Model Predictive Control — ME-425

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Exercise sheet 5
Tracking MPC

Consider the discrete-time system

$$x_{k+1} = \begin{pmatrix} 0.7115 & -0.4345 \\ 0.4345 & 0.8853 \end{pmatrix} x_k + \begin{pmatrix} 0.2173 \\ 0.0573 \end{pmatrix} u_k \tag{1}$$
$$y_k = \begin{pmatrix} 0 & 1 \end{pmatrix} x_k + d$$

where d is an *unknown* constant disturbance and x_0 is unknown. The goal of this exercise sheet is to design a controller able to track a constant output reference while fulfilling input constraints

$$-3 \leq u_k \leq 3 \ .$$

Exercise 1 (Observer design)

Since state and disturbances are unknown at time zero, we need to design an observer to estimate them. We call $\hat{x}$ and $\hat{d}$ the estimate of x and d, respectively. Design an observer for the given system, and test it for the condition $x_0 = [1; 2]$, $\hat{x}_0 = [3; 0]$, $\hat{d}_0 = 0$, $d = 0.2$ and $u = 0$.

Hints :

- Similarly to the previous exercise sheets, you can use YALMIP to implement the MPC controller
- To estimate the disturbance you will have to augment the state as seen in class
- Note the eigenvalious of $(A + LC)$ are the same of $(A' + C'L')$
- The matlab function `K = place(A,B,F)` computes a state-feedback matrix K such that the eigenvalues of $A - BK$ are those specified in the vector F

Deliverables :

- Matlab code computing the observer
- A plot showing that the estimates converge to the true values

Exercise 2 (Steady-state target computation)

Given the system above, and a reference r, use YALMIP to compute a steady state for the system that minimizes u^2.

Exercise 3 (MPC tracking)

Implement an MPC controller to track an output reference signal r.

Hints :

- Use a terminal set of $\mathcal{X}_f = \mathbb{R}^n$ and a terminal cost of $V_f(x_N) = \Delta x_N' P \Delta x_N$ where P is the solution of $P - A'PA = Q$
  These correspond to a terminal controller of $u = 0$. Note that this is a valid terminal controller because the system is stable.
  You can use the function `P = dlyap(A,Q)` to compute P.
- Good values for the horizon and stage costs are: $N = 5$, $Q = I$, $R = 1$
- In the previous exercise you designed an observer specifying the eigenvalues of the estimation error state-update matrix. Eigenvalues with a small norm will speed up the estimation process, but may increase the initial overshoot of the estimate $\hat{d}$. A large $\hat{d}$ can cause the problem of computing the set-point to be infeasible. Use moderate eigenvalues (e.g. $0.5, 0.6, 0.7$).

Deliverables :

- Matlab code computing your controller
- Plots showing for the references $r = 1$ and $r = 0.5$:
  - the estimates converge to the true values
  - the output converges to the reference
  - the input does not violate the constraints