Table of Contents


```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%             Model Predictive Control - Exercise 5
%               EPFL - Spring semester 2017 -
%
%             Huber Lukas - Zgraggen Jannik
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear variables;
addpath(genpath('../tbxmanager'))
```

System initialization

```
A = [0.7115, -0.4345; 0.4345, 0.8853];
B = [0.2173; 0.0573];

C = [0, 1];

% Augmented system
B_d = zeros(2,1);
C_d = [1];

A_augm = [A,B_d;zeros(1,2),1];
B_augm = [B;0];
C_augm = [C, 1];


% Make sure that eigenvalues of (A+LC) are in unit circle
L = (place(A_augm',-C_augm',[0.5,0.6,0.7]))';

% Initial Estimation
x0_est = [3;0];
d0_est = [0];

%Initial Conditions - Real system
x0_r = [1;2];
d_r = 0.2;
```

Exercise 1 - Observer Design

```
% Error between real system and estimation
```

```
deltaX = [x0_r-x0_est];
deltaD = [d_r-d0_est];

obsError = [deltaX; deltaD];


% Rund the integral disturbance dynamics

MAXITER = 50; minTol = 1e-2;

for i = 2:MAXITER
    obsError(:,i) = (A_augm + L *C_augm)*obsError(:,i-1);
    if(norm(obsError(i)) < minTol)
        fprintf('Problem converged after iteration %d \n',i);
        break;
    end
end

% Plot the results

figure('Position',[0 0 1000 600]); grid on;
plot(sqrt(sum(obsError(1:2,:).^2,1))); hold on;
plot(obsError(3,:)); grid on;
legend('Error x', 'Error d')
xlabel('Step [s]'); ylabel('Error')
title('Estimator error');


% Estimation converges very nicely towards the real value. The error
 in x
% and d converges below the minimum Tolerance in less than 100
 iterations.

% Initialize vectores
xVal_est = [x0_est]; xVal_r = [x0_r];
dVal_est = [d0_est]; %dVal_r = [d_r];


% Define loop
MAXITER= MAXITER; minTol = 1e-2;

for i = 2:MAXITER
    u = 0; % No control or input
    y = C*xVal_r(:,i-1)+d_r;

    xVal_r(:,i) = A*xVal_r(:,i-1) + B*u;

    dh_hat2 = A_augm*[xVal_est(:,i-1); dVal_est(i-1)]+B_augm*u ...
            +  L*(C*xVal_est(:,i-1)+C_d*dVal_est(i-1)-y);
    xVal_est(:,i) = dh_hat2(1:2);
    dVal_est(i) = dh_hat2(3);

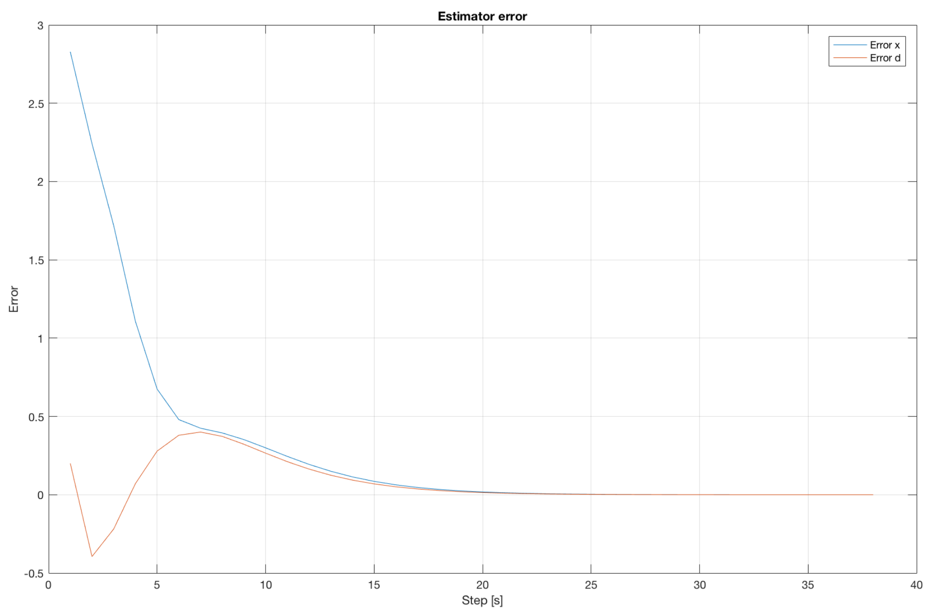
    if(and(norm(xVal_est(i)-xVal_r(i)) < minTol, ...
            norm(xVal_r(:,i)-xVal_r(:,i-1)) < minTol))
```

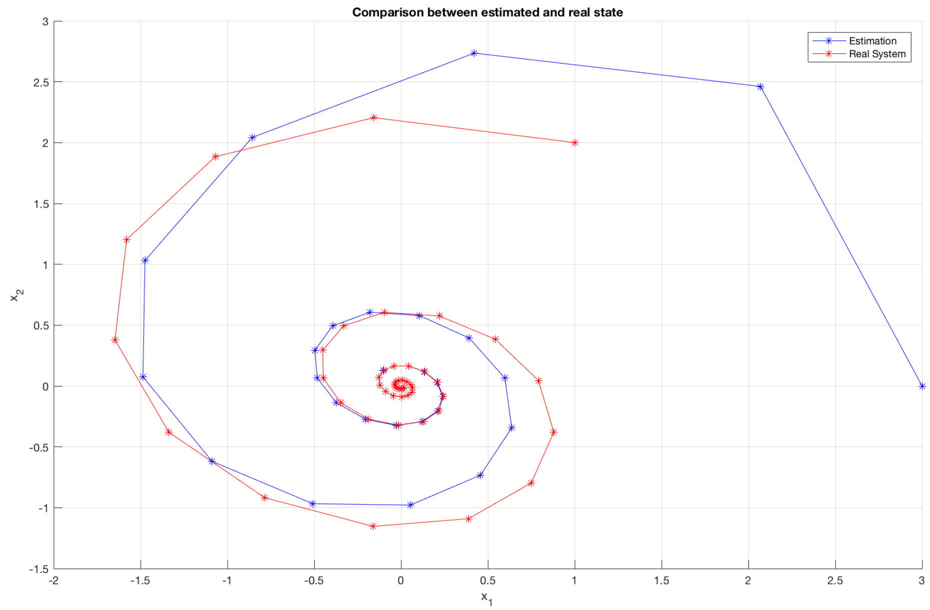
```
        fprintf('Problem converged after iteration %d \n',i);
        break;
    end
end

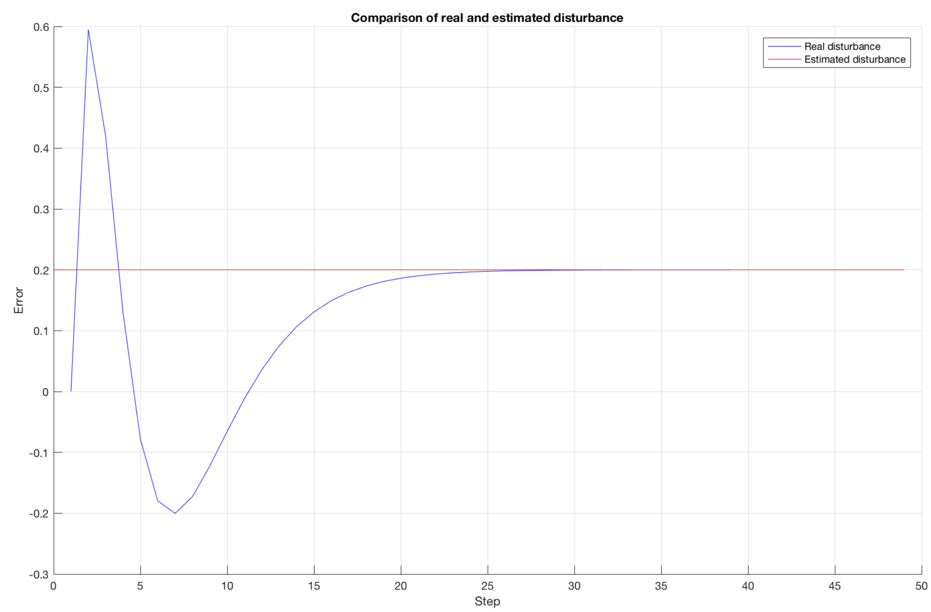

figure('Position',[0 0 1000 600]); hold on; grid on;
plot(xVal_est(1,:), xVal_est(2,:),'b-*')
plot(xVal_r(1,:),xVal_r(2,:), 'r-*')
xlabel('x_1'); ylabel('x_2')
legend('Estimation','Real System')
title('Comparison between estimated and real state');

figure('Position',[0 0 1000 600]); hold on; grid on;
plot(dVal_est,'b'); plot([0, length(dVal_est)], [d_r, d_r], 'r')
xlabel('Step'); ylabel('Error')
legend('Real disturbance', 'Estimated disturbance')
title('Comparison of real and estimated disturbance');
```

Problem converged after iteration 38
Problem converged after iteration 49







Exercise 2 & 3 - Controller Design

```
N =5; % Horizon length

% Define optimization variables
x = sdpvar(2,N,'full');
u = sdpvar(1,N,'full');

% Constraints
```

```
h = [3; 3];              %Input constraint
H = [1; -1];             %Input constraint

% Stage cost
%Weights Controller that is able to track constant output reference
Q = eye(size(A,1));
R = 1;
I=eye(2);


% Weight of final cost
P = dlyap(A,Q);

% Solver settings
opt = sdpsettings;
opt.solver = 'quadprog';
opt.quadprog.TolCon = 1e-16;


% Initial conditions
xi = x0_est; % try to controll estimation
xd_est = [x0_est; d0_est]
y_est = [C*xd_est(1:2,1)+d0_est];
r_val = [0.5, 1];

for r = r_val
    % Real conditions
    y_r = [0];
    u_r = [0];

    x_r = [x0_r];
    y_r = [C*xi+d_r];

    t = [0];

    tolX = 1e-8;
    % Can now compute the optimal control input using
    for i = 2:MAXITER

        % Exercise 2 - Optimize u^2 (Target tracking)
        x_s = sdpvar(2,1,'full');
        u_s = sdpvar(1,1,'full');
        obj_ss = u_s*R*u_s;
        con_ss =[I-A,-B;C,0]*[x_s;u_s] == [0;0;r-C_d*xd_est(3,i-1)]; %
 System dynamics
        con_ss = [con_ss, H*u_s <= h];                        % Input
 constraint
        solvesdp(con_ss, obj_ss, opt);
        x_s=double(x_s);
        u_s=double(u_s);

        % Define constraints and objective for MPC-controller
        con = [];
        obj = 0;
```

```
        %con = [con, x(:,1) == x0];
        for j = 1:N-1
            obj = obj + (x(:,j)-x_s)'*Q*(x(:,j)-x_s) + (u(:,j)-
u_s)'*R*(u(:,j)-u_s); % Cost function
            con = [con, x(:,j+1) == A*x(:,j) + B*u(:,j)]; % System
 dynamics
            con = [con, H*u(:,j) <= h];                   % Input
 constraints
        end
        obj = obj + (x(:,N)-x_s)'*P*(x(:,N)-x_s);      % Terminal
 weight
        ctrl = optimizer(con, obj, opt, x(:,1), u(:,1));
        [u_opt,infeasible] = ctrl{xi};

        if(infeasible); fprintf('Problem infeasible at i=%d
 \n',i); break; end;

        u_r(i) = u_opt;
        t(i) = i;

        % Real sytem
        x_r(:,i) = A*x_r(:,i-1) + B*u_opt;
        y_r(i) = C*x_r(:,i)+d_r;

        % Estimated system
        xd_est(:,i) = [A_augm*xd_est(:,i-1)+B_augm*u_opt ...
                      + L*(C*xd_est(1:2,i-1) + C_d*xd_est(3,i-1) -
 y_r(i-1))];

        xi = xd_est(1:2,i);

        if(norm(abs(y_r(i)-r)) < tolX);
            fprintf('System converged at after %d steps. \n',i);
            break
        end

    end

    % Plot results
    figure('Position',[0 0 1000 600]);
    plot(xd_est(1,:),xd_est(2,:),'b-*');
    grid on; hold on;
    plot(x_r(1,:),x_r(2,:),'r-*');
    legend('Estimation','Real System')
    xlabel('x_1'), ylabel('x_2')
    title(['Controller performance for r=',num2str(r)]);


    figure('Position',[0 0 1000 600]); grid on;
    plot(0:length(u_r)-1, u_r,'b'); hold on; grid on;
    plot(0:length(u_r)-1,y_r,'r');
    plot([0,length(u_r)-1],[r,r],'r--')
    plot([0,length(u_r)-1],[-3,-3],'k--')
    plot([0,length(u_r)-1],[3,3],'k--')
```

```
    xlim([0,length(u_r)-1])
    ylim([-3.1,3.1])
    title(['Controller performance for r=',num2str(r)]);

    legend('u(t)','y(t)','reference','input constraints')
    xlabel('Steps')


end
```

```
xd_est =

     3
     0
     0


Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-
decreasing in
feasible directions, to within the default value of the optimality
 tolerance,
and constraints are satisfied to within the selected value of the
 constraint tolerance.

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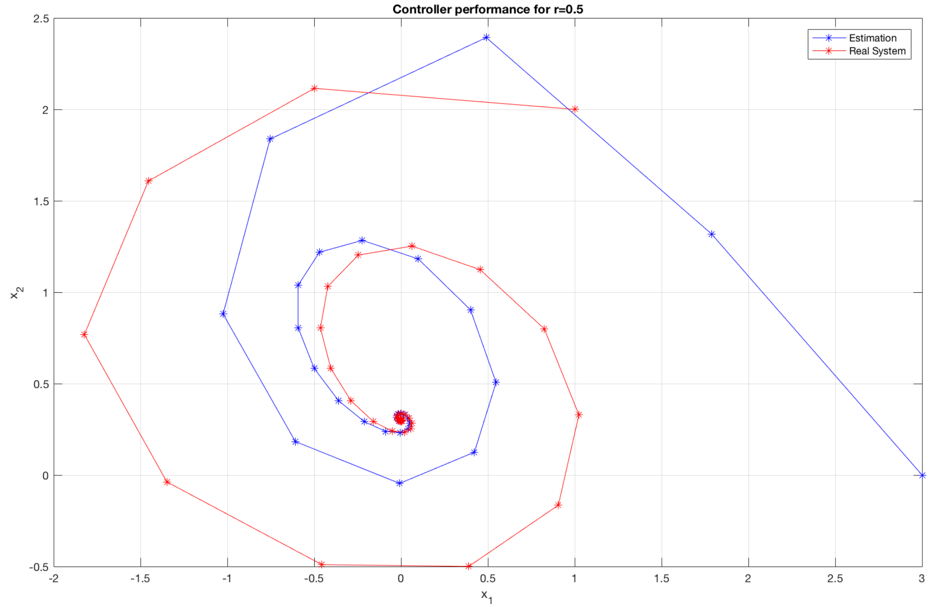

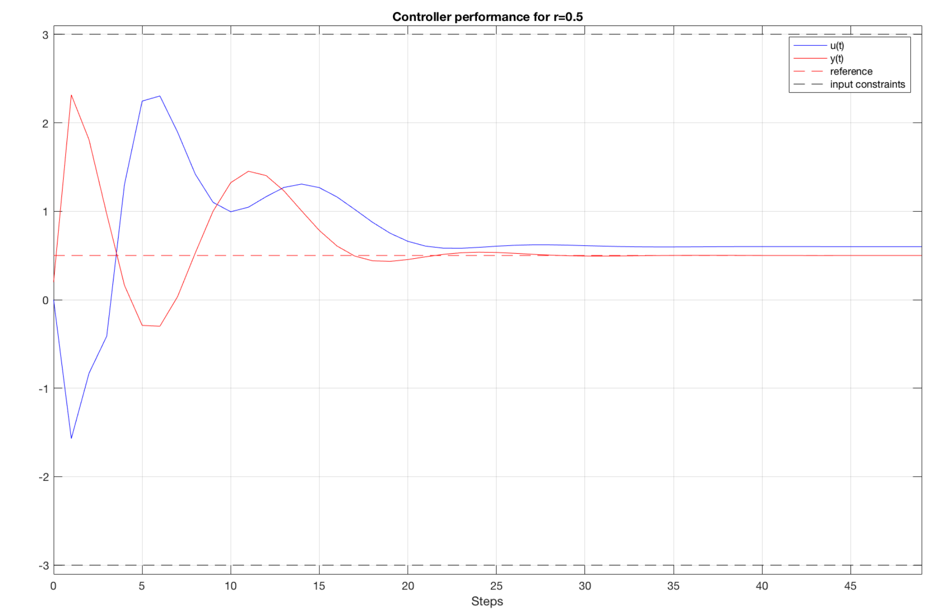

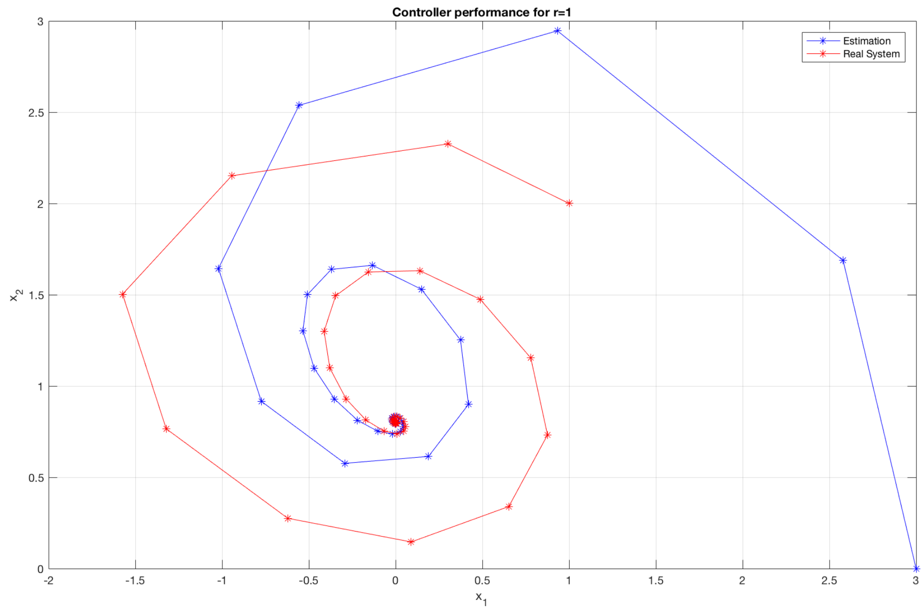

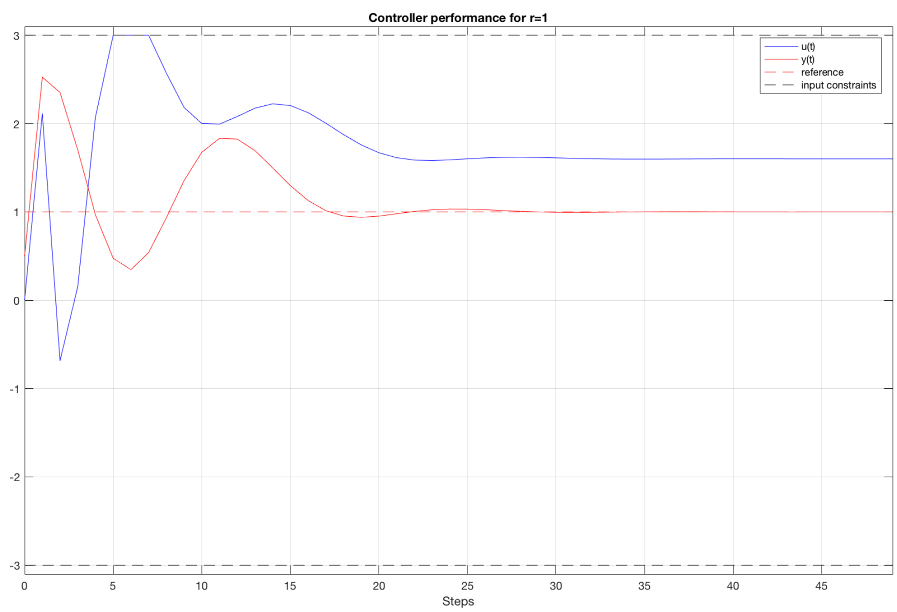


```
fprintf('Programm terminated. \n')
```

Programm terminated.

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