Nicole Fleming
A02079019

STAT 5810 Homework #3

I worked with: on problems:

Code will follow the written prompts. It will be labeled appropriately for the respective problem it models.

---

① Consider the following optimization problem:

$$\max_{\vec{x}} f(\vec{x}) = (x_1-1)^2 + (x_2-2)^2$$

s.t.

$$g_i(x) = x_1 + x_2 \leq 10$$
$$h_j(x) = x_1, x_2 \geq 0$$

a) Is this a convex optimization problem? Why or why not?
b) Use the KKT conditions, solve the optimization problem.
c) Using the programming language of your choice, plot the feasible region given by the constraints alongside the contour plot of $f(x)$.

---

a) $f(\vec{x}) = -[(x_1-1)^2 + (x_2-2)^2]$ Since $f(x)$ = max, to test convexity $f(x)$ = min, so test $-f(x)$ for convexity.

$\nabla f(\vec{x}) = -[2(x_1-1)\cdot 1 + 2(x_2-2)\cdot 1]$

$= -2x_1 + 2 - 2x_2 + 4$

$\nabla^2 f(\vec{x}) = -2 - 2 = -4.$

Since the Hessian of $-f(\vec{x})$ is less than 0, so this is not a convex problem. The minimum is $-$max, so $-f(x)$ was used. This is also a constrained optimization problem with the given $g(\vec{x}_i)$ + $h(\vec{x}_j)$ functions. But the KKT conditions won't provide 1 global point, Rather they will provide many points that could be possible maximums.

b) KKT ① $\nabla_{x^*} L(\vec{x}^*, \vec{\lambda}^*, \vec{v}^*) = 0$ ② $g_i(\vec{x}^*) \leq 0 \ \forall i$ ③ $h_j(\vec{x}^*) = 0 \ \forall j$

④ $\vec{\lambda}_i^* \geq 0$ ⑤ $\vec{\lambda}_i^* g_i(\vec{x}) = 0$

$\vec{x} \geq 0$ i.e. $x_1, x_2 \geq 0$ $(h_j(x))$ $x_1 + x_2 \leq 10$ $(g_i(x))$

In this constrained problem, $g(\vec{x}) \leq 10$

$g_i(x) = x_1 + x_2 \leq 10 \implies g_i(x) = x_1 + x_2 - 10 \leq 0$ (condition 2)

and $h_j(x) \geq \vec{0}$ or $x_1, x_2 \geq 0$ so $h_j(x) = 0$ (condition 3)

In saying $f(\vec{x})$ is convex and positive definite $\vec{\lambda}_i^*$ can be assumed to be: $\vec{\lambda}^* \geq 0$ (condition 4)

In the equation $\nabla_{x^*} L(\vec{x}^*, \vec{\lambda}^*, \vec{r}^*)$.

$$= \min_{\vec{x}} -\left(f(x) + \sum_{i=0}^{m} \vec{\lambda}_i^* g_i(\vec{x}^*) + \sum_{j=0}^{n} \vec{r}_j^* h_j(\vec{x}^*)\right)$$

If at $\vec{x}^*$, $\vec{\lambda}_i^* g_i(\vec{x}^*) = 0$ then

$$L = -\min_{\vec{x}} \left[(x_1-1)^2 + (x_2-2)^2 + \underbrace{\sum_{i=0}^{m} \vec{\lambda}_i^* [x_1 + x_2 - 10 \leq 0]}_{\text{at } \vec{x}^* \text{ this is } 0.} + \underbrace{\sum_{j=0}^{n} \vec{v}^* h_j(\vec{x}^*)}_{} \right]$$

$h_j(\vec{x})$ is defined to be ≥ 0.
if at $\vec{x}^*$ $h_j(\vec{x}^*) = 0$, then

$$L = \min_{\vec{x}^*} -\left[(x_1-1)^2 + (x_2-2)^2 \right] + \sum \lambda_i^* g(x^*) + \sum v_j^* h_j(x^*)]$$

$$L = \min_{\vec{x}^*} -\left[(x_1-1)^2 + (x_2-2)^2 \right] + \lambda_1 (x_1 + x_2 - 10) - (x_1)\lambda_2 - (x_2)\lambda_3$$

$$\nabla L = \left[-2(x-1)\cdot 1 + \lambda_1 \cdot 1 - \lambda_2 \cdot 1 \ , \ -2(x_2-2)\cdot 1 + \lambda_1 \cdot 1 - \lambda_3 \cdot 1 \right] = 0$$ (condition 1)

$-2(x_1-1)^2 + \lambda_1 - \lambda_2 = 0$, $-2(x_2-2)^2 + \lambda_1 - \lambda_3 = 0$

$\lambda_i^* g_i(\vec{x}^*) = 0$ (condition 5)

$$\begin{cases} \lambda_1 (x_1 + x_2 - 10) = 0 & X \quad X \\ \lambda_2 (x_1) = 0 & X \\ \lambda_3 (x_2) = 0 & \quad\ \ X \end{cases}$$

if $x_2 = 0$ $2x_2 - 4 + \lambda_3 = 0$
$\lambda_3 = 4$

if $\lambda_1 + \lambda_2 = 0$
$-2x_1 + 2 = 0$
$\underline{x_1 = 1}$

if $\lambda_1 + \lambda_3 \neq 0$
$-2x_2 + 4 = 0$
$\underline{x_2 = 2}$

meaning $(1, 2)$ is a possible maximum.

b) continued...

$\lambda_1(x_1+x_2-10)=0$
$\lambda_2(x_1)=0$
$\lambda_3(x_2)=0$

$-2(x_1-1)+\lambda_1-\lambda_2=0$ $\quad$ $-2(x_2-2)+\lambda_1-\lambda_3=0$

for $\lambda_1 \neq 0$

$-2x_1+2+\lambda_1=0$
$x_1+x_2-10=0$
$-2x_2-4+\lambda_1=0$

$-2x_1+2=-\lambda_1$
$x_1+x_2-10=0$
$-2x_2-4=-\lambda_1$

$-2x_1+2=-2x_2-4$
$-2x_1+2x_2+6=0$

ADD $2^*(x_1+x_2-10=0)$

$-2x_1+2x_2+6=0$
$+2x_1+2x_2-20=0$
$4x_2-14=0$
$x_2=\frac{14}{4}\simeq 3.5$

$x_1+3.5-10=0$
$x_1=6.5$

$-13+2=-\lambda_1$
$-11=-\lambda_1$
$11=\lambda_1$ is positive.

for $\lambda_2 \neq 0$

$-2x_1+2=+\lambda_2$
$\lambda_2(x_1)=0$

$-2x_1+2=+\lambda_2$
$x_1=0$ $\quad$ $2=\lambda_2$

for $\lambda_3 \neq 0$

$-2x_2+4=\lambda_3$
$x_2=0$
$4=\lambda_3$

$2(x_1)=0$ $\quad$ if $x_1\ x_2=0$
$4(x_2)=0$

$x_1=0$
$x_2=0$ $\quad$ so $(0,0)$ is apoint.

Cases:

$x_1=x_2=0$ ✓ $\quad$ $x_1\neq 0,\ x_2\neq 0$ ✓
$x_1=0\quad x_2\neq 0$ ✓ $\quad$ $x_1+x_2=10$ ✓
$x_1\neq 0\quad x_2=0$ ✓ $\quad$ $x_1+x_2<10$ —

if $x_2=0$ & $x_1\neq 0$

$\frac{\lambda_1(x_1+x_2-10)=0}{\lambda_1}$
$x_1+x_2-10=0$
$x_2=0 \rightarrow x_1=10$

possible point = $(10,0)$

if $x_1=0$ & $x_2\neq 0$

$-2x_2+4+\lambda_1-\overset{\lambda_3}{0}=0$ $\quad$ $\lambda_3=0$ $\quad$ $x_1=0$
$\lambda_1(\overset{x_1}{0}+x_2-10)=0$
$(2x_2-4)(x_2-10)=0$ $\quad$ $2x_2^2+24x_2+40$
$-2x_2+\lambda_1+4=0$ $\quad$ $2(x_2\quad)(x_2\quad)$

$\lambda_1\neq 0\quad \lambda_2\neq 0 \rightarrow x_2=0\quad x_1=10$
$\lambda_1\neq 0\quad \lambda_3\neq 0 \rightarrow x_1=10\quad x_2=0$
$\lambda_2\neq 0\quad \lambda_3\neq 0 \longrightarrow x_1=0\quad x_2=0$

This point $(0,10)$ follows the constraints.

$x_1+x_2<10$

$-2x_1+2-11+2=0$ $\quad$ $x_1=\frac{7}{2}$
$-2x_2+4-11+4$ $\quad$ $x_2=\frac{3}{2}$
$x_1+x_2<10$

$\underbrace{(-3.5,-1.5)}_{\text{Not valid due to constraints}}$

x_1+x_2
$-\frac{7}{2}+-\frac{3}{2}<10 \Rightarrow -5<10$

Test to see if $(1,2)$ is possible maxima.

$$(x_1-1)^2+(x_2-2)^2 \longrightarrow 0$$

Test to see if $(0,0)$ is possible maxima

$$(0-1)^2+(0-2)^2$$
$$1 + 4 = 5$$

Test to see if $(0,10)$ is possible maxima

$$(-1)^2+(10-2)^2 = 64+1=65$$

Test to see if $(3.5, 6.5)$ is a possible maxima

$$(5.5)^2+(1.5)^2$$
$$30.25 + 2.25 = \underline{32.5}$$

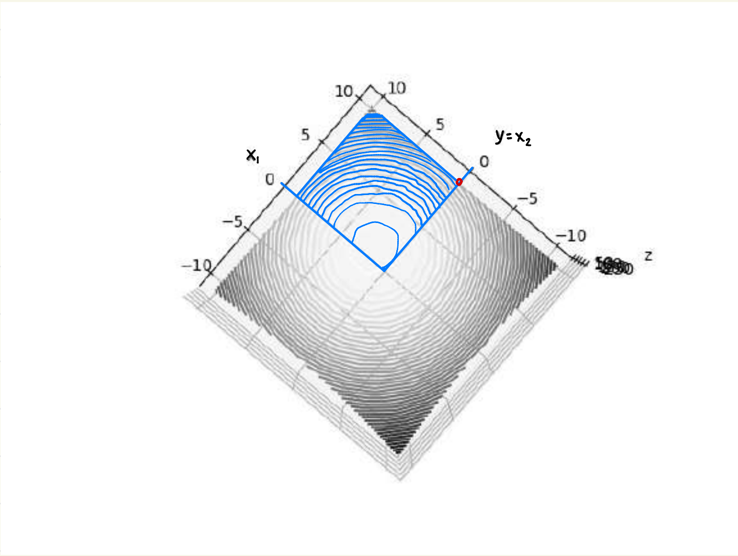
Test to see if $(10,0)$ is possible maxima:

$$10-1=9^2+-2^2 = \underline{\underline{85}}$$

$(-3.5,-1.5)$ – is not valid due to constraints.

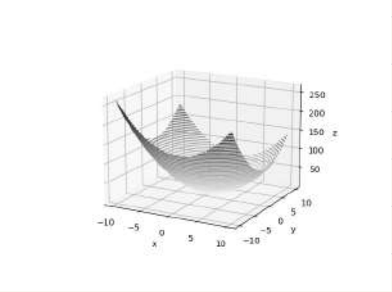
Thus, the $\max_{\vec{x}} f(\vec{x})$ is at $x=(10,0)$.

c) The plot is below. The code to get the plot is included in the index at the end of this assignment.



The area with constraints is in blue with the max point in Red.

In 3D



② Prove that for a convex set $X \subset \mathbb{R}^n$, the range of the function $f(\vec{x}) = A\vec{x} + \vec{b}$ for $\vec{x} \in X$ is also a convex set. Note that A is a rectangular matrix in $\mathbb{R}^{m \times n}$ and $\vec{b} \in \mathbb{R}^m$. Thus, $f: \mathbb{R}^n \to \mathbb{R}^m$.

Consider the range of $f(x) := C = AX + b$ where $\exists\, y \in \mathbb{R}^m$ such that

$$C = AX + b = \{ y \in \mathbb{R}^m : y = Ax + b, \text{for some } x \in X \},$$

then $\exists\, y_1, y_2 \in C$.

$$z = \theta y_1 + (1-\theta) y_2 \quad \in C, \ \forall\, \theta \in [0,1] \text{ ; from the definition of a convex function.}$$

Then there is a need to find a corresponding $\vec{x}$ such that $f(x) = z$, where $x \in X$.

this means $\exists\, x_1, x_2 \in X$ where

$$y_1 = Ax_1 + b \quad \text{and} \quad y_2 = Ax_2 + b$$

This means that $z = \theta(Ax_1 + b) + (1-\theta)(Ax_2 + b)$

$$= \theta(Ax_1 + b) + (1-\theta)(Ax_2 + b)$$

$$= A\underbrace{(\theta x_1 + (1-\theta)x_2)}_{\substack{x \in X \text{ where} \\ X \text{ is convex} \\ \text{by definition.}}} + b\underbrace{(\theta + 1 - \theta)}_{=1}$$

$$= Ax + b$$

Since x is convex by definition.

<u>Thus, $Ax + b = f(x)$ where $x \in X$ is convex.</u>

Next page for #3.

③ Solve problems 7.3 and 7.4 from the book, providing a convincing arguement to justify your response, i.e. a proof or a counterexample.

$\boxed{7.3}$ Consider whether the following statements are true or false.

a) The intersection of any 2 convex sets is convex.

b) The union of any two convex sets is convex.

c) The difference of a convex set A from another convex set B is convex.

---

a) True.

Consider A and B being convex sets. To show $A \cap B$ is also convex, take $x_1, x_2 \in A \cap B$, and let x lie on the line segment between these two points. Then $x \in A$ since A is convex and $x \in B$ due to B also being convex. Thus, $x \in A \cap B$ as desired.

b) False.

Consider the convex intervals $[0,1]$ and $[2,3]$. These are both convex sets, but $[0,1] \cup [2,3]$ is not convex because $t \cdot 1 + (1-t) \cdot 2$ is not in $[0,1] \cup [2,3]$ $\forall t \in (0,1)$. Intuitvly, if two sets have an area between them, then a point in one space and a point in the other won't have points in either.

c) False.

Along the same lines of b), Consider the convex set $[0,3]$. In this set is the interval $[1,2]$, that is also convex. However, if one removes $[1,2]$ from $[0,3]$, we are left with $[0,1) \cup (2,3]$ which is not a convex set since we can find $t \in (0,1)$ where $\frac{1}{2}t + \frac{5}{2}(1-t) \notin$ of this new set

---

$\boxed{7.4}$ Consider whether the following statements are true or false:

a) The sum of any two convex functions is convex.

b) The difference of any two convex functions is convex.

c) The product of any two convex functions is convex.

d) The maximum of any two convex functions is convex.

---

a) True.

Using the definition of function convexity in a summation one gets

$$(f+g)(\theta x + (1-\theta)y) = f(\theta x + (1-\theta)y) + g(\theta x + (1-\theta)y)$$

$$\Rightarrow \leq \theta f(x) + (1-\theta) f(y) + \theta g(x) + (1-\theta) g(y)$$

$$\Rightarrow \leq \theta(f(x) + g(x)) + (1-\theta)(f(y) + g(y))$$

, thus this is true.

b) False.

This is not always necissarily the case. A counter-example is $f(x)=-x^2$ and $g(x)=-2x^2$. These don't impose convexity directly on f or g, such that $f-g$ is convex, but f and g alone are concave, not convex. In other terms, $f \geq g$ is not sufficient for this statement to be true.

c) False.

Let $h(x)=f(x)\cdot g(x)$. Then $h(x)$ would always be convex for this to be true, but consider $f(x)=x$ and $g(x)=x^2$, both of which are convex. However, $f\cdot g = x\cdot x^2 = x^3 = h(x)$ is not convex. Also consider $f(x)=1-x$ and $g(x)=1+x$, $h(x)=1-x^2$. Where f and g are both convex h is not. Thus, by these two counter examples, this is false, since it is not necissarily always true.

d) True.

A function is convex if and only if the space above it (or if thinking graphically the space above its graph) is convex. This means the region above $h(x)$ where $h(x)=\max[f(x),g(x)]$, is the intersection of the area above f and the area above g. The intersection of convex sets is convex, thus $h(x)$ is convex.

---

④ Show that the polytope in n dimensions defined by:

$$P_n = \{\vec{x} \in \mathbb{R}^n : x_i \geq 0, \sum_{i=1}^{n} x_i = 1\}$$

is convex. Note that this describes the set of all probability mass functions with no more than n possible outcomes.

Prove Set Convexity
← use the definition →

The convexity of a set is defined by:

$$\theta x + (1-\theta)y \in P_n \qquad \theta \in [0,1]$$

Let $z = \theta x + (1-\theta)y$ $\quad x,y \in P_n$. For P_n to be convex z must satisfy each of the conditions for P_n, specifically:

1) $z \in \mathbb{R}^n$, 2) $z_i \geq 0$ 3) $\sum_{i=1}^{n} z_i = 1$.

1)
$z \in \mathbb{R}^n$ is convex and true. For all $x,y \in P_n$ since P_n is in $x_n \in \mathbb{R}^n$ space, this is true. This condition is convex as well since no matter what equation with $x,y \in \mathbb{R}^n$ the solution stays in the $\mathbb{R}^n$ space.

continues on next page.

2) $z_i \geq 0 \Rightarrow \theta x_i + (1-\theta)y_i \geq 0$, $\theta \in [0,1]$. For this to be true for any x_i, y_i, Consider the following cases:

① $\theta = 0$

$$0(x_i) + (1-0)y_i$$
$$= 0 + 1 \cdot y_i = y_i \geq 0, \text{ if } y_i \geq 0 \text{ this is true.}$$

② $\theta = 1$

$$1(x_i) + (1-1)y_i$$
$$= x_i \geq 0, \text{ if } x_i \geq 0 \text{ this is true.}$$

Thus $z_i = \theta x_i + (1-\theta)y_i \geq 0$ $\forall x_i, y_i \geq 0$, and $\theta \in [0,1]$, which meets the condition. Thus, 2) is convex as z exists in 2).

3) $\sum_{i=1}^{n} z_i = 0 \Rightarrow \sum_{i=1}^{n} \theta x_i + (1-\theta)y_i = 1$

There are 2 cases:

$$\begin{cases} \theta = 0 & \sum_{i=1}^{n} 0 \cdot x_i + (1-0)y_i = \sum_{i=1}^{n} y_i = 1 \\ \theta = 1 & \sum_{i=1}^{n} 1 \cdot x_i + (1-1)y_i = \sum_{i=1}^{n} x_i = 1 \end{cases}$$

for either case to be true, $x_i < 1$, and $y_i < 1$; it is strictly less than 1 since if $x_0 = 1$, and $x_2 = 0.1$, $\sum_{i=1}^{2} x_i = 1.1$ is not ≤ 1.
Thus, this case is true $\forall x_i, y_i < 1$ when the $\sum_{i=1}^{n} z_i$ will sum to 1 if for every x_i and y_i being less than 1.

For the conditions of P_n, z meets all these conditions for P_n as long as $0 \leq x_i \leq 1$, $0 \leq y_i \leq 1$.

Next page for problem 5

⑤ Solve problems 7.6, 7.7, and 7.8 in the book. Using the software of your preference find the solution for 7.6 and 7.7 primal and dual formulations. You may consider **R** implementations for solving linear and quadratic programming as solve.Lp and solve.Qp but are not limited to those. The code will follow the handwritten portion of this problem.

$\boxed{7.6}$ Consider the linear program:

$$\min_{x \in \mathbb{R}^2} -\begin{bmatrix} 5 \\ 3 \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow -\vec{c}^T\vec{x}$$

subject to:

$$\begin{bmatrix} 2 & 2 \\ 2 & -4 \\ -2 & 1 \\ 0 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 33 \\ 8 \\ 5 \\ -1 \\ 8 \end{bmatrix} \Rightarrow A\vec{x} \leq \vec{b}$$

Derive the dual Linear program using Lagrange duality.

$$\min_{x \in \mathbb{R}^2} -\vec{c}^T\vec{x} \qquad \text{s.t.} \quad A\vec{x} \leq \vec{b}$$

dual Legrangian

Note: λ is of the same dimensions as b, in this case $\lambda \in \mathbb{R}^5$

$$L(\vec{x},\vec{\lambda}) = -\vec{c}^T\vec{x} + \vec{\lambda}^T(A\vec{x} - \vec{b})$$

$$\Rightarrow -(c + A^T\lambda)^T x - \lambda^T b$$

$$L_D(\lambda) = -\lambda^T b \qquad \text{s.t.} \quad -c - A^T\lambda = \vec{0}, \quad \lambda \geq \vec{0}$$

$$L_D = \vec{\lambda}^T \begin{bmatrix} 33 \\ 8 \\ 5 \\ -1 \\ 8 \end{bmatrix}$$

so the dual Linear program is:

$$\text{Dual} = \max_{\lambda} \vec{\lambda}^T \begin{bmatrix} 33 \\ 8 \\ 5 \\ -1 \\ 8 \end{bmatrix} \quad \text{subject to} \quad -[5\ 3] - \begin{bmatrix} 2 & 2 & -2 & 0 & 0 \\ 2 & -4 & 1 & -1 & 1 \end{bmatrix}\vec{\lambda} = \vec{0}$$

Next page for 7.7

7.7 Consider the quadratic program

$$\min_{x \in \mathbb{R}^2} \frac{1}{2}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 5 \\ 3 \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\text{subject to: } \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Derive the dual quadratic program using Lagrange duality.

Using the derivation from class OR equation 7.52 in the textbook, the dual optimization problem is given by

$$L_D = \max_{\lambda \in \mathbb{R}^m} -\frac{1}{2}(c + A^T\lambda)^T \cdot Q^{-1} \cdot (c + A^T\lambda) - \lambda^T b. \quad \text{subject to } \lambda \geq 0.$$

Let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $Q = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$, $c = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, and $\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix}$. Also $Q^{-1} = \frac{1}{7}\begin{bmatrix} 4 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{7} & \frac{-1}{7} \\ -\frac{1}{7} & \frac{2}{7} \end{bmatrix}$.

Thus the dual program is:

$$\max_{\lambda \in \mathbb{R}^m} -\frac{1}{2}\left(\begin{bmatrix} 5 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}\vec{\lambda}\right)^T \cdot \begin{bmatrix} \frac{4}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{2}{7} \end{bmatrix} \cdot \left(\begin{bmatrix} 5 \\ 3 \end{bmatrix} + \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}\vec{\lambda}\right) - \vec{\lambda}^T \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

subject to $\vec{\lambda} \geq 0$.

7.8 Consider the following convex optimization problem

$$\min_{\omega \in \mathbb{R}^D} \frac{1}{2}\omega^T\omega \quad \text{subject to: } \omega^T x \geq 1.$$

Derive the dual quadratic program using Lagrange duality.

Dual quadratic program: $-\frac{1}{2}(c + A^T\lambda)^T \cdot Q^{-1} \cdot (c + A^T\lambda) - \lambda^T b$

$\frac{1}{2}w^T I w + \vec{c}w \qquad I = Q,\ c = \vec{0} \qquad -w^T x \leq 1$

$-w^T x - 1 \leq 0$

The dual Quadratic Program is:

$$L_D = -\frac{1}{2}(0 + w^T\lambda)^T \cdot I^{-1} \cdot (0 + w^T\lambda) - \lambda^T$$

$$L_D(\lambda) = -\frac{1}{2}(w^T\lambda)^T(w^T\lambda) - \lambda^T$$

index for code on Next Page.

Index:

i) Code for problem 1 and results.

ii) Code for problem 5 and Results.

I have included the source code either in the .zip file, or as additional files

Code for HW3 Question 1c:

```
import math
import inline as inline
import matplotlib
from mpl_toolkits
 import mplot3d
import numpy as np
import matplotlib.pyplot as plt
#from lpsolve55 import * import pip
 #path = 'C:\Users\nefle\Downloads\lpsolve55-5.5.2.5-cp35-cp35m-win_amd64.whl'


def f(x, y):
        return (x-1)**2 + (y-2)**2
x = np.linspace(-10, 10, 10)
y = np.linspace(-10, 10, 10)
X, Y = np.meshgrid(x, y)
 Z = f(X, Y) fig = plt.figure()
ax = plt.axes(projection='3d')
ax.contour3D(X, Y, Z, 50, cmap='binary')
#ax = plt.axes(projection='3d')
#ax.plot_surface(X, Y, Z, rstride=1, cstride=1, # cmap='viridis', edgecolor='none')
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('z')
plt.show()
```

Code for Question 5: 7.6 Linear Programming and the results:

```
# Import lpSolve package
library(lpSolve)

# Set coefficients of the objective function
f.obj <- c(-5, -3)

# Set matrix corresponding to coefficients of constraints by rows
# Do not consider the non-negative constraint; it is automatically assumed
f.con <- matrix(c(2,  2,
              2, -4,
              -2, 1,
              0, -1,
              0,  1), nrow = 5, byrow = TRUE)

# Set unequality signs
f.dir <- c("<=",
        "<=",
        "<=",
        "<=",
        "<=")

# Set right hand side coefficients
f.rhs <- c(33,
        8,
        5,
       -1,
        8)

# Final value (z)
lp("min", f.obj, f.con, f.dir, f.rhs)

# Variables final values
lp("min", f.obj, f.con, f.dir, f.rhs)$solution

# Sensitivities
lp("min", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$sens.coef.from
lp("min", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$sens.coef.to

# Dual Values (first dual of the constraints and then dual of the variables)
# Duals of the constraints and variables are mixed
lp("min", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$duals

# Duals lower and upper limits
```

lp("min", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$duals.from
lp("min", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$duals.to

This returned the following:

```
Success: the objective function is -74.16667
[1] 12.333333  4.166667
[1] -1e+30 -5e+00
[1] -3 10
[1] -2.1666667 -0.3333333  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000
[1]  1.4e+01 -1.5e+01 -1.0e+30 -1.0e+30 -1.0e+30 -1.0e+30 -1.0e+30
```

Value of z
Values of variable final values
Values of Coefficients to/from
Values of the dual function
Values of dual lower/upper limit

Code for 5: 7.7 Quadratic Programming Code:

```
# Import quadprog library
library(quadprog)

# Matrix appearing in the quadratic function
Dmat <- matrix(c(2, 4, 1, 4), 2, 2)

# Vector appearing in the quadratic function
dvec <- c(5, 3)

# Matrix defining the constraints
Amat <- t(matrix(c(1, 0,
            -1, 0,
            0, 1,
            0, -1), 4, 2))

# Vector holding the value of b_0
bvec <- c(1, 1, 1, 1)

# Only the first constraint is equality
qp <- solve.QP(Dmat, dvec, Amat, bvec, meq = 1)
qp
```

For this problems case, the result gave this error:

```
Error in solve.QP(Dmat, dvec, Amat, bvec, meq = 1) (QuadProgram.R#26): constraints are
 inconsistent, no solution!
```

The error was in the matrix in the constraint with the alternating signs on the 1s: [1, 0; -1, 0; 0, 1; 0,-1]. To verify this program still functioned, I took the negatives off the ones and ran the program, and it returned the following:

$solution (primal solution)

[1] 1 1

$value (of objective function)

[1] -4

$unconstrained.solution

[1] 2.4285714 0.1428571

$Lagrangian (dual program solution)

[1] 2 2 0 0