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(*3.1a*)
(*How many fixed points does the Lorenz system have and how many of them
are stable for the parameter values given above?Give your answer as
the vector[number of fixed points,number of stable fixed points]*)

xDot[x_, y_, z_] :=  $\sigma$  * (y - x);
yDot[x_, y_, z_] :=  $r$  * x - y - x * z;
zDot[x_, y_, z_] := x * y - b * z;
fixedP = Solve[{xDot[x, y, z] == 0, yDot[x, y, z] == 0, zDot[x, y, z] == 0}, {x, y, z}];
(*Jacobian to check the fixed points*)
Eqs = Transpose[{{xDot[x, y, z]}, {yDot[x, y, z]}, {zDot[x, y, z]}}];
M = Grad[Eqs, {x, y, z}];
(*Add Lorenz attractor*)
 $\sigma$  = 10;
b = 8/3;
r = 28;
(*Check stability, with eigenvalue calc.*)
eigVal = Eigenvalues[M] /. fixedP;
(*We get 3 fixed points and all are unstable*)

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(*So the answer is [3,0]*)

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(*3.1b*)
(*Solve the equations (1) numerically using the parameters stated above for
some initial condition close to the origin. Plot an approximation of
Lorenz attractor obtained by discarding the initial part of the solution.*)

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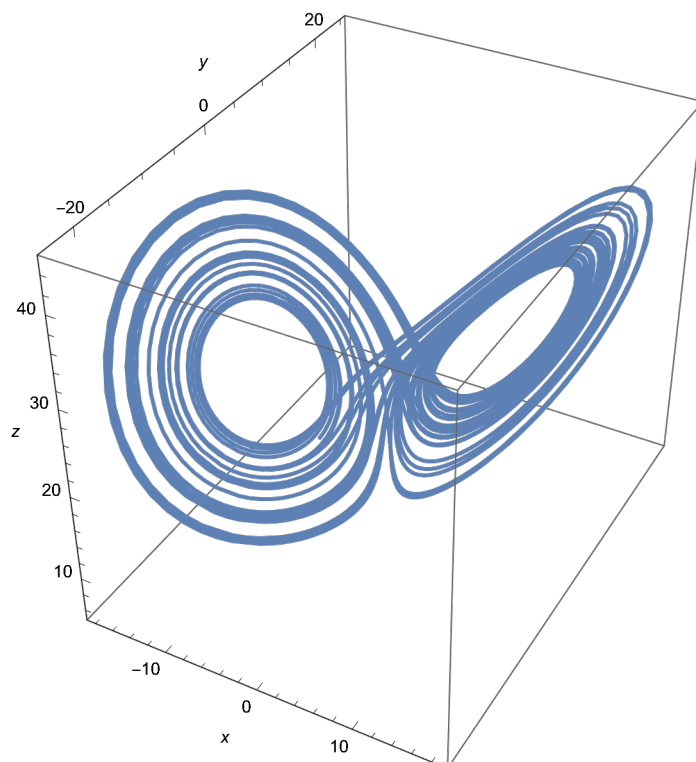
(* 3.1b *)
(* Pick initial values close to origin, I picked 1 *)
(*Define Lorenz flow equations*)
lorenz = {{x'[t] ==  $\sigma$  * (y[t] - x[t])},
  {y'[t] ==  $r$  * x[t] - y[t] - x[t] * z[t]}, {z'[t] == x[t] * y[t] - b * z[t]}};
(*Define parameters*)
 $\sigma$  = 10;
b = 8 / 3;
r = 28;
(*Define initial conditions*)
ics = {{x[0] == 1}, {y[0] == 1}, {z[0] == 1}};

(*Solve numerically the system with given parameters and initial conditions*)
solution = NDSolve[Join[lorenz, ics], {x, y, z}, {t, 0, 50}];

(*Discard initial part of the solution*)
discardedSolution = {x[t], y[t], z[t]} /. solution;

(*Plot the Lorenz attractor*)
ParametricPlot3D[discardedSolution,
  {t, 15, 50}, AxesLabel -> {x, y, z}, ImageSize -> Medium]

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(*3.1c*)
(* (c) Compute the stability matrix. *)
 $\sigma = .;$   $b = .;$   $r = .;$ 
M
{{{ $-\sigma$ ,  $\sigma$ , 0}}, { $r - z$ , -1, -x}, {y, x, -b}}}

(*3.1d*)
(* Confirm that the trace of the stability matrix,
is independent of the coordinates (x,y,z). Compute
the sum of Lyapunov exponents for the Lorenz system *)

(* The Lyapunov exponents is equal
to the trace of the stability matrix(J11,J22,J33);
→ we get the answer  $-\sigma-1-b$  *)

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