```
(*4.1a*)
(*Analytically find an expression for the boxcounting
 dimension of the fractal obtained by evolving the unit
 square according to way (a) in the figure above.*)

(*One can see that figure (a) is a Cantor Dust (2D);
  But we can ignore it for now and solve it analytically;

  First we can make a table from the
    figure with the information presented in the problem;

  #######################;
  # n   #   N(ϵ) #   ϵ  #;
  #######################;
  # 0   #    1   #  1   #;
  # 1   #    4   #  1/3 #;
  # 2   #   16   #  1/9 #;
  #######################;
```

From above table we can recognize a pattern that $N(\epsilon) = 4^n$ and $\epsilon = \left(\frac{1}{3}\right)^n$;

Now we use information from lecture 13(12.1) or in the book page 416 to find the expression for the boxcounting dimension ;
This is the final answer:

$$d_0 = \lim_{\epsilon\to 0} \frac{\ln(N(\epsilon))}{\ln(1/\epsilon)} = \frac{\ln(4^n)}{\ln(3^n)} = \frac{n*\ln(4)}{n*\ln(3)} = \frac{\ln(4)}{\ln(3)};$$

*)

(*4.1b*)
(*Analytically find an expressioin for the
 boxcounting dimension of the fractal obtained by evolving
 the unit square according to way (b) in the figure above*)

```
(*This is also an asymmetric Cantor Dust(2D)
  but to solve this we need a different approach than in a);
We can see from the figure that we will obtain two relative length scales λ;

We can use self-similarity and scaling argument to compute N(ϵ);
```
First iteration → $\lambda_a = \frac{1}{2}$ and $\lambda_b = \frac{1}{4}$;

With the information above and by looking at figure (b) we conclude that;

$N(\epsilon) = 1*N_a(\epsilon) + 4*N_b(\epsilon)$ (eq.1);

From self-similarity → $N_a(\epsilon) = N\left(\frac{\epsilon}{\lambda_a}\right)$ (eq.2) and $N_b(\epsilon) = N\left(\frac{\epsilon}{\lambda_b}\right)$ (eq.3);

From the scaling argument → $N(\epsilon) = A\epsilon^{-d_0}$ (eq.4);

We can now put eq 2,3 & 4 in eq 1;

$A\epsilon^{-d_0} = 1*A\left(\frac{\epsilon}{\lambda_a}\right)^{-d_0} + 4*A\left(\frac{\epsilon}{\lambda_b}\right)^{-d_0}$;

$\epsilon^{-d_0} = \epsilon^{-d_0}*\lambda_a{}^{d_0} + 4*\epsilon^{-d_0}*\lambda_b{}^{d_0}$;

$1 = \lambda_a{}^{d_0} + 4*\lambda_b{}^{d_0}$ (eq.5);

We can rewrite eq 5 by inserting $\lambda_a = \frac{1}{2}$ and $\lambda_b = \frac{1}{4}$ and solve d_0 with Mathematica;*)

```
λa = 1 / 2;
λb = 1 / 4;
s = Solve[1 == λa^d0 + 4 λb^d0, d0, Reals] // Simplify
```

$$\left\{\left\{d_0 \to -1 + \frac{\mathrm{Log}\left[1+\sqrt{17}\right]}{\mathrm{Log}[2]}\right\}\right\}$$

```
(*Note: log in mathetmatica is the natural logarithm, ln.*)
```