Assignment 1: Martingale

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Abstract—This report presents an empirical analysis of a gambling simulator based on the Martingale betting strategy. The Martingale strategy is a popular betting system in gambling, particularly in roulette, where the player doubles their bet after every loss, with the expectation of recouping all previous losses and gaining a small profit when they eventually win. These two experiments were designed under different circumstance test the effectiveness and risks inherently.

1 INTRODUCTION

This report presents an empirical analysis of a gambling simulator based on the Martingale betting strategy. The Martingale strategy is a popular betting system in gambling, particularly in roulette, where the player doubles their bet after every loss, with the expectation of covering all previous losses and gaining a small profit eventually. These two experiments were designed under different circumstance testing the effectiveness and risks of this strategy inherently. [1]

2 EXPERIMENTAL HYPOTHESIS AND DESIGN

All the episodes and each bet spinning within the episode are independent and randomly happened. All experiments involve running a large size of episodes of sequential bets, and the winning probability is about $\frac{18}{38}$, [2] each follows the Martingale strategy:

2.1 Experiment1(Unlimited Bankroll)

There is no limitation on the bankroll, and the player continues to double their bet after each loss set target as $80, if the target is reached, stop betting and allow $80 to persist. In addition, the max number of spinning of each episode sets to 1000 times.

1 refer to https://gatech.instructure.com/courses/372882/assignments/1610458
2 refer to https://en.wikipedia.org/wiki/Roulette

2.2 Experiment2(Limited Bankroll)

This experiment introduces a realistic element by giving the gambler a $256 bankroll and still set target as $80 as long as the gambler gains $80 or loses $256, this player must be out of this game. Additionally, the gambler can only bet the remaining bankroll if it is less than the next required bet. Also the betting amount cannot more than accumulated episode_winning plus the initial bankroll. In addition, the max number of spinning of each episode sets to 1000 times.

3 EXPERIMENTAL FINDINGS ANALYSIS

3.1 Experiment 1 Results

The analysis of 10 and 1000 simulated episodes reveal a very certain likelihood of achieving the $80 target. However, it also shows that the player might experience huge losses at some point, even though finally the loss can be covered.

3.2 Experiment 2 Results

With a bankroll limit of $256, the expected value of outcomes decreased notably. Considering before reaching the max number of bets, the player already lost the entire bankroll or reach the cap, addressing the increased risk as the financial constraints introduced in the real world. As bankruptcy happens, the player has to stop the betting strategy, increasing the probability of final loss.

3.3 Question 1–estimated probability in Experiment 1

As we assumed a fix win_probability=$\frac{18}{38}$ the Experiment1 shows that as the sequential bets increase until 1000, the almost all episode_winning reach the cap ($80). In fact, the gambler could earn $1 once he or she wins in one bet, since the current bet amount can cover no matter how much they lose before, according to the original strategy with no limitation (unlimited bankroll,etc).(Williams., 1991) In the very rare case, we assume the gambler can not winning $80, that means the gambler might lost more than 920 times , the probability of this situation is about (1-$\frac{18}{38}$)$\hat{9}$20, almost close to 0, therefore, the probability of winning more than $80 should be 1 minus this worse case probability , which equals almost to 1. This is also illustrated by the Figure1 as below.

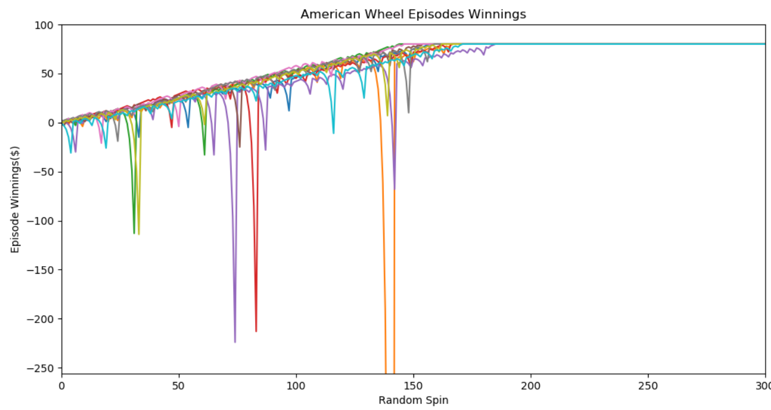

Figure 1—American Wheel Episodes Winnings

3.4 Question 2–estimated expected value in Experiment 1

Figure2 shows the expected value of winnings after 1000 bets would be 80. The reason is the same as question 1, the estimated probability of winning 80 within 1000 bets is almost 100%, that means within 1000 bets, the episode_winning will reach 80 and remain at 80 afterward since $80 is set as cap. As all episode_winning plot (after 1000 bets) remain at $80, the expected value will be approaching 80, which also align with the theoretical formula-An expectation can be written as:

$$E[X] = \sum_{i=1}^{n} p_i \cdot x_i$$

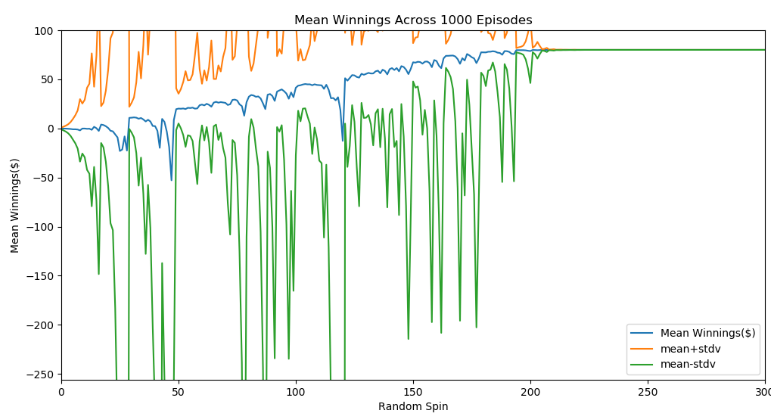

Figure 2—Mean Winnings Across 1000 Episodes

3.5 Question 3 –standard deviation trend In Experiment 1

In experiment1(assume the win prob=$\frac{18}{38}$, cap=$80), As the Figure 2 and Figure 3 shows that upper and lower standard deviation line reach a maximum value and they all become more stabilization as the bets increase. Especially the high volatility happens when the number of spins is small but the standard deviation lines converge later. Through 1000 times simulation, almost all episode shows that the episode_winning would reach at $80 before 200 spins, that is the small amount compared with total max spin. Once the episode_winning reaches $80, the rest of episode_winning will remain $80 forever. Therefore, after a cutoff point, the mean is approaching expected value ($80) until it equals to expected value of episode_winning. As the huge amount data sample drops at expect value, the standard deviation closes to a constant number, so it will converge as the number bets increases.This result align with the theoretical formula- Standard Deviation.

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

(Murphy., 2022b)

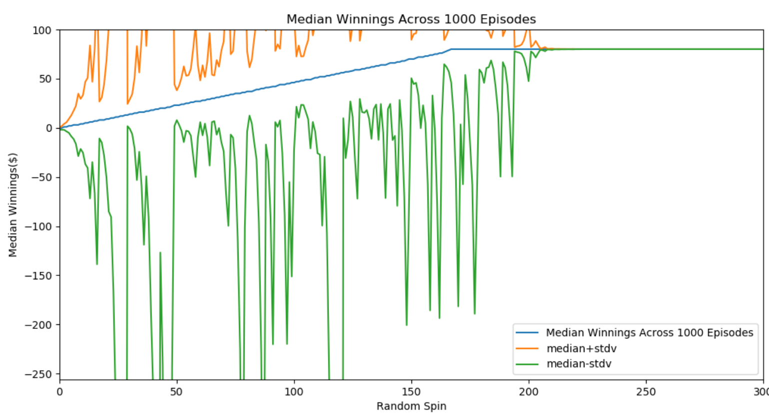

Figure 3—Median Winnings Across 1000 Episode

3.6 Question 4 –estimated probability in Experiment 2

According to the Experiment2's result, the estimated probability of winning$80 within 1000 sequential bets is about 65.3%. (see Figure 4). We assume 1000 episode as a big sample size, in an independent randomly process, there are around

650 episodes having $80 winning finally, about 34% chance the gambler gets bankrupt.(Murphy., 2022a)

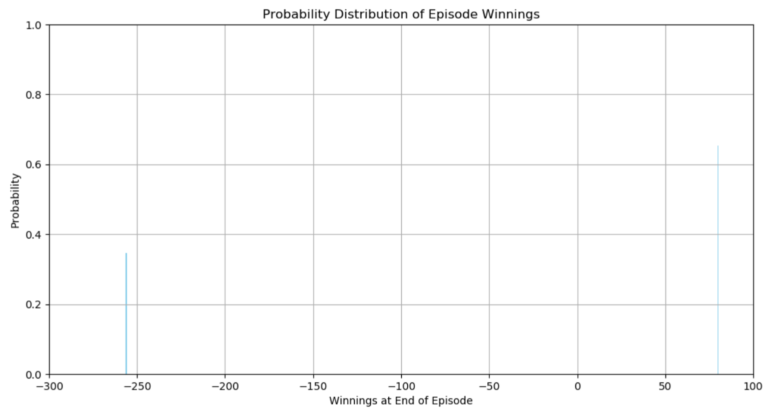

Figure 4—Probability Distribution of Episode Winning with Limitation

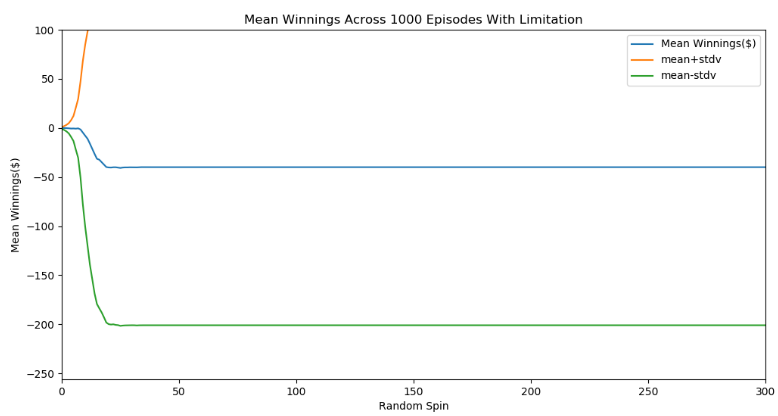

Figure 5—Mean Winnings Across 1000 Episodes with Limitation

3.7 Question 5 –estimate expected value in Experiment 2

The Figure 5 illustrate the expected-value of winning will be around $ -40.2 after a large number of episode simulations. In fact, according the result of Experiment2 and the estimated probability of episode_winning outcome, we can calculate the expected value =p1* outcome1+p2*outcome1. . . . +p(n)*outcome(n). Since we estimated the probability of $80 is about 65.3%, and probability of -$256 is about 34%, and the probability of outcome range from (-256,80) is around 0.7% (1-65.3%-34%=0.7%) which have little impact on the expected value. Therefore, theoretically the expected value is around 65.3% * $ 80+ 34%*(-$256) = -$35, as we extend more bets the mean in Figure 5 will be more close to the expected value.

3.8 Question 6 –standard deviation trend In Experiment 2

the upper line reaches a maximum value and get stabilized and converge as the bets number increases. Considering 1000 episodes, In the most case,over (65.3%+34%=99.3%) chances, the gambler either reaches the $80 goal or bankrupt. In addition, according to standard deviation formula

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

, the outcomes will be either $80 or -$256 as the simulation time increases, the standard deviation tends to be a constant number. See Figure 5 and Figure 6 as below.

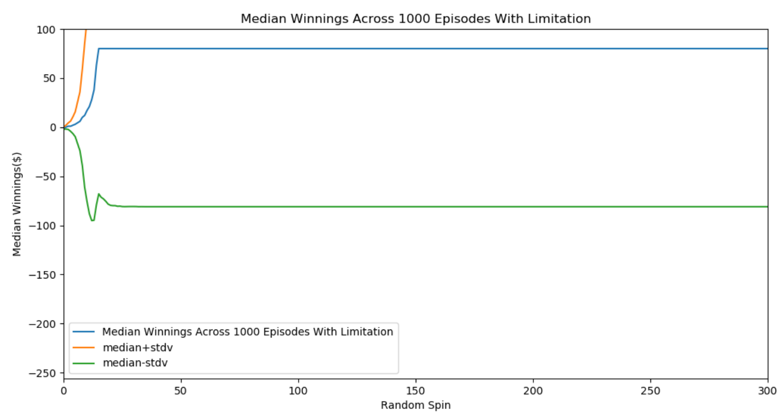

Figure 6—Median Winnings Across 1000 Episodes with Limitation

3.9 Question 7 –the advantages of applying expected values instead of one specific random episode

Expected values are based on the mean outcome of a huge numbers of trials. Considering the Law of Big Number and central limit theorem: the sample mean (expected value) equals to population mean as the sample size tends to infinite. Using the experiments' expected values could reduce the volatility and possibility of outliers which impact the final result.

4 CONCLUSION

The empirical results from the simulations align with the theoretical understanding of the Martingale system. While it may provide short-term gains, this strategy is inherently risky and unsustainable over an extended period or under financial constraints.

5 REFERENCES

1. Williams., David (1991). "Probability with Martingales". In: *Probability with Martingales*. Cambridge, United Kingdom: Cambridge University Press.
2. Murphy., Kevin P. (2022a). "Probability". In: *Probabilistic Machine Learning*. Massachusetts, United States.
3. Murphy., Kevin P. (2022b). "Statistics". In: *Probabilistic Machine Learning*. Massachusetts, United States.