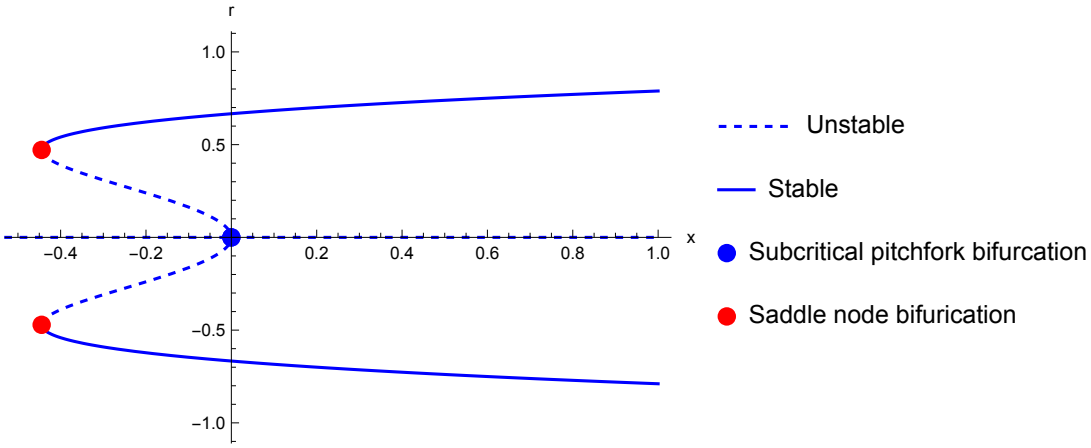
```
(*2a*)
(*Make a bifurcation diagram of x* vs r using dotted lines to indicate
 unstable fixed points and solid lines to indicate stable fixed
 points. Identify the different bifurcations and label them in the plot*)

ClearAll["Global`*"];
Remove["Global`*"]
SetOptions[$FrontEnd, ShowCellLabel → False]
f[x] := r * x + 4 * x^3 - 9 * x^5;
xsolution = Solve[f[x] == 0, x];
x = xsolution[[All, 1, 2]];
p = Plot[x, {r, -0.5, 0.5}];
(*Bifurcation points*)
limits = FunctionDomain[x, {r}];
xLimitsMin = x /. r → limits[[1]];
xLimitsMax = x /. r → limits[[5]];
(*By using the plot and the point we can find the bifurcation points*)
bifurcationP = List[{limits[[5]], xLimitsMax[[1]]},
    {limits[[1]], xLimitsMin[[2]]}, {limits[[1]], xLimitsMin[[3]]}];
(*Final plot*)
p1 =
  Plot[x[[1]], {r, -1, 0}, PlotStyle → {Blue, Dashed}, PlotLegends → {"Unstable"}];
p2 = Plot[x[[1]], {r, 0, 1}, PlotStyle → {Blue, Dashed}];
p3 = Plot[x[[2]], {r, -1, 1}, PlotStyle → {Blue, Dashed}];
p4 = Plot[x[[3]], {r, -1, 1}, PlotStyle → {{Blue, Dashed}, Blue}];
p5 = Plot[x[[4]], {r, -1, 1}, PlotStyle → Blue, PlotLegends → {"Stable"}];
p6 = Plot[x[[5]], {r, -1, 1}, PlotStyle → Blue];
p7 = ListPlot[{bifurcationP[[1]]}, PlotMarkers → {Automatic, 10},
  PlotStyle → Blue, PlotLegends → {"Subcritical pitchfork bifurcation"}];
p8 = ListPlot[{bifurcationP[[2]], bifurcationP[[3]]}, PlotMarkers → {Automatic, 10},
  PlotStyle → Red, PlotLegends → {"Saddle node bifurication"}];
p9 = Show[p1, p2, p3, p4, p5, p6, p7, p8, Axes → {True, True},
  AxesLabel → {"x", "r"}, PlotRange → {{-0.5, 1}, {-1, 1}}]
Export["1.2a_plot.png", p9]
```




```
(*1.2b*)
(*Find two saddle-node bifurcations. Analytically calculate the point rc
 where these occur*)
(*To find the r_c point we can look at the bifurcation diagram and for r=
 rc we can see that rc is the point on the negative x-axis*)
rc = limits[[1]];
```

$$-\frac{4}{9}$$