An introduction to mixture modelling for unsupervised clustering

Mini-tutorial

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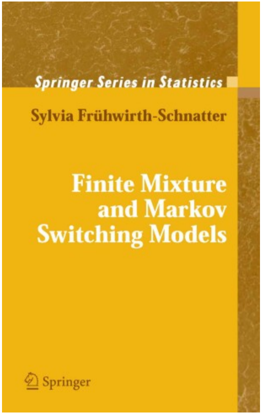
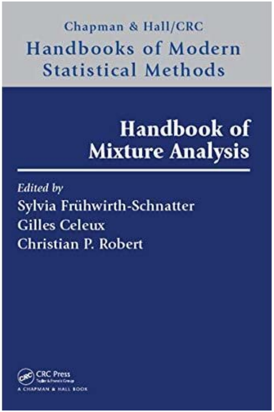
Further reading

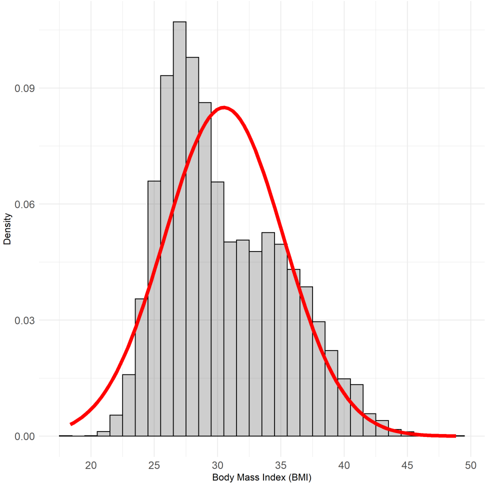




https://github.com/nicolemwhite/anzsc-mixture-modelling

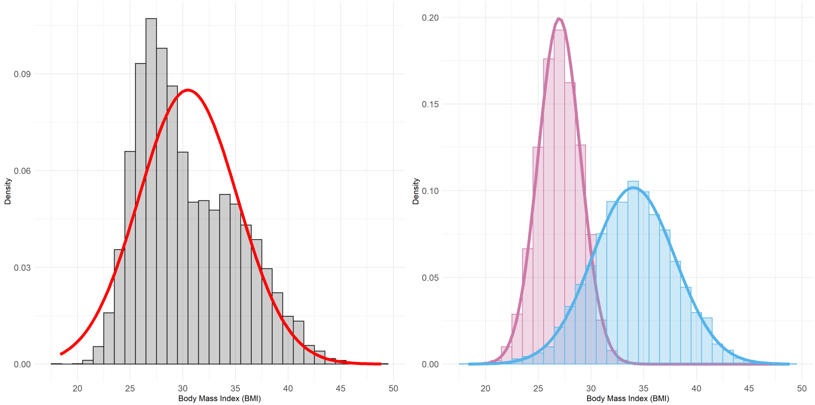
A motivating example

Distribution of body mass index (BMI) for 10,000 participants.



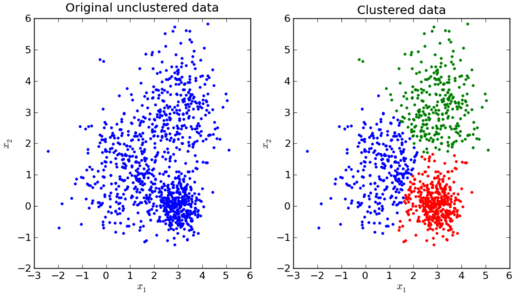
A motivating example

Distribution of body mass index (BMI) for 10,000 participants



Defining clustering

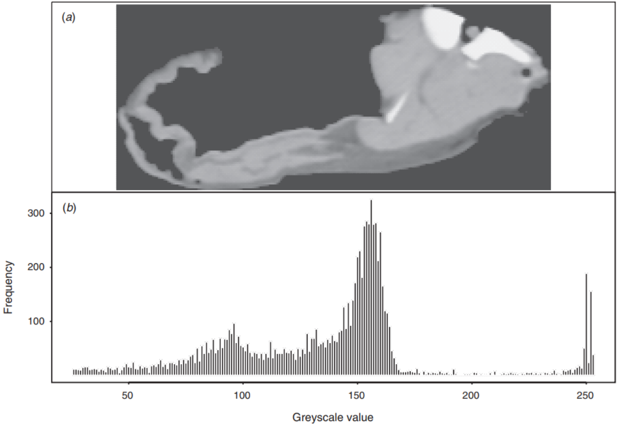
Unsupervised clustering $\leftrightarrow$ Identifying subgroups



Common approaches:

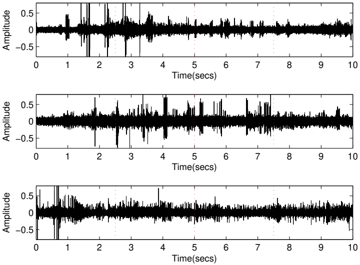
- Hierarchical clustering, K-means
- Mixture models

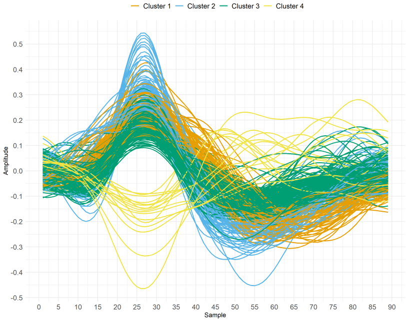
Examples of clustering using mixture models: Image classification



Alston et al (2005) DOI: 10.1071/AR04211

Examples of clustering using mixture models: Spike sorting





Mixture model ingredients

Data are drawn from a *convex combination of components*

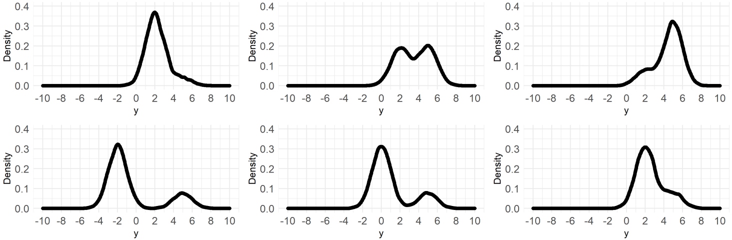
$$p(y) = \eta_1 f(y|\boldsymbol{\theta}_1) + \ldots + \eta_K f(y|\boldsymbol{\theta}_K)$$
$$= \sum_{k=1}^{K} \eta_k f(y|\boldsymbol{\theta}_k)$$

Unknown parameters: $\boldsymbol{\nu} = (\boldsymbol{\eta}, \boldsymbol{\theta})$

- $\boldsymbol{\eta} = (\eta_1, \ldots, \eta_K)$: Mixture weights; $\sum_{k=1}^{K} \eta_k = 1$
- $f(y|\boldsymbol{\theta}_k)$: k^{th} Mixture component; same parametric family

A simple 2-component mixture model

$$y_i \sim \eta_1 \mathcal{N}(\mu_1, 1) + \eta_2 \mathcal{N}(\mu_2, 1)$$



Mixture model examples

General formulation:

$$p(y_i) = \sum_{k=1}^{K} \eta_k f(y_i|\boldsymbol{\theta}_k)$$

Latent class analysis (J items)

$$f(\mathrm{y}_i|\boldsymbol{\theta}_k) = \prod_{j=1}^{J} f(y_{ij}|\theta_{jk})$$

Latent class regression: $\eta_k \rightarrow \eta_k(\mathrm{x}_i)$

$$\eta_k(\mathrm{x}_i) = \frac{exp\left(\mathrm{x}_i^T \beta_k\right)}{\sum_{l=1}^{K} exp\left(\mathrm{x}_i^T \beta_l\right)}$$

Mixture model examples

Focus of mini-tutorial: cross-sectional data

- Finite mixture model
- Dirichlet Process mixture model
- Profile regression

Bayesian approaches to inference: Markov chain Monte Carlo (MCMC)

1. Finite mixture models
2. Dirichlet Process Mixture models
3. Profile regression

Finite mixture model: Setup

Assume:

- K is fixed *a priori*
- Each observation has a probability of belonging to components $1, \ldots, K$

Likelihood for $\boldsymbol{y} = (y_1, \ldots, y_n)$

$$p(\boldsymbol{y}) = \prod_{i=1}^{n} \sum_{k=1}^{K} \eta_k f(y_i | \boldsymbol{\theta}_k)$$

Aim is to learn $\boldsymbol{\nu} = (\boldsymbol{\eta}_{1,\ldots,K}, \boldsymbol{\theta}_{1,\ldots,K})$

Finite Mixture Model: Setup

Likelihood:

$$p(\boldsymbol{y}|\boldsymbol{\nu}) = \prod_{i=1}^{n} \sum_{k=1}^{K} \eta_k f(y_i|\boldsymbol{\theta}_k)$$

Priors:

$$(\eta_1, \ldots, \eta_K) \sim \mathcal{D}(\gamma_1, \ldots, \gamma_K)$$
$$\theta_k \sim p(\theta_k|\delta)$$

Finite Mixture Model: Setup

Example: Finite mixture of Normal distributions

$$p(\boldsymbol{y}|\boldsymbol{\nu}) = \prod_{i=1}^{n} \sum_{k=1}^{K} \eta_k \mathcal{N}(y_i|\mu_k, \sigma_k^2)$$

$$\mu_k = \mathcal{N}\left(\mu_0, \frac{\sigma_k^2}{N_0}\right)$$

$$\sigma_k^2 = \mathcal{IG}\left(c_0, C_0\right)$$

How to estimate when membership of y_i to components $1, \ldots, K$ is not known?

Finite Mixture Model: Estimation

Enter data augmentation! (Tanner and Wong, 1987; *JASA*)
The idea:

- Introduce z_i = cluster membership for y_i and treat as missing data

$$p(y_i|\boldsymbol{\nu}) = \sum_{k=1}^{K} p(y_i|z_i = k, \boldsymbol{\nu})Pr(z_i = k|\boldsymbol{\nu})$$

$$Pr(z_i = k|\boldsymbol{\nu}) = \eta_k$$

$$p(y_i|z_i = k, \boldsymbol{\nu}) = f(y_i|\theta_k)$$

- Inference on z_i provides information on clustering

Finite Mixture Model: Estimation

Inference based on the complete-data likelihood

$$\begin{aligned} p(\mathrm{y}, \mathrm{z}|\boldsymbol{\nu}) &= \prod_{k=1}^{K} \prod_{i:z_i=k} \eta_k p(y_i|\theta_k) \\ &= \prod_{k=1}^{K} \eta_k^{\sum_{i=1}^{n} \mathbb{I}(z_i=k)} \prod_{i:z_i=k} p(y_i|\theta_k) \end{aligned}$$

Finite Mixture Model: Estimation by MCMC

1. Sample $\boldsymbol{z}$ (Bayes' rule)

$$Pr(z_i = k|y_i, \boldsymbol{\nu}) = \frac{\eta_k f(y_i|\boldsymbol{\theta}_k)}{\sum_{j=1}^{K} \eta_j f(y_i|\boldsymbol{\theta}_j)}$$

$$z_i \sim MN(1, Pr(z_i = 1|y_i, \boldsymbol{\nu}), \ldots, Pr(z_i = K|y_i, \boldsymbol{\nu}))$$

2. Conditional on $\boldsymbol{z}$: Update $\eta_1, \ldots, \eta_K$

$$\eta_1, \ldots, \eta_K \sim \mathcal{D}(\delta_1 + \sum_{i=1}^{n} \mathbb{I}(z_i = 1), \ldots, \delta_K + \sum_{i=1}^{n} \mathbb{I}(z_i = K))$$

3. Conditional on $\boldsymbol{z}$: Update $\theta_1, \ldots, \theta_K$

$$\theta_k \sim p(\boldsymbol{\theta}_k|\delta) \prod_{i:z_i=k} f(y_i|\boldsymbol{\theta}_k)$$

Finite Mixture Model: Estimation by MCMC

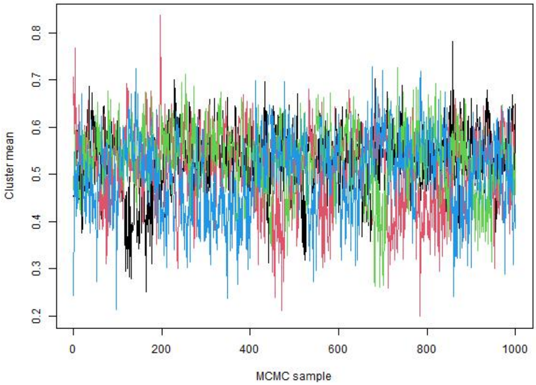
Available approaches in R:

- `R2OpenBUGS` (see `fmm_BUGS.R`)
- `bayesmix`
- Maximum likelihood: `mixtools`, `mclust`

Or code from scratch:

- `fmm_mvn.R`
- `fmm_multinomial.R`
- `fmm_pois.R`

Example MCMC output



Label switching

Issue arises as likelihood is invariant to permutations of k e.g. $K = 3$

$$
\begin{aligned}
p(y_i|\boldsymbol{\nu}) &= \eta_1 p(y_i|\boldsymbol{\theta}_1) + \eta_2 p(y_i|\boldsymbol{\theta}_2) + \eta_3 p(y_i|\boldsymbol{\theta}_3) \\
&= \eta_3 p(y_i|\boldsymbol{\theta}_3) + \eta_2 p(y_i|\boldsymbol{\theta}_2) + \eta_1 p(y_i|\boldsymbol{\theta}_1) \\
&= \eta_2 p(y_i|\boldsymbol{\theta}_2) + \eta_3 p(y_i|\boldsymbol{\theta}_3) + \eta_1 p(y_i|\boldsymbol{\theta}_1)
\end{aligned}
$$

When sampling z_i, components can be relabelled $\rightarrow$ affects posterior inference

Exploring posterior space $\leftrightarrow$ Posterior inference on components

Label switching: Possible solutions

Prior constraints (not a good idea):

$$\eta_1 < \eta_2 < \ldots < \eta_K$$

Relabelling algorithms:

- Loss functions: minimise over all MCMC samples of z (Stephens, 2000)
- MAP estimate $\widehat{z}$ as 'pivot' (Marin et. al, 2005)

Label switching: Possible solutions

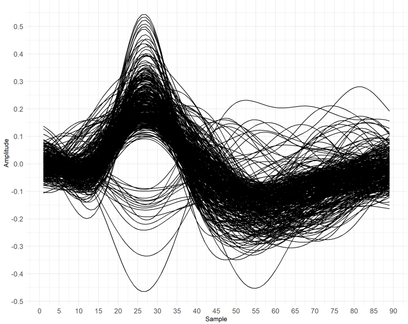
Similarity matrix, **S**:

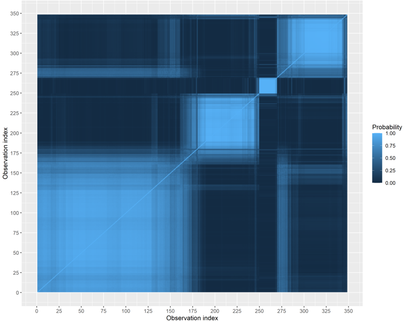
$$S_{ii'}^{(d)} = \begin{cases} 1 & \text{if } z_i^{(d)} = z_{i'}^{(d)} \\ 0 & \text{otherwise.} \end{cases}$$

$$\overline{S} = \frac{1}{D} \sum_{d=1}^{D} S^{(d)}$$

Label switching

Example: Spike sorting, $K = 4$ clusters





Loss functions to obtain likely classification z: e.g Least squares (Dahl, 2006) R packages: `mcclust`, `label.switching`

Choosing K

Common information criteria:

- Akaike's Information Criterion (AIC)

$$AIC_K = -\log p(\mathrm{y}|\boldsymbol{\eta}^*, \boldsymbol{\theta}^*) + 2p_k$$

- Bayesian Information Criterion (BIC)

$$BIC_K = -\log p(\mathrm{y}|\boldsymbol{\eta}^*, \boldsymbol{\theta}^*) + p_k \log n$$

- Deviance Information Criterion (DIC)

$$DIC_K = -4E_{\boldsymbol{\eta},\boldsymbol{\theta}|\mathrm{y}}\left[\log p(\mathrm{y}|\boldsymbol{\eta}, \boldsymbol{\theta})\right] + 2\log f(\mathrm{y})$$

$$f(\mathrm{y}) = \prod_{i=1}^{n} \frac{1}{D} \sum_{d=1}^{D} \sum_{k=1}^{K} \eta_k^{(d)} f(y_i|\theta_k^{(d)})$$

1. Finite mixture models
2. Dirichlet Process Mixture models
3. Profile regression

Dirichlet Process mixture model: Motivation

General formulation:

$$p(y_i|\boldsymbol{\nu}) = \sum_{k=1}^{K} \eta_k f(y_i|\boldsymbol{\theta}_k)$$

In a finite mixture - assume K as fixed $\rightarrow$ model comparison problem
Alternative: Infer K as part of modelling

$$p(y_i|\boldsymbol{\nu}) = \sum_{k \geq 1} \eta_k f(y_i|\boldsymbol{\theta}_k)$$

Dirichlet Process mixture model: Setup

- Nonparametric approach to mixture modelling
- Does not estimate K directly; focus on clustering of individual parameters, θ_i

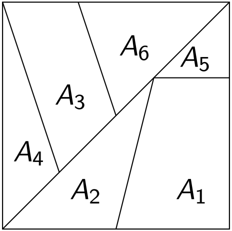
Dirichlet process (DP) prior:

$$\begin{aligned} G &\sim DP(\alpha, G_0) \\ G_0 &= E(G); \qquad \text{Base distribution} \\ \alpha &> 0; \qquad \text{Concentration parameter} \end{aligned}$$

Dirichlet Process mixture model: Setup

Each draw from a DP is itself a distribution:

$$G(A_1), \ldots, G(A_k) | \alpha, G_0 \sim D(\alpha G_0(A_1), \ldots, \alpha G_0(A_k))$$



"Discreteness property": multiple draws from $DP(\alpha, G_0)$ can take the same value; induces clustering behaviour

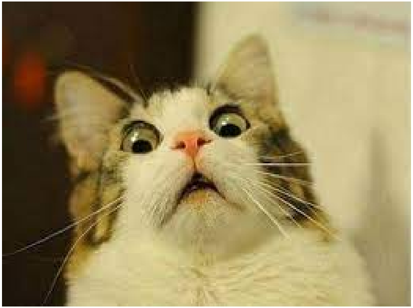
Dirichlet Process mixture model: Setup

DP as prior within mixture setting:

$$
\begin{aligned}
y_i|\boldsymbol{\theta}_i &\sim p(y_i|\boldsymbol{\theta}_i) \\
\boldsymbol{\theta}_i|G &\sim G \\
G &\sim DP(\alpha, G_0)
\end{aligned}
$$

- G is the mixing distribution
- G_0 prior distribution on unknown components
- α controls variation around G_0

Dirichlet Process mixture model: Estimation



How to sample from $DP(\alpha, G_0)$?!
What happened to η_k?!

- Stick breaking representation
- Pòlya Urn scheme/Chinese restaurant process

Stick breaking representation

G replaced by an infinite sum of weighted point masses:

$$\begin{aligned} G &\sim DP(\alpha, G_0) \\ G &= \sum_{k=1}^{\infty} \eta_k \delta_{\theta_k} \\ \theta_k &\sim G_0. \end{aligned}$$

$\eta_k; k = 1, \ldots$ are the "stick breaking weights".
Weights are drawn sequentially:

$$\begin{aligned} w_k|\alpha &\sim Beta(1, \alpha) \\ \eta_1 &= w_1 \\ \eta_k &= w_k \prod_{l=1}^{k-1}(1 - w_l). \end{aligned}$$

Stick breaking process

1

- Draw $w_1 \sim Beta(1, \alpha)$ and set $\eta_1 = w_1$

w_1	$1 - w_1$

- Draw $w_2 \sim Beta(1, \alpha)$ and compute $\eta_2 = w_2(1 - w_1)$

w_1	w_2	$(1 - w_2)(1 - w_1)$

- Draw $w_3 \sim Beta(1, \alpha)$ and compute $\eta_3 = w_3(1 - w_1)(1 - w_2)$

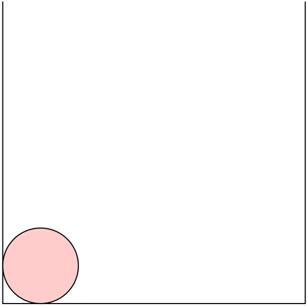
w_1	w_2	w_3	$(1 - w_3)(1 - w_2)(1 - w_1)$

As $K \to \infty$

$$p(y_i|\boldsymbol{\nu}) = \sum_{k=1}^{K} \eta_k p(y_i|\boldsymbol{\theta}_k)$$

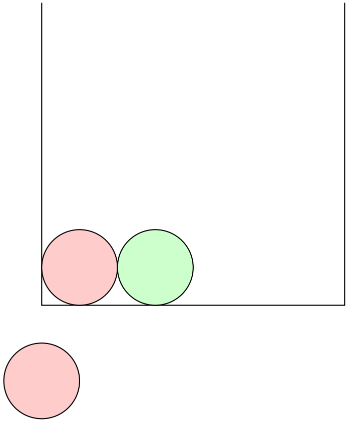
The Pòlya Urn scheme

Good analogy for implied clustering behaviour
Begin with α red balls in an urn:



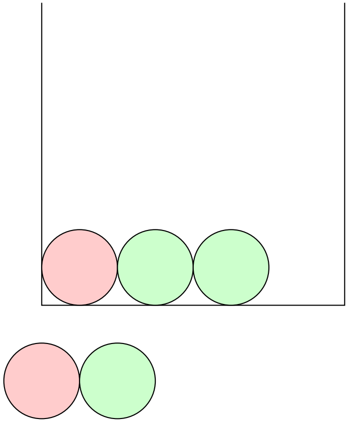
The Pòlya Urn scheme

If a red ball is drawn, record colour and replace with a ball of a new colour:

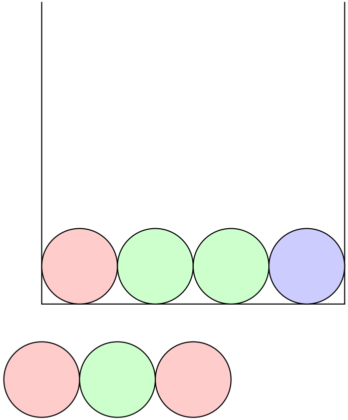


The Pòlya Urn scheme

If a non-red ball is drawn, record colour and replace with a ball of the same colour:



The Pòlya Urn scheme



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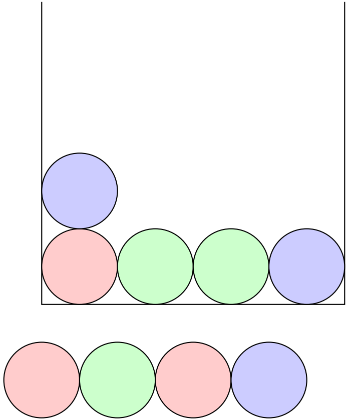

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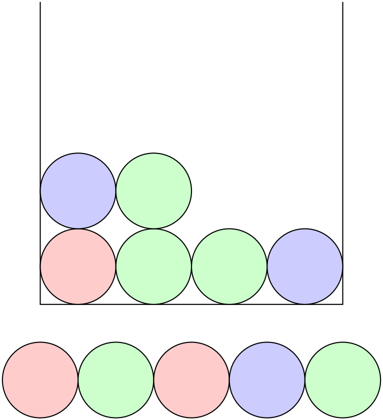
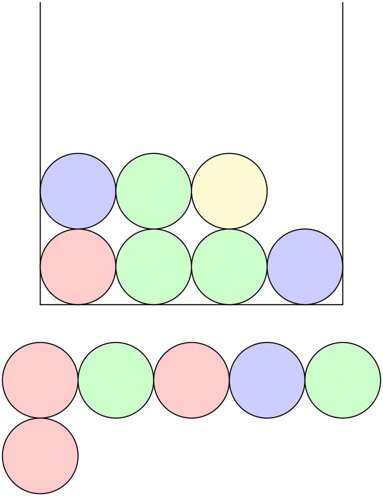


The Pòlya Urn scheme



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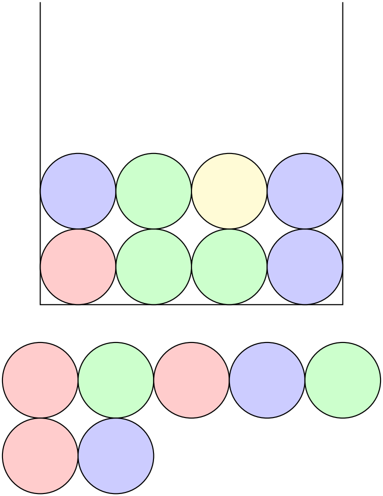

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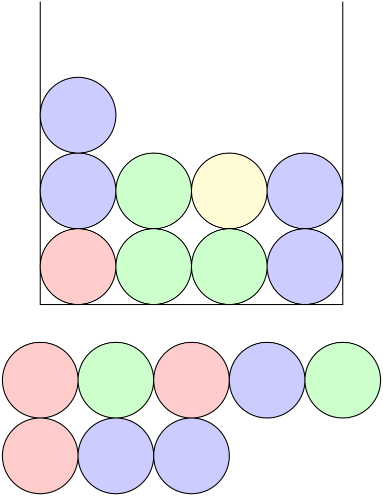

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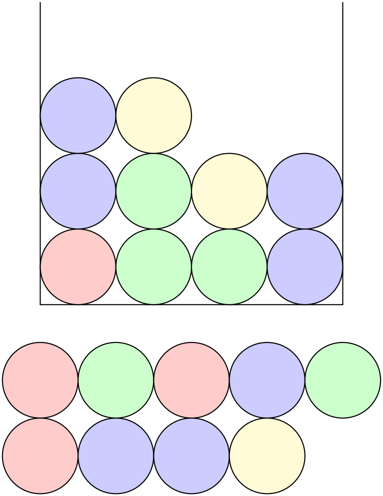
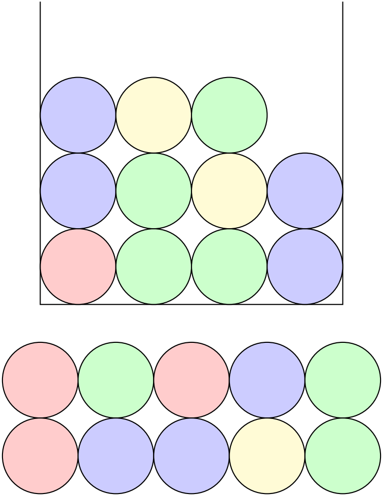


The Pòlya Urn scheme



The Pòlya Urn scheme

The more often a colour is drawn, the more likely it is to appear - induced clustering effect.
G is integrated out of the DP (Blackwell and MacQueen, 1973)

$$
\begin{aligned}
p(y_i|\theta_i) &\sim p(y_i) \\
\theta_1 &\sim G_0 \\
\theta_i|\theta_{i-1},\ldots,\theta_1,\alpha,G_0 &= \sum_{j=1}^{i-1}\frac{\delta_{\theta_j}}{\alpha+i-1}+\frac{\alpha G_0}{\alpha+i-1}
\end{aligned}
$$

The Pòlya Urn scheme

In terms of unobserved cluster memberships, z_1, z_2, etc

$$z_i = k | z_1, \ldots, z_{i-1}, \alpha = \begin{cases} \frac{N_k}{i-1+\alpha} & 1 \leq k \leq K \\ \frac{\alpha}{i-1+\alpha} & k = K+1. \end{cases}$$

For each observation:

- Chance of being assigned to a cluster $\propto$ number of observations already assigned (N_k)
- Always a chance of a new cluster being generated (α)

Dirichlet Process Mixture model: Role of α

α controls the level of variation about G_0

$$G \sim DP(\alpha, G_0)$$

- as $\alpha \to 0$, no clustering
- as $\alpha \to \infty$, singleton clusters

Assume α unknown and assign Gamma prior (Escobar and West, 1995)

$$\alpha \sim G(a, b)$$

Dirichlet Process Mixture model: Estimation

Stick breaking representation

- Blocked Gibbs sampler (Ishwaran and James, 2002)

$$p(\mathrm{y}|\boldsymbol{\nu}) = \prod_{i=1}^{n} \sum_{k=1}^{L} \eta_k p(y_i|\boldsymbol{\theta}_k); \qquad L >> E(K)$$

- Slice sampler (Walker, 2007); "adaptive truncation"

$$\begin{aligned} u_i &\sim \mathcal{U}(0, \eta_{z_i}) \\ \sum_{k \geq 1} \eta_k &> 1 - \min\{u_1, \ldots, u_n\} \end{aligned}$$

- R packages: `PReMiuM`
- see `spike_sorting_DPM.R`

Dirichlet Process Mixture model: Estimation

The Pòlya Urn scheme: collapsed Gibbs sampler
Each z_i is updated sequentially given current values for remaining observations, $\mathrm{z}^{(-i)}$

$$p(z_i = k|\mathrm{z}^{-i}, \mathrm{y}, \xi) = \begin{cases} \frac{N_k^{-i}}{n-1+\alpha} p(y_i|\mathrm{y}^{-i}, \mathrm{z}^{-i} = k, \xi) & 1 \leq k \leq K, \\ \frac{\alpha}{n-1+\alpha} p(y_i|\xi) & k = K+1. \end{cases}$$

$$p(y_i|\mathrm{y}^{-i}, \mathrm{z}^{-i} = k, \xi) = \int p(y_i|\theta, z_i = k) p(\theta|\mathrm{y}^{-i}, \mathrm{z}^{-i} = k, \xi)\, d\theta$$

see R package `dirichletprocess`

Dirichlet Process Mixture model: An example

Subgroup identification in Parkinson's disease based on symptom severity (White et al, 2012)

- Data collected using Unified Parkinson's Disease Rating Scale (UPDRS)
- Symptom scores: Activities of daily living (ADL), Tremor, Rigidity and Akinesia
- Responses coded as Low/Moderate/High Progression based on UPDRS score

Model specification

- For J symptoms, $y_i = (y_{i1}, \ldots, y_{iJ})$ is Multinomial with $R = 3$ levels

$$
\begin{aligned}
p(\mathrm{y}_i|\boldsymbol{\theta}_k &\sim \prod_{j=1}^{J} Mult(y_{ij}; \boldsymbol{\theta}_{jk}) \\
\boldsymbol{\theta}_{jk} &= (\theta_{jk1}, \ldots, \theta_{jkR})
\end{aligned}
$$

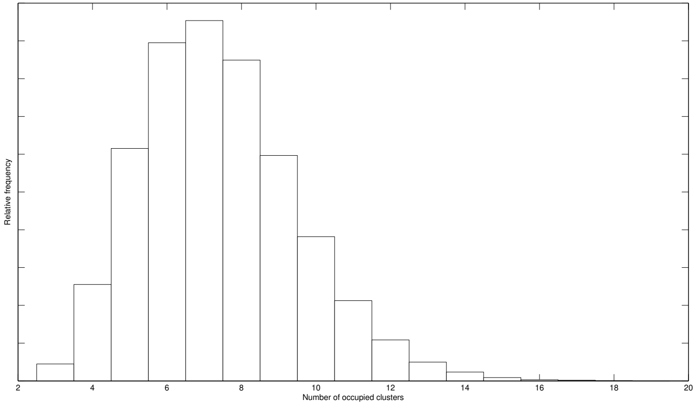
- Base distribution, G_0

$$
\begin{aligned}
G_0 &= p(\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_J) \\
&= \prod_{j=1}^{J} D\left(\frac{1}{R}, \ldots, \frac{1}{R}\right)
\end{aligned}
$$

- Assume stick breaking representation with $\alpha \sim G(2, 2)$

Results - Clustering inference

Blocked Gibbs sampler: 100,000 iterations (50,000 burnin), and $L = 50$



Parameter inference

ADL	Cluster						
	1	2	3	4	5	6	7*
Low	0.09(0.04)	0.16(0.05)	0.85(0.06)	0.46(0.10)	0.62(0.17)	0.11(0.16)	0.17(0.21)
Moderate	0.20(0.06)	0.60(0.07)	0.11(0.05)	0.31(0.09)	0.19(0.14)	0.78(0.21)	0.66(0.27)
High	0.71(0.06)	0.24(0.06)	0.04(0.03)	0.23(0.07)	0.19(0.14)	0.11(0.18)	0.17(0.21)
Tremor	Cluster						
	1	2	3	4	5	6	7*
Low	0.10(0.04)	0.43(0.07)	0.69(0.08)	0.01(0.02)	0.05(0.07)	0.11(0.18)	0.17(0.21)
Moderate	0.26(0.06)	0.24(0.06)	0.30(0.08)	0.88(0.06)	0.62(0.17)	0.11(0.16)	0.66(0.27)
High	0.64(0.07)	0.33(0.07)	0.01(0.02)	0.11(0.06)	0.33(0.17)	0.78(0.21)	0.17(0.21)
Rigidity	Cluster						
	1	2	3	4	5	6	7*
Low	0.01(0.01)	0.52(0.07)	0.59(0.09)	0.06(0.05)	0.90(0.10)	0.44(0.25)	0.66(0.27)
Moderate	0.13(0.05)	0.30(0.06)	0.33(0.08)	0.93(0.05)	0.05(0.07)	0.12(0.16)	0.17(0.21)
High	0.86(0.05)	0.18(0.05)	0.08(0.05)	0.01(0.02)	0.05(0.07)	0.44(0.25)	0.17(0.21)
Akinesia	Cluster						
	1	2	3	4	5	6	7*
Low	0.07(0.04)	0.01(0.01)	0.95(0.04)	0.49(0.10)	0.90(0.10)	0.78(0.21)	0.17(0.21)
Moderate	0.05(0.03)	0.88(0.05)	0.04(0.03)	0.23(0.08)	0.05(0.07)	0.11(0.16)	0.17(0.21)
High	0.88(0.05)	0.11(0.04)	0.01(0.02)	0.28(0.09)	0.05(0.07)	0.11(0.16)	0.66(0.27)
η	0.30(0.04)	0.30(0.04)	0.19(0.03)	0.15(0.03)	0.04(0.01)	0.01(0.01)	0.01(0.01)

Cluster 1 - High severity

ADL	Cluster						
	1	2	3	4	5	6	7*
Low	0.09(0.04)	0.16(0.05)	0.85(0.06)	0.46(0.10)	0.62(0.17)	0.11(0.16)	0.17(0.21)
Moderate	0.20(0.06)	0.60(0.07)	0.11(0.05)	0.31(0.09)	0.19(0.14)	0.78(0.21)	0.66(0.27)
High	**0.71(0.06)**	0.24(0.06)	0.04(0.03)	0.23(0.07)	0.19(0.14)	0.11(0.18)	0.17(0.21)
Tremor	Cluster						
	1	2	3	4	5	6	7*
Low	0.10(0.04)	0.43(0.07)	0.69(0.08)	0.01(0.02)	0.05(0.07)	0.11(0.18)	0.17(0.21)
Moderate	0.26(0.06)	0.24(0.06)	0.30(0.08)	0.88(0.06)	0.62(0.17)	0.11(0.16)	0.66(0.27)
High	**0.64(0.07)**	0.33(0.07)	0.01(0.02)	0.11(0.06)	0.33(0.17)	0.78(0.21)	0.17(0.21)
Rigidity	Cluster						
	1	2	3	4	5	6	7*
Low	0.01(0.01)	0.52(0.07)	0.59(0.09)	0.06(0.05)	0.90(0.10)	0.44(0.25)	0.66(0.27)
Moderate	0.13(0.05)	0.30(0.06)	0.33(0.08)	0.93(0.05)	0.05(0.07)	0.12(0.16)	0.17(0.21)
High	**0.86(0.05)**	0.18(0.05)	0.08(0.05)	0.01(0.02)	0.05(0.07)	0.44(0.25)	0.17(0.21)
Akinesia	Cluster						
	1	2	3	4	5	6	7*
Low	0.07(0.04)	0.01(0.01)	0.95(0.04)	0.49(0.10)	0.90(0.10)	0.78(0.21)	0.17(0.21)
Moderate	0.05(0.03)	0.88(0.05)	0.04(0.03)	0.23(0.08)	0.05(0.07)	0.11(0.16)	0.17(0.21)
High	**0.88(0.05)**	0.11(0.04)	0.01(0.02)	0.28(0.09)	0.05(0.07)	0.11(0.16)	0.66(0.27)
η	**0.30(0.04)**	0.30(0.04)	0.19(0.03)	0.15(0.03)	0.04(0.01)	0.01(0.01)	0.01(0.01)

Cluster 2 - ADL/Akinesia

ADL	Cluster						
	1	2	3	4	5	6	7*
Low	0.09(0.04)	0.16(0.05)	0.85(0.06)	0.46(0.10)	0.62(0.17)	0.11(0.16)	0.17(0.21)
Moderate	0.20(0.06)	**0.60(0.07)**	0.11(0.05)	0.31(0.09)	0.19(0.14)	0.78(0.21)	0.66(0.27)
High	0.71(0.06)	0.24(0.06)	0.04(0.03)	0.23(0.07)	0.19(0.14)	0.11(0.18)	0.17(0.21)
Tremor	Cluster						
	1	2	3	4	5	6	7*
Low	0.10(0.04)	**0.43(0.07)**	0.69(0.08)	0.01(0.02)	0.05(0.07)	0.11(0.18)	0.17(0.21)
Moderate	0.26(0.06)	0.24(0.06)	0.30(0.08)	0.88(0.06)	0.62(0.17)	0.11(0.16)	0.66(0.27)
High	0.64(0.07)	0.33(0.07)	0.01(0.02)	0.11(0.06)	0.33(0.17)	0.78(0.21)	0.17(0.21)
Rigidity	Cluster						
	1	2	3	4	5	6	7*
Low	0.01(0.01)	**0.52(0.07)**	0.59(0.09)	0.06(0.05)	0.90(0.10)	0.44(0.25)	0.66(0.27)
Moderate	0.13(0.05)	0.30(0.06)	0.33(0.08)	0.93(0.05)	0.05(0.07)	0.12(0.16)	0.17(0.21)
High	0.86(0.05)	0.18(0.05)	0.08(0.05)	0.01(0.02)	0.05(0.07)	0.44(0.25)	0.17(0.21)
Akinesia	Cluster						
	1	2	3	4	5	6	7*
Low	0.07(0.04)	0.01(0.01)	0.95(0.04)	0.49(0.10)	0.90(0.10)	0.78(0.21)	0.17(0.21)
Moderate	0.05(0.03)	**0.88(0.05)**	0.04(0.03)	0.23(0.08)	0.05(0.07)	0.11(0.16)	0.17(0.21)
High	0.88(0.05)	0.11(0.04)	0.01(0.02)	0.28(0.09)	0.05(0.07)	0.11(0.16)	0.66(0.27)
η	0.30(0.04)	**0.30(0.04)**	0.19(0.03)	0.15(0.03)	0.04(0.01)	0.01(0.01)	0.01(0.01)

Cluster 3 - Low severity

ADL	Cluster						
	1	2	3	4	5	6	7*
Low	0.09(0.04)	0.16(0.05)	**0.85(0.06)**	0.46(0.10)	0.62(0.17)	0.11(0.16)	0.17(0.21)
Moderate	0.20(0.06)	0.60(0.07)	0.11(0.05)	0.31(0.09)	0.19(0.14)	0.78(0.21)	0.66(0.27)
High	0.71(0.06)	0.24(0.06)	0.04(0.03)	0.23(0.07)	0.19(0.14)	0.11(0.18)	0.17(0.21)
Tremor	Cluster						
	1	2	3	4	5	6	7*
Low	0.10(0.04)	0.43(0.07)	**0.69(0.08)**	0.01(0.02)	0.05(0.07)	0.11(0.18)	0.17(0.21)
Moderate	0.26(0.06)	0.24(0.06)	0.30(0.08)	0.88(0.06)	0.62(0.17)	0.11(0.16)	0.66(0.27)
High	0.64(0.07)	0.33(0.07)	0.01(0.02)	0.11(0.06)	0.33(0.17)	0.78(0.21)	0.17(0.21)
Rigidity	Cluster						
	1	2	3	4	5	6	7*
Low	0.01(0.01)	0.52(0.07)	**0.59(0.09)**	0.06(0.05)	0.90(0.10)	0.44(0.25)	0.66(0.27)
Moderate	0.13(0.05)	0.30(0.06)	0.33(0.08)	0.93(0.05)	0.05(0.07)	0.12(0.16)	0.17(0.21)
High	0.86(0.05)	0.18(0.05)	0.08(0.05)	0.01(0.02)	0.05(0.07)	0.44(0.25)	0.17(0.21)
Akinesia	Cluster						
	1	2	3	4	5	6	7*
Low	0.07(0.04)	0.01(0.01)	**0.95(0.04)**	0.49(0.10)	0.90(0.10)	0.78(0.21)	0.17(0.21)
Moderate	0.05(0.03)	0.88(0.05)	0.04(0.03)	0.23(0.08)	0.05(0.07)	0.11(0.16)	0.17(0.21)
High	0.88(0.05)	0.11(0.04)	0.01(0.02)	0.28(0.09)	0.05(0.07)	0.11(0.16)	0.66(0.27)
η	0.30(0.04)	0.30(0.04)	**0.19(0.03)**	0.15(0.03)	0.04(0.01)	0.01(0.01)	0.01(0.01)

Cluster 4 - Tremor/Rigidity

ADL	Cluster						
	1	2	3	4	5	6	7*
Low	0.09(0.04)	0.16(0.05)	0.85(0.06)	**0.46(0.10)**	0.62(0.17)	0.11(0.16)	0.17(0.21)
Moderate	0.20(0.06)	0.60(0.07)	0.11(0.05)	0.31(0.09)	0.19(0.14)	0.78(0.21)	0.66(0.27)
High	0.71(0.06)	0.24(0.06)	0.04(0.03)	0.23(0.07)	0.19(0.14)	0.11(0.18)	0.17(0.21)
Tremor	**Cluster**						
	1	2	3	4	5	6	7*
Low	0.10(0.04)	0.43(0.07)	0.69(0.08)	0.01(0.02)	0.05(0.07)	0.11(0.18)	0.17(0.21)
Moderate	0.26(0.06)	0.24(0.06)	0.30(0.08)	**0.88(0.06)**	0.62(0.17)	0.11(0.16)	0.66(0.27)
High	0.64(0.07)	0.33(0.07)	0.01(0.02)	0.11(0.06)	0.33(0.17)	0.78(0.21)	0.17(0.21)
Rigidity	**Cluster**						
	1	2	3	4	5	6	7*
Low	0.01(0.01)	0.52(0.07)	0.59(0.09)	0.06(0.05)	0.90(0.10)	0.44(0.25)	0.66(0.27)
Moderate	0.13(0.05)	0.30(0.06)	0.33(0.08)	**0.93(0.05)**	0.05(0.07)	0.12(0.16)	0.17(0.21)
High	0.86(0.05)	0.18(0.05)	0.08(0.05)	0.01(0.02)	0.05(0.07)	0.44(0.25)	0.17(0.21)
Akinesia	**Cluster**						
	1	2	3	4	5	6	7*
Low	0.07(0.04)	0.01(0.01)	0.95(0.04)	**0.49(0.10)**	0.90(0.10)	0.78(0.21)	0.17(0.21)
Moderate	0.05(0.03)	0.88(0.05)	0.04(0.03)	0.23(0.08)	0.05(0.07)	0.11(0.16)	0.17(0.21)
High	0.88(0.05)	0.11(0.04)	0.01(0.02)	0.28(0.09)	0.05(0.07)	0.11(0.16)	0.66(0.27)
η	0.30(0.04)	0.30(0.04)	0.19(0.03)	**0.15(0.03)**	0.04(0.01)	0.01(0.01)	0.01(0.01)

Cluster 5 to 7 - small weights

ADL	Cluster						
	1	2	3	4	5	6	7*
Low	0.09(0.04)	0.16(0.05)	0.85(0.06)	0.46(0.10)	0.62(0.17)	0.11(0.16)	0.17(0.21)
Moderate	0.20(0.06)	0.60(0.07)	0.11(0.05)	0.31(0.09)	0.19(0.14)	0.78(0.21)	0.66(0.27)
High	0.71(0.06)	0.24(0.06)	0.04(0.03)	0.23(0.07)	0.19(0.14)	0.11(0.18)	0.17(0.21)
Tremor	Cluster						
	1	2	3	4	5	6	7*
Low	0.10(0.04)	0.43(0.07)	0.69(0.08)	0.01(0.02)	0.05(0.07)	0.11(0.18)	0.17(0.21)
Moderate	0.26(0.06)	0.24(0.06)	0.30(0.08)	0.88(0.06)	0.62(0.17)	0.11(0.16)	0.66(0.27)
High	0.64(0.07)	0.33(0.07)	0.01(0.02)	0.11(0.06)	0.33(0.17)	0.78(0.21)	0.17(0.21)
Rigidity	Cluster						
	1	2	3	4	5	6	7*
Low	0.01(0.01)	0.52(0.07)	0.59(0.09)	0.06(0.05)	0.90(0.10)	0.44(0.25)	0.66(0.27)
Moderate	0.13(0.05)	0.30(0.06)	0.33(0.08)	0.93(0.05)	0.05(0.07)	0.12(0.16)	0.17(0.21)
High	0.86(0.05)	0.18(0.05)	0.08(0.05)	0.01(0.02)	0.05(0.07)	0.44(0.25)	0.17(0.21)
Akinesia	1	2	3	4	5	6	7
Low	0.07(0.04)	0.01(0.01)	0.95(0.04)	0.49(0.10)	0.90(0.10)	0.78(0.21)	0.17(0.21)
Moderate	0.05(0.03)	0.88(0.05)	0.04(0.03)	0.23(0.08)	0.05(0.07)	0.11(0.16)	0.17(0.21)
High	0.88(0.05)	0.11(0.04)	0.01(0.02)	0.28(0.09)	0.05(0.07)	0.11(0.16)	0.66(0.27)
η	0.30(0.04)	0.30(0.04)	0.19(0.03)	0.15(0.03)	**0.04(0.01)**	**0.01(0.01)**	**0.01(0.01)**

Comparison with finite mixture modelling

- DIC provided support for a $K = 4$

K	3	4	5
DIC	1248.7	1241.1	1247.9

- Identified subgroups had same characteristics as DPM Clusters 1-4
- DPM versus FMM: 1 model versus comparison of many models

1. Finite mixture models
2. Dirichlet Process Mixture models
3. Profile regression

Profile regression: Motivation

- Extends Dirichlet Processes to account for joint clustering in outcomes and covariates
- Developed as a solution for dealing with multi-collinearity within standard regression framework

Profile regression: Setup

Assignment submodel:

$$p\left(\mathrm{x}_i|\boldsymbol{\xi}\right) = \sum_{k\geq 1} \eta_k \prod_{j=1}^{J} p(x_{ij}|\phi_k)$$

Outcome submodel:

$$p\left(y_i|\boldsymbol{\theta}, z_i = k\right) = f\left(y_i|\boldsymbol{\theta}_k\right)$$

Profile regression: An example

Assessing falls in early stage Parkinson's disease (Sarini, 2018)
Assignment submodel: Disease severity profile

$$p(x_i|z_i = k) = \prod_{j=1}^{J} \mathcal{N}(x_{ij}|\mu_k, \sigma_k^2)$$

Outcome submodel: Poisson distribution

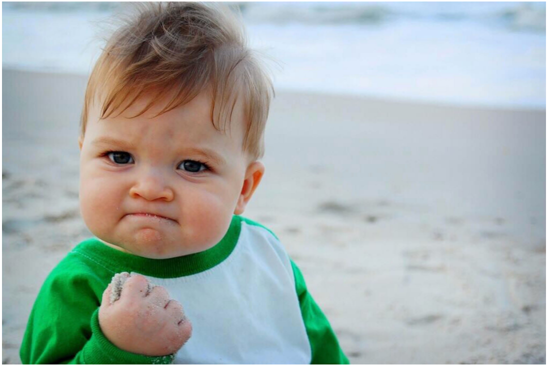
$$p(y_i|z_i = k) = Po\left(\theta_k\right))$$

Profile regression: The PReMiuM package

https://cran.r-project.org/web/packages/PReMiuM/index.html

- Outcome submodel: Continuous (Normal), binary/categorical (Multinomial), count (Poisson)
- Assignment submodel: Continuous (Normal), binary/categorical (Multinomial)
- Estimation based on stick-breaking representation
- Simulated datasets available as part of package

Done!



https://github.com/nicolemwhite/anzsc-mixture-modelling

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