```
(*2.3a*)
(*What is ω for the two systems (1)and(2),respectively?*)
(*System (1)*)
x1[x, y] = μ * x - 6 * y - 2 * x ^ 3;
y1[x, y] = 6 * x + μ * y + 4 * y ^ 3;
(*System (2)*)
x2[x, y] = μ * x + y - x ^ 2;
y2[x, y] = - x + μ * y + 2 * x ^ 2;
(*Hopf bif. at origin for μ=0 with ẏ = ωx, ẋ = -ωy:*)
Eqs = {{x1[x, y], y1[x, y]}, {x2[x, y], y2[x, y]}} /. μ → 0;
Eqs〚1〛
```

$\{-2x^3 - 6y, 6x + 4y^3\}$

```
(*For system one we can see that-6y=-ωy and 6x=ωx → ω1=6;*)
Eqs〚2〛
```

$\{-x^2 + y, -x + 2x^2\}$

```
(*For system two we get y=-ωy and-x=ωx,thus ω2=-1;*)


(*2.3b) Determine f and g for the systems (1) and (2)*)
(* x=-ωy+f(x,y), y=ωx+g(x,y);
and the computed equations(Eqs) we get f(x,y) and g(x,y) by comparison;
From system one → f1(x,y)=-2x^3 and g1(x,y)=4y^3;
From system two → f2(x,y)=-x^2 and g2(x,y)=2x^2;*)
```

```
(*2.3c) Determine a for the two systems (1)and (2)*)
(*System one*)
Clear["Global`*"]
ω = {6, -1};
fx = D[{-2 x^3, -x^2}, x];
fxx = D[fx, x];
fxxx = D[fxx, x];
fxy = D[fx, y];
fxyy = D[fxy, y];
fy = D[{-2 x^3, -x^2}, y];
fyy = D[fy, y];
gx = D[{4 y^3, 2 x^2}, x];
gxx = D[gx, x];
gxxy = D[gxx, x];
gxy = D[gx, y];
gy = D[{4 y^3, 2 x^2}, y];
gyy = D[gy, y];
gyyy = D[gyy, y];
Solve[
 16 a == fxxx[[1]] + fxyy[[1]] + gxxy[[1]] + gyyy[[1]] + 1 / ω[[1]] * (fxy[[1]] * (fxx[[1]] + fyy[[1]]) -
      gxy[[1]] * (gxx[[1]] + gyy[[1]]) - fxx[[1]] * gxx[[1]] + fyy[[1]] * gyy[[1]]), a]
Solve[
 16 a == fxxx[[2]] + fxyy[[2]] + gxxy[[2]] + gyyy[[2]] + 1 / ω[[2]] * (fxy[[2]] * (fxx[[2]] + fyy[[2]]) -
      gxy[[2]] * (gxx[[2]] + gyy[[2]]) - fxx[[2]] * gxx[[2]] + fyy[[2]] * gyy[[2]]), a]
```

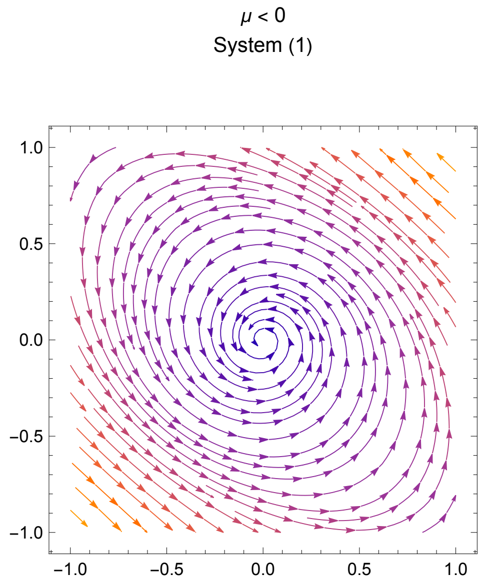
$\left\{\left\{a \to \frac{3}{4}\right\}\right\}$

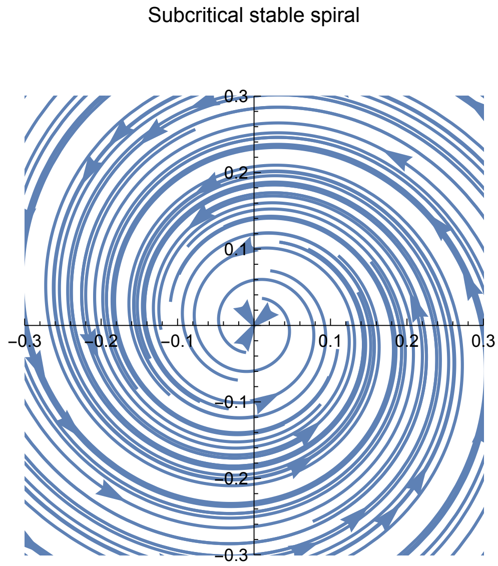
$\left\{\left\{a \to -\frac{1}{2}\right\}\right\}$

```
(*2.3d*)
(*Draw phase portraits of the global dynamics for positive and
 negative μ for each of the systems (1) and (2). Make sure that
 these phase portraits verify the criteria you found in subtask*)

(*From 2.3c we see that system (1) is subcritical (a<0);
and system (2) is supercritical (a<0)*)
(*subcritical (a<0)*)
(*First case where μ < 0 *)
xDot[x_, y_, μ_] := μ x - 6 y - 2 x^3;
yDot[x_, y_, μ_] := 6 x + μ y + 4 y^3;
mu1 = {-1, 1};
Eqs1 = Transpose[{x1[x, y], y1[x, y]} /. μ → mu1];
p1 =
  StreamPlot[Eqs1[[1]], {x, -1, 1}, {y, -1, 1}, PlotLabel → "μ < 0\nSystem (1)"];
minx = -0.3; maxx = 0.3; miny = -0.3; maxy = 0.3;
s[x0_, y0_] = NDSolve[{x'[t] == xDot[x[t], y[t], μ], y'[t] == yDot[x[t], y[t], μ],
```

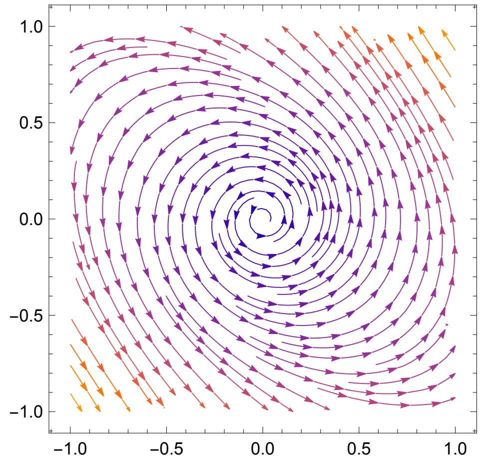
```
      x[0] == x0, y[0] == y0} /. μ → -1, {x, y}, {t, -1, 1}];
initialCondition = Join[Table[{0, y}, {y, miny, maxy, 0.1}],
    Table[{minx, y}, {y, miny, maxy, 0.1}], Table[{maxx, y}, {y, miny, maxy, 0.1}],
    Table[{x, miny}, {x, minx, maxx, 0.1}], Table[{x, maxy}, {x, minx, maxx, 0.1}]];
p12 =
  Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /. s[initialCondition[[i, 1]],
         initialCondition[[i, 2]]]], {t, -1, 1}, PlotRange →
       {{minx, maxx}, {miny, maxy}}], {i, Length[initialCondition]}] /.
     Line[x_] :> {Arrowheads[{0., 0.05, 0.05, 0.05, 0.}], Arrow[x]},
    PlotLabel → "Subcritical stable spiral"];

(*Second case where μ > 0 *)
p2 =
  StreamPlot[Eqs1[[2]], {x, -1, 1}, {y, -1, 1}, PlotLabel → "μ > 0\nSystem (1)"];
s[x0_, y0_] = NDSolve[{x'[t] == xDot[x[t], y[t], μ], y'[t] == yDot[x[t], y[t], μ],
      x[0] == x0, y[0] == y0} /. μ → 1, {x, y}, {t, -1, 1}];
initialCondition = Join[Table[{0, y}, {y, miny, maxy, 0.1}],
    Table[{minx, y}, {y, miny, maxy, 0.1}], Table[{maxx, y}, {y, miny, maxy, 0.1}],
    Table[{x, miny}, {x, minx, maxx, 0.1}], Table[{x, maxy}, {x, minx, maxx, 0.1}]];
p22 =
  Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /. s[initialCondition[[i, 1]],
         initialCondition[[i, 2]]]], {t, -1, 1}, PlotRange →
       {{minx, maxx}, {miny, maxy}}], {i, Length[initialCondition]}] /.
     Line[x_] :> {Arrowheads[{0., 0.05, 0.05, 0.05, 0.}], Arrow[x]},
    PlotLabel → "Subcritical unstable spiral"];

(*System (2): supercritical case*)
(*First case where μ < 0 *)
xDot2[x_, y_, μ_] := μ x + y - x^2;
yDot2[x_, y_, μ_] := -x + μ y + 2 x^2;

mu2 = {-0.1, 0.1};
Eqs2 = Transpose[{x2[x, y], y2[x, y]} /. μ → mu2];
p3 = StreamPlot[Eqs2[[1]], {x, -1, 1}, {y, -1, 1}, PlotLabel → "μ < 0\nSystem (2)"];
s[x0_, y0_] = NDSolve[{x'[t] == xDot[x[t], y[t], μ],
      y'[t] == yDot[x[t], y[t], μ], x[0] == x0, y[0] == y0} /. μ → -1,
    {x, y}, {t, -1.5, 1.5}];
initialCondition = Join[Table[{0, y}, {y, miny, maxy, 0.1}],
    Table[{minx, y}, {y, miny, maxy, 0.1}], Table[{maxx, y}, {y, miny, maxy, 0.1}],
    Table[{x, miny}, {x, minx, maxx, 0.1}], Table[{x, maxy}, {x, minx, maxx, 0.1}]];
p32 =
  Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /. s[initialCondition[[i, 1]],
         initialCondition[[i, 2]]]], {t, -1.5, 1.5}, PlotRange →
       {{minx, maxx}, {miny, maxy}}], {i, Length[initialCondition]}] /.
     Line[x_] :> {Arrowheads[{0., 0.05, 0.05, 0.05, 0.}], Arrow[x]},
    PlotLabel → "Supercritical stable spiral"];
(*Second case where μ > 0 *)
```

```
p4 =
  StreamPlot[Eqs2[[2]], {x, -1, 1}, {y, -1, 1}, PlotLabel → "μ > 0\nSystem (2)"];
s[x0_, y0_] = NDSolve[{x'[t] == xDot[x[t], y[t], μ],
      y'[t] == yDot[x[t], y[t], μ], x[0] == x0, y[0] == y0} /. μ → 1,
    {x, y}, {t, -1.5, 1.5}];
initialCondition = Join[Table[{0, y}, {y, miny, maxy, 0.1}],
    Table[{minx, y}, {y, miny, maxy, 0.1}], Table[{maxx, y}, {y, miny, maxy, 0.1}],
    Table[{x, miny}, {x, minx, maxx, 0.1}], Table[{x, maxy}, {x, minx, maxx, 0.1}]];
p42 =
  Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /. s[initialCondition[[i, 1]],
          initialCondition[[i, 2]]]], {t, -1.5, 1.5}, PlotRange →
        {{minx, maxx}, {miny, maxy}}], {i, Length[initialCondition]}] /.
     Line[x_] :→ {Arrowheads[{0., 0.05, 0.05, 0.05, 0.}], Arrow[x]},
    PlotLabel → "Supercritical unstable spiral"];
GraphicsRow[{p1, p12}]
GraphicsRow[{p2, p22}]
GraphicsRow[{p3, p32}]
GraphicsRow[{p4, p42}]
(*From the plots we can draw the conclustion that they agree with
  what we predicted thus verifying our criteria in 2.3c (see page 254-
 255 in the book,fig 8.2.3 and 8.2.5).*)
```

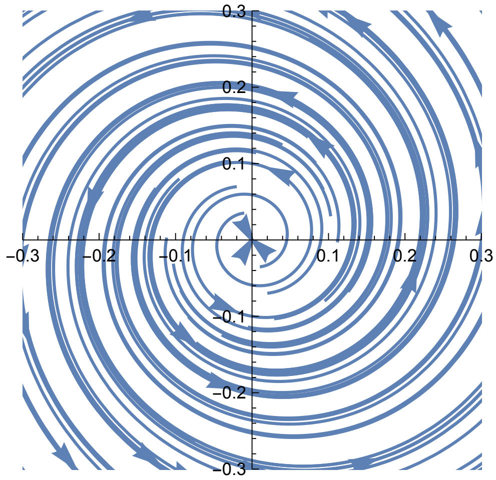




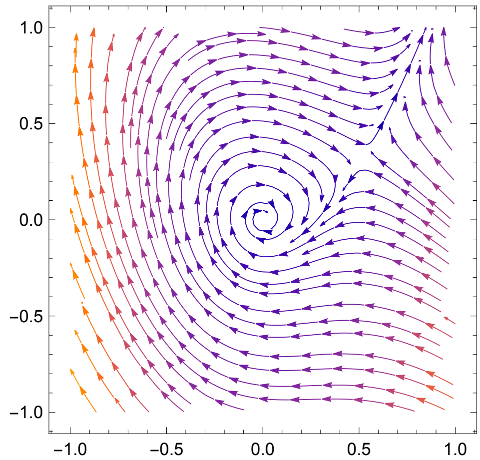
$\mu > 0$
System (1)



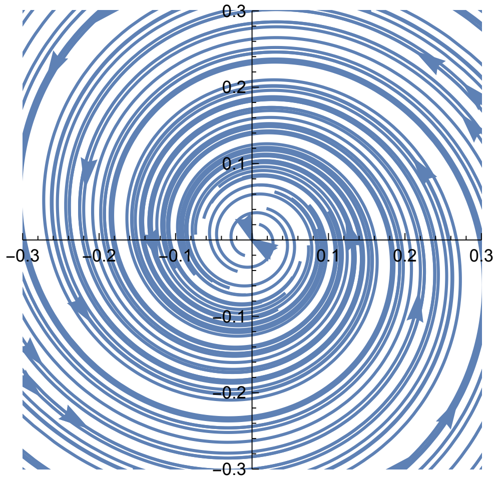
Subcritical unstable spiral



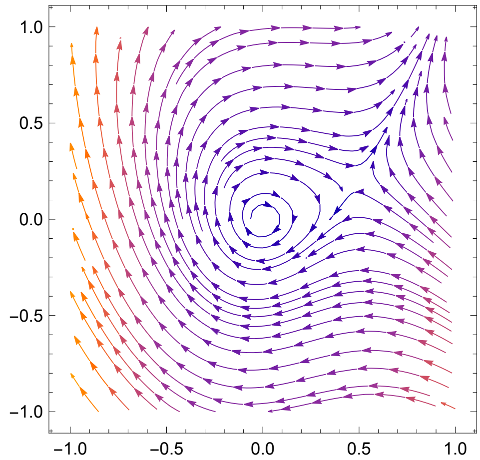
$\mu < 0$
System (2)



Supercritical stable spiral



$\mu > 0$
System (2)



Supercritical unstable spiral