PIEZOELECTRIC MATERIALS

PIEZOELECTRICITY · property of some crystalline solids to generate electrical charges on their surfaces following a mechanical stress including a deformation } opposite polarities on opposite sides determined by measuring voltage btw electrodes on surfaces

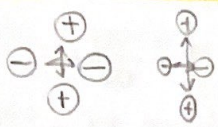
DIRECT PIEZOELECTRIC EFFECT: mechanical stress (forces) $\xrightarrow{\text{elastic material}}$ mechanical deformation → electrical charge } instruments (accel./force/pressure)

vs

INVERSE PIEZOELECTRIC EFFECT: external electric field $\xrightarrow{\text{dielectric material}}$ polarization (electrical charge) → mechanical deformation } US systems (mech. waves)

energy conversion (direction sensitive)

NON PIEZOELECTRIC MATERIALS

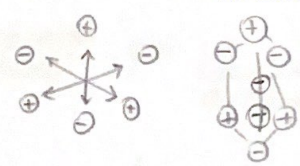


· symmetric charge distribution (average location of ⊖ and ⊕ charges matches)
→ no electrical response to mechanical force

PIEZOELECTRIC MATERIALS

· asymmetrical charge distribution (average locations do not match) / deformation determines DIPOLE
↓
opposite polarities on the 2 sides

[unperturbed condition → average location matches]

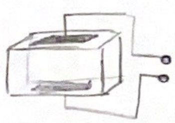


or $\bar{E}$ applied → charge separation → deform. of the structure

crystalline solid ≡ repetition of structural unit (UNIT/ELEMENTARY CELLS) of simple geometrical shape

- NATURAL CRYSTALS (quartz)
- SYNTHETIC CRYSTALS
- POLARIZED FERROELECTRIC CERAMICS (ultrasound transducer

Behavior depending on < material / cut of slab wrt crystalline solid }
- THICKNESS EXPANSION
- TRANSVERSE EXPANSION
- THICKNESS SHEAR
- FACE SHEAR



METAL ELECTRODES placed over selected surfaces of p.e. → ~ capacitor's plates with dielectric material within it

+ LEAD WIRES bringing in or leading out electrical charges

MECHANICAL INPUT (mechanical deformations) → charge Q is generated → voltage drop

$$V = \frac{Q}{C_s}$$

C_s: capacitance of p.e

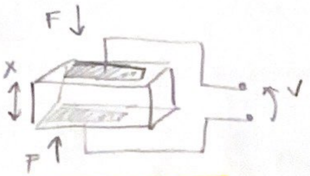
PIEZOELECTRIC DISPLACEMENT SENSORS

AIM: measure F, accelerations and pressure

DISPLACEMENT X due to a force (≡ thickness out) ⟶ charge Q is generated ⟶ voltage drop V across the 2 faces

$$Q = K_q x$$

$$V = \frac{Q}{C_s} = \frac{K_q}{C_s} x$$

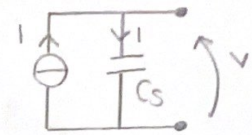


K_p [C/cm] constant characteristic of the p. sensor depending on material

EQUIVALENT ELECTRICAL CIRCUIT of electrical behaviour of the pe. sensor: charge generator // capacitance

$\underbrace{i(t)}_{\text{charge} \propto \text{displacement}}$ ⟶ converted into current generator

$\underbrace{C_s}_{\text{charge converted into voltage}}$



$$i(t) = \frac{dQ(t)}{dt} = K_p \frac{dx(t)}{dt}$$

current entirely flow across Cs $\quad i(t) = C\frac{dV(t)}{dt}$

$$i(t) = \frac{C_s\, dV(t)}{dt} = K_p \frac{dx(t)}{dt}$$

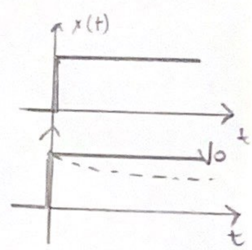
$$\frac{dV(t)}{dt} = \frac{K_p}{C_s} \frac{dx(t)}{dt}$$

$$V(t) = \frac{K_p}{C_s} \int dx(t) \longrightarrow V(t) = \frac{K_p}{C_s} x(t)$$

Voltage across 2 faces ∝ displacement of the 2 faces

INPUT: step (dynamic performances)
OUTPUT perfectly reproduces the step } perfect sensor

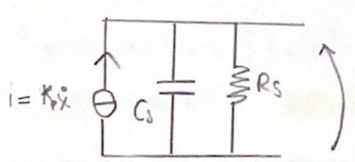
BUT not complete description



Piezoelectric element is not a perfect dielectric material with ∞ resistivity (= no conductivity) (high but not ∞) [GΩ - hundreds GΩ] } output tends to decade over time

⟶ QUASI-STATIC RESPONSE

LEAKAGE RESISTANCE Rs // Cs // current generator



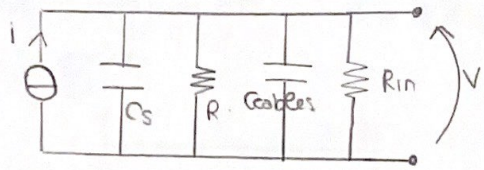
VOLTAGE AMPLIFIER

displacement sensor connected to **CONDITIONING BLOCK** through **cables** (connecting c)

→ introduction of capacitans

C_{cables} parasitic capacitance

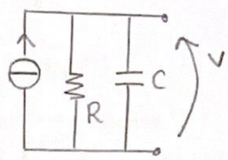
R_{in} input resistance $\ll R_s$
(impedance matching: R_{in} high to not loss energy)



$R = R_{in} // R_s$

$C = C_s // C_{cables} = C_{in} + C_{ables}$

overall circuit



$$Z(s) = \frac{1}{sC + \frac{1}{R}} = \frac{R}{1+sRC}$$

$$V(s) = \frac{R}{1+sRC} I(s)$$

$i(t) = k_p \dot{x}(t)$

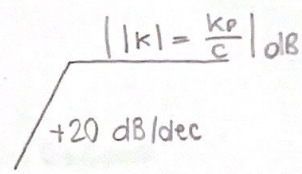
$I(s) = k_p s X(s)$

$$V(s) = \frac{R k_p s}{1+sRC} X(s)$$

$$G(s) = \frac{V(s)}{X(s)} = \frac{k_p R s}{1+sRC} \cdot \frac{C}{C} =$$

$$= \frac{k_p}{C} \frac{sRC}{1+sRC}$$ where $\tau = RC$ TIME CONSTANT

1st order HP filter



$K = \frac{k_q}{C}$ sensitivity / passband gain

$$V_u(s) = \overset{K}{\frac{k_p}{C}} \frac{s\tau}{1+s\tau} X(s)$$

$(1+s\tau) V_u(s) = k s\tau X(s)$

$V_u + s\tau V_u(s) = k s\tau X(s) \longrightarrow V_u(t) + \tau \frac{dV_u(t)}{dt} = k\tau \frac{dx(t)}{dt}$

$$\tau \frac{dV_u(t)}{dt} + V_u(t) = k\tau \frac{dx(t)}{dt}$$ non homogeneous 1st order diff. eq

$x(t), v(t) = 0 \; [t<0]$

$x(t)$ = forcing action

STEP INPUT: $x(t) = L$ for $0<t<T \Rightarrow \frac{dx}{dt} = 0$ (sudden change of x btw 0^- and 0^+)

↓ HIGH - PASS BEHAVIOR
$\Rightarrow$ sudden change of the output

$$\tau \frac{dV_u(t)}{dt} + V_u(t) = 0$$ homogeneous 1st order diff. eq

IC: $V_u(0^+) = KL$

$V_u(t) = A e^{-t/\tau}$ $\quad A = kL$

$$V_u(t) = kL e^{-t/\tau}$$ $0<t<T$ exponential decrease from VL to zero

$t = T^-$ $\quad x(T^-) = L$
$t = T^+$ $\quad x(T^+) = 0$ } sudden change of L $\rightarrow$ V_u: sudden change of $-kL$

IC: $V(T^+) = kLe^{-T/\tau} - kL$

$V_u(t) = Be^{-t/\tau}$

NB: x constant = 0 for $t>0$

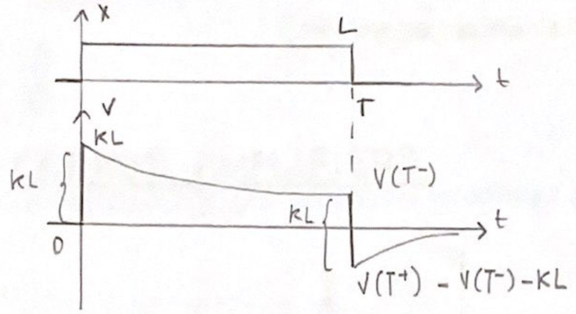
$\frac{dx}{dt} = 0$

$Be^{-T/\tau} = kLe^{-T/\tau} - kL$

$B = e^{\frac{T}{\tau}}(kLe^{-T/\tau} - kL)$

$\rightarrow$ $V_u(t) = (kLe^{-T/\tau} - kL)e^{-\frac{t-T}{\tau}}$

exponential increase from $V(T^+)$ to zero



τ large : faithful reproduction of the input

$\tau\uparrow \equiv \omega_t\downarrow$ improving response at low freq

$\hookrightarrow$ $C\uparrow$, $R\uparrow$, both $\uparrow$

INCREASE C : capacitance C_{par} inserted in parallel

$C' = C + C_{par}$

$\stackrel{!}{=} C_s + C_{cables} + C_{par}$

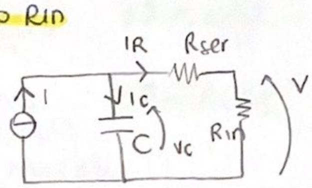
$\tau' = RC'$

$k' = \frac{k_p}{C'}$ } NB. $k'\downarrow$ if $C\uparrow$ loss in sensitivity

INCREASE R : $R = \frac{R_{in}R_S}{R_{in}+R_S} \simeq R_{in}$ since $R_S \gg R_{in}$

- $R_{in}\uparrow$ changing amplifier
- R_{ser} inserted in series to R_{in}

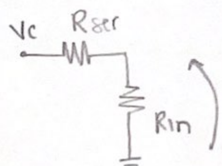
$R_S \cong \infty$ open circuit



$i_C(t) = C\frac{dV_C(t)}{dt}$

$i_R(t) = \frac{V(t)}{R_{in}}$

$i(t) = k_q\frac{dx(t)}{dt}$



voltage divider $V = \frac{R_{in}}{R_{in}+R_{ser}} V_C$

$V_C = V\frac{R_{in}+R_{ser}}{R_{in}} = V\left(1+\frac{R_{ser}}{R_{in}}\right)$

$k_q\frac{dx(t)}{dt} = C\frac{dV_C(t)}{dt} + \frac{V(t)}{dt}\frac{1}{R_{in}}$

$C\left(1+\frac{R_{ser}}{R_{in}}\right)\frac{dV(t)}{dt} + \frac{1}{R_1}\frac{V(t)}{dt} = k_q\frac{dx(t)}{dt}$

$\frac{C}{R_{in}}(R_{ser}+R_{in})sV(s) + \frac{V(s)}{R_{in}} = k_q s X(s)$

$k_q s X(s) = \frac{V(s)}{R_{in}}\left(1 + sC(R_{ser}+R_{in})\right)$

$\frac{V(s)}{X(s)} = \frac{sk_qR_{in}}{1+sC(R_{in}+R_{ser})}\cdot\frac{C}{C} = \frac{k_q}{C}\cdot\frac{sR_{in}C}{1+sC(R_{in}+R_{ser})}\frac{(R_{in}+R_{ser})}{(R_{in}+R_{ser})}$

$= \frac{k}{C}\frac{R_{in}}{R_{ser}+R_{in}}\frac{sC(R_{in}+R_{ser})}{(1+sC(R_{in}+R_{ser}))}$

$\Rightarrow$ $G(s) = \frac{k_q}{C} \frac{R_{in}}{R_{in}+R_{ser}} \frac{sC(R_{in}+R_{ser})}{1+sC(R_{in}+R_{ser})}$

$\tau'' = (R_{in}+R_{ser})C$

$K'' = \frac{k_q}{C} \frac{R_{in}}{R_{in}+R_{ser}}$

loss in sensitivity

LIMITATIONS : $\tau\uparrow$ enlarging passband zone to lower freq
BUT UNABLE to respond to STATIC / LOW-FREQ PHENOMENA

1st order HP filter (sensor + cables + amplifier)

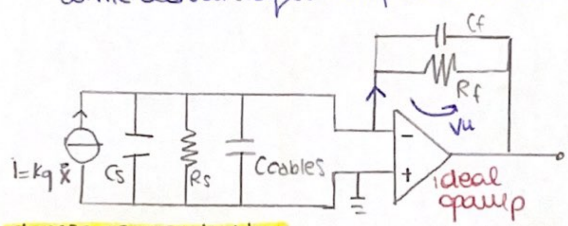
sensitivity / τ always depend on C, including C_{cables} (parasitic parameter)

[NB longer cables $\downarrow$ K]

$\Downarrow$

CHARGE AMPLIFIERS

some advantages compared to voltage amplifier



V_u output of charge amplifier due to C_f in feedback branch who determines the dynamics of the system

$C = C_s + C_{cables}$

charge generated by piezoelectric element totally transferred on C_f and discharged through R_f

$V^+ = V^- = 0$ rule 1
$V_d = 0$

$\rightarrow$ $i = k_q \dot{x}$ flow only across feedback branch
$I_C = I_R = 0$

} why called charge amplifier

$$Z_F(s) = \frac{1}{sC_f + \frac{1}{R_f}} = \frac{R_F}{1+sC_fR_f}$$

$$V_u(s) = -I(s)\frac{R_f}{1+sC_fR_f} = -k_q \frac{sR_f}{1+sC_fR_f} X(s) \left[\frac{C_f}{C_f}\right] = -\frac{k_q}{C_f} \cdot \frac{sR_fC_f}{1+sR_fC_f} X(s)$$

$I(s) = k_q s X(s)$

$$\Rightarrow G(s) = -\frac{k_q}{C_f} \frac{sC_fR_f}{1+sC_fR_f}$$

constant < 0

zero in origin + real pole

$k = -\frac{k_q}{C_f}$ sensitivity (high-freq gain)

$\tau = R_fC_f$ time constant

same transfer function

$\downarrow$

(NB) HIGH FREQ RESPONSE is not modified
(with inability to respond to static/low freq phenomena)

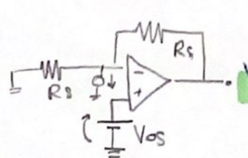
BUT K/τ do not depend on C_{cables}
(only on design parameters)

ω_t small if $\tau\uparrow$ < $R_f\uparrow$, $C_f\uparrow$ } or both
(enlarge low freq. passband zone)

but $C_f\uparrow \equiv K\downarrow$ => more convenient $R\uparrow$ to $\uparrow\tau$

DC PARAMETERS

C = open circuits



$V_{u\_offset} = V_{os}\left(1+\frac{R_f}{R_s}\right) + I^- R_f$

$R_f\uparrow$ => $\uparrow$ effect of I^- bias current

$\uparrow$ NOISE