Table of Contents

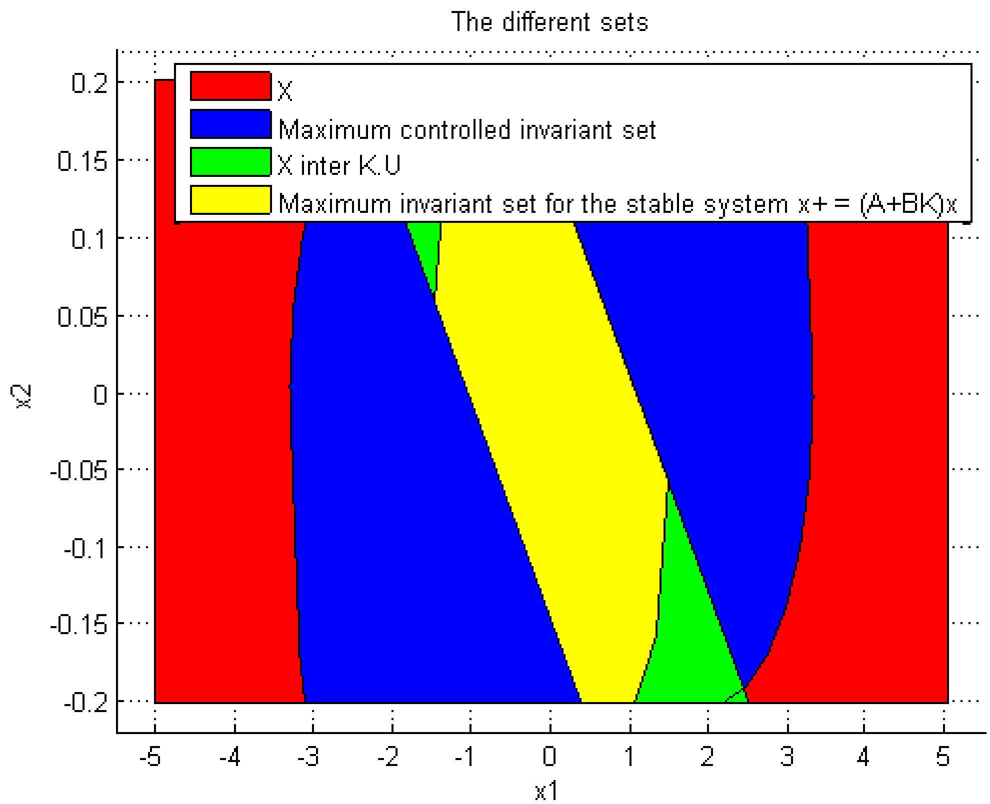

```
clear all;
close all;
clc;
warning('off'); % in order to avoid warnings in the report / it does not
 % the working of the code
```

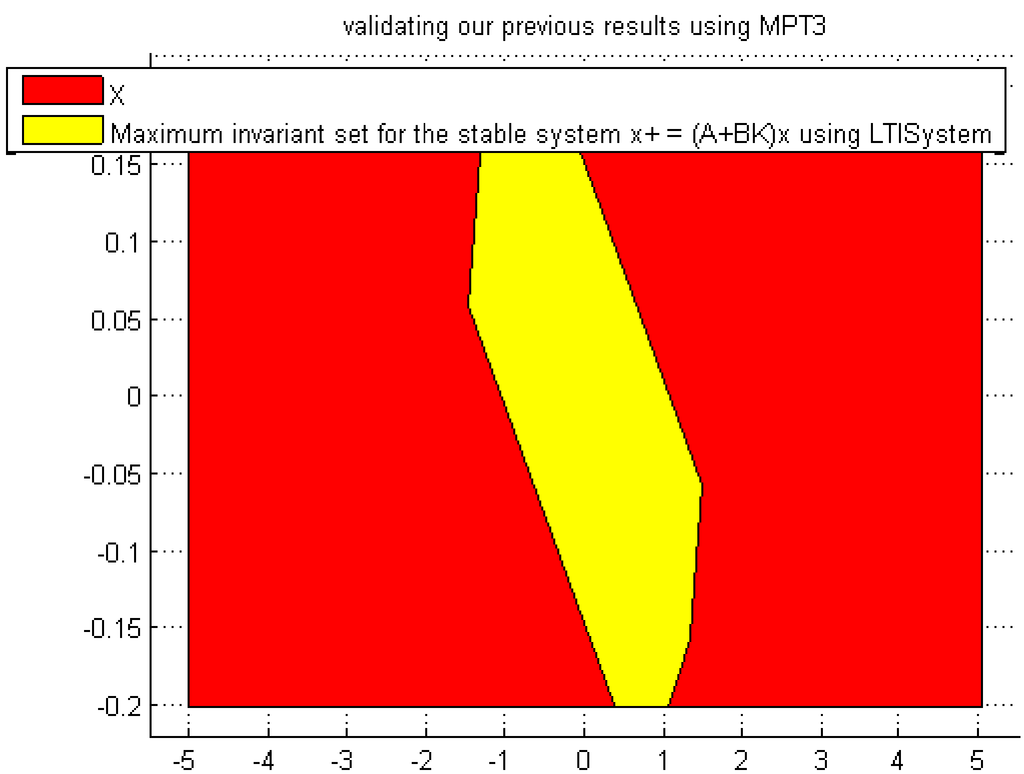
Question 1.1

```
A = [ 0.9752  1.4544;
     -0.0327  0.9315];
B = [0.0248;
     0.0327];
H = [ 1  0;
      -1  0;
       0  1;
       0 -1] ;
h = [5; 5; 0.2; 0.2];
X = Polyhedron(H, h);
hu = [1.75; 1.75];
Hu = [1; -1];

% Computing of the largest controlled invariant set of the constrained system
C = X;
while 1
    C_prev=C;
    preS = projection(Polyhedron([C.A*A C.A*B; zeros(2,2) Hu],...
        [C.b; hu]), (1:2));
    C = Polyhedron([preS.A;C.A],[preS.b;C.b]);
    if C == C_prev
        break
    end;
end;

% Computing the infinite-horizon LQR controller
R = 1;
Q = 10*eye(2);
[K,~,~] = dlqr(A,B,Q,R,zeros(size(B)));
K=-K; % In our case, u=Kx and not u=-Kx

% Computing the maximum invariant set for the stable system x+ = (A+BK)x
newA = A + B*K;
newX = Polyhedron([H;Hu*K], [h; hu]);
```

```
omega = newX;
while 1
    omega_prev=omega;
    omega = Polyhedron([omega.A*newA;omega.A],[omega.b;omega.b]);
    if omega == omega_prev
        break
    end;
end;

% Plotting the feasible and of largest controlled invariant set of the constrained
figure;
xlabel('x1');
ylabel('x2');
title('The different sets');
hold on;
plot(X, 'color', 'r');
plot(C, 'color', 'b');
plot(newX, 'color', 'g');
plot(omega, 'color', 'y');
legend('X','Maximum controlled invariant set','X inter K.U','Maximum invariant set

% Computing of the largest controlled invariant set using LTISystem
sys = LTISystem('A',A,'B',B);
sys.x.max = [5; 0.2];
sys.x.min = [-5; -0.2];
sys.u.max = [1.75];
sys.u.min = [-1.75];
sys.x.penalty = QuadFunction(Q);
sys.u.penalty = QuadFunction(R);
sys.LQRGain
sys.LQRPenalty.weight
figure();
hold on;
plot(X,'color','r');
plot(sys.LQRSet,'color','y');
title('validating our previous results using MPT3');
legend('X','Maximum invariant set for the stable system x+ = (A+BK)x using LTISyst
```

```
ans =

   -1.6478  -11.8344


ans =

   37.0252   68.3850
   68.3850  407.1177

Iteration 1...
Iteration 2...
Iteration 3...
```


The different sets
X
Maximum controlled invariant set
X inter K.U
Maximum invariant set for the stable system x+ = (A+BK)x
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
x2
-5
-4
-3
-2
-1
0
1
2
3
4
5
x1


validating our previous results using MPT3
X
Maximum invariant set for the stable system x+ = (A+BK)x using LTISystem
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-5
-4
-3
-2
-1
0
1
2
3
4
5

Question 1.2: Opimize using quadprog

```
% defining all the parameters
N = 10; % the number of horizons
x0 = [3; 0]; % the initial point
A = [ 0.9752  1.4544;
     -0.0327  0.9315];
B = [0.0248;
     0.0327];

F = [ 1  0;
     -1  0;
      0  1;
      0 -1];
f = [5; 5; 0.2; 0.2];
m = [1.75; 1.75];
M = [1; -1];
R = 1;
Q = 10*eye(2);

%Defining the different matrices needed for running quadprog
Ff=F;
ff=f;
G = blkdiag(kron(eye(N-1),F), Ff, kron(eye(N),M));
g = [repmat(f,N-1,1);
    ff;
    repmat(m,N,1)];
Qf = 10*Q;
H = blkdiag(kron(eye(N-1),Q), Qf, kron(eye(N),R));
h = [zeros(2*(N-1),1);
     zeros(2,1);
     zeros(N,1)];
T1 = [zeros(2,2*(N-1))  zeros(2)
      kron(eye(N-1),A)  zeros(2*(N-1),2)];
T1 = T1 + kron(eye(N),eye(2));
T2 = kron(eye(N),-B);
T  = [T1,T2];

x = zeros(2,100);
x(:,1) = x0;
for i=2:100
    t  = [A*x(:,i-1);
      zeros(2*(N-1),1)];
    [zopt, fval, flag] = quadprog(H, h, G, g, T, t);
    u = zopt((N*size(A,2) + 1):(N*size(A,2) + size(B,2)));
    if flag==1
        x(:,i) = A*x(:,i-1)+B*u;
    else
        break
    end
end

% Plotting
```

```
figure;
xlabel('x1');
ylabel('x2');
title('Optimal trajectory using quadprog');
hold on;
plot(X, 'color', 'r');
plot(C, 'color', 'b');
plot(newX, 'color', 'g');
plot(omega, 'color', 'y');
plot(x(1,:),x(2,:), 'color','k');
legend('X','Maximum controlled invariant set','X inter K.U','Maximum invariant set
```

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Optimization terminated.

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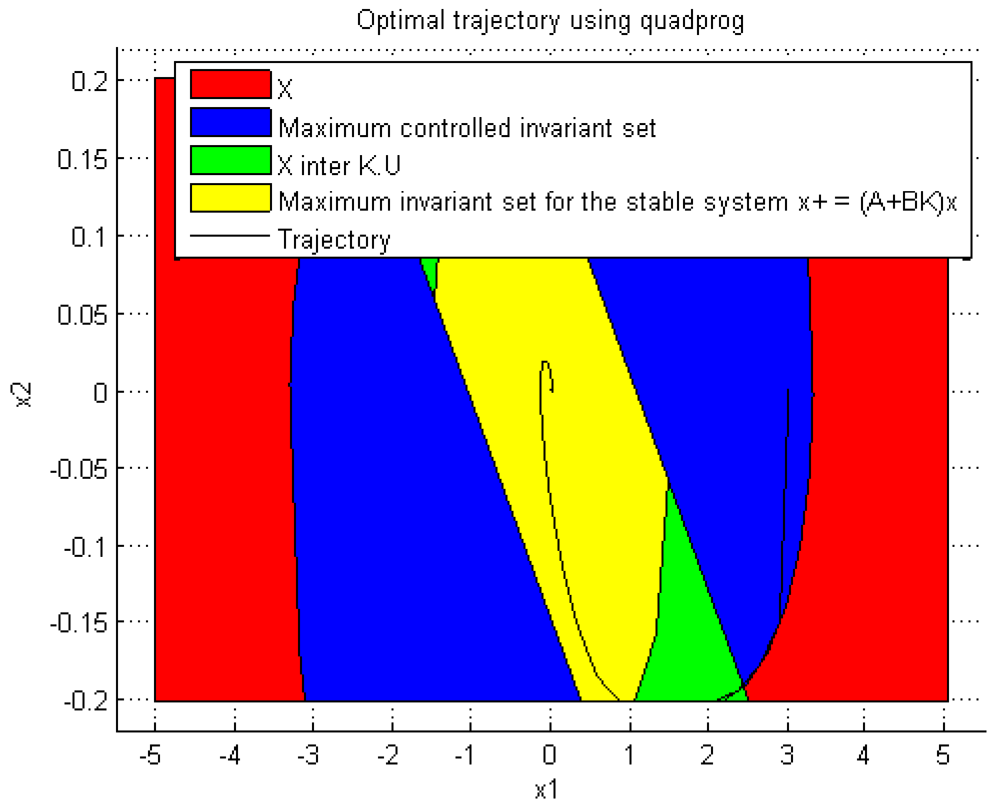


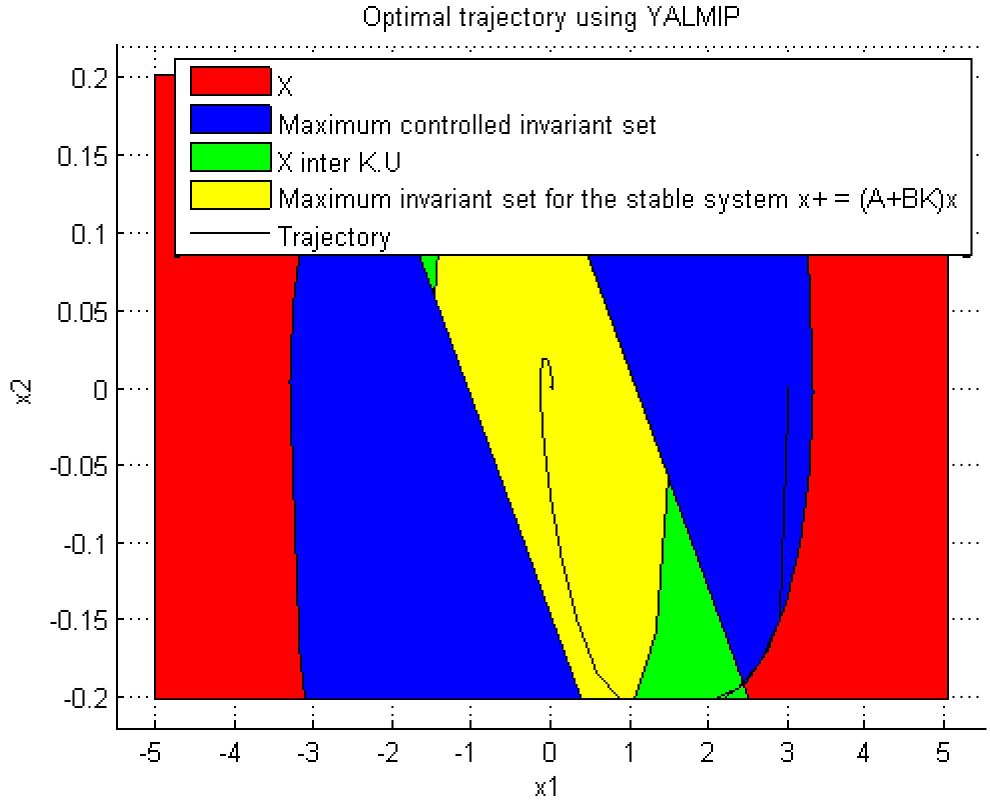
Question 2: Opimize using Yalmip

```
% defining all the parameters
N = 10;
x0 = [3; 0];
A = [ 0.9752  1.4544;
     -0.0327  0.9315];
B = [0.0248;
     0.0327];
F = [ 1  0;
     -1  0;
      0  1;
      0 -1];
f = [5; 5; 0.2; 0.2];
m = [1.75; 1.75];
M = [1; -1];
R = 1;
Q = 10*eye(2);
Ff=F;
ff=f;
Qf = 10*Q;
```

```
x = zeros(2,100);
x(:,1) = x0;

% Define optimization variables
z = sdpvar(2,N,'full');
u = sdpvar(1,N-1,'full');
X0 = sdpvar(2,1,'full');
% Define constraints and objective
con = [];
obj = 0;
con = [con, z(:,1) == X0];
for i = 1:N-1
    con = [con, z(:,i+1) == A*z(:,i) + B*u(:,i)]; % System dynamics
    con = [con, F*z(:,i) <= f]; % State constraints
    con = [con, M*u(:,i) <= m]; % Input constraints
    obj = obj + z(:,i)'*Q*z(:,i) + u(:,i)'*R*u(:,i); % Cost function
end
con = [con, Ff*z(:,N) <= ff]; % Terminal constraint
obj = obj + z(:,N)'*Qf*z(:,N); % Terminal weight
% Defining the optimizer
ops = sdpsettings('solver','sedumi','verbose',0); % choosing the solver
% We saw that the results of the optimazation depends a lot on the choice
% of the solver
ctrl = optimizer(con, obj, ops, X0, u(:,1));
% Runing the loop and computing the optimal input à each step
% using the optimizer defined above
for j=2:100
    [uopt,isfeasible] = ctrl{x(:,j-1)};
    % isfeasible == 1 if the problem was solved successfully
    x(:,j) = A*x(:,j-1)+B*uopt; % state equation
end

% Plotting
figure;
xlabel('x1');
ylabel('x2');
title('Optimal trajectory using YALMIP');
hold on;
plot(X, 'color', 'r');
plot(C, 'color', 'b');
plot(newX, 'color', 'g');
plot(omega, 'color', 'y');
plot(x(1,:),x(2,:), 'color','k');
legend('X','Maximum controlled invariant set','X inter K.U','Maximum invariant set
% We can see that we have the same trajectory using the 2 optimizers
```



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