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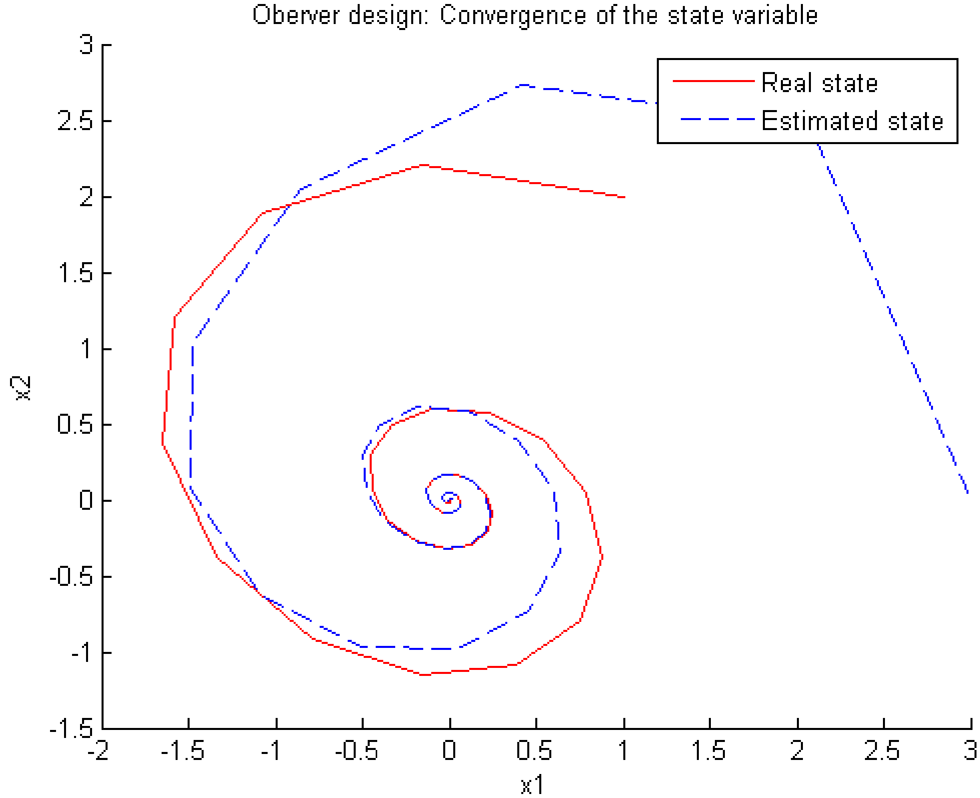
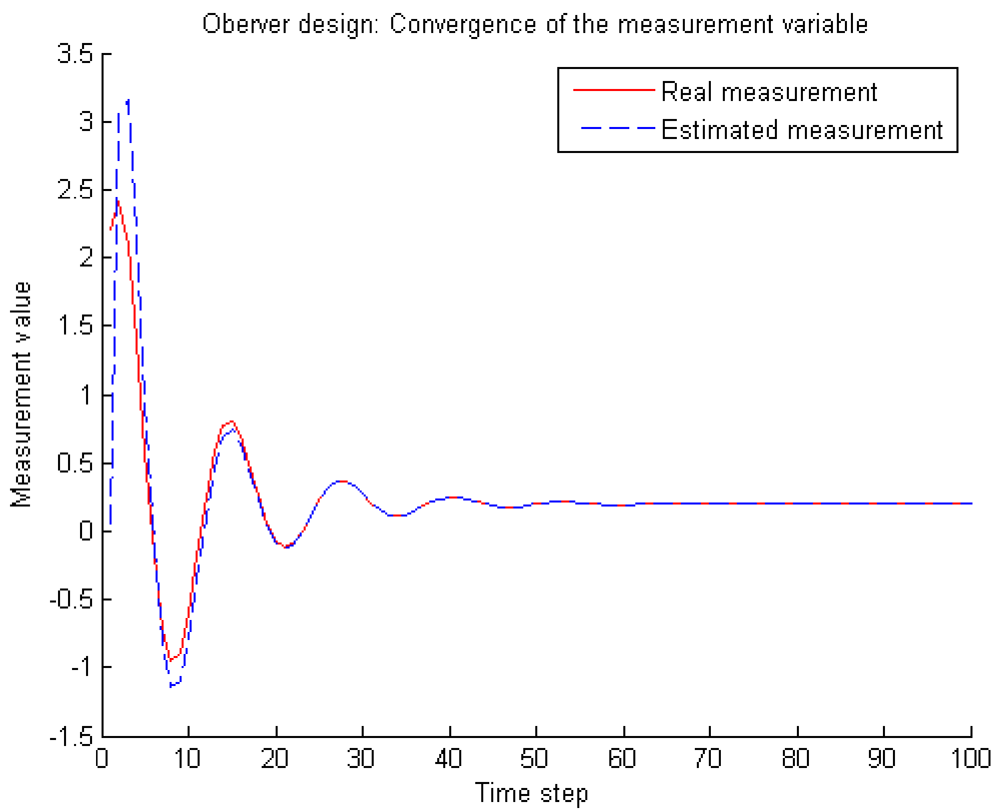
```
clc;
clear all;
close all;
warning('off');
% in order to avoid warnings in the report / it does not affect
 % the working of the code
```

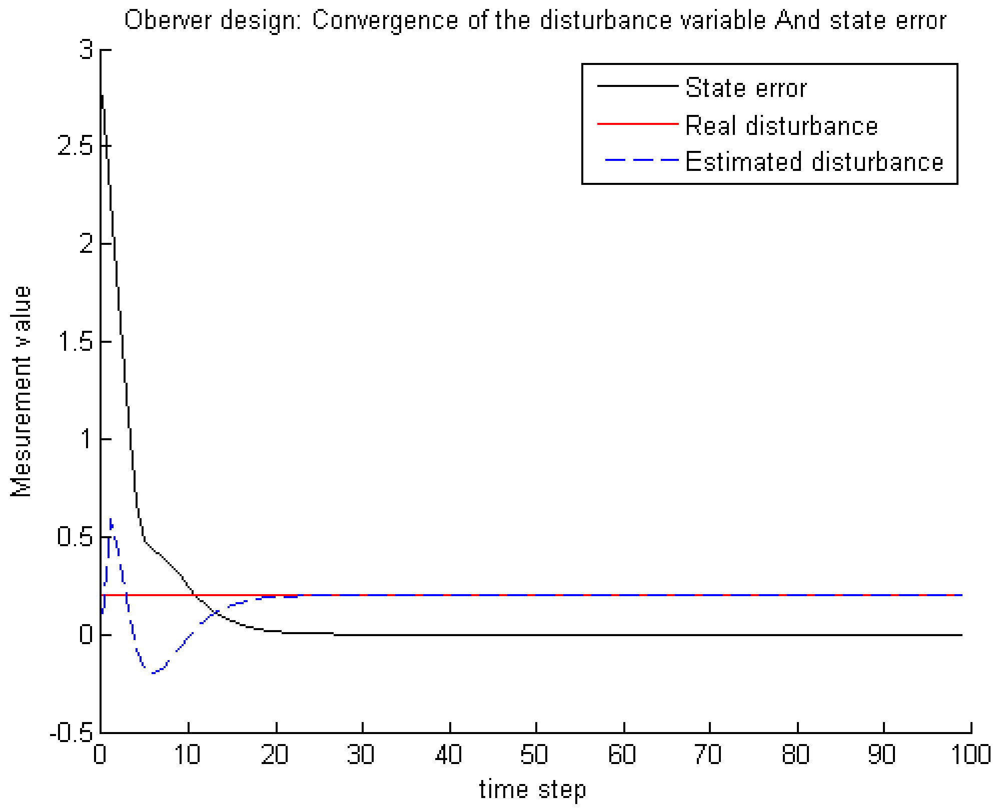
------------ EXERCISE 1 -------------

```
% ---------- System nitialization ----------

A=[0.7115  -0.4345;
   0.4345   0.8853];
B=[0.2173;
   0.0573];
Bd=[0;0];

C=[0 1];
Cd=1;

x0=[1;2];
x0_=[3;0];
d0=0.2;
d0_=0;
u=0;

% ---------- Variables initializations ----------

pN = 100;           % number of points to be computed

x = zeros(2,pN);    % state
x(:,1) = x0;
x_ = zeros(2,pN);   % state estimate
x_(:,1) = x0_;

errX = zeros(1,pN); % state error
errX(1)=sqrt((x0(1)-x0_(1))*(x0(1)-x0_(1))+(x0(2)-x0_(2))*(x0(2)-x0_(2)));

d = d0;             % disturbance
d_ = zeros(1,pN);   % disturbance estimate
d_(1) = d0_;

y = zeros(1,pN);    % augmented model
y_ = zeros(1,pN);
```

```
% Choosing L such that the error dynamics are stable and converge to zero
L = (place([A Bd; 0 0 1]', -[C'; Cd],[0.5;0.6;0.7]))';
Lx = L(1:2);
Ld = L(3);

% ---------- Disturbance estimator based on augmented model ----------

for i=1:pN-1
    y(i) = C*x(:,i)+Cd*d;
    y_(i) = C*x_(:,i)+Cd*d_(i);
    x_(:,i+1) = [A Bd]*[x_(:,i);d_(i)] + B*u + Lx*(C*x_(:,i)+Cd*d_(i)-y(i));
    d_(i+1) = [0 0 1]*[x_(:,i);d_(i)] + Ld*(C*x_(:,i)+Cd*d_(:,i)-y(i));
    x(:,i+1) = A*x(:,i);
    errX(i+1)=sqrt((x(1,i+1)-x_(1,i+1))*(x(1,i+1)-x_(1,i+1))+(x(2,i+1)-x_(2,i+1))*
end;
y(pN) = C*x(:,pN)+Cd*d;
y_(pN) = C*x_(:,pN)+Cd*d_(pN);

% ---------- Plotting results ----------

figure();
hold on;
plot(x(1,:),x(2,:),'r');
plot(x_(1,:),x_(2,:),'b--');
xlabel('x1');
ylabel('x2');
legend('Real state','Estimated state');
title('Oberver design: Convergence of the state variable');

figure();
hold on;
plot(y(:),'r');
plot(y_(:),'b--');
xlabel('Time step');
ylabel('Measurement value');
legend('Real measurement','Estimated measurement');
title('Oberver design: Convergence of the measurement variable');

timeStep=linspace(0,pN-1,pN);
figure;
hold on;
plot(timeStep,errX,'k');
plot(timeStep,d*ones(1,pN),'r');
plot(timeStep,d_,'b--');
xlabel('time step');
ylabel('Mesurement value');
legend('State error','Real disturbance','Estimated disturbance');
title('Oberver design: Convergence of the disturbance variable And state error');
```


Oberver design: Convergence of the state variable
Real state
Estimated state
x2
x1
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3


Oberver design: Convergence of the measurement variable
Real measurement
Estimated measurement
Measurement value
Time step
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
10
20
30
40
50
60
70
80
90
100



------------ EXERCISE 2 -------------

```
u=0;
ku = [3; 3];
Hu = [1; -1];
d_ = d;

% YALMIP
Rs = 1;

r = 1;
%r = 0.5;

% Define optimization variables
xs = sdpvar(2,1);
us = sdpvar(1,1);
% Define constraints and objective
con = [[eye(2)-A -B; C 0]*[xs; us]==[Bd*d_; r-Cd*d_], Hu*us<=ku];
obj = us*Rs*us; % Terminal weight
% Defining the optimizer
ops = sdpsettings('solver','quadprog','verbose',0); % choosing the solver
% We saw that the results of the optimazation depends a lot on the choice
% of the solver
diagnosis = solvesdp(con, obj, ops);
xs = double(xs)
```

```
us = double(us)
```

```
xs =

    0.0002
    0.8000


us =

    1.5999
```

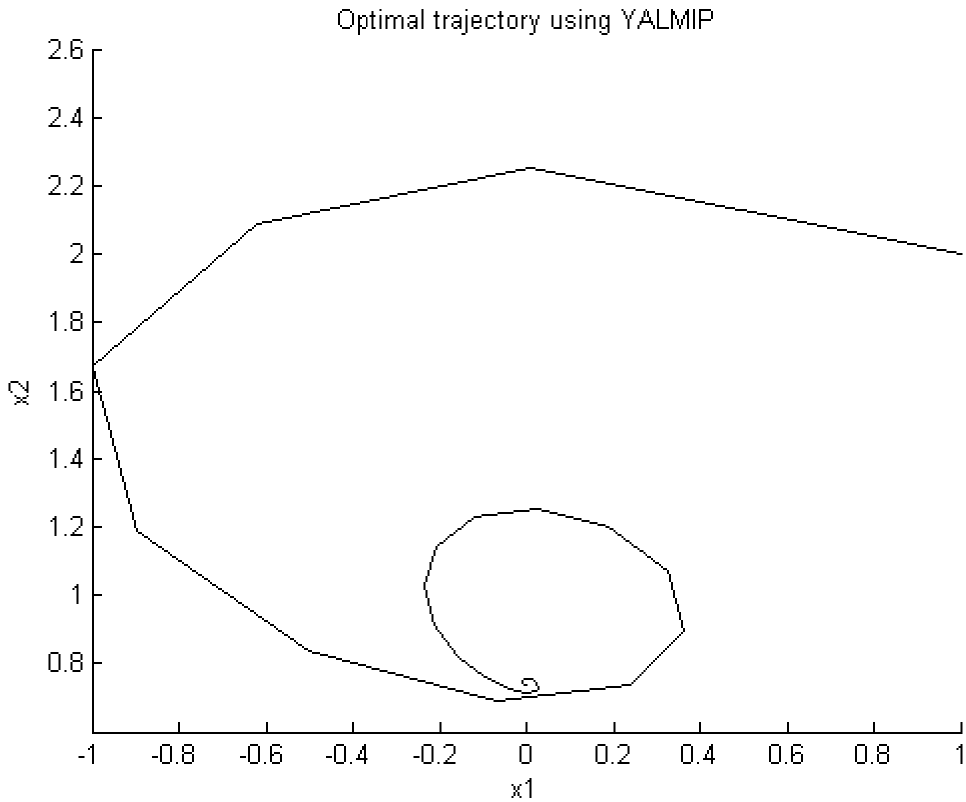
------------ EXERCISE 3 -------------

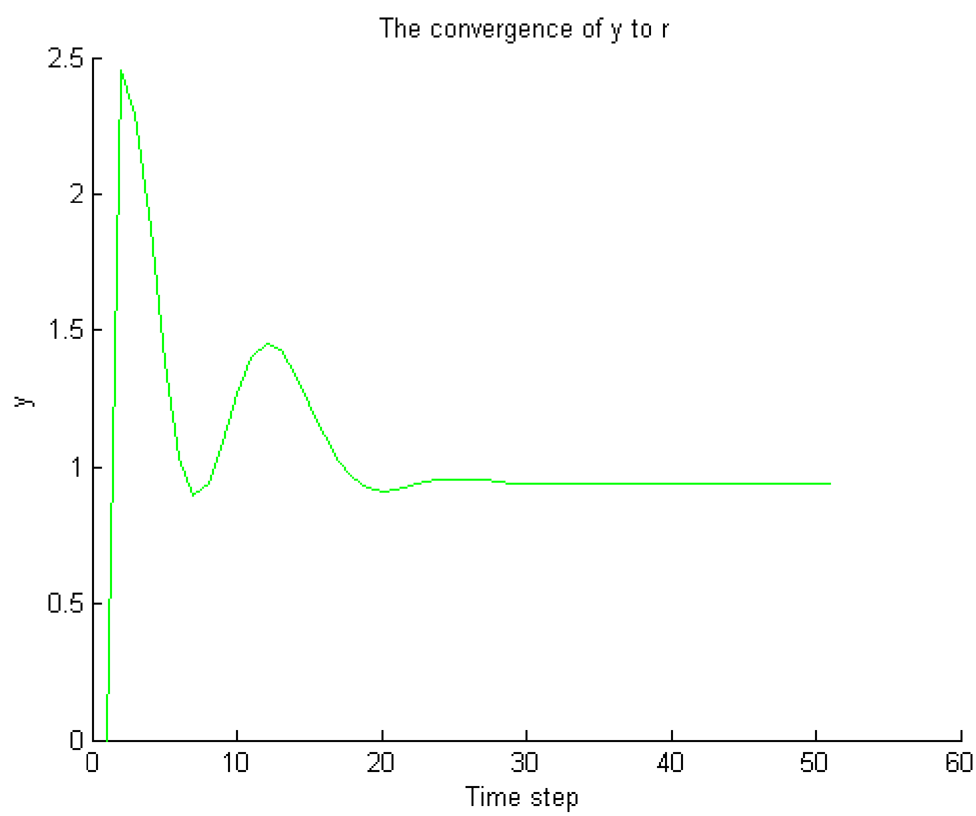
```
% ---------- Initialization ----------
r = 1;
%r = 0.5;
% The stage costs
Q = eye(2);
R = 1;
% The horizon
N = 5;
P = dlyap(A,Q);

pN3 = 50;
x = zeros(2,pN3);
x(:,1) = x0;
x_ = zeros(2,pN3);
d = d0;
d_ = zeros(1,pN3);
y = zeros(1,pN3);
u = zeros(1,pN3);


% Defining the optimizer for MPC
opsMPC = sdpsettings('solver','sedumi','verbose',0);
% Define optimization variables for MPC
delZMPC = sdpvar(2,N,'full');
delUMPC = sdpvar(1,N-1,'full');
x0MPC = sdpvar(2,1,'full');
% Define the cost function for MPC
objMPC = 0;
for i = 1:N-1
    objMPC = objMPC + delZMPC(:,i)'*Q*delZMPC(:,i) + delUMPC(:,i)'*R*delUMPC(:,i);
end
objMPC = objMPC + delZMPC(:,N)'*P*delZMPC(:,N); % Terminal weight



for i=1:pN3
    if i == 1 % First step: Initialization
```

```
        x_(:,i) = x0_;
        d_(i) = d0_;
        y_(i) = C*x_(:,i)+Cd*d_(i);
    else
        % Estimate the system's state, disturbance and measurment
        x_(:,i) = [A Bd]*[x_(:,i-1);d_(i-1)] + B*u(i-1) + Lx*(C*x_(:,i-1)+Cd*d_(i-
        d_(i) = [0 0 1]*[x_(:,i-1);d_(i-1)] + Ld*(C*x_(:,i-1)+Cd*d_(:,i-1)-y(i-1))
        y_(i) = C*x_(:,i)+Cd*d_(i);
    end
    % Obtain (xs,us) using disturbance estimate
    % Define optimization variables for steady-state identification
    % Define optimization variables
    xs = sdpvar(2,1);
    us = sdpvar(1,1);
    % Define constraints and objective for steady-state identification
    con = [[eye(2)-A -B; C 0]*[xs; us]==[Bd*d_(i); r-Cd*d_(i)], Hu*us<=ku];
    obj = us*Rs*us; % Terminal weight
    % Defining the optimizer for steady-state identification
    ops = sdpsettings('solver','quadprog','verbose',0);
    con = [[eye(2)-A -B; C 0]*[xs; us]==[Bd*d_(i); r-Cd*d_(i)], Hu*us<=ku];
    sol = solvesdp(con, obj, ops);
    if sol.problem == 1
        brake;
    end
    xs = double(xs);
    us = double(us);
    % Initiating the state for the controller
    delX_ = x_(:,i)-xs;


    % ---- Solve the MPC problem for tracking ----
    % Define constraints
    conMPC = [];
    conMPC = [conMPC, delZMPC(:,1) == x0MPC];
    for j = 1:N-1
        conMPC = [conMPC, delZMPC(:,j+1) == A*delZMPC(:,j) + B*delUMPC(:,j)]; % Sy
        conMPC = [conMPC, Hu*delUMPC(:,j) <= ku - Hu*us]; % Input constraints
    end
    conMPC = [conMPC, isreal(delZMPC(:,N))]; % Terminal constraint
    ctrl = optimizer(conMPC, objMPC, opsMPC, x0MPC, delUMPC(:,1));
    [uopt,unfeasible] = ctrl{x(:,i)};
    if unfeasible
        disp('The initial position is unfeasible')
        break;
    end
    u(i) = uopt+us;

    % run the system
    if i<pN
        x(:,i+1) = A*x(:,i)+B*u(i)+Bd*d;
        y(i+1) = C*x(:,i+1)+Cd*d;
    end
end
```

```
% ---------- Plotting ----------
figure;
xlabel('x1');
ylabel('x2');
title('Optimal trajectory using YALMIP');
hold on;
plot(x(1,:),x(2,:), 'color','k');

figure;
xlabel('Time step');
ylabel('y');
title('The convergence of y to r');
hold on;
plot(y,'g');

timeStep3=linspace(0,pN3-1,pN3)
figure;
xlabel('Time step');
ylabel('u')
title('Input respecting the constraints');
hold on;
plot(timeStep3,3,'r-');
plot(timeStep3,-3,'r-');
plot(u,'g');
```

```
timeStep3 =

  Columns 1 through 13

     0     1     2     3     4     5     6     7     8     9    10    11

  Columns 14 through 26

    13    14    15    16    17    18    19    20    21    22    23    24

  Columns 27 through 39

    26    27    28    29    30    31    32    33    34    35    36    37

  Columns 40 through 50

    39    40    41    42    43    44    45    46    47    48    49
```


Optimal trajectory using YALMIP
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
x2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
x1


The convergence of y to r
2.5
2
1.5
1
0.5
0
y
0
10
20
30
40
50
60
Time step

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