2.4d*)
(*Find an analytical expression for u suitable for the system in Eq.(1)*)
(*We can obtain u from the eigenvalue *)
eig2 = eigVal[[2]]
```

$$\frac{-1+2\,\mu+\sqrt{5+9\,\mu^2+4\,\mu^3+\mu^4}}{2+\mu}$$

```
(*2.4e*)
(*Give your estimate for aa with one digit accuracy.*)
(*With the help of c,d and e we can rewrite the function:*)
ab = Normal[NonlinearModelFit[
   Table[Abs[μ - 0.066], {μ, 0, 0.066}], Log[A * x^a], {a, A}, x]]
ab2 = ab /. x → 1
(*A=0.7 and a=1*)
```

$\mathrm{Log}\left[1.06823\, x^{1.}\right]$

0.066

```
(*2.4f*)
(*Numerically evaluate the period time
  of the periodic orbit and plot it against μ-μc*)
μc = 0.066;
(* 2.4f *)
func[x_, a_, A_] = -Log[A * x^a] / eig2 /. μ → {μc - x};
Plot[{func[x, a, A] /. {a → 1, A → 0.7}},
 {x, 0, μc}, AxesLabel → {"|μ-μc|", "Tμ"}]
```