(*4.2*)
(*Calculate analytically the Renyi dimension
spectrum $D_q$of the weighted Cantor set. Make sure that for q=
0 you recover the box counting dimension of the Cantor set*)

(*
The general formula of Rényi dimension spectrum is;

$D_q = \frac{1}{1-q} \lim_{\epsilon\to 0} \frac{\ln(I_q(\epsilon))}{\ln\left(\frac{1}{\epsilon}\right)}$ (eq.1);

where;

$I_q(\epsilon) = \sum_{j=1}^{N_{box}} P_j{}^q(\epsilon)$ (eq.2);

$P_j(\epsilon) = \frac{N_j(\epsilon)}{N_{total}}$ (eq.3);

$\left(P_j(\epsilon)\to\right.$ the fraction of points in the j:th box of size $\left.\epsilon\right)$;

$\left(N_j(\epsilon)\right.$ → number of points in the j:th box of size $\left.\epsilon\right)$;

To obtain D_q we need to first compute $P_j(\epsilon)$;

We can look at the figure to find a pattern. The
pattern that can be discerned in the figure is that the
right side is doubled while the left side remain the same.;

We can visualize this like a P and 1-P for the first level;

We can now find expressions for all the levels(L);

L 0 - no expression;

L 1 $\quad P \qquad 1-P$;

L 2 $\quad P^2 \quad P(1-P) \quad P(1-P) \quad (1-P)^2$;

L 2 - can be rewritten to;

L 2 $\quad P^2 \qquad 2P(1-P) \qquad (1-P)^2$;

L 3 $P^3 \quad P^2(1-P) \quad 2P^2(1-P) \quad 2P(1-P)^2 \quad P(1-P)^2 \quad (1-P)^3$;

L 3 - can also be rewritten as;

L 3 $P^3 \quad 3P^2(1-P) \quad 3P(1-P)^2 \quad (1-P)^3$;

By looking at level 3 we can see that the
pattern is Pascal's triangle and Binomial Coefficient;

$P_j(\epsilon) = P^k(1-P)^{n-k}$ with n =
the level & multiplicity is given by binomial coefficient;

We can now replace (eq.2)to;

$I_q(\epsilon) = \sum_{k=1}^{n} \text{Binomial}(n,k) * \left(P^k(1-P)^{n-k}\right)^q$;

We have now obtained an expression in
terms of q and can solve the rest in mathematica*)

```
P = 1/3;
ϵ = 3^-n;
I_q = FullSimplify[Sum[Binomial[n, k] * (P^k (1 - P)^(n-k))^q, {k, 1, n}]];
D_q = 1/(1 - q) Limit[Log[I_q]/Log[1/ϵ], {n → Infinity}];
expr = Numerator[D_q[[1]]];
s = FullSimplify@Log@Exp[expr]
```

$\text{Log}\left[3^{-q}\left(1+2^{q}\right)\right]$

(* The answer above can be rewritten as ;

$\text{Log}\left[\left(\frac{1}{3}\right)^{q} + \left(\frac{2}{3}\right)^{q}\right]$;

And we can now insert this in the original D_q;

$D_q = \frac{\text{Log}\left[\left(\frac{1}{3}\right)^{q} + \left(\frac{2}{3}\right)^{q}\right]}{(1-q)\,\text{Log}[3]} = \frac{1}{1-q} * \frac{\text{Log}\left[\left(\frac{1}{3}\right)^{q} + \left(\frac{2}{3}\right)^{q}\right]}{\text{Log}[3]}$;

Lastly we can check if q=0;

$D_q = \frac{1}{1-0} * \frac{\text{Log}[1 + 1]}{\text{Log}[3]} = \frac{\text{Log}[2]}{\text{Log}[3]}$; → the box counting dimension of the Cantor set

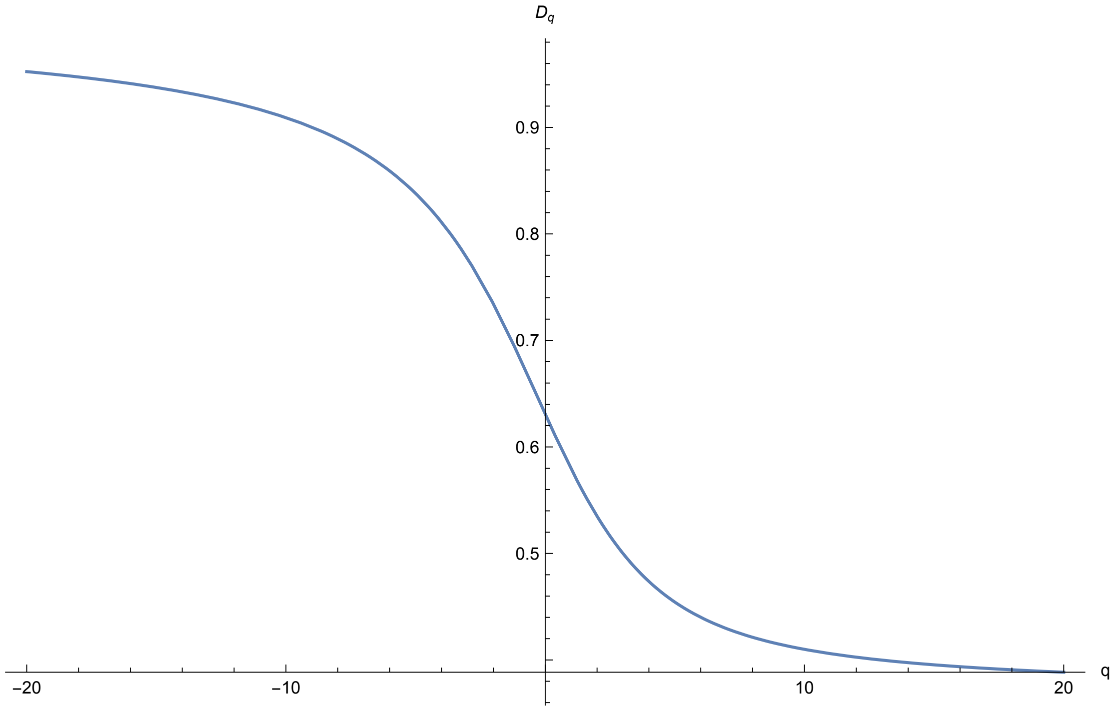
so the answer is → $D_q = \frac{1}{1-q} * \frac{\text{Log}\left[\left(\frac{1}{3}\right)^{q} + \left(\frac{2}{3}\right)^{q}\right]}{\text{Log}[3]}$;

*)

(*4.2b*)
(*Using the expression derived in (a)
make a plot of D_q as a function of q for q∈[-20,20].*)

$$D_{q2} = \frac{1}{1-q} * \frac{\text{Log}\left[\left(\frac{1}{3}\right)^q + \left(\frac{2}{3}\right)^q\right]}{\text{Log}[3]};$$

```
Plot[Dq2, {q, -20, 20}, AxesLabel → {"q", "Dq"}]
```




```
(*4.2c*)(*Note: log in mathematica is the natural logarithm ln*)
(*Using the expression derived in (a)
 compute explicitly D1 (information dimension) and D2
 (correlation dimension) of the weighted Cantor set.*)
```

```
s1 = {Limit[Dq2, {q → 1}], Limit[Dq2, {q → 2}]}
```

$$\left\{\frac{\text{Log}\left[\frac{27}{4}\right]}{\text{Log}[27]}, \frac{\text{Log}\left[\frac{9}{5}\right]}{\text{Log}[3]}\right\}$$

```
(*4.2d*)
(*Using the expression derived in (a),
compute explicitly D-∞=limq→-∞Dq and D∞=limq→∞Dq of the weighted Cantor set.*)
```

```
s2 = {Limit[Dq2, {q → -∞}], Limit[Dq2, {q → ∞}]}
```

$$\left\{1, \frac{\text{Log}\left[\frac{3}{2}\right]}{\text{Log}[3]}\right\}$$