Model Predictive Control | ME-425
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Exercise sheet 6
Operator Splitting Methods for Fast MPC

Part 1 Consider the discrete-time LTI system defined by

$$x_{i+1} = Ax_i + Bu_i$$

with

$$A = \begin{pmatrix} 1.988 & -0.998 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0.125 \\ 0 \end{pmatrix}$$

We want to compute the optimal control law that minimizes the following cost

$$V(x,u) = \sum_{i=0}^{N-1} (1/2)\left(x_i'Qx_i + u_i'Ru_i\right) + (1/2)x_N'Sx_N$$

with

$$Q = I_{2\times 2}, \quad R = 15 \ ,$$

and S being the terminal cost associated to the LQR controller. The horizon is set to $N = 30$. Furthermore, we have box constraints on states and inputs,

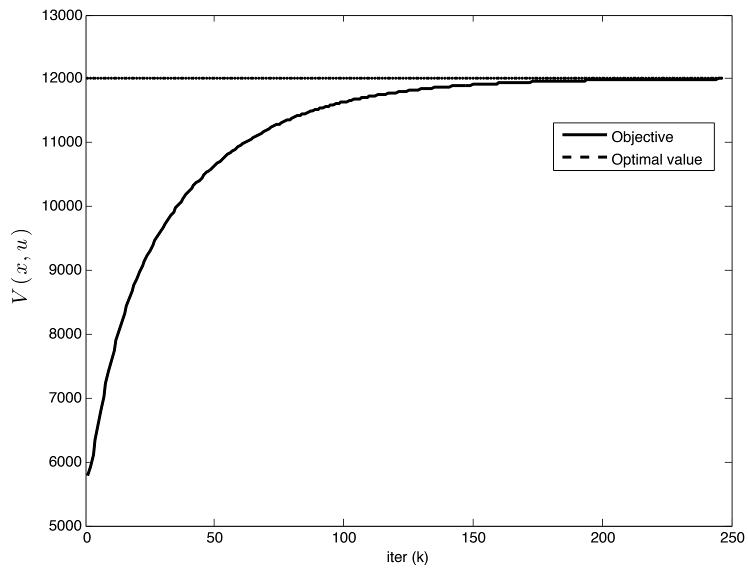
$$x_{\min} \leq x_i \leq x_{\max}, \quad |u_i| \leq u_{\max} \ .$$

We are going ot use the *Alternating direction method of multipliers (ADMM)* to compute the optimal control law.

- Implement the three steps of ADMM as presented in the course. The penalty parameter ρ is specified as $\rho = 5$ and the boxes' bounds as $x_{\max} = 10$, $u_{\max} = 10$.
- In the *admm.m* routine set the variable *plot_ traj* to 1. This will output the state trajectories as the algorithm iterates towards convergence, for the iterates $k = 5, 100, 200, 300$. Depict the state sequences on the states' constraint set. What do you observe?
- Shrink the state constraints by setting $x_{\max} = 9$. How does the algorithm behave?

Part 2 (Optional) By setting the variable *system=2* in the code we consider a different, randomly generated discrete-time LTI system with $n = 20$ states, $m = 5$ inputs, $N = 20$ and the penalty parameter $\rho = 10$. The constraints are now set to $x_{\max} = 9$, $u_{\max} = 1$.

- Run ADMM for this problem. If your code is correct, you should get that the cost $V(x, u)$ converges to the optimal one as shown in the figure below. What you observe is that, although we solve a minimization



  problem, the cost increases until it converges to the optimal one. Why do we get such a behavior?

- Change the input bound to $u_{\max} = 4$. What do you observe in terms of the number of iterations? *Hint: Check the number of active constraints in comparison to the previous case.*

- The most expensive step of the algorithm is the linear system solve (matrix inversion) for the primal sequence z. Can you think of a way to perform this step that could save us some time? Implement the alternative and compare the timings.