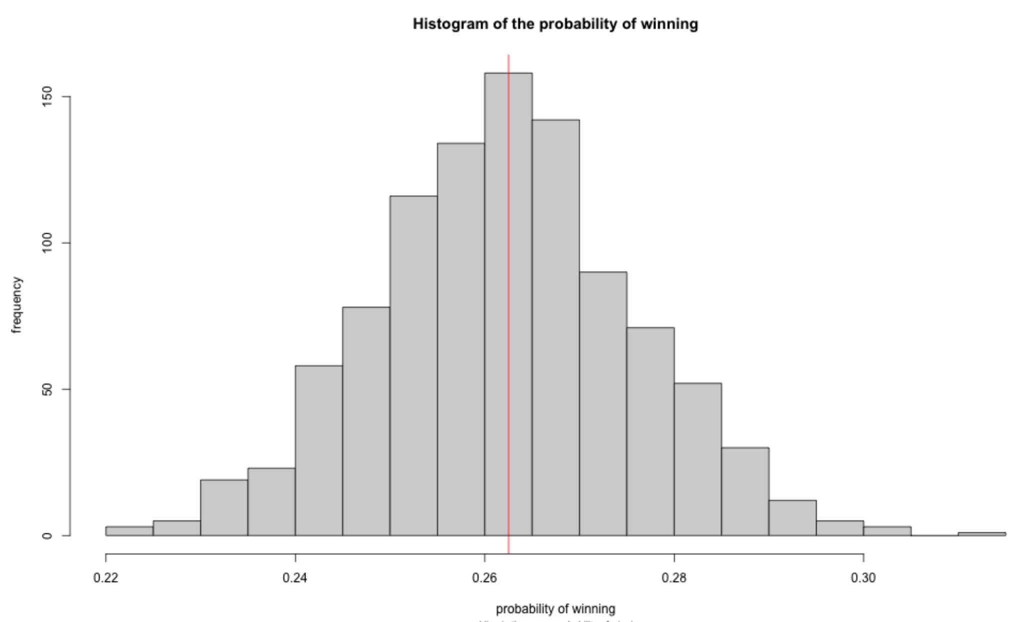
Crazy 8s is a card game that can be played with 2-5+ players and a standard deck of cards (jokers excluded). Each player is dealt 5 cards, and the goal of the game is to be the first player to get rid of all the cards in your hand. Players can only play cards from their hand that match the suit or rank of the card on top of the discard pile. If a player has an 8 in their hand, they can play that regardless of what is on top of the discard pile and change the play to the suit of their choosing. If a player cannot place a card with matching suit, rank, or an 8 the player draws cards from the deck until they can play. The first player to get rid of all their cards wins and the game is over. The official game page can be found here.

For this game I identified three strategies centered around when the player will play an 8. If a player has an 8, should they immediately play it to change the current suit to the one that fits their hand the best, or save it until they would have to draw a card and then change the suit to one that they have, or should they only play the 8 when it is the only card(s) left in the players hand? I named these strategies play8s, save8s, and keep8s respectively. When a player does not have an 8, then the style of play is similar for all strategies. If a player can play a card, play one that matches the suit if possible otherwise play one that matches the rank.

I coded the game in java and found confidence intervals for the probability of winning for each strategy when playing 3 players with a different strategy. The code and output can be found in my github repository with this url: `https://github.com/nicolegooding/math381-a6`

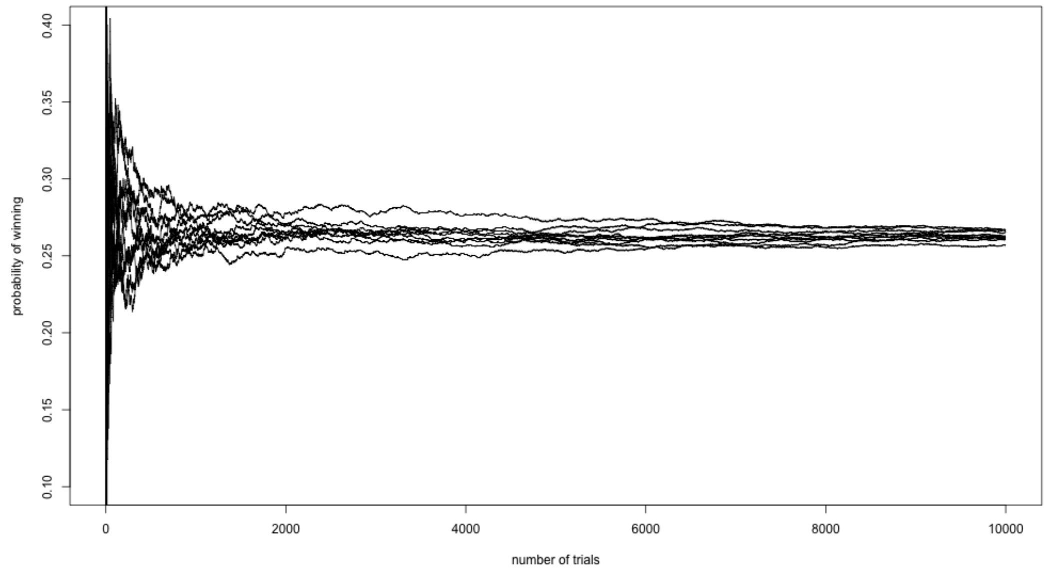
The Central Limit Theorem "states, roughly, that if we sample many values from an unknown distribution and average them, and do this multiple times, the averages will be approximately normally distributed around the mean of the distribution." (Conroy) We can apply this to the game, where a sample with size n will be n simulations. Each simulation will be represented by 0 or 1 (0 if the player in question loses and 1 if the player in question wins). Taking the average of the n 0s and 1s will give the probability of winning, so we can say the mean of the sample is equivalent to the probability of winning. Then taking multiple samples, (doing multiple runs of n simulations of the game) will create a normal distribution centered around the true probability of winning.

To test this idea, we will look at the probability of winning for 1,000 runs of a simulation with 1,000 games to confirm that the calculated probabilities will be approximately normal centered about the mean. Running the Create_figs.R file in the repository will output the probabilities_histogram.png which shows the histogram below.



In the following simulations we will run the game 10,000 times, calculate the probability of winning from those games, then reset and do that 10 times. This will give us 10 probabilities of winning that are approximately normally distributed by the Central Limit Theorem, meaning we can assume they are symmetric around the true probability of winning. If the true probability was outside the range of calculated probabilities, every probability would have to fall on one side of the true probability. The probability that one estimate falls above the true probability is $\frac{1}{2}$ - so the probability that all of them fall on one side is $\frac{1}{2} * \frac{1}{2} * ... * \frac{1}{2} = \frac{1}{2^{10}}$. Then the probability that all estimates fall above or all estimates fall below is $\frac{1}{2^{10}} + \frac{1}{2^{10}} = \frac{2}{2^{10}} = \frac{1}{2^9} = \frac{1}{512} \approx 0.001953125$. So the probability that the true probability of winning is outside our range of estimates is approximately 0.001953125. This means we can be 99.8% confident that the true value of the probability of winning is within our range of 10 calculated probabilities.

To confirm that the probability of winning converges to a single value as more games are run, we will look at the probability of winning after each game for 10,000 games and do this 10 times. I used R to run the Create_figs.R file in the repository and create the convergence_plot.png which shows this below.



We can see that for every run, in the beginning when we only simulate a few games, there is a lot of variation in the probability when we run another game. The more games we do the straighter the line gets, so the probability of winning is staying constant and converging to a single value.

Both of these previous plots confirm our assumptions we make from the Central Limit Theorem that allow us to create the confidence intervals.

Since the game is random, if every player uses the same strategy - they should have an equal chance of winning (25%). However when we do 10 runs of 10,000 games with all the players using the same strategy, the probability that the first player wins is greater than 25%. If we repeat this three times having all players use the keep8, save8, or play8 strategy - we get the following 99.8% confidence intervals (.2568, .2692) for keep8, (.2722, .2828) for save8, and (.2569, .267) for play8. This means the probability of the first player winning is higher than it should be regardless of strategy. This makes sense because the first player to get rid of all their cards wins - so if player 1 and player 3 could both win in 4 turns, player 1 will win and player 3 will lose simply because player 1 got to place their card first. To avoid this first player bias, we will rotate which strategy goes first(i.e. if we are testing 1 keep8 strategy vs 3 save8 strategies, we will have an equal number of games where the keep8 strategy goes 1st, 2nd, 3rd, and 4th). To confirm this, I did the same test from before where all players use the same strategy, but this time rotated which player counted as a win. So there are 2,5000 games counting if player 1 wins, 2,500 games counting if player 2 wins, and the same for player 3 and 4. This is just to ensure that the idea of rotating the players will create an even playing field. When we calculate the probabilities this way we get the following 99.8% confidence intervals (.2409, .2569) for keep8, (.2434, .2566) for save8, and (.2428, .2635) for play8. Since all of the intervals contain .25, we can now reduce the first player bias by rotating which strategy goes first.

Running the Main.java file from the github repository will compare strategies against each other gives the following intervals in the confidence-intervals.txt file:

$$\begin{array}{c c}
 & \begin{array}{ccc} \quad keep8 \quad & \quad play8 \quad & \quad save8 \quad \end{array} \\
\begin{array}{c} keep8 \\ play8 \\ save8 \end{array} & \begin{pmatrix} (.2409, .2569) & (0.1884, 0.1994) & (0.1509, 0.1614) \\ (0.2895, 0.2993) & (0.2428, 0.2635) & (0.193, 0.204) \\ (0.3451, 0.356) & (0.2903, 0.3062) & (0.2434, 0.2566) \end{pmatrix}
\end{array}$$

Each row represents the one strategy we are testing and each column represents the other 3 strategies we are comparing against. For example, the confidence interval in the second row, first column (0.2895, 0.2993) represents the confidence interval of the probability that a player using the play8 strategy wins against 3 players using the keep8 strategy.

Looking at these strategies shows that using the save8 strategy generally tends to have a higher probability of winning than play8 and keep8. When testing save8 vs keep8 and save8 vs play8 both intervals are in the mid to lower 30%. Keeping 8's seems to be the worst strategy, where both intervals against other strategies are less than 20%. The play8 strategy seems to do well against the keep8 strategy, but not as good against the save8 strategy. I think the biggest takeaway from this is that you have a better chance of winning when you only play your 8s when you would have to draw instead. Playing the 8 immediately can be effective, but you are not taking advantage of the fact that you can use the 8 in critical times to draw. The keep8 strategy is definitely the worst. The only advantage this strategy has is if you can get down to only having 8's in your hand you can guarantee you can play all of them regardless of what is in on top of the discard pile. If you get really unlucky you can end up drawing cards unnecessarily when you can play an 8. Once you draw several cards, it can be difficult to come back and win if other players are not picking up as frequently.

Works Cited